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Adaptive Synchronization and Parameter Estimation of a 5D Hyperchaotic System with Unknown System Parameters

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***Abstract*— This paper presents the non-trivial synchronization and parameter estimation of identical 5-dimensional hyperchaotic systems based using adaptive controllers. The 5-dimensional hyperchaotic system exhibits extremely complex attractors with corresponding hypersensitivity to perturbations in their system and algebraic structures. An elegant adaptive control technique was used to synchronize the dense state trajectories in finite time and also estimate the unknown parameter vectors of the response system as the parameter update law satisfies some stringent Lyapunov stability criteria whose solutions are asymptotically stable in the sense of Lyapunov. Numerical confirmation via MATLAB proved the effectiveness of the method.**

Keywords-adaptive synchronization; hyperchaotic system; Lyapunov stability; parameter estimation

I. INTRODUCTION

A hyperchaotic system is a nonlinear dynamic system which has two or more positive Lyapunov exponents, in addition to a null exponent along the chaotic flow and one or more negative exponent that ensures boundedness of the solution. A continuous-time hyperchaotic flow has a minimum of four dimensions, although there are reported cases of three-dimensional systems exhibiting hyperchaotic behaviours [1], [2]. Since the proposition of the first hyperchaotic system [3], extensive research has been focused on hyperchaotic phenomena, resulting in the evolution of several 4D hyperchaotic systems with varying topological characteristics in the literature [4]–[11]. Five-dimensional systems [12], six-dimensional system [13], seven-dimensional system [14], eight-dimensional system [15] and nine-dimensional experimental system[16] have also been evolved. Corollary to this development, plethora of works have appeared on the control and synchronization of the dynamics of 3D and 4D chaotic systems using various control strategies [17], [18], leading to widespread applications in non-engineering and engineering systems [19]–[21] . Chaos synchronization occurs when two identical or non-identical chaotic systems are coupled such that, in spite of exponential divergence of their nearby trajectories, synchrony can be achieved in finite time or as $t \to \infty$. Synchronization depend on a number of conditions such as the coupling strength, parameter region of the system and the degree of divergence of the two chaotic systems while the necessary condition for master-slave synchronization is that the non-driven slave subsystem must be asymptotically stable in the sense of Lyapunov. Various control strategies have been used to synchronize chaotic systems. These include adaptive control [22], sliding mode control [23], nonlinear control [24], active control [18] , feedback and hybrid feedback control strategies among others [25], [26]. Different approaches to adaptive controller design for control, antisynchronization and synchronization of chaotic systems have been proposed in the literature and applied to nonlinear chaotic systems by various researchers. Although some of these methods are effective in tackling the control and synchronization objectives, they are often computationally complex. Hence, the justification for the use of the method proposed in [27]. This method bye-passes the rigorous mathematical analysis associated with the design of many controllers for adaptive control and synchronization of complex chaotic systems.

II. THE 5D HYPERCHAOTIC SYSTEM

The 5D hyperchaotic [28] evolves extremely complex attractors with corresponding hypersensitivity to perturbations in their system and algebraic structures. The algebraic structure of the system is given by

$$\begin{cases} \dot{x}_1 = \beta_1(x_2 - x_1) + x_2x_3x_4x_5 \\ \dot{x}_2 = \beta_2(x_1 + x_2) - x_1x_3x_4x_5 \\ \dot{x}_3 = -x_3 + 0.1x_1^2 \\ \dot{x}_4 = -\beta_3x_4 + x_1x_2x_3x_5 \\ \dot{x}_5 = -\beta_4(x_5 - x_4) + \beta_5x_1 + x_1x_2x_3x_4 \end{cases} \quad (1)$$

Where $[\beta_1, \beta_2, \beta_3, \beta_4, \beta_5] = [37, 14.5, 10.5, 15, 9.5]$ positive constants are, whose values determines the evolution of the chaotic attractors and trajectories in time space. A sample of the chaotic attractors evolved from the positive constants are given in Figure 1 and Figure 2.

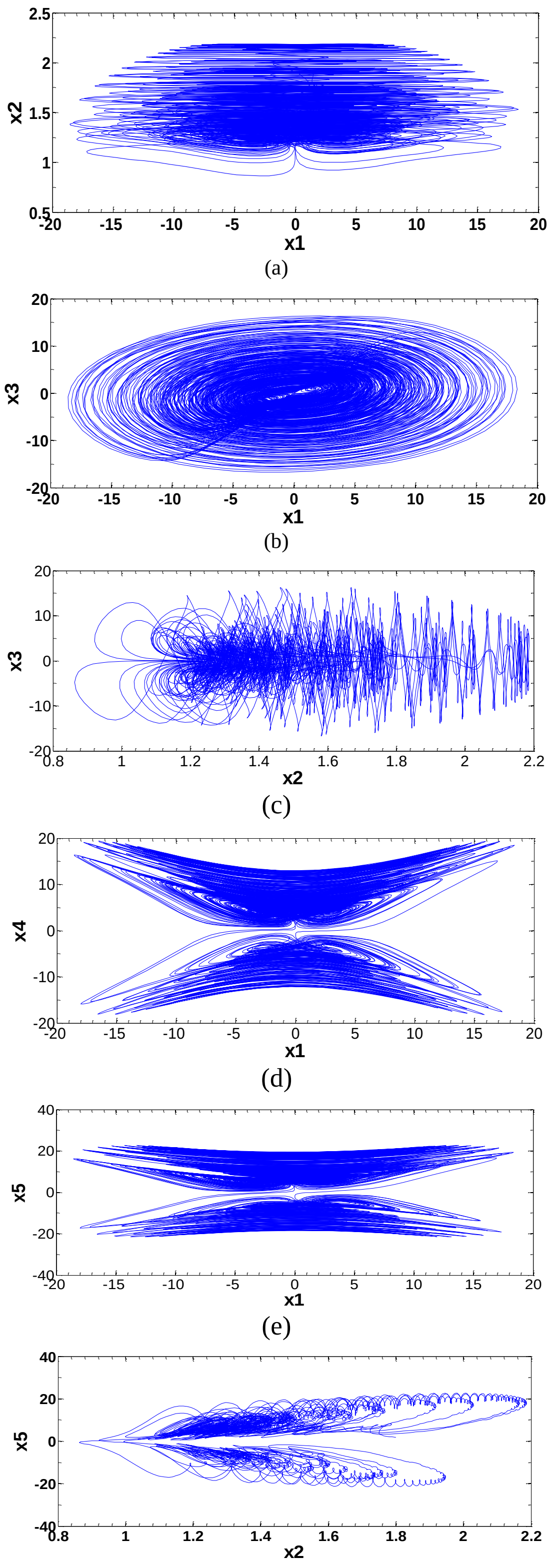


Figure 1. Phase portraits of the 5D hyperchaotic system

III. THEORETICAL ANALYSIS OF THE ADAPTIVE CONTROLLER

In this section, the master and slave systems, adaptive controllers and parameter estimation subsystem are designed according the method proposed in [23]. Let a Master chaotic system of the form $\dot{x} = F(x)$, where $F(x): R^n \rightarrow R^n$ is the vector field, be decomposed into the following form:

$$\dot{x} = A_m(x) + B_m(x)\beta \qquad (2)$$

A_m and B_m are parameters of the Master system and β is a vector of the system parameter, $x = (x_1, x_2, \ldots x_n)^T \in R^1$ are vectors of the state variables and n is the dimension of the system. Similarly, Let a controlled Slave system $\dot{y} = G(y)$, where $G(y): R^n \rightarrow R^n$ is the vector field, $y = (y_1, y_2, \ldots y_n)^T \in R^1$ are vectors of the state variables, be decomposed into the following form:

$$\dot{y} = A_s(y) + B_s(x)\hat{\beta} + L_i(x, y) \qquad (3)$$

$A_s \in R^n$ and $B_s \in R^{n \times m}$ are nonlinear and linear functions in matrix decomposition forms. $\hat{\beta}$ is the estimate of the parameter β and $K_i(x, y) \in R^m$ are adaptive controllers to be designed. Let the synchronization error be given as

$$e = (e_1, e_2, \ldots e_n)^T = (y_1 - x_1, y_2 - x_2, \ldots, y_n - x_n)^T \qquad (4)$$

While the parameter estimate error be $\phi(t) = \hat{\beta}(t) - \beta$ to be determined. If there exist some adaptive controllers $K_i(x, y) \in R^m$, $i = 1, 2 \ldots n$ such that

$$\lim_{t \to 0} \|e\| = \lim_{t \to 0} \|y(t) - x(t)\| = 0 \qquad (5)$$

then the master and slave systems can be synchronized and the uncertain parameters can be estimated simultaneously for $t \geq 0$. From (1) and (2), the error dynamic system can be given as:

$$\dot{e} = \dot{y} - \dot{x} = A_s(y) - A_m(x) + (B_s(y) - B_m(x))\hat{\beta} + B_m(x)\phi + K_i(x, y) \qquad (6)$$

And the adaptive controller will be of the form:

$$K = A_m(x) - A_s(y) - (B_s(y) - B_m(x))\hat{\beta} - \Psi \qquad (7)$$

Where $\Psi \in R^1$; Ψ_i, $i = 1, 2, \ldots n; n = 5$ is a matrix of the synchronization error variables and is of the form:

$$\begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \\ \Psi_5 \end{bmatrix} = \square \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} \tag{8}$$

□ is a diagonal matrix whose diagonal elements $diag[\lambda_{11}, \lambda_{22}, ..., \lambda_{55}]$ constitutes the positive coefficients of the adaptive controller, such that

$$\begin{bmatrix} \Psi_1 \\ \Psi_2 \\ \Psi_3 \\ \Psi_4 \\ \Psi_5 \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 0 & . & . & 0 \\ 0 & \lambda_{22} & 0 & . & 0 \\ 0 & 0 & \lambda_{33} & . & 0 \\ 0 & . & . & \lambda_{44} & 0 \\ 0 & . & . & 0 & \lambda_{55} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \end{bmatrix} \tag{9}$$

By substituting (7) into (6), the synchronization error system and parameter estimate error system is obtained as

$$\begin{cases} \dot{e} = B_m(x)\phi - \Psi \\ \dot{\hat{\beta}} = -B_m^T(x)e \end{cases} \tag{10}$$

Where B_m^T is the transpose of B_m.

Theorem [23]:

By appropriate selection of the controller coefficients of (8), the Slave system (3) can be regulated by the adaptive controller (7) to achieve synchrony with the Master system (1), such that the parameter estimate error system (10) satisfies

$$\lim_{t\to 0}\|\phi(t)\| = \lim_{t\to 0}\left\|\hat{\beta}(t) - \beta(t)\right\| = 0 \tag{11}$$

Proof:

Adopt a Lyapunov function candidate

$$V = \frac{1}{2}(e^T e + \phi^T \phi) \tag{12}$$

By making use of (10)

$$\begin{aligned} \dot{V} &= e^T \dot{e} + \phi^T \dot{\phi} = e^T (B_m(x)\phi - \Psi) + \phi^T \dot{\hat{\beta}} \\ &= -e^T \Psi + e^T B_m(x)\phi + \phi^T \dot{\hat{\beta}} \\ &= -e^T \Psi \le 0 \end{aligned} \tag{13}$$

It can be inferred from (13), based on the Lyapunov stability criteria [21], that the parameter estimation error system and synchronization error system are globally asymptotically stable for $t \ge 0$.

IV. APPLICATION TO THE 5D HYPERCHAOTIC SYSTEM

In this section, the theory discussed above is applied to synchronize two identical 5-D hyperchaotic systems with different initial conditions. Let (1) and (2) be the master and controlled slave systems respectively. Firstly, the nonlinear part A_m and linear parts B_m of the Master system are separated into matrix decomposition forms as follows:

$$A_m(x) = \begin{bmatrix} x_2x_3x_4x_5 \\ -x_1x_3x_4x_5 \\ 0.1x_1^2 \\ x_1x_2x_3x_5 \\ x_1x_2x_3x_4 \end{bmatrix} \tag{14}$$

$$B_m = \begin{bmatrix} x_2 - x_1 & 0 & 0 & 0 & 0 \\ 0 & x_1 + x_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -x_4 & 0 & 0 \\ 0 & 0 & 0 & x_4 - x_5 & x_1 \end{bmatrix} \tag{15}$$

Secondly, the matrix (15) is transposed as follows:

$$B_m^T(x) = \begin{bmatrix} x_2 - x_1 & 0 & 0 & 0 & 0 \\ 0 & x_1 + x_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -x_4 & 0 \\ 0 & 0 & 0 & 0 & x_4 - x_5 \\ 0 & 0 & 0 & 0 & x_1 \end{bmatrix} \tag{16}$$

By using (10), the parameter update law was then computed and reduced to the matrix structure given by (17).

$$\begin{bmatrix} \dot{\hat{\beta}}_1 \\ \dot{\hat{\beta}}_2 \\ \dot{\hat{\beta}}_3 \\ \dot{\hat{\beta}}_4 \\ \dot{\hat{\beta}}_5 \end{bmatrix} = \begin{bmatrix} -e_1(x_2 - x_1) \\ -e_2(x_1 + x_2) \\ e_4x_4 \\ -e_5(x_4 - x_5) \\ -e_5x_1 \end{bmatrix} \tag{17}$$

The adaptive control law is then solved by using (7) and (9) and presented in the following matrix form:

$$K(x,y)=\begin{bmatrix}K_1\\K_2\\K_3\\K_4\\K_5\end{bmatrix}$$

$$=\begin{bmatrix}x_2x_3x_4x_5-y_2y_3y_4y_5-\hat{\beta}_1(e_2-e_1)-\lambda_{11}e_1\\-x_1x_3x_4x_5+y_1y_3y_4y_5-\hat{\beta}_2(e_1+e_2)-\lambda_{22}e_2\\0.1x_1^2-0.1y_1^2-\lambda_{33}e_3\\x_1x_2x_3x_5-y_1y_2y_3y_5+\hat{\beta}_3e_4-\lambda_{44}e_4\\x_1x_2x_3x_4-y_1y_2y_3y_4-\hat{\beta}_4(e_4-e_5)-\hat{\beta}_5e_1-\lambda_{55}e_5\end{bmatrix} \quad (18)$$

Where $\lambda_{11},\lambda_{22}...,\lambda_{55}>0$ are appropriately chosen.

V. NUMERICAL SIMULATION RESULTS

The numerical confirmation of the control strategy was carried out in the MATLAB numerical simulation environment, for the following initial conditions: Master system, $x(0)\big|_{x_1(0),x_2(0),x_3(0),x_4(0),x_5(0)}=[1.0,2.0,3.0,4.0,5.0]^T$ and the Slave system, $y(0)\big|_{y_1(0),y_2(0),y_3(0),y_4(0),y_5(0)}=[2.0,3.0,4.0,5.0,6.0]^T$ which gives $e(0)\big|_{e_1(0),e_2(0),e_3(0),e_4(0),e_5(0)}=[1.0,1.0,1.0,1.0,1.0]^T$ and the parameter estimate system $\dot{\hat{\beta}}(0)\big|_{\dot{\hat{\beta}}_1(0),\dot{\hat{\beta}}_2(0),\dot{\hat{\beta}}_3(0),\dot{\hat{\beta}}_4(0),\dot{\hat{\beta}}_5(0)}=[1.0,2.0,3.0,2.0,1.0]^T$.

The results are depicted in the following plots of Figure 2.

The adaptively synchronized dynamics of the controlled master-slave systems are depicted in Figure 2.

The synchronized dynamics of the controlled master and slave systems are depicted in Figure 3.

Remark 1

Figure 2 shows the stabilized error state dynamics of the synchronized master and slave systems. It is known in control system design that the error state dynamics will converge asymptotically at the origin as the systems synchronize in finite time. Figure 3 depicted the synchronized dynamics of the two systems. All the five state variables synchronized in finite time. Figure 4 shows how the states of the slave system settle to the estimated values of the unknown parameters. A closer look at the plot shows that the unknown parameters were accurately estimated, as they share the same values with the known parameters of the master system. By implication, the designed controller is robust in the presence of uncertainty.

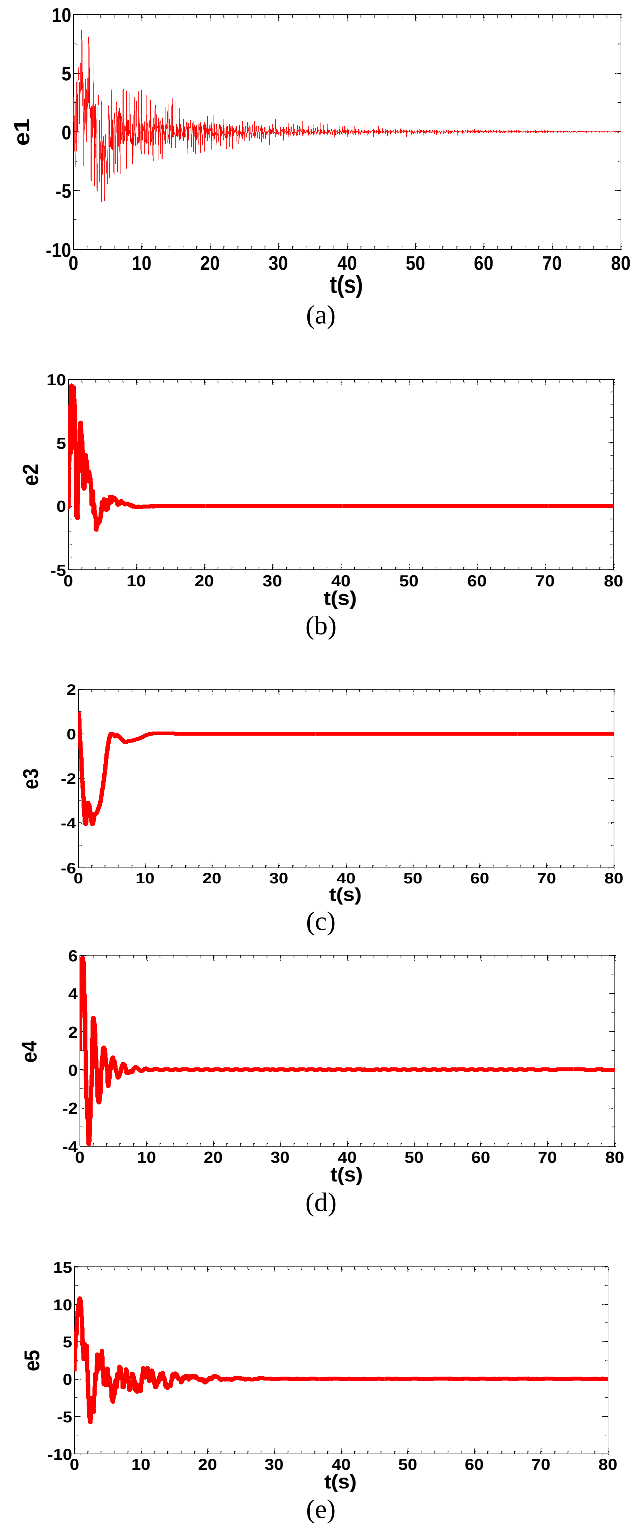
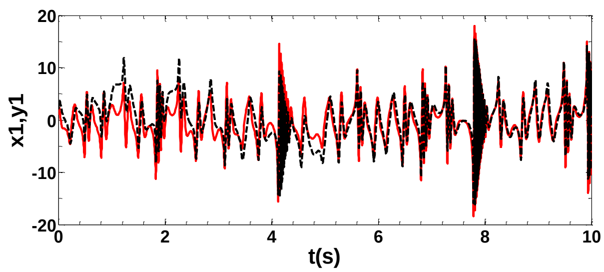
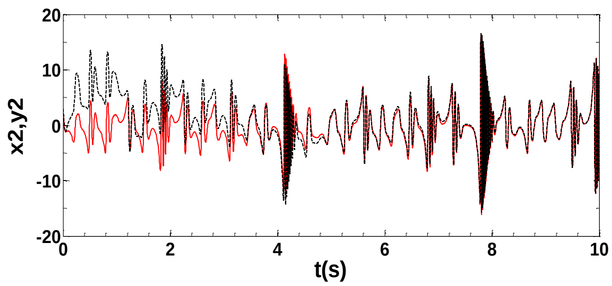


Figure 2. Asymptotically stable synchronization error dynamics



(a)



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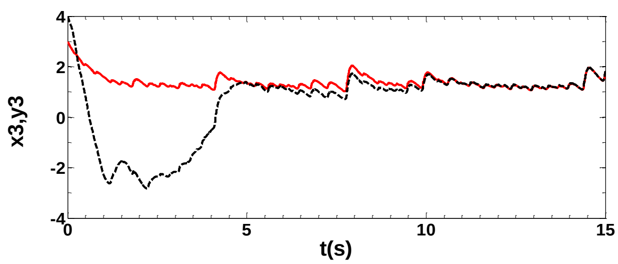

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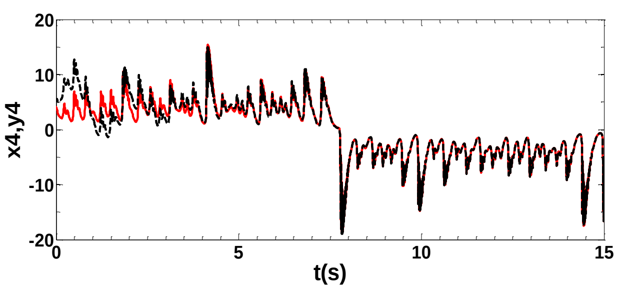
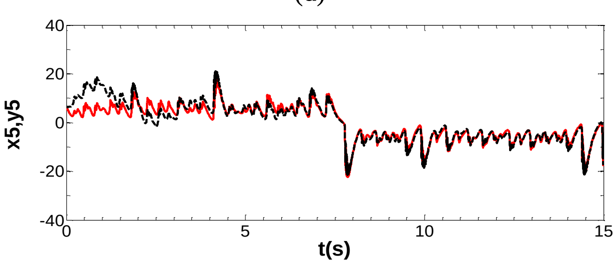


(d)



(e)

Figure 3. Synchronized trajectories of the master-slave systems

The trajectories of the estimated parameters of the slave's error system are plotted collectively in Figure 4.

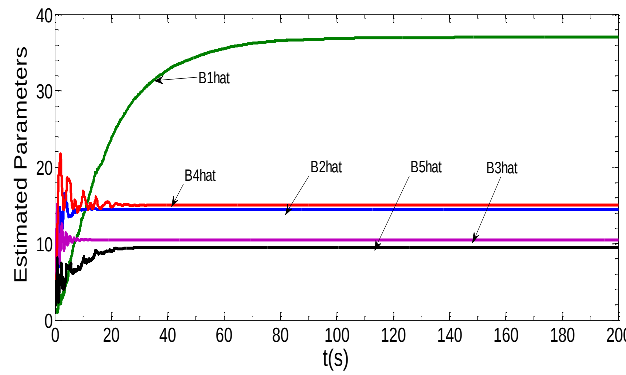


Figure 4. Converged parameter estimates of the slave system

VI. APPLICATION TO SYSTEM MODELLING

Chaos relate to the notion of nonlinearity, which imply a loss of causality relationship between perturbation and effect propagated over time. Thus, it provides an alternate method that explains the random behaviours of complex systems. Chaos plus mathematical tools is a framework for studying different models from different fields. Although the use of chaos in the field of computing has a short history, however, it has led to a new way of thinking computing among computer sciences.

A. Chaos computing

Chaos challenges the precision of mathematics and computing in general, by introducing a new computing technique that uses a network of chaotic elements to produce solutions, without the precision inherent in conventional computing. [29]. Chaos computing is the idea of using chaotic systems for computing. Modern computers perform computation based upon digital logic operations which are implemented with logic gates. Chaos can be used to produce logic gates where the dynamic characteristics of chaos are used for switching functions. For example, the sensitive dependence of chaos on initial conditions has been used as switching functions to generate patterns (change of states, analogous to "ON" and OFF") [30]. Logic gates in conventional computers uses logic elements tp perform singular or combinational functions which are predictable and limited. However, chaotic elements could assume an infinite number of behaviours that can be used to represent different values. This flexibility will allow a single chaotic computer to perform a variety of computations, using its inherent self-organization, in contrast to conventional computer in which computing is more specialized. [39]. A "*ChaoGate*" is a chaos variant of the popular digital logic gate and consists of a generic nonlinear circuit that exhibits patterns that are caused by chaotic dynamics [31].

B. Computer vision

Chaos has been applied to improve computer vision algorithms for "smart" autonomous machines such as mobile robots, drones and submersibles. This has increased the viewing field of these machines [36].

C. Economic and financial models

Chaos is increasingly finding relevance in modelling of complex economic and financial phenonmena that defies linear solution paradigms [32]. For example, in currency exchange, chaos has been applied to model and simulate volatile behaviours of currency exchange rates and stock market dynamics. The purpose is to derive control strategies that can stabilize the volatile behaviours and possibly synchronizes the tumbling rates with the realities in the markets. [33]. In addition, chaos has been embedded in the modelling of the nonlinear feedback mechanisms of profits of firms where spending do not always depend on profits [34].

D. Manufacturing information systems

The increasingly complex challenges confronting manufacturing systems nowadays, caused by uncertain structural and dynamic complexity in industrial markets, has pedestalled chaos as an alternate solution paradigm to convention approaches. Thus, chaos has been applied to enhance the response and performance characteristics of manufacturing information systems. [38].

E. Data traffic in computer networks

Computer scientists have applied chaos to construct mathematical models of the dynamics of traffics in networks, for the purpose of analyzing bottlenecks in the structure of the system and provide guaranteed quality of service (Q_0S) [37].

F. Secure communication systems

Synchronization of chaos first found application in the modelling of secure communication systems, where the chaotic dynamics were used to mask information as they stream through public communication network. The broadband characteristics of chaos, coupled with its features are useful in communication systems.

G. Chaos-based cryptosystems

Chaos has found numerous applications in the design of cryptographic systems. Chaotic dynamics are used in the shuffling and ciphering of text or multimedia information.

VII. CONCLUSION

The numerical confirmation of the effectiveness of the designed controller shows that the synchronized error dynamics quickly converged asymptotically at $t < 20s$, while the controlled trajectories of the master-slave systems coupled in synchrony at $t < 5s$. This is a measure of the effectiveness of the controller. The parameter estimates equally converged to the true values of the system parameter measured with reference to the master system. The dynamics of two identical 5D hyperchaotic systems have been synchronized and the uncertain parameters of the controlled slave system were accurately estimated via the designed adaptive controllers. The simulated results confirmed that the error systems satisfy the Lyapunov stability criteria. It is worth noting that the hyperchaotic system can find applications in the modeling of various engineering and non-engineering systems such as in image processing, secure communications, robot mobility and in complex medical science modeling and control.

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