OPTIMAL CHARACTERIZATION OF THE TRANSFORMATION PROCEDURE OF KARMARKAR ALGORITHM

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Abstract

This research aims at proposing NyorRauf algorithm that standardizes the choice of arbitrary control U in the transformation procedure of Karmarkar algorithm. The NyorRauf transformation algorithm has not only provided a standard optimal U in any given problem, it has also yielded a better result when compared with the result of simplex method which is used as our exact solution..

Indexing terms/Keywords

Linear Programming, Karmarkar algorithm, simplex method, inequality constraints & NyorRauf algorithm.

Academic Discipline and Sub-Disciplines

Provide examples of relevant academic disciplines for this journal: E.g., History; Education; Sociology; Psychology; Cultural Studies;

SUBJECT CLASSIFICATION

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TYPE (METHOD/APPROACH)

Optimization; Analysis

INTRODUCTION

Karmarkar algorithm also known as Interior Point Method is used to solve Linear Programming Problems (LPP). Linear Programming (LP) has several advantages such as helping in dealing with the problem of allocation of limited resources among different competitive activities in the most optimal manner; determining the optimal allocation of scarce resources to meet certain objectives; providing practical and better quality of decision that reflect very precisely the limitations of the system. i.e various restrictions under which the system must operate for the solution to be optimal; and being an adaptive and flexible mathematical technique, hence, been used in analyzing a variety of multi-dimensional problems quite successfully. Given such advantages which LP offers, it is academically expedient to explore the method from different angles in order to apply it in solving modern world problems

LITERATURE REVIEW

Linear Programming (LP) is a tool for optimal allocation of scarce resources among a number of competing activities. It is the problem of optimizing (i.e. minimizing or maximizing) a linear function subject to linear inequality constraints with the function being optimized as the objective function. It is an optimal decision making tool in which the objective is a linear function and the constraints on the decision problem are linear equalities and/or inequalities. It is the most commonly applied form of constrained optimization. The four main elements of any constrained optimization are decision variables, objective function, constraints and variable bounds. In LP, all the mathematical expressions for the objective function and constraints are linear. One might imagine that the restriction to linear models severely limits the ability to model real-world problems; but this is not so. An amazing range of problems can be modelled using LP from airline scheduling to least cost petroleum processing and distribution, see [3] and [5].

The popular Simplex method of solving LP problems obtains the optimum solution by moving along edges of the solution space from one extreme point to another. According to [12], although in practice simplex method has served well in solving large problems, theoretically the number of iterations needed to reach the optimal solution can grow exponentially.

Narendra Karmarkar, an Indian mathematician, proposed a new method in 1984 called Karmarkar Algorithm for solving large-scale linear programming problems very efficiently. The method is also known as an interior point method since it finds improved search directions strictly in the interior of the feasible space. This is in contrast with the simplex method, which searches along the boundary of the feasible space by moving from one feasible vertex to an adjacent one until the optimum point is found. For large LP problems, the number of vertices will be quite large and hence the simplex method would become very expensive in terms of computer time. Along with many other applications, Karmarkar's method has been applied to aircraft route scheduling problems. It was reported that Karmarkar's method solved problems involving 150,000 design variables and 12,000 constraints in 1 hour while the simplex method required 4 hours for solving a smaller problem involving only 36,000 design variables and 10,000 constraints. In fact, it was found that Karmarkar's method is as much as 50 times faster than the simplex method for large problems, see [13] and [16]. Karmarkar algorithm is in the group of polynomial time algorithms with other methods such as ellipsoid method and the affine scaling algorithm. According to [1], An algorithm 'A' is said to be a polynomial-time algorithm for a problem 'P', if the number of steps (i. e. iterations) required to solve 'P' on applying 'A' is bounded by a polynomial function O(m,n, L) of dimension and input length of the problem.

The motivation for the choice of Karmarkar Interior Point method is predicated upon the following three points:

1. It is effective in solving extremely large LP problems which are typical of the kind of problems confronting analyst in the present dispensation
2. It is faster in reaching the optimal point. "Unlike the ellipsoid method (another polynomial time algorithm), Karmarkar's method appears to solve many LPs faster than does the simplex algorithm" [15].
3. Most LP software that are developed today are based on Karmarkar algorithm, and as such, one would like to be abreast with current issues

The transformation procedure of Karmarkar algorithm from the original LP problem to Karmarkar form requires the choice of U (which is $\sum y_{ij}$; i = 1; 2; … m; j = 1; 2; … n+1) to be sufficiently large, see [12] and [13]. How much sufficiently large is not suggested by Karmarkar algorithm. This means that, the choice of U is at the discretion of the problem solver. It is arbitrarily and does results to different values of the objective function coefficients when solved by different analysts.

In this work, we propose an algorithm that will standardize the choice of U which is arbitrary (at the discretion of the problem solver) in the transformation procedure of Karmarkar algorithm

KARMARKAR TRANSFORMATION PROCEDURE

The following steps are the existing transformation procedure by Karmarkar:

1. Standardize the constraint inequalities
2. a) Set the result of step 1 and add another variable as

   $\sum Y_{ij} \leq U$; i = 1, 2, …, m; j = 1, 2, …, n+1 Satisfying 1Y = 1

   b) Select U sufficiently large. (This is an arbitrarily choice at the discretion of the analyst)
3. Standardize step 2a by adding a slack variable Y_{ij} to obtain

   $\sum Y_{ij} = U$; i = 1, 2, …, m; j =1, 2, …, n+2
4. Homogenize the RHS of the result in step 1 by $\frac{\sum Y_{ij}}{U}$. Simplify to obtain the homogeneous form of the constraint
5. Introduce artificial variables (where necessary) to ensure that the coefficient of the result in step 4 is zero.
6. Define new variable for the objective function as $X_i = \frac{Y_i}{U}$ and substitute the new variables X_i into the constraints
7. Penalize the artificial variables in the objective function as appropriate.

NYORRAUF TRANSFORMATION ALGORITHM

Consider the original Linear Programming (LP) Problem:

Maximize Z = CX

Subject to AX ≤ b

X ≥ 0.

We propose the following steps for the transformation:

1. Convert the constraints inequalities of the original LP problem into equations by augmenting slack or surplus variables appropriately

   $\sum AX_{ij} = b;$ $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$

   Where n is the number of X variables in the original LP

2. Define the augmented equations in 1 above leaving out the coefficient of X_j but considering all the X_{js} as

   $$\sum_{j=1}^{n+1} X_{ij} \leq U; \qquad i = 1, 2, \ldots, m$$

   Where m is the number of constraint equations,

   U is the value that is sufficiently large so as not to eliminate any feasible point from the solution space.

3. For each standardized constraint i, obtain

   X_j = L.U.I. $\left|\frac{b_i}{a_{ij}}\right|$ if fractional part; by setting other n variables equal to zero.

   Where L.U.I. is Least Upper Integers.

4. Obtain $U_i = \sum_{j=1}^{n+1} X_j$

5. Obtain U_{val} from

   U_{val} = G.L.I. $\left(\frac{\sum_{i=1}^{m} U_i}{m}\right)$; whether fractional parts or not.

   Where G.L.I. is Greatest Lower Integers.

6. Thus, step 2 becomes,

   $\sum_{j=1}^{n+1} X_j \leq U_{val}$

7. Augment again the defined constraint in 6 above to have

   $$\sum_{j=1}^{n+2} X_j = U_{val}$$

8. Homogenize the Right Hand Size (RHS) of each augmented constraints in 1 by

   $$\frac{\sum_{j=1}^{n+2} X_j}{U_{val}}$$

   Hence

   $$\left(\sum_{j=1}^{n+1} a_{ij} X_{ij}\right) U_{val} = b_i \left(\sum_{j=1}^{n+2} X_j\right)$$

   Where 'a' is the coefficient of X in the original LP problem

   'b' is the RHS constant of the original LP

   Thus,

   $$\left(\sum_{j=1}^{n+1} a_{ij} X_{ij}\right) U_{val} - b_i \left(\sum_{j=1}^{n+2} X_j\right) = 0$$

9. Ensure that the sum of the coefficient of the LHS equals zero by adding artificial variables where necessary
10. Penalize the artificial variables introduced in step 7 in the objective function
11. Define new variables for the objective function as

    $$Y_i = \frac{X_i}{U_{val}}$$

12. Substitute the new variables as defined in step 11 into the constraints to maintain consistency; hence the transformed karmarkar algorithm

SOME EXAMPLES OF STANDARD LP PROBLEMS

The table below shows some selected LP problems to demonstrate NyorRauf transformation algorithm and compare the result with that of Karmarkar's algorithm.

Table1. Selected Examples of some Standard LP Problems

PROBLEM NO.	ORIGINAL LP PROBLEM	PROBLEM TYPE	SOURCE
1	**Maximize** $Z = 5x_1+3x_2$ **Subject to:** $3x_1+5x_2 \leq 15$ $5x_1+2x_2 \leq 10$ $x_1, x_2 \geq 0$	Two Variables and Two Constraints	Aderibigbe A.Y. (1998). Operations Research for Science and Management. Best way Nig. Ilorin, Nigeria.
2	**Maximize** $Z = 3x_1+2x_2$ **Subject to:** $x_1+x_2 \leq 6$ $x_1, x_2 \geq 0$	Two Variables and One Constraint	Taha H. (2007). Operations Research- An Introduction. Prentice Hall. USA
3	**Maximize** $Z = 3x_1+2x_2$ **Subject to:** $x_1+2x_2 \leq 6$ $2x_1+x_2 \leq 8$ $-x_1+x_2 \leq 1$ $x_1, x_2 \geq 0$	Two Variables and Three Constraints	Taha H. (2007). Operations Research- An Introduction. Prentice Hall. USA
4	**Maximize** $Z = x_1 - x_2$ **Subject to:** $x_1+2x_2 \leq 2$ $2x_1- x_2 \leq 1$ $x_1, x_2 \geq 0$	Two Variables and two Constraints	Taha H. (2007). Operations Research - An Introduction. Prentice Hall. USA
5	**Maximize** $Z = x_1 + x_2 + 3x_3$ **Subject to:** $3x_1+2x_2+ x_3 \leq 3$ $2x_1+x_2+2x_3 \leq 2$ $x_1, x_2, x_3 \geq 0$	Three Variables and Two Constraints	Kalavathy S. (2000). Operations Research. Vikas Publishing House PVT LTD. Delhi.

SOLVING THE LP PROBLEMS USING KARMARKAR TRANSFORMATION PROCEDURE

Using the steps under Karmarkar Transformation Procedure, we solved the problems given in Table1. Problem 5 is demonstrated below:

PROBLEM 5

Maximize $Z = x_1 + x_2 + 3x_3$

Subject to: $3x_1+2x_2+ x_3 \leq 3$

$2x_1+x_2+2x_3 \leq 2$

$x_1, x_2, x_3 \geq 0$

Standardize constraints inequalities by adding slack variables,

$$3X_1+ 2X_2 + X_3 + X_4 = 3 \quad (1)$$

$$2X_1+ X_2 + 2X_3 + X_5 = 2 \quad (2)$$

Define

$$X_1+ X_2 + X_3 + X_4+ X_5 \leq U$$

Choosing U sufficiently large to be 8

$$X_1+ X_2 + X_3 + X_4+ X_5 \leq 8$$

Augmenting

$$X_1+ X_2 + X_3 + X_4 + X_5+ X_6= 8 \quad (3)$$

Multiplying the RHS of equations (1) and (2) respectively by $\frac{X1 + X2 + X3 + X4 + X5+ X6}{8}$,

From Equation (1),

$$3X_1+ 2X_2 + X_3+ X_4= 3\left(\frac{X1 + X2 + X3 + X4 + X5+X6}{8}\right)$$

$$24X_1+ 16X_2 + 8X_3 + 8X_4 = 3X_1+ 3X_2 + 3X_3 + 3X_4 + 3X_5 + 3X_6$$

$$21X_1+13X_2 + 5X_3+5X_4 - 3X_5– 3X_6= 0$$

But

$$21+13+5+5-3-3 = 38$$

Thus,

$$21X_1+13X_2 + 5X_3 + 5X_4 - 3X_5 – 3X_6 – 38X_7 = 0 \quad (4)$$

From equation (2),

$$2X_1+ X_2 + 2X_3 + X_5 = 2\left(\frac{X1 + X2 + X3 + X4 + X5+ X6}{8}\right)$$

$$16X_1+8X_2 + 16X_3 + 8X_5 = 2X_1+ 2X_2 + 2X_3 + 2X_4 + 2X_5 + 2X_6$$

$$14X_1+ 6X_2+14X_3-2X_4+4X_5 -2X_6 = 0$$

But

$$14+6+14-2+4-2 = 34$$

Thus,

$$14X_1+ 6X_2 + 14X_3 - 2X_4 + 4X_5 -2X_6 – 34X_8= 0 \quad (5)$$

The Transformed Objective Function

$$Y_i = \frac{X_i}{U}$$

Our U is 8, thus

$$Y_i = \frac{X_i}{8} \quad ; \quad X_i = 8Y_i$$

Hence, the original objective function becomes

$$1(8Y_1)+ 1(8Y_3)+ 3(8Y_2)= 8Y_1+8Y_2 + 24Y_3$$

Equations (4) and (5) are the transformed constraints. Penalizing the artificial variables X_6 and X_7 in equations (4) and (5) respectively in the objective function using the big-M method gives us the transformed problem as

Minimize $Z = 8Y_1 + 8Y_2 + 24Y_3 - 100Y_7 – 100Y_8$

Subject to: $21Y_1+13Y_2 + 5Y_3 + 5Y_4 - 3Y_5 – 3Y_6 – 38Y_7 = 0$

$$14Y_1 + 6Y_2 + 14Y_3 - 2Y_4 + 4Y_5 - 2Y_6 - 34Y_8 = 0$$

$$Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + Y_7 + Y_8 = 1$$

TORA RESULT

$Z^* = 3.0000$

$X_1^* = 0.0000$

$X_2^* = 0.0000$

$X_3^* = 0.1250$

SOLVING THE SELECTED LP PROBLEMS USING NYORRAUF TANSFORMATION ALGORITHM

Using the steps under NyorRauf Transformation Algorithm, we solved the problems given in Table 1. Problem 5 is demonstrated thus:

PROBLEM 5

Maximize $Z = x_1 + x_2 + 3x_3$

Subject to: $3x_1 + 2x_2 + x_3 \leq 3$

$2x_1 + x_2 + 2x_3 \leq 2$

$x_1, x_2, x_3 \geq 0$

Convert constraint inequalities to equations by adding slack variables

$$3X_1 + 2X_2 + X_3 + X_4 = 3 \quad (1)$$

$$2X_1 + X_2 + 2X_3 + X_5 = 10 \quad (2)$$

Define

$$X_1 + X_2 + X_3 + X_4 + X_5 \leq U \quad (3)$$

From equation (1),

If $X_2 = X_3 = X_4 = 0$, then $X_1 = \frac{3}{3} = 1.$

If $X_1 = X_3 = X_4 = 0$, then $X_2 = \frac{3}{2} = 1.5;$

LUI = 2.

If $X_1 = X_2 = X_4 = 0$, then $X_3 = 3.$

If $X_1 = X_2 = X_3 = 0$, then $X_4 = 3$

$U_1 = 1+2+3+3 = 9$

From equation (2),

If $X_2 = X_3 = X_5 = 0$, then $X_1 = \frac{2}{2} = 1.$

If $X_1 = X_3 = X_5 = 0$, then $X_2 = 2;$

If $X_1 = X_2 = X_5 = 0$, then $X_3 = \frac{2}{2} = 1.$

If $X_1 = X_2 = X_3 = 0$, then $X_5 = 2$

$U_1 = 1+2+1+2 = 6$

But $U_{val} = \text{G.L.I.}\left(\frac{\sum_{i=1}^{m} U_i}{m}\right) = \text{G.L.I.}\left(\frac{9+6}{2}\right) = 7$

Equation (3) becomes,

$$X_1 + X_2 + X_3 + X_4 + X_5 \leq 7$$

Augmenting

$$X_1 + X_2 + X_3 + X_4 + X_5 + X_6 = 7$$

Multiplying the RHS of equations (1) and (2) respectively by $\frac{X1 + X2 + X3 + X4 + X5 + X6}{7}$,

From Equation (1),

$$3X_1+ 2X_2 + X_3+ X_4 = 3\left(\frac{X1 + X2 + X3 + X4 + X5 + X6}{7}\right)$$

$$21X_1+ 14X_2 + 7X_3 + 7X_4 = 3X_1+ 3X_2 + 3X_3 + 3X_4 + 3X_5+ 3X_6$$

$$18X_1+ 11X_2 + 4X_3+4X_4 - 3X_5- 3X_6 = 0$$

But

$$18+11+4+4-3-3 = 31$$

Thus,

$$18X_1+ 11X_2 + 4X_3+4X_4 - 3X_5 – 3X_6-31X_7= 0 \quad (4)$$

From equation (2),

$$2X_1 + X_2 + 2X_3+ X_4= 2\left(\frac{X1 + X2 + X3 + X4 + X5 + X6}{7}\right)$$

$$14X_1+ 7X_2 + 14X_3+ 7X_5 = 2X_1+ 2X_2 + 2X_3 + 2X_4 + 2X_5 + 2X_6$$

$$12X_1+ 5X_2+12X_3-2X_4+5X_5- 2X_6= 0$$

But

$$12+5+12-2+5-2 = 30$$

Thus,

$$12X_1+ 5X_2+12X_3- 2X_4+ 5X_5 – 2X_6– 30X_8= 0 \quad (5)$$

Transforming Objective Function

$$Y_i = = \frac{X_i}{U_{val}}$$

But U_{val} is 7, thus

$$Y_i = \frac{X_i}{7} \quad ; \quad X_i = 7Y_i$$

Hence, the original objective function becomes

$$1(7Y_1) + 1(7Y_2)+ 3(7Y_3) = 7Y_1 + 7Y_2 + 21Y_3$$

Equations (4) and (5) are the transformed constraints. Penalizing the artificial variables X_7 and X_8 in equations (4) and (5) respectively in the objective function using the big-M method gives us the transformed problem as

Minimize $Z = 7Y_1 + 7Y_2 + 21Y_3 - 100Y_7 – 100Y_8$

Subject to: $18Y_1+ 11Y_2 + 4Y_3 + 4Y_4 – 3Y_5 – 3Y_6 -31Y_7= 0$

$12Y_1+ 5Y_2 +12Y_3 – 2Y_4 + 5Y_5 – 2Y_6 – 30Y_8 = 0$

$Y_1+ Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + Y_7+ Y_8 = 1$

TORA RESULT

$Z^* = 3.0000$

$X_1^* = 0.0000$

$X_2^* = 0.0000$

$X_3^* = 0.1429$

SUMMARY TABLE

Table 2: LP Problems in Simplex, Karmarkar and NyorRauf forms

PROBLEM NO.	ORIGINAL LP PROBLEM	KARMARKAR FORM	NYORRAUF ALGORITHM FORM
1	Maximize $Z = 5x_1+3x_2$ Subject to: $3x_1+5x_2 \le 15$ $5x_1+2x_2 \le 10$ $x_1, x_2 \ge 0$	Minimize $Z = 100Y_1 + 60Y_2 - 150Y_6 - 150Y_7$ Subject to: $45Y_1 + 85Y_2 + 5Y_3 - 15Y_4 - 15Y_5 - 105Y_6 = 0$ $90Y_1 + 30Y_2 - 10Y_3 + 10Y_4 - 10Y_5 - 110Y_7 = 0$ $Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + Y_7 = 1$	Minimize $Z = 95Y_1 + 57Y_2 - 150Y_6 - 150Y_7$ Subject to: $42Y_1 + 80Y_2 + 4Y_3 - 15Y_4 - 15Y_5 - 96Y_6 = 0$ $85Y_1 + 28Y_2 - 10Y_3 + 9Y_4 - 10Y_5 - 102Y_7 = 0$ $Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + Y_7 = 1$
2	Maximize $Z = 3x_1+2x_2$ Subject to: $x_1+ x_2 \le 6$ $x_1, x_2 \ge 0$	Minimize $Z = 57Y_1 + 38Y_2 - 100Y_5$ Subject to: $13Y_1 + 13Y_2 + 13Y_3 - 6Y_4 - 33Y_5 = 0$ $Y_1 + Y_2 + Y_3 + Y_4 + Y_5 = 1$	Minimize $Z = 51Y_1 + 34Y_2 - 100Y_5$ Subject to: $11X_1 + 11X_2 + 11X_3 - 6X_4 - 27X_5 = 0$ $Y_1 + Y_2 + Y_3 + Y_4 + Y_5 = 1$
3	Maximize $Z = 3x_1+2x_2$ Subject to: $x_1+2 x_2 \le 6$ $2x_1+ x_2 \le 8$ $-x_1+ x_2 \le 1$ $x_1, x_2 \ge 0$	Minimize $Z = 45Y_1 + 30Y_2 - 100Y_7 - 100Y_8 - 100Y_9$ Subject to: $9Y_1 + 24Y_2 + 9Y_3 - 6Y_4 - 6Y_5 - 6Y_6 - 24Y_7 = 0$ $22Y_1 + 7Y_2 - 8Y_3 + 7Y_4 - 8Y_5 - 8Y_6 - 12Y_8 = 0$ $-16Y_1 + 14Y_2 - Y_3 - Y_4 + 14Y_5 - Y_6 - 9Y_9 = 0$ $Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + Y_7 + Y_8 + Y_9 = 1$	Minimize $Z = 36Y_1 + 24Y_2 - 100Y_7 - 100Y_8$ Subject to: $6Y_1 + 18Y_2 + 6Y_3 - 6Y_4 - 6Y_5 - 6Y_6 - 12Y_7 = 0$ $16Y_1 + 4Y_2 - 8Y_3 + 4Y_4 - 8Y_5 - 8Y_6 = 0$ $-13Y_1 + 11Y_2 - Y_3 - Y_4 + 11Y_5 - Y_6 - 6Y_8 = 0$ $Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + Y_7 = 1$
4	Maximize $Z = x_1 - x_2$ Subject to: $x_1+2x_2 \le 2$ $2x_1- x_2 \le 1$ $x_1, x_2 \ge 0$	Minimize $Z = 5Y_1 - 5Y_2 - 50Y_6 - 50Y_7$ Subject to: $3Y_1 + 6Y_2 + 3Y_3 - 2Y_4 - 2Y_5 - 8Y_6 = 0$ $9Y_1 - 6Y_2 - Y_3 + 4Y_4 - Y_5 - 5Y_7 = 0$ $Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + Y_7 = 1$	Minimize $Z = 3Y_1 - 3Y_2 - 50Y_6 - 50Y_7$ Subject to: $Y_1 + 4Y_2 + Y_3 - 2Y_4 - 2Y_5 - 2Y_6 = 0$ $5Y_1 - 4Y_2 - Y_3 + 2Y_4 - Y_5 - Y_7 = 0$ $Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + Y_7 = 1$
5	Maximize $Z = x_1 + x_2 + x_3$ Subject to: $3x_1+2x_2+ x_3 \le 3$ $2x_1+ x_2+2x_3 \le 2$ $x_1, x_2, x_3 \ge 0$	Minimize $Z = 8Y_1 + 8Y_2 + 24Y_3 - 100Y_7 - 100Y_8$ Subject to: $21Y_1 +13Y_2 + 5Y_3 + 5Y_4 - 3Y_5 - 3Y_6 - 38Y_7 = 0$ $14Y_1 + 6Y_2 + 14Y_3 - 2Y_4 + 4Y_5 - 2Y_6 - 34Y_8 = 0$ $Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + Y_7 + Y_8 = 1$	Minimize $Z = 7Y_1 + 7Y_2 + 21Y_3 - 100Y_7 - 100Y_8$ Subject to: $18Y_1 + 11Y_2 + 4Y_3 + 4Y_4 - 3Y_5 - 3Y_6 - 31Y_7 = 0$ $12Y_1 + 5Y_2 + 12Y_3 - 2Y_4 + 5Y_5 - 2Y_6 - 30Y_8 = 0$ $Y_1 + Y_2 + Y_3 + Y_4 + Y_5 + Y_6 + Y_7 + Y_8 = 1$

Table 3: Summary of Results

PROBLEM	SIMPLEX METHOD			EXISTING KARMARKAR			NYORRAUF ALGORITHM		
	Z^*	X^*_1	X^*_2 X^*_3	Z^*	X^*_1	X^*_2 X^*_3	Z^*	X^*_1	X^*_2 X^*_3
1	12.3684	1.0526	2.3684	12.3684	0.0526	0.1184	12.3684	0.0554	0.1247
2	18.0000	6.0000	0.0000	18.0000	0.3158	0.0000	18.0000	0.3529	0.0000
3	12.6667	3.3333	1.3333	12.6667	0.2222	0.0889	12.6667	0.2778	0.1111
4	0.5000	0.5000	0.0000	0.5000	0.1000	0.0000	0.5000	0.1667	0.0000
5	3.0000	0.0000	0.0000 1.0000	3.0000	0.0000	0.0000 0.1250	3.0000	0.0000	0.0000 0.1429

DISCUSSION OF RESULTS

Observe from table 3 above that the result of NyorRauf algorithm is closer to the simplex method than the existing Karmarkar method. The result of Simplex Method is used as our basis for comparism because simplex method performs better in small problems which have few constraints and variables. However, for very large scale LP problems, Karmarkar performs far better than simplex method because its strength lies in extremely large problems, see [14], [2], [4] and [6-11].

CONCLUSION

Based on the above results, our transformation algorithm is feasible and valid. It does not only provide a standard procedure for obtaining U, it also yields an improved result over the existing Karmarkar's algorithm.

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