UNSTEADY HYDROMAGNETIC FREE CONVECTIVE FLOW THROUGH A POROUS MEDIUM IN AN INFINITE VERTICAL POROUS PLATES

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Abstract

The paper investigates analytically two dimensional flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate in a porous medium in the presence of uniform transverse magnetic field and constant heat source. Physical properties are assumed as constant and Fluid particles are assumed as electrically conducting. The equations describing the heat and mass transfer were transformed using self-similar solution and solved analytically using Frobenius method. The effects of various flow parameters on the velocity and temperature, for the case of Grashof number, $Gr < 0$ (that is heating of the plate) are shown with the aid of graphs and discussed. The results obtained revealed that velocity decreases along distance but increases as permeability number and Grashof number increase while maximum temperature increases as permeability number, Prandtl number and Eckert number increase.

Keywords and phrases**:** Convective flow, porous medium, self-similar solution and Frobenius method.

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1. Introduction

Convective flows are important in the context of process involving high temperatures. In many engineering areas such as nuclear power plants, gas, turbines and various propulsion devices for aircraft, missiles and space vehicles. The effect of free convection on accelerated flow of a viscous incompressible fluid past an infinite vertical plate with suction has many important technological applications in the astrophysical, geophysical and engineering problems .The study of the flow of an electrically conducting fluid over porous media has been studied due to its numerous applications such applications include MHD pumps, induction pumps, MHD generators , oil exploration, nuclear power plants ,gas turbines, air crafts and space vehicles among many others. Seigel (1958) first studied transient free convection flow past a semi-infinite vertical plate by an integral method. Since then many researchers have been published papers on free convection flow past a semi-infinite vertical plate.

A few other works of interest in this area include the works of Ogulu and Prakash (2006), Kim (2000), Makinde (2005) and Ogulu and Makinde (2009). Anand *et al*. (2014) used finite element method (FEM) to obtain the solution of heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity. Sharma *et al*. (2012) investigated the flow of a viscous incompressible electrically conducting fluid along a porous vertical isothermal non-conducting plate with variable suction and internal heat generation in the presence of transverse magnetic field. Mohammed *et al*. (2015) presented an analytical method to describe the heat and mass transfer in the flow of an incompressible viscous fluid past an infinite vertical plate. With the governing equations accounting for the viscous dissipation effect and mass transfer with chemical reaction of constant reaction rate. The couple differential equations were transformed using similarity transformation and solved analytically using iteration perturbation method. Hamad *et al*. (2011) investigated the unsteady magneto hydrodynamic flow of a Nano fluid past an oscillatory moving vertical permeable semi-infinite flat plate with constant heat source in a rotating frame of reference. The velocity along the plate (slip velocity) is assumed to oscillate on time with a constant frequency. Das and Jana (2010) investigated the effect of heat and mass transfer on the unsteady free convection flow of a viscous, electrically conducting incompressible fluid near an infinite vertical plate embedded in porous medium which moves with time dependent velocity under the influence of uniform magnetic field applied normal to the plate. An exact solution of the governing partial differential equation is obtained by using Laplace transform technique. Maina *et al*. (2015) studied the effects of heat transfer on unsteady MHD free convective flow past a vertical porous plate in a porous medium with heat source and constant injection. Crank-Nicolson method (FDM) was used to solve the governing coupled differential equations.

In this paper, a mathematical study of heat and mass transfer of the two dimensional flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate in a porous medium in the presence of uniform transverse magnetic field and constant heat source is presented. We simulate the flow analytically, using self-similar solution and Frobenius method.

2. Model Formulation

Consider the two dimensional flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate in a porous medium in presence of uniform transverse magnetic field $\left(B_0\right)$ and constant heat source $\left(Q\right)$. The x-axis is measured along vertical plate and y-axis normal to it as shown figure 1. The surface of the vertical plate is at uniform temperature T and concentration C. The temperature and concentration far away from the plate are T_∞ *and* C_∞ respectively. A magnetic field of strength B_0 acts normal to the plate that is, along the y-axis. The analysis of this study is based on following assumptions:

*Physical properties are assumed as constant.

*Fluid particles are assumed as electrically conducting.

The physical sketch and geometry of the problem is shown in figure 1:

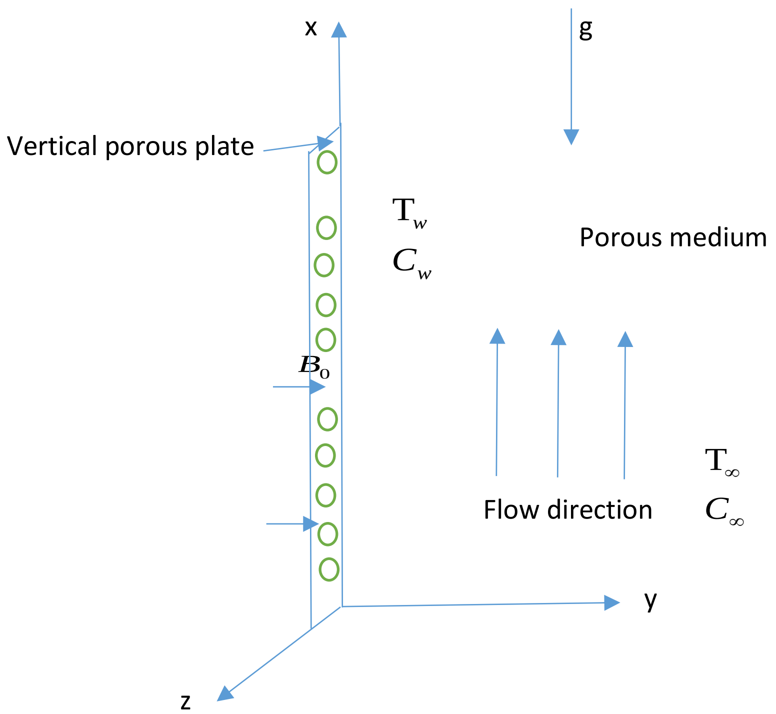

Figure 1: The flow configuration

Using these assumptions together with usual boundary layer approximations and following Maina *et al*. (2015) and Mohammed *et al*. (2015) we get the two dimensional equations describing the phenomenon as:

Continuity equation

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

Momentum equation

$$\left.\begin{aligned}\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}&=\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\upsilon}{K}u-\frac{\sigma B_0^2}{\rho}u+g\beta'(C-C_\infty)\\ \frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}&=\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\upsilon}{K}v-\frac{\sigma B_0^2}{\rho}v+g\beta'(C-C_\infty)\end{aligned}\right\} \tag{2}$$

Energy equation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)-\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\right)^2+Q(T-T_\infty) \tag{3}$$

The equation for species concentrations

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \tag{4}$$

Where $u,\ v$ are the dimensionless velocity components along the $x-$ and $y-$ directions respectively, υ is the kinematic viscosity, k thermal conductivity, σ is the electrical conductivity, B_0 the constant applied magnetic field, ρ the fluid density, g gravity acceleration, β^1 the concentration expansion coefficient, C and C_∞ are the concentration of solute at the plate and far away from the plate respectively. T is the temperature of the fluid in the boundary layer T_∞ the temperature of fluid far away from the plate, c_p is the specific heat capacity at constant pressure, Q additional heat source, and D_m is the molecular diffusivity.

The problem is two-dimensional**.** Since the plate is an infinite, the velocity vector

$\bar{q}=(\mathrm{u},0)$, $\quad \mathrm{u}=\mathrm{u}(\mathrm{x},\mathrm{y},\mathrm{t})$, $v=v(\mathrm{x},\mathrm{y},\mathrm{t})$

By symmetry and from continuity equation (1)

$u=u(y,t)$, $\quad T=T(y,t)$ and $C=C(y,t)$

Then, equations (1) – (4) reduce to

$$\frac{\partial u}{\partial t}=\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\upsilon}{K}u-\frac{\sigma B_0^2}{\rho}u+g\beta'(C-C_\infty) \tag{5}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 + Q(T - T_\infty) \tag{6}$$

$$\frac{\partial C}{\partial t} = D_m \frac{\partial^2 C}{\partial y^2} \tag{7}$$

With initial and boundary conditions

$$\left.\begin{array}{lll} \mathrm{u}(\mathrm{y},0) = U_\infty & u(0,t) = U_w & u(\infty,t) = 0 \\ T(\mathrm{y},0) = \mathrm{T}_\infty & T(0,\mathrm{t}) = T_w & T(\infty,t) = T_\infty \\ C(y,0) = \mathrm{C}_\infty & C(0,t) = C_w & C(\infty,t) = C_\infty \end{array}\right\} \tag{8}$$

3. Method of Solution

3.1 Non-dimensionalisation

We introduce dimensionless variables for space and time,

$$t' = \frac{D_m t}{L^2}, \qquad y' = \frac{y}{L} \tag{9}$$

We also introduce dimensionless variables for velocity, temperature and concentration;

$$u' = \frac{u}{U_\infty}, \qquad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \tag{10}$$

Using (9) and (10), and after dropping the prime, the equations (5) - (8) become

$$\frac{\partial u}{\partial t} = Sc\frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K_p}\right)u + Gr_\phi \phi \tag{11}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\left(\frac{\partial u}{\partial y}\right)^2 + q\theta \tag{12}$$

$$\frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} \tag{13}$$

With initial and boundary conditions

$$\left.\begin{array}{lll} u(y,0)=1, & u(0,t)=\alpha, & u(\infty,t)=0 \\ \theta(y,0)=0, & \theta(0,t)=1, & \theta(\infty,t)=0 \\ \phi(y,0)=0, & \phi(0,t)=1, & \phi(\infty,t)=0 \end{array}\right\} \tag{14}$$

Where

$M = \dfrac{\sigma B_0^2 L^2}{D_m \rho}$ magnetic parameter, $K_p = \dfrac{kD_m}{\upsilon L^2}$, permeability parameter

$Sc = \dfrac{\upsilon}{D_m}$, schmidt number $Gr_\phi = \dfrac{g\beta' L^2 (C_w - C_\infty)}{U_\infty D_m}$ grashof number

$\Pr = \dfrac{\rho C_p D_m}{K}$ prantdl number, $Ec = \dfrac{U_\infty^2 \upsilon}{C_p (T_w - T_\infty) D_m}$ eckert number

$q = \dfrac{QL^2}{D_m}$ heat source parameter

3.2 Solution via Frobenius Method

Here, we seek self-similar solutions (Olayiwola 2015) as

$$\left.\begin{array}{l} u(y,t) = t^{-\alpha} f(\eta), \ \eta = y t^{-\alpha} \\ \theta(y,t) = t^{-\alpha} g(\eta), \ \eta = y t^{-\alpha} \\ \phi(y,t) = t^{-\alpha} h(\eta), \ \eta = y t^{-\alpha} \end{array}\right\} \tag{15}$$

and this self-similar solutions exist when $\alpha = 0$ so that equations (11) - (14) reduce to

$$Scf'' - \left(M + \frac{1}{K_p}\right) f + Gr_\phi h = 0 \tag{16}$$

$$\frac{1}{\Pr} g'' + Ecf^2 + qg = 0 \tag{17}$$

$$h'' = 0 \tag{18}$$

Together with the boundary conditions:

$$\left.\begin{array}{ll} f(0)=\alpha, & f(\infty)=0 \\ g(0)=1, & g(\infty)=0 \\ h(0)=1, & h(\infty)=0 \end{array}\right\} \quad (19)$$

Using the transformation

$$m = 2e^{-\eta} - 1 \quad (20)$$

we change equations (16) – (19) from infinite plane $(0 \le \eta < \infty)$ to finite plane $(-1 \le m \le 1)$ and we get

$$Sc(m+1)^2 f'' + Sc(m+1)f' - \left(M + \frac{1}{K_p}\right)f + Gr_\phi h = 0 \quad (21)$$

$$\frac{1}{\Pr}(m+1)^2 g'' + \frac{1}{\Pr}(m+1)g' + Ec(m+1)^2 f'^2 + qg = 0 \quad (22)$$

$$(m+1)^2 h'' + (m+1)h' = 0 \quad (23)$$

$$\left.\begin{array}{ll} f(-1)=0, & f(1)=\alpha \\ g(-1)=0, & g(1)=1 \\ h(-1)=0, & h(1)=1 \end{array}\right\} \quad (24)$$

Then we solve equations (21) – (24) using Frobenius method and we obtain

$$h(m) = 2\left(1 + m - \frac{1}{2}m^2 + \frac{1}{3}m^3 + \cdots\right) - \frac{260}{99}\left(m - \frac{1}{6}m^3 + \frac{5}{24}m^4 - \frac{13}{60}m^5 + \cdots\right) \quad (25)$$

$$f(m) = P_1(1 + m + P_2m^2 + P_3m^3 + P_4m^4 + \ldots) + P_5(m + P_6m^3 + P_7m^4 + P_8m^5 + \ldots) \quad (26)$$

$$g(m) = q_1\left(1 + m + q_2m^2 + q_3m^3 + q_4m^4 + \cdots\right) + q_5\left(m + q_6m^3 + q_7m^4 + q_8m^5 + \cdots\right) \quad (27)$$

Where,

$$P_1 = -\frac{B\alpha}{AE - BC}$$

$$P_2 = \frac{\left(M + \frac{1}{K_p}\right) - Sc - 2Gr}{2Sc}$$

$$P_3 = \frac{3Sc\left[\left(M + \frac{1}{K_p}\right) - Sca_1 - 2\,\mathrm{Gr}\right] + \left[\left(M + \frac{1}{K_p}\right) - Sc\right] + \frac{62}{99}Gr}{6Sc}$$

$$P_4 = \left(\frac{5Sc\left[3Sc(\left(M + \frac{1}{K_p}\right) - Sc - 2\,\mathrm{Gr})\right] + \left[\left(M + \frac{1}{K_p}\right) - Sc\right] + \frac{62}{99}Gr}{24Sc} + \frac{\left[\left(M + \frac{1}{K_p}\right) - Sc - 2Gr\right]\left[\left(M + \frac{1}{K_p}\right) - 4Sc\right]}{24Sc} + \frac{Gr}{12} \right)$$

$$P_5 = \frac{A\alpha}{AE - BC}$$

$$P_6 = \left(\frac{\left[\left(M + \frac{1}{K_p}\right) - Sc\right] + \frac{62}{99}Gr}{6Sc} \right)$$

$$P_7 = \left(\frac{\left[\left(M + \frac{1}{K_p}\right) - Sc\right] + \frac{62}{99}Gr}{24Sc} + \frac{Gr}{12Sc} \right)$$

$$P_8 = \left(\frac{\left[\left(M + \frac{1}{K_p}\right) - Sc\right] + \frac{62}{99}Gr}{40Sc} + \frac{\left[\left[\left(M + \frac{1}{K_p}\right) - Sc\right] + \frac{62}{99}Gr\right]\left[\left(M + \frac{1}{K_p}\right) - 9Sc\right]}{120Sc} - \frac{2Gr}{660Sc} \right)$$

$$q_1 = \frac{-G}{(FJ - HG)}$$

$$q_2 = \left(\frac{-1 - \Pr q - \Pr Ec(P_1 + P_5)^2}{2} \right)$$

$$q_3 = \left(\frac{2 + 2\Pr q + 3\,\mathrm{Ec}(P_1 + P_5)^2 - 2\Pr Ec\left((P_1 + P_5)^2 + 2P_1 P_2 (P_1 + P_5)\right)}{6} \right)$$

$$q_4 = \left(\frac{1}{12} \left[\begin{array}{l} -\frac{5}{2}\left[2 + 3\Pr q - \Pr q a_1 + 3Ec\left(P_1 + P_5\right)^2 - 2\Pr Ec \left(\begin{array}{l} \left(P_r + P_5\right)^2 + \\ 2P_1P_2(\mathrm{P}_1 + P_5) \end{array} \right) \right] \\ -2\left(-1 - \Pr q - Ec\left(P_1 + P_5\right)^2\right) - \Pr q \left(\frac{-1 - \Pr q - Ec\left(P_1 + P_5\right)^2}{2} \right) - \\ \Pr Ec\left(\left(P_1 + P_5\right)^2 + 8P_1P_2\left(P_1 + P_5\right) + 2P_1^2\left(3P_3 + 2P_2^2\right) + \left(6P_5P_6\left(2P_1 + P_5\right)\right) \right) \end{array} \right] \right)$$

$$q_5 = \frac{F}{\left(FJ - HG\right)}$$

$$q_6 = \left(\frac{-(1 - \Pr q) - 2Ec\Pr\left(\left(P_1 + P_5\right)^2 - 2P_1P_2\left(P_1 + P_5\right) \right)}{6} \right)$$

$$q_7 = \left(\frac{\frac{5}{2}\left[-\left(1 + \Pr q\right) - 2Ec\Pr\left(P_1 + P_5\right)^2 + 8P_1P_2\left(P_1 + P_5\right) \right] - Ec\Pr\left(\left(P_1 + P_5\right)^2 + 8P_1P_2\left(P_1 + P_5\right) \right)}{12} \right)$$

$$q_8 = \left(-\frac{1}{20} \times \left[\begin{array}{l} -\frac{9}{4}\left[-\frac{5}{2}\left[-\left(1 + \Pr q\right) - 2Ec\Pr\left(\left(P_1 + P_5\right)^2 - 2P_1P_2\left(P_1 + P_5\right) \right) \right] - Ec\Pr\left(\left(P_1 + P_5\right)^2 - 8P_1P_2\left(P_1 + P_5\right) \right) \right] \\ -\frac{1}{6}\left[-\left(1 + \Pr q\right) - 2Ec\Pr\left(\left(P_1 + P_5\right)^2 - 2P_1P_2\left(P_1 + P_5\right) \right) \right]\left(9 + \Pr q\right) - Ec\Pr 4P_1P_2\left(P_1 + P_5\right) - \\ Ec\Pr\left(8P_1^2P_2^2 + 12P_1P_3\left(P_1 + P_5\right) + 12P_5P_6\left(P_1 + P_5\right) \right) - \Pr Ec \left(\begin{array}{l} 8P_1P_4\left(P_1 + P_5\right) + 8P_5P_7\left(P_1 + P_5\right) + \\ 12P_1P_2\left(P_1 + P_5P_6\right) \end{array} \right) \end{array} \right] \right)$$

$$A = \left[\begin{array}{l} \frac{\left(M + \frac{1}{K_p}\right) - Sc - 2Gr}{2Sc} - \left(\frac{3Sc\left[\left(M + \frac{1}{K_p}\right) - Sca_1 - 2\mathrm{Gr}\right] + \left[\left(M + \frac{1}{K_p}\right) - Sc\right] + \frac{62}{99}Gr}{6Sc} \right) + \\ \left(\frac{5Sc\left[3Sc\left(M + \frac{1}{K_p}\right) - Sc - 2\mathrm{Gr}\right] + \left[\left(M + \frac{1}{K_p}\right) - Sc\right] + \frac{62}{99}Gr}{24Sc} + \frac{\left[\left(M + \frac{1}{K_p}\right) - Sc - 2G_r\right]\left[\left(M + \frac{1}{K_p}\right) - 4Sc\right] + }{24Sc} \frac{Gr}{12} \right) + \cdots \end{array} \right]$$

$$B = \left[-1 - \left(\frac{\left[\left(M + \frac{1}{K_p}\right) - Sc\right] + \frac{62}{99}Gr}{6Sc} \right) + \left(\frac{\left[\left(M + \frac{1}{K_p}\right) - Sc\right] + \frac{62}{99}Gr}{24Sc} + \frac{Gr}{12Sc} \right) - \left(\begin{array}{l} \frac{\left[\left(M + \frac{1}{K_p}\right) - Sc\right] + \frac{260}{99}Gr}{40Sc} + \\ \frac{\left[\left[\left(M + \frac{1}{K_p}\right) - Sc\right] + \frac{66}{99}Gr\right]\left[\left(M + \frac{1}{K_p}\right) - 9Sc\right]}{120Sc} - \frac{2Gr}{660Sc} \end{array} \right) + \cdots \right]$$

$$C=\left[\begin{array}{l}2+\left(\dfrac{\left(M+\dfrac{1}{K_p}\right)-Sc-2Gr}{2Sc}\right)+\left(\dfrac{3Sc\left[\left(M+\dfrac{1}{K_p}\right)-Sc-2\,\mathrm{Gr}\right]+\left[\left(M+\dfrac{1}{K_p}\right)-Sc\right]+\dfrac{62}{99}Gr}{6Sc}\right)+\\ \left(\dfrac{5Sc\left[3Sc(\left(M+\dfrac{1}{K_p}\right)-Sc-2\,\mathrm{Gr})]+\left[\left(M+\dfrac{1}{K_p}\right)-Sc\right]+\dfrac{62}{99}Gr\right]}{24Sc}+\dfrac{\left[\left(M+\dfrac{1}{K_p}\right)-Sc-2Gr\right]\left[\left(M+\dfrac{1}{K_p}\right)-4Sc\right]+}{24Sc}\dfrac{Gr}{12}\right)+\cdots\end{array}\right]$$

$$E=\left[1+\left(\dfrac{\left[\left(M+\dfrac{1}{K_p}\right)-\mathrm{Sc}\right]+\dfrac{62}{99}Gr}{6Sc}\right)+\left(\dfrac{\left[\left(M+\dfrac{1}{K_p}\right)-\mathrm{Sc}\right]+\dfrac{260}{99}Gr}{24Sc}\right)+\left(\begin{array}{l}\dfrac{\left[\left(M+\dfrac{1}{K_p}\right)-Sc\right]+\dfrac{66}{99}Gr}{40Sc}+\\ \dfrac{\left[\left[\left(M+\dfrac{1}{K_p}\right)-Sc\right]+\dfrac{66}{99}Gr\right]\left[\left(M+\dfrac{1}{K_p}\right)-9Sc\right]}{120Sc}-\dfrac{2Gr}{660Sc}\end{array}\right)+\cdots\right]$$

$$F=\left[\begin{array}{l}\left(-\dfrac{1-\Pr q-\Pr Ec(\mathrm{P}_1+P_5)^2}{2}\right)+\\ \left(\dfrac{2+3\Pr q-\Pr qa_1+3\,\mathrm{Ec}(\mathrm{P}_1+\mathrm{P}_5)^2-2\Pr Ec\left((\mathrm{P}_1+\mathrm{P}_5)^2+2\,\mathrm{P}_1\,\mathrm{P}_2(\mathrm{P}_1+\mathrm{P}_5)\right)}{6}\right)+\\ \left(\dfrac{1}{12}\left[\begin{array}{l}-\dfrac{5}{2}\left[2+3\Pr q-\Pr qa_1+3Ec\left(P_1+P_5\right)^2-2\Pr Ec\left(\left(P_r+P_5\right)^2+2P_1P_2(\mathrm{P}_1+P_5)\right)\right]\\ -2\left(-1-\Pr q-Ec\left(P_1+P_5\right)^2\right)-\Pr q\left(\dfrac{-1-\Pr q-Ec\left(P_1+P_5\right)^2}{2}\right)-\\ \Pr Ec\left(\left(P_1+P_5\right)^2+8P_1P_2\left(P_1+P_5\right)+2P_1^2\left(3P_3+2P_2^2\right)+\left(6P_5P_6\left(2P_1+P_5\right)\right)\right)\end{array}\right]\right)+\cdots\end{array}\right]+$$

$$G=\left[\begin{array}{l} -1+\left(\dfrac{-(1+\Pr q)-2Ec\Pr\left(\left(P_1+P_5\right)^2-2P_1P_2\left(P_1+P_5\right)\right)}{6}\right)m^3 \\ +\left(\dfrac{\frac{5}{2}\left[-\left(1+\Pr q\right)-2Ec\Pr\left(P_1+P_5\right)^2+8P_1P_2\left(P_1+P_5\right)\right]-Ec\Pr\left(\left(P_1+P_5\right)^2+8P_1P_2\left(P_1+P_5\right)\right)}{12}\right)+ \\ \left(-\dfrac{1}{20}\times\left[\begin{array}{l} -\dfrac{9}{4}\left[-\dfrac{5}{2}\left[-\left(1+\Pr q\right)-2Ec\Pr\left(\left(P_1+P_5\right)^2-2P_1P_2\left(P_1+P_5\right)\right)\right]-Ec\Pr\left(\left(P_1+P_5\right)^2-8P_1P_2\left(P_1+P_5\right)\right)\right] \\ -\dfrac{1}{6}\left[-\left(1+\Pr q\right)-2Ec\Pr\left(\left(P_1+P_5\right)^2-2P_1P_2\left(P_1+P_5\right)\right)\right]\left(9+\Pr q\right)-Ec\Pr 4P_1P_2\left(P_1+P_5\right)- \\ Ec\Pr\left(8P_1^2P_2^2+12P_1P_3\left(P_1+P_5\right)+12P_5P_6\left(P_1+P_5\right)\right)-\Pr Ec\left(8P_1P_4\left(P_1+P_5\right)+8P_5P_7\left(P_1+P_5\right)+12P_1P_2\left(P_1+P_5P_6\right)\right) \end{array}\right]\right) \end{array}\right]$$

$$H=\left[\begin{array}{l} \left(2+\left(\dfrac{-1-\Pr q-\Pr Ec(P_1+P_5)^2}{2}\right)\right)+ \\ \left(\dfrac{2+2\Pr q+3\mathrm{Ec}(\mathrm{P}_1+\mathrm{P}_5)^2-2\Pr Ec\left((\mathrm{P}_1+\mathrm{P}_5)^2+2\mathrm{P}_1\mathrm{P}_2(\mathrm{P}_1+\mathrm{P}_5)\right)}{6}\right)+ \\ \left(\dfrac{1}{12}\left[\begin{array}{l} -\dfrac{5}{2}\left[2+3\Pr q-\Pr q a_1+3Ec\left(P_1+P_5\right)^2-2\Pr Ec\left(\left(P_1+P_5\right)^2+2P_1P_2(\mathrm{P}_1+P_5)\right)\right] \\ -2\left(-1-\Pr q-Ec\left(P_1+P_5\right)^2\right)-\Pr q\left(\dfrac{-1-\Pr q-Ec\left(P_1+P_5\right)^2}{2}\right)- \\ \Pr Ec\left(\left(P_1+P_5\right)^2+8P_1P_2\left(P_1+P_5\right)+2P_1^2\left(3P_3+2P_2^2\right)+\left(6P_5P_6\left(2P_1+P_5\right)\right)\right) \end{array}\right]\right)+\cdots \end{array}\right]$$

$$
J = \left[\begin{array}{l}
1 + \left(\dfrac{-(1+\Pr q) - 2Ec\Pr\left(\left(P_1+P_5\right)^2 - 2P_1P_2\left(P_1+P_5\right) \right)}{6} \right) \\
+ \left(\dfrac{\frac{5}{2}\left[-\left(1+\Pr q\right) - 2Ec\Pr\left(P_1+P_5\right)^2 + 8P_1P_2\left(P_1+P_5\right) \right] - Ec\Pr\left(\left(P_1+P_5\right)^2 + 8P_1P_2\left(P_1+P_5\right) \right)}{12} \right) + \\
\left(-\dfrac{1}{20} \times \left[\begin{array}{l}
-\frac{9}{4}\left[-\frac{5}{2}\left[-\left(1+\Pr q\right) - 2Ec\Pr\left(\left(P_1+P_5\right)^2 - 2P_1P_2\left(P_1+P_5\right) \right) \right] - Ec\Pr\left(\left(P_1+P_5\right)^2 - 8P_1P_2\left(P_1+P_5\right) \right) \right] \\
-\frac{1}{6}\left[-\left(1+\Pr q\right) - 2Ec\Pr\left(\left(P_1+P_5\right)^2 - 2P_1P_2\left(P_1+P_5\right) \right) \right]\left(9+\Pr q\right) - Ec\Pr 4P_1P_2\left(P_1+P_5\right) - \\
Ec\Pr\left(8P_1^2P_2^2 + 12P_1P_3\left(P_1+P_5\right) + 12P_5P_6\left(P_1+P_5\right) \right) - \Pr Ec\left(8P_1P_4\left(P_1+P_5\right) + 8P_5P_7\left(P_1+P_5\right) + 12P_1P_2\left(P_1+P_5P_6\right) \right)
\end{array} \right] \right) + \cdots
\end{array} \right]
$$

The computations were done using computer symbolic algebraic package MAPLE.

4. Results and Discussion

In order to study the behavior of velocity $f\left(x\right)$ and temperature $g\left(x\right)$ fields, a comprehensive numerical computation is carried out for various values of the parameters that describe the flow characteristics, and the results are reported in terms of graphs as shown in Figures 2 to 6. To be more realistic, the value of the Prandtl number is chosen to be Pr = 0.71, which corresponds to air.

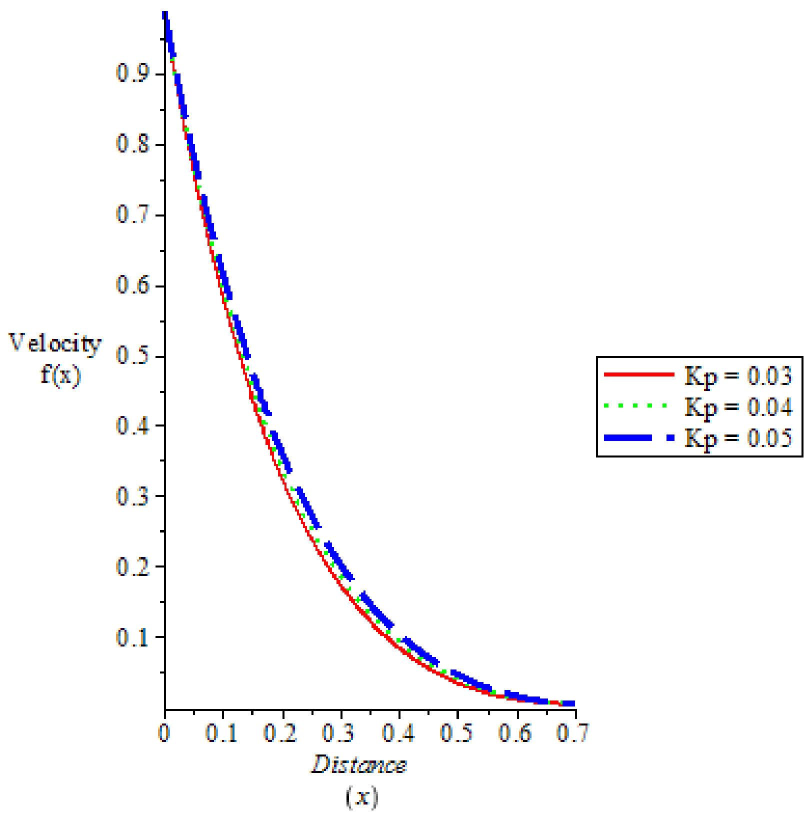

Figure 2: Variation of Velocity f(x) with Permeability Parameter K_p

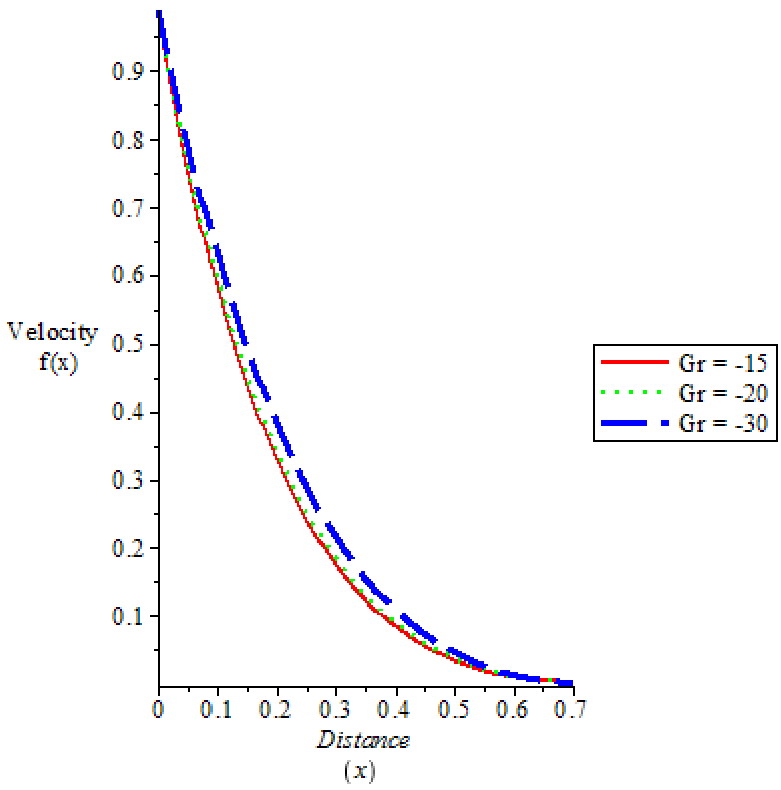

Figure 3: Variation of Velocity f(x) with Grashof number Gr

Figures 2 and 3 shows an exponential decrease in the fluid velocity from the plate surface to the free stream value away from the plate. From figure 2, it is observed that the velocity increases as Permeability parameter increases while the velocity increases as Grashof number decreases in figure 3.

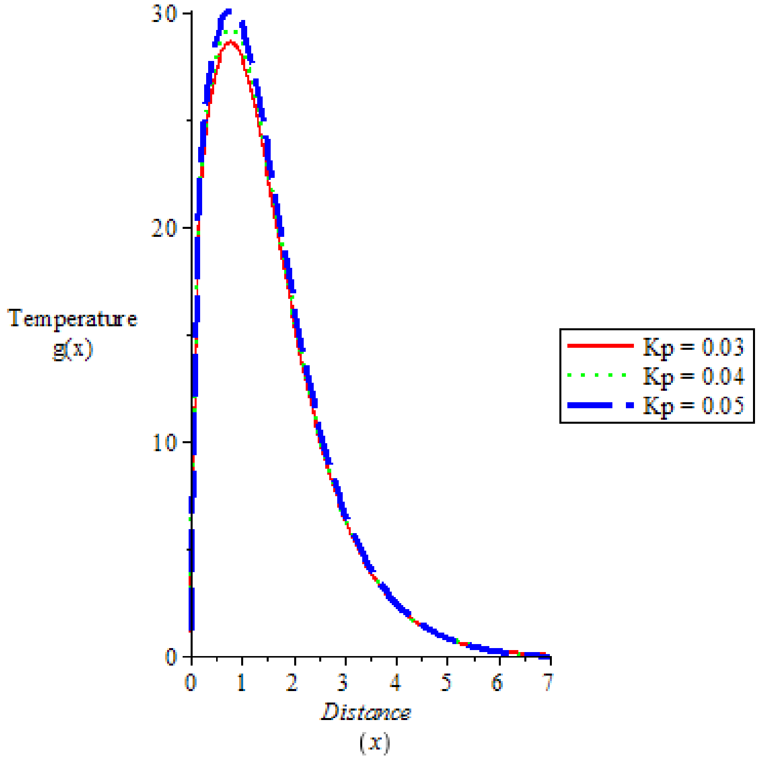

Figure 4: Variation of Temperature g(x) with Permeability Parameter K_p **.**

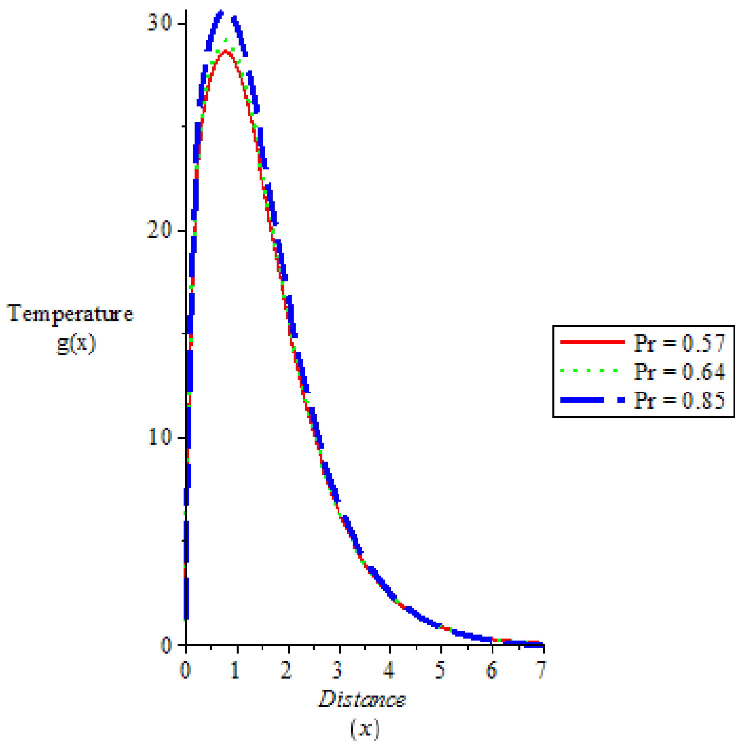

Figure 5: Variation of Temperature g(x) with Prandtl number Pr **.**

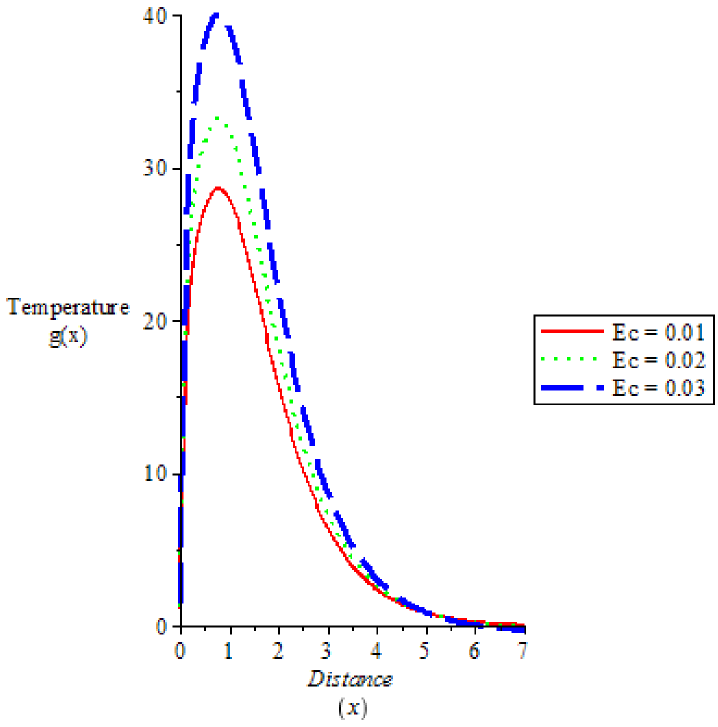

Figure 6: Variation of Temperature g(x) with Eckert number Ec **.**

From figures. 4, 5 and 6 the fluid temperature rises rapidly from unity at the plate, attains a maximum near the plate and decreases to free stream value away from plate. It is observed that the maximum fluid temperature increases respectively as Permeability parameter, Prandtl number and Eckert number increases.

However, it is interesting to note that as the permeability parameter $\left(K_p\right)$ increases, both velocity and thermal boundary layer thickness decrease when the plate is heated by free convection current $\left(Gr < 0\right)$.

5. Conclusion

We can therefore conclude that for heating of the plate by free convection current $\left(Gr < 0\right)$

(i). Increase in K_p results in a decrease in the velocity boundary layer thickness.

(ii). Increase in K_p, $\Pr$ and Ec lead to a decrease in the thermal boundary layer thickness.

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