Frequency Characteristics of the Compliant Constant-Force Mechanism Based on the Pseudo-Rigid-Body Model

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Abstract

A very practical and important alternative approach to the analysis of compliant mechanisms is the frequency response method. Frequency characteristics analysis of a system is important to gain an understanding of the dynamic performance of a compliant mechanism especially when it has a wide range of working frequencies. The resonance phenomenon can be examined through the amplitude-frequency characteristics of the system. As shown on the amplitude-frequency diagram, it is obvious that one must avoid driving this system at or near its natural frequency.

Keyword: *Compliant mechanisms, frequency response method, dynamic performance, resonance phenomenon, amplitude-frequency diagram, natural frequency.*

Introduction

A constant-force mechanism can be defined as one that generates a constant, unidirectional force at any given point on a hinged lever, for all positions of the lever (Nathan 1985). Alternatively, it can be defined as a mechanism that produces a constant output force for a range of input displacements (Nahar and Sugar 2003). Such mechanisms are important in applications with varying displacements, but requiring a constant resultant output force. The mechanism is typically displacement driven, the input is a displacement at the slider and the output is a force. Unlike a linear spring where the force increases as the displacement increases, the reaction force remains constant for various displacements. Compliant constant-force mechanisms have specific geometry and stiffness that cause the combination of energy storage and mechanical advantage to produce a constant-force.

The constant-force mechanism can be configured in a variety of ways for applications such as: electronic connectors that maintain a constant-force regardless of part tolerances; a constant-force spring in a hospital bed that will allow the same force to be applied evenly over a person’s body to reduce bed sores; a gripping device to hold delicate parts of varying size; wear testing, where a constant-force needs to be applied to a surface as the surface is worn down; manufacturing processes that involve tool diameter changes such as grinding and honing; motor brush wear improvement; and safety return spring to cause valves or other devices to go to a specified position when power is lost while minimizing actuator size. In these and other applications, the constant-force mechanism eliminates the need for expensive and elaborate force control, replacing it with a simple mechanical device (Evans and Howell 1999).

A very practical and important alternative approach to the analysis of compliant mechanisms is the frequency response method. The frequency response of a system is defined as the steady-state response of the system to a sinusoidal input. The sinusoid is a unique input and the resulting output for a linear system, as well as throughout the system, is sinusoidal in the steady state, it differs from input waveform only in amplitude and phase angle (Dorf and Bishop 1998). Basic frequency spectrum includes the amplitude-frequency characteristic (dynamic compliance) and the phase-frequency characteristic of a system. Frequency characteristic analysis of a system is important to gain an understanding of the dynamic performance of a compliant mechanism especially when it has a wide range of working frequencies. The resonance phenomenon can be examined through the amplitude-frequency characteristics of the system. Even if the amplitude of the system output matches the design specification, the phase angle between the input and output of the system may not meet the desired performance.

Design Analysis

Dynamic Equations of Motion

The dynamic differential equations of motion for the constant-force mechanism shown in Fig. 1 can be derived from Lagrange's equation given as

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{U}_i}\right) - \frac{\partial T}{\partial U_i} + \frac{\partial U}{\partial U_i} = Q_i \,, \qquad (1)$$

where $i = 1, 2,\ldots, 25$.

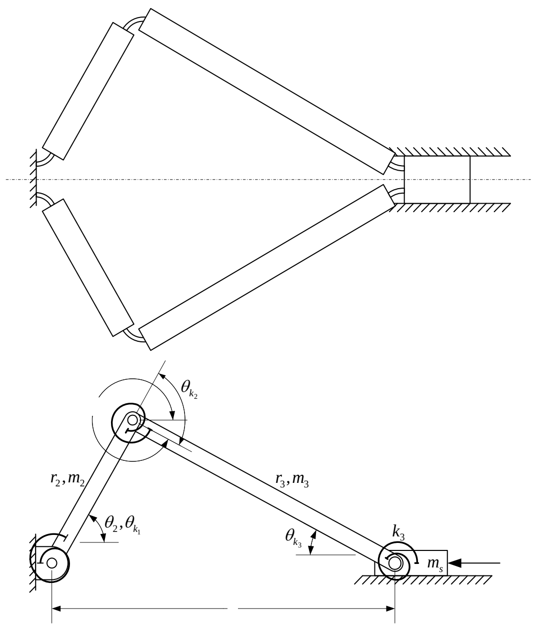

Fig. 1. (a) The compliant constant-force mechanism; and (b) Its Pseudo-rigid-body model.

In the absence of damping forces in the mechanism and of external forces on the slider, the equations of motion may be expressed in matrix form as

$$[M]_{n\times n}\left\{\ddot{U}_a\right\}_{n\times 1} + [K]_{n\times n}\{U\}_{n\times 1} = \{Q\}_{n\times 1}\,. \qquad (2)$$

When damping forces are included, the equation of motion becomes

$$[M]_{n\times n}\left\{\ddot{U}\right\}_{n\times 1} + [C]_{n\times n}\left\{\dot{U}\right\}_{n\times 1} + [K]_{n\times n}\{U\}_{n\times 1} = \{Q\}_{n\times 1}\,, \qquad (3)$$

where n = number of generalized coordinates (elastic degrees of freedom of mechanism).

The coefficient matrices $[M]$, $[C]$ and $[K]$ are system mass, damping and stiffness matrix respectively and $\{U\}$ is the set of generalized coordinates representing the translation and rotation deformations at each element node in a global coordinate system. They are functions of the mechanism geometry and vary as input angle is varied. These values are repeated after each motion cycle of the mechanism.

Mechanisms Mass and Stiffness matrices

The element mass matrix $[m]$ and stiffness matrix $[k]$ can be written as follows:

$$[m]_e = \rho A l_e \begin{bmatrix} \frac{1}{3} & 0 & 0 & \frac{1}{6} & 0 & 0 \\ 0 & \frac{13}{35} & \frac{11 l_e}{210} & 0 & \frac{9}{70} & -\frac{13 l_e}{420} \\ 0 & \frac{11 l_e}{210} & \frac{l_e^2}{105} & 0 & \frac{13 l_e}{420} & -\frac{l_e^2}{140} \\ \frac{1}{6} & 0 & 0 & \frac{1}{3} & 0 & 0 \\ 0 & \frac{9}{70} & \frac{13 l_e}{420} & 0 & \frac{13}{35} & -\frac{11 l_e}{210} \\ 0 & -\frac{13 l_e}{420} & -\frac{l_e^2}{140} & 0 & -\frac{11 l_e}{210} & \frac{l_e^2}{105} \end{bmatrix}, \qquad (4)$$

$$[k]_e = \begin{bmatrix} \frac{E_e A_e}{l_e} & 0 & 0 & -\frac{E_e A_e}{l_e} & 0 & 0 \\ 0 & \frac{12 E_e I_e}{l_e^3} & \frac{6 E_e I_e}{l_e^2} & 0 & -\frac{12 E_e I_e}{l_e^3} & \frac{6 E_e I_e}{l_e^2} \\ 0 & \frac{6 E_e I_e}{l_e^2} & \frac{4 E_e I_e}{l_e} & 0 & -\frac{6 E_e I_e}{l_e^2} & \frac{2 E_e I_e}{l_e} \\ -\frac{E_e A_e}{l_e} & 0 & 0 & \frac{E_e A_e}{l_e} & 0 & 0 \\ 0 & -\frac{12 E_e I_e}{l_e^3} & -\frac{6 E_e I_e}{l_e^2} & 0 & \frac{12 E_e I_e}{l_e^3} & -\frac{6 E_e I_e}{l_e^2} \\ 0 & \frac{6 E_e I_e}{l_e^2} & \frac{2 E_e I_e}{l_e} & 0 & -\frac{6 E_e I_e}{l_e^2} & \frac{4 E_e I_e}{l_e} \end{bmatrix}, \qquad (5)$$

where A_e = the cross sectional area of the element.

The transformation matrix is given as

$$[R]=\begin{bmatrix} \lambda & \mu & 0 & 0 & 0 & 0 \\ -\mu & \lambda & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda & \mu & 0 \\ 0 & 0 & 0 & -\mu & \lambda & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (6)$$

where:

$$\lambda = \cos\theta \text{ and } \mu = \sin\theta. \quad (7)$$

With the transformation matrix, the following vector transformations may be expressed:

$$[u]=[R]\{U\}, \quad (8)$$

$$[\dot{u}]=[R]\{\dot{U}\}, \quad (9)$$

$$[\ddot{u}]=[R]\{\ddot{U}\}. \quad (10)$$

The element mass matrix [*m*] and stiffness matrix [*k*] given in Eqs. (4) and (5) may be transformed from the elements local (element-oriented) coordinates to global (system-oriented) coordinates using Eq. (6). Typically, a compliant mechanism is discretized into many elements as in finite element analysis. Each element is associated with a mass and a stiffness matrix. Each element has its own local coordinate system. As shown in Fig. 2, half of the symmetric compliant mechanism is discretized into eight planar frame elements. When the element mass and stiffness matrices of all elements are combined and coordinate transformation necessary to transform the element local coordinate system to a global coordinate system is carried out, it gives the system mass [*M*] and stiffness [*K*] matrices.

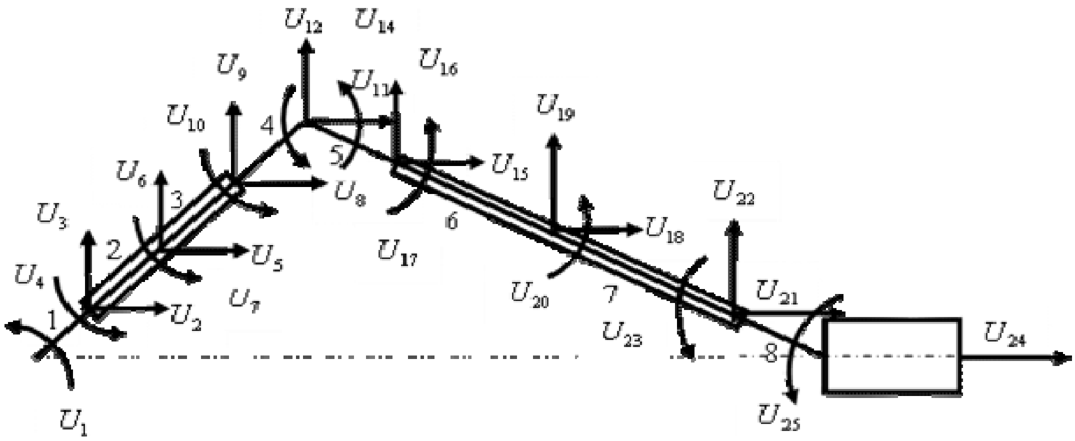

Fig. 2. Generalized coordinate in system-oriented coordinates with nodal compatibility.

Natural Frequencies and Modes of Mechanism

For most structures, the exact form of damping matrix is unknown since the sources of energy loss are complicated. Also, in most cases, the effect of damping on the vibration mode shapes of the structure is small (Sandor and Erdman 1988). Therefore, an assumption as to the form of this matrix is justifiable and since a mechanism during its motion is regarded as a structure in numerous positions, the adaptation of the conventional structural damping matrices to mechanism problems is deemed appropriate (Sandor and Erdman 1988). In order to obtain the natural frequencies and natural modes of a system, undamped free vibration equation is used because the damping has very little influence on the natural frequencies of a system. From the free vibration of the system, the following modal equation is obtained,

$$([K]-\lambda_i[M])\{X_i\}=\{0\},\ i = 1, 2,\dots, n. \quad (11)$$

The condition of non-zero solution of equation is

$$|([K]-\lambda_i[M])| = 0,\ i = 1, 2,\dots, n. \quad (12)$$

From Eq. (12), the eigenvalues λ_i, i = 1, 2,…, n, of the system can be obtained:

$$\lambda_i = \omega_{ni}^2 . \qquad (13)$$

The circular natural frequency ω_n (rad/sec) can be converted to cycles per second (hertz) by noting that there are 2π radians per revolution and one revolution per cycle:

$$f_n = \frac{1}{2\pi}\omega_n \text{ Hertz.} \qquad (14)$$

By substituting each eigenvalue λ_i into Eq. (11), the eigenvector $\{X_i\}$ of the system can be determined. The modal matrix, whose columns are the natural modes of the system, is defined as

$$[\Phi] = [\{X_1\} \quad \{X_2\}.......\{X_n\}]. \qquad (15)$$

Frequency Characteristics of Mechanism

The equation of motion for damped force vibrations is given as

$$M\ddot{u} + C\dot{u} + Ku = F_0 \sin \omega t . \qquad (17)$$

The solution for steady-state vibration of the system (after the initial transient behavior) is given as (Sandor *et al*. 1999)

$$u = A\sin(\omega t - \phi), \qquad (18)$$

where: A = amplitude, ω = angular velocity of forcing function, and ϕ = phase angle between applied force and displacement.

The amplitude and phase angle are obtained from the following expressions

$$A = \frac{F_0}{\sqrt{\left(K - M\omega^2\right)^2 + \left(C\omega\right)^2}}, \qquad (19)$$

$$\phi = \arctan\left[\frac{C\omega}{\left(K - M\omega^2\right)^2}\right]. \qquad (20)$$

It should be understood that this phase angle is limited to the range 0 < θ < 180° (Clough and Penzien 2003). Also:

$$\frac{A}{\left(\frac{F_0}{K}\right)} = \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}, \qquad (21)$$

$$\phi = \arctan\left[\frac{2\xi\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right]. \qquad (22)$$

It is convenient to define the frequency ratio β as

$$\beta = \frac{\omega}{\omega_n} \text{ and } A_0 = \frac{F_0}{K}. \qquad (23)$$

Then Eq. (21) becomes

$$\frac{A}{(A_0)} = \frac{1}{\sqrt{\left(1 - (\beta)^2\right)^2 + (2\xi\beta)^2}} \qquad (24)$$

and Eq. (22) becomes

$$\phi = \arctan\left[\frac{2\xi\beta}{1 - (\beta)^2}\right]. \qquad (25)$$

Results and Discussion

The natural frequency (and its overtones) is of great importance to the design of compliant mechanisms as they define the frequencies at which the system will resonate (Norton 2004). Any system which contains more than one energy storage device such as a mass and a spring will possess at least one natural frequency. If such a system is excited at its natural frequency, a condition called resonance is set up, in which the energy stored in the system's elements will oscillate from one element to the other at that frequency. The result can be violent oscillations in the displacements of the movable elements in the system as the energy moves from potential to kinetic form and vice versa (Norton 2004). Fig. 3 shows the amplitude-frequency characteristic of the output displacement, from which a non-linear relationship of the amplitude versus input frequency can be seen. These plots normalize the forcing frequency as a frequency ratio of the input frequency over the fundamental frequency. Likewise, the amplitude is normalized as the ratio of the amplitude of the output displacement over the static displacement. The maximum value of the amplitude displacement at a given operation frequency can be quantitatively determined from the amplitude-frequency characteristic curve. Thus, at a frequency of zero, the output is one, equal to the static displacement at the amplitude of the input force. As the forcing frequency increases toward the natural frequency, the amplitude of the output motion increases rapidly towards maximum at resonance. Beyond this point, the amplitude decreases rapidly and asymptotically toward zero at high frequency ratios. Dangerously, large amplitudes may occur at resonance and at other frequency ratios near the resonant frequency (Sandor, *et al.* 1999). It is therefore obvious as shown in Fig. 3, that one must avoid driving this system at or near its natural frequency. The designer has a degree of control over resonance, in that the system's mass and stiffness can be tailored to move its natural frequency away from any required operating frequencies. A common rule of thumb is to design the system to have a fundamental natural frequency at least ten times the highest forcing frequency expected in service, thus keeping all operations well below the resonance point. This is often difficult to achieve in mechanical systems.

Fig. 4 shows the spectrum of the phase difference between the output and input. In static and low speed situations, the phase difference is near 90^o, that is, the input and output are moving in directions opposite to each other. But when the frequency ratio is over 0.9, the phase difference reduces quickly. At the resonant state, the phase difference is near zero. The input and output move in the same direction. The analysis indicates that for the mechanism to function properly, it must operate at frequencies far away from the resonant state, either at relatively low frequency or at very high frequencies.

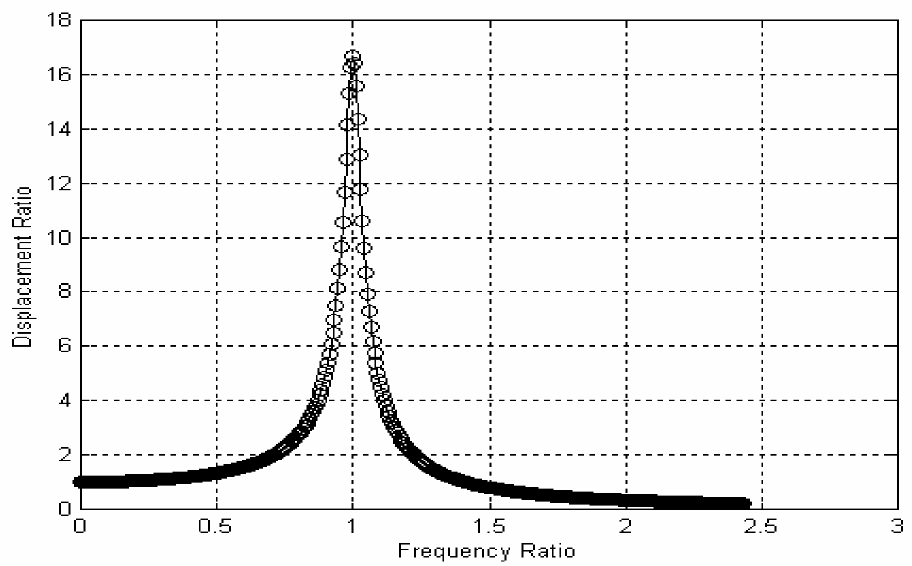

Fig. 3. Amplitude-frequency characteristic of mechanism.

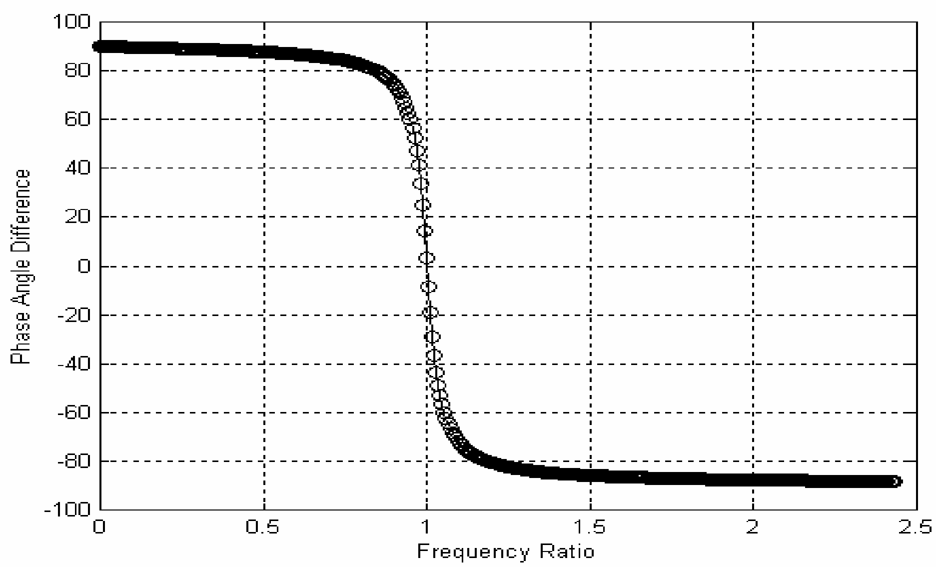

Fig. 4. Phase-frequency characteristic of mechanism.

Conclusion

Frequency characteristic analysis of a compliant mechanism is important to gain an understanding of the dynamic performance of compliant mechanisms especially when it has a wide range of working frequencies. The resonance phenomenon can be examined through the amplitude-frequency characteristics of the system. Results show that large amplitudes may occur at resonance and at other frequency ratios near the resonant frequency which makes it obvious, that one must avoid driving this system at or near its natural frequency. The designer has a degree of control over resonance, in that the system's mass and stiffness can be tailored to move its natural frequency away from any required operating frequencies.

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