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ORIGINAL ARTICLE

Evaluation of the various orthometric height systems and the Nigerian scenario – A case study of Lagos State

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Abstract Though considered the easiest in 3-Dimensional Point Positioning, the choice of a height system especially in areas with spatially-vast land mass is rather a complicated choice. Orthometric heights are naturally and fully referenced to the actual earth gravity field but laborious to compute considering the required approximations of gravity variation along the plumb-line from the surface to the geoid. Normal heights on the other hand are less laborious to compute and do not require actual gravity observations. Unfortunately, natural height systems are the only systems that can efficiently predict fluid flows. This paper has therefore examined the theory and practical possibility of replacing the use of Helmert Orthometric Heights with Normal Orthometric heights within Lagos State. A maximum height discrepancy of 1 mm was obtained in the comparison of both systems and thus their replaceability found suitable to within 3rd order geodetic accuracy.

1. Introduction

Height is the distance of a point above a specified surface of constant potential; the distance measured along the direction of gravity between the point and the surface (Meyer et al., 2005). It is the third component of a spatial point position when expressed in Cartesian forms. It is a measure of the elevation of a point above a predetermined reference surface. For ease of assessment, natural heights are measured as elevation differences (vertical distance) between points of interest and the Mean Sea Level (MSL) using either direct or indirect measurement techniques (El-Hassan and Ali, 2011). The elevation differences thus measured could be mathematically expressed as:

$$\Delta H = H_{\mathrm{A}} - H_{\mathrm{MSL}} \quad (1)$$

$$H_{\mathrm{A}} = \Delta H + H_{\mathrm{MSL}} \quad (2)$$

However, the MSL does not have equal elevation at all points for several reasons, therefore, in the real sense of it, *natural heights* are referenced to gravity and can be expressed in the form:

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$$H = \frac{C}{g} \quad (3)$$

where c is the geopotential number and it represents the difference in potential between the constant value at the geoid (w_0) and the potential at the point p on the surface (w_p):

$$c = w_0 - w_p \quad (4)$$

g is measured gravity value at observation points.

This shows that for any height system to be truly natural and sufficient to effectively solve engineering problems and provide necessary geospatial solutions; the gravity field of the earth must be taken into consideration.

Depending on the availability of gravity data and reference surface from which the measurements stem, heights could be Orthometric, Dynamic, Normal or ellipsoidal. Of greatest concern to engineering and environmental activities is the orthometric height because of its physical geometric meaning and ability to predict fluid flows.

The Nigerian Geodetic Levelling network consists of over 250 lines covering a total distance of over 20,000 km (Isioye et al., 2010). With its origin at the Apapa Datum, the Nigerian Levelling Network stretches across the entire country with the aim of providing unified height system across the entire nation.

Contrary however to contemporary national levelling networks, the Nigerian network consists of only "mere elevation differences" that have not been corrected for the effect of gravity thus, the Normal Orthometric height has been adopted for vertical control points rather than the Helmert Orthometric Heights.

Isioye et al. (2010) evaluated the use of normal gravity instead of observed gravity and its distortion on the Nigerian Levelling network and discovered that the use of Normal heights instead of Normal Orthometric heights has shown no statistical significance.

This paper therefore attempts to evaluate the level of reliability of replacing Helmert Orthometric Heights with the Normal Orthometric heights in Nigeria using Lagos State as case study.

1.1. Orthometric heights

The orthometric height is defined as the length of the curved plumbline from a point P (on the earth surface), to its intersection with the geoid at P_0 (Matt, 2010), as shown in Fig. 1 and is given by Fotopoulos (2003) as Eqs. (5) and (6):

$$H^{ortho} = \frac{c}{g_{mean}} \quad (5)$$

where c = geopotential number, g is the mean value of actual gravity along the Earth's (curved) plumb line between the geoid and topographic surface:

$$g_{mean} = \frac{1}{\mathrm{H}_0}\int_{p_0}^{\mathrm{p}} \mathrm{gdH} \quad (6)$$

where p = point on topographic surface, g_r = measured gravity value at points along the plumbline, p_0 = Point on geoid, dH = differential element along the plumbline between the geoid and the point on the earth surface, H_0 = Orthometric Height.

Since evaluation of Eq. (6) requires orthometric height, Eq. (5) is therefore solved by iteration (Fotopoulos, 2003).

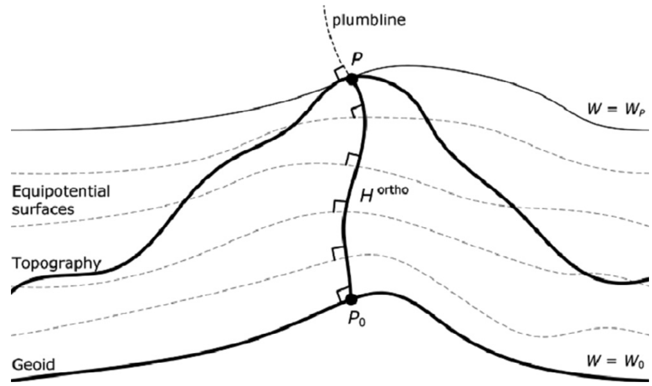

Figure 1 The Orthometric height (adapted from Featherstone and Kuhn, 2006).

More commonly, it is often defined as the perpendicular distance from a point to the Mean Sea Level along the direction of gravity. Due to the complexities involved in determination of gravity for all points within the levelling network, several variations of the orthometric height exist with each particular variation differing from the other in accuracy and computational reliability.

Orthometric height (OH) can be obtained by spirit levelling; if obtained height differences are corrected for non-parallel equipotential surfaces using the orthometric correction to obtain orthometric height (Heiskanen and Moritz, 1967). This requires that the gravity be measured at the level stations.

1.1.1. Forms of orthometric height

Owing to the complexities and rigours involved in precise determination of the mean gravity value along the plumbline, several approximations of the mean gravity have being used resulting in different forms of orthometric heights. Three basic forms of orthometric heights are described below:

Helmert Orthometric Heights: are based on the Poincare–Prey relationship for mean gravity and the Bouguer shell gravity expression that accounts for the topographic mass above the geoid but neglects the terrain effects. Therefore, the adopted mean gravity becomes (Heiskanen and Moritz, 1967):

$$g^n = g^s + \frac{1}{2}\mathrm{FH}_0 - 2\pi \mathrm{G}\rho \mathrm{H}_0 \quad (7a)$$

where g^s = Observed Gravity at the topographic Surface, F = Linear vertical "Free-air" gravity gradient, G = Universal gravitational constant and ρ = Constant Topographic density, H_0 = Elevation Difference.

Thus, Helmert Orthometric Heights is given as:

$$H^{Helmert\ ortho} = \frac{C}{g^n} \quad (7b)$$

Though several mathematical formulations abound for determining Orthometric Correction required for converting spirit levelling elevation differences to Helmert Orthometric Heights, the model given by Hwang and Hsiao (2003) is considered in this paper for simplicity and since distance between the successive controls used does not exceed 2 km:

$$OC_{AB} = \frac{1}{\overline{g}_B}\left(\frac{g_A + g_B}{2} - \overline{g}_B\right)\Delta n_{AB} + H_A\left(\frac{\overline{g}_A}{\overline{g}_B} - 1\right) \quad (8)$$

where g_A and g_B = Surface Gravity Values at A and B, Δn_{AB} = Sum of all geometric height differences between A and B measured directly from Spirit Levelling, H_A = Approximate orthometric height at A, $\bar{g}_A$ and $\bar{g}_B$ = Mean gravity values along the plumbline (g + 0.0424H).

Mader Orthometric Height: This takes into account the mean value of the gravimetric terrain correction within the topography by taking a simple mean of the terrain effect at the geoid and the earth surface (Tenzer et al., 2005; Santos et al., 2006). The Mader Mean gravity is therefore given by (Mader, 1954; Krakiwsky, 1965) as:

$$g^m = g^s + \frac{1}{2}\mathrm{FH}_0 - 2\pi\mathrm{G}\rho\mathrm{H}_0 + \frac{1}{2}(\delta g^T - \delta g_0^T) \tag{9a}$$

$$g^m = g^s + \frac{1}{2}\mathrm{FH}_0 - \frac{1}{2}(g^T - g_0^T) \tag{9b}$$

where g^s = Observed Gravity at the topographic Surface, F = Linear vertical "Free-air" gravity gradient, G = Universal gravitational constant, ρ = Constant Topographic density, H_0 = Elevation Difference, g^T and g_0^T = vertical components of gravity due to topographic masses at the ground surface and at the geoid, respectively.

Mader Orthometric Heights can therefore be written as:

$$H^{Mader\ ortho} = \frac{C}{g^m} \tag{9c}$$

Although, this gives a closer approximation of the true orthometric height than Helmert heights, its computational complexity has seen them used less frequently in practice (Matt, 2010).

Niethammer Orthometric Heights: Niethammer (1932) computed mean gravity considering the integral mean of terrain effects evaluated at discrete points along the plumbline rather than just at the geoid and earth surface as Mader (Santos et al., 2006). Therefore, the computation is based on mean gravity as given by (Niethammer, 1932; Rapp, 1961; Krakiwsky, 1965):

$$g^N = g^s + \frac{1}{2}\mathrm{FH}_0 - g^T + g_0^T \tag{10a}$$

where g^T and g_0^T = vertical components of gravity due to topographic masses at the ground surface and at the geoid, respectively.

The Nethammer Orthometric height can therefore be expressed as:

$$H^{Nethammer\ ortho} = \frac{C}{g^N} \tag{10b}$$

Dennis and Feathestone (2002)evaluated these three approximations, and showed that the accuracy standards drop from Niethammer, Mader then to Helmert, which reflects the levels of approximation used.

1.2. Normal heights

Normal height systems replace the mean actual gravity within the topography by the mean normal gravity between the reference ellipsoid and the telluroid (Heiskanen and Moritz, 1967; Tenzer et al., 2005). The normal gravity field is the gravity field defined by an Earth-fitting ellipsoid that contains the total mass of the Earth (including its atmosphere), and rotates at a constant angular velocity more or less equivalent to that of the Earth (Moritz, 1980). The normal gravity field can be used to define a height that avoids assumptions about the shape and density of the topographic masses needed to compute g (Matt, 2010).

The normal height, H^N, replaces g in Eq. (5) (which was measured along the plumbline) with normal gravity, γ, measured along the curved ellipsoidal normal (of the reference ellipsoid) hence (Jekeli, 2000):

$$H^N = \frac{C}{\gamma} \tag{11}$$

Because normal heights have no physical meaning (being defined by a gravity model), they are not as applicable to the real Earth as the Orthometric Height (Featherstone and Kuhn, 2006). Besides, they cannot predict fluid flows universally, though they give a reasonable approximation in many situations.

1.2.1. Normal orthometric heights

In situations where gravity values are not available at levelling stations, the use of *Normal-Orthometric Height*, H^{N-O} was developed (Rapp, 1961; Heck, 2003). In this system the geopotential number, C, is replaced with the spheropotential number, C', which is wholly derived from the normal gravity and thus given as:

$$H^{N-O} = \frac{C'}{\gamma} \tag{12}$$

However, Normal Orthometric heights could be derived from spirit levelling elevation differences by applying the Normal Orthometric Correction (NOC) given by (Sneeuw, 2013):

$$\mathrm{NOC} = -0.0053\mathrm{H_{AB}}\Delta\emptyset_{\mathrm{AB}}\mathrm{Sin}2\emptyset_{\mathrm{AB}} \tag{13}$$

where $\emptyset_{\mathrm{A}}$ = Latitude of reference station, $\emptyset_{\mathrm{B}}$ = Latitude of computation station:

$$\mathrm{H_{AB}} = 0.5(\mathrm{H_A} + \mathrm{H_B})$$

$$\Delta\emptyset_{\mathrm{AB}} = (\emptyset_{\mathrm{B}} - \emptyset_{\mathrm{A}})$$

$$\Delta\emptyset_{\mathrm{AB}} = 0.5(\emptyset_{\mathrm{A}} + \emptyset_{\mathrm{B}})$$

The consequence of not using surface gravity observations is that while Normal Orthometric Heights are easy to compute, they are even less likely to predict fluid flows correctly than Normal Heights (see Fig. 2).

2. Materials and methods

2.1. Data used

A total of 216 control points being part of the newly established Lagos State second-order control network were used in this paper. The control points selected being part of the ZTT 14–30 Series covering most parts of Lagos State has an even spatial distribution across the state. The information was obtained from the office of the Surveyor General of Lagos State.

The control network consists of the 3-dimensional co-ordinates of the control points referenced to the WGS84 ellipsoid and also the "Spirit levelled height differences".

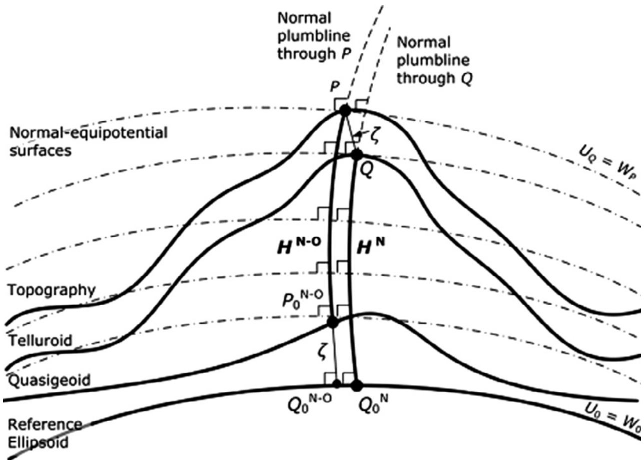

Figure 2 The Normal and Normal-Orthometric heights (Adapted from Featherstone and Kuhn, 2006).

The spatial distribution of some of the control points across the study area is shown in Fig. 3.

Because the model used for computing the Helmert Orthometric correction is optimum between controls with successive distances that do not exceed 2 km, only 153 Control Points out of the 216 were used for the analysis.

2.2. Study area

Lagos State is a Low-lying coastal state having a fairly stable terrain with minimal undulation and an approximate land-mass area of about 3600 Sq km. Bounded in the South by the Atlantic Ocean and the Lagoon; several other tributaries from the Lagoon extend into the state some of which include the five cowries, the Iddo Port, Apapa port amongst others.

Being the host state where the nation's vertical datum (Apapa Datum) is established, most control points within the state have spirit levelled elevation differences observed on them.

2.3. Methodology

Further to the completed First Order spirit levelling – determined height values of the control points, the previously illustrated mathematical models were applied as appropriate to determine the Helmert Orthometric and Normal Orthometric heights, respectively.

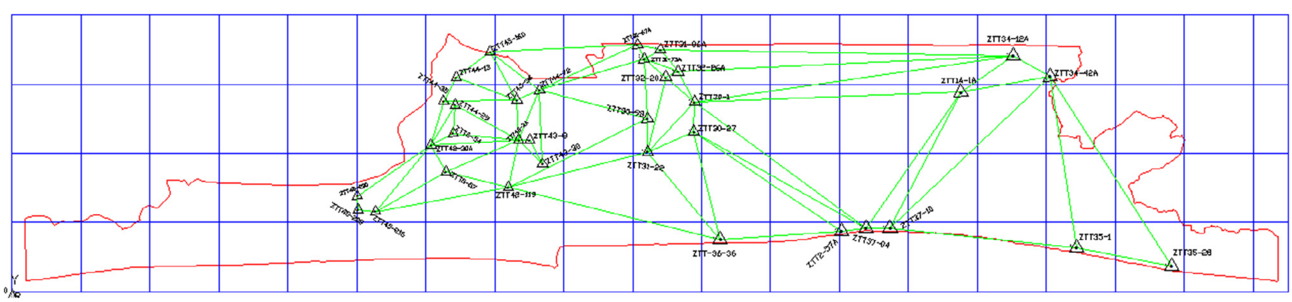

Figure 3 Study area/spatial distribution of control points used for analysis.

Table 1 Summary table showing extract of final results.

Control no.	Spirit levelling elevation	Normal Orthometric HTS		Helmert Orthometric HTS		Diff (NOH–HOH)
		Normal OC	Normal HTS	Helmert OC	Helmert HTS	
ZTT30-09A	25.072					
ZTT30-1	15.811	−0.0011	15.8099	-0.0004	15.8106	-0.0007
ZTT30-13A	22.845	0.0003	22.8453	0.0001	22.8451	0.0002
ZTT30-14A	22.531	0.0002	22.5312	0.0001	22.5311	0.0001
ZTT30-15A	17.552	0.0001	17.5521	0.0000	17.5520	0.0001
ZTT30-17A	19.839	0.0002	19.8392	0.0001	19.8391	0.0001
ZTT30-18A	19.851	0.0001	19.8511	0.0000	19.8510	0.0001
ZTT30-19A	17.827	0.0001	17.8271	0.0000	17.8270	0.0001
ZTT30-18A	17.736	0.0001	17.7361	0.0000	17.7360	0.0001
ZTT30-21	19.685	0.0001	19.6851	0.0000	19.6850	0.0001
ZTT30-22	18.892	0.0000	18.8920	0.0000	18.8920	0.0000
ZTT30-23	13.482	0.0000	13.4820	0.0000	13.4820	0.0000
ZTT30-24	19.061	−0.0001	19.0609	0.0000	19.0610	−0.0001
ZTT30-25A	19.897	0.0000	19.8970	0.0000	19.8970	0.0000
ZTT30-26A	18.495	0.0000	18.4950	0.0000	18.4950	0.0000
ZTT30-27	11.093	0.0000	11.0930	0.0000	11.0930	0.0000
ZTT30-5	27.315	0.0004	27.3154	0.0001	27.3151	0.0003
ZTT30-6A	27.258	0.0002	27.2582	0.0001	27.2581	0.0001
ZTT30-73A	3.15	0.0000	3.1500	0.0000	3.1500	0.0000
ZTT30-74A	1.782	0.0000	1.7820	0.0000	1.7820	0.0000

Where: OC = Orthometric Correction, HOH = Helmert Orthometric Height, NOH = Normal Orthometric Height.

3. Results and discussions

Sample points for some of the results obtained are as shown in Table 1. Twenty Points are herein provided; the remaining values could be obtained from the office of the Surveyor General of Lagos State.

Also shown (Figs. 4b, 5b and 6b) are Digital Terrain Models of the study area plotted with the three Different height Systems. This will help investigate if there are any conspicuous irregularities in the relief pattern (surface undulation) of the study area in the various height systems. This is especially useful in terms of prediction of fluid flows and mitigating the possibility of flooding within the area.

The contour Plots (Figs. 4(a), 5(a) and 6(a)) reveal a clear difference between the "Spirit Levelled Elevations", the Normal Orthometric and Helmert Orthometric Heights. While it conspicuously reveals that the use of "Mere Elevation Differences" though fairly close, cannot efficiently predict fluid flows and terrain undulation as it is completely inconsistent with both Normal and Helmert Orthometric Heights, it also suggests that within the study area, the interchangeable use of Normal Orthometric Heights and the Helmert Orthometric Height is suitable. These assertions are further substantiated by the 3-D surface model plot generated from the three height Systems (Figs. 4(b), 5(b) and 6(b)).

3.1. Statistical test

The degree of variance between the three height systems (i.e. spirit levelled elevation differences, Normal Orthometric Heights and Helmert Orthometric Heights) was computed using the one way ANOVA statistical test in order to further investigate the statistical acceptability of the interchangeability in their use. The results obtained from the ANOVA test as shown (Table 2) indicate that there is no statistically significant difference between all three data sets at 95% confidence interval.

The One-Way ANOVA test was conducted using the following hypothesis:

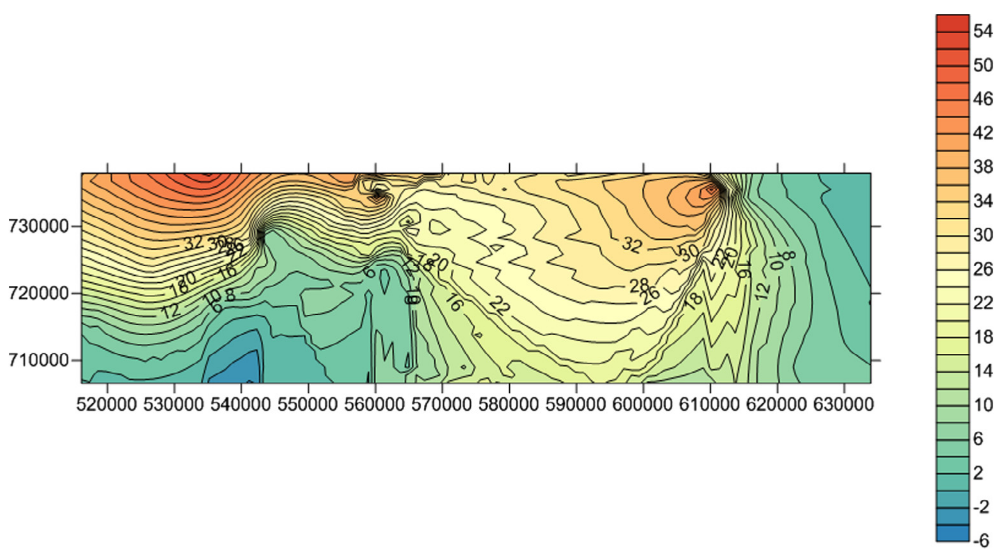

Figure 4(a) Contour plot of study area plotted from spirit levelling elevation differences.

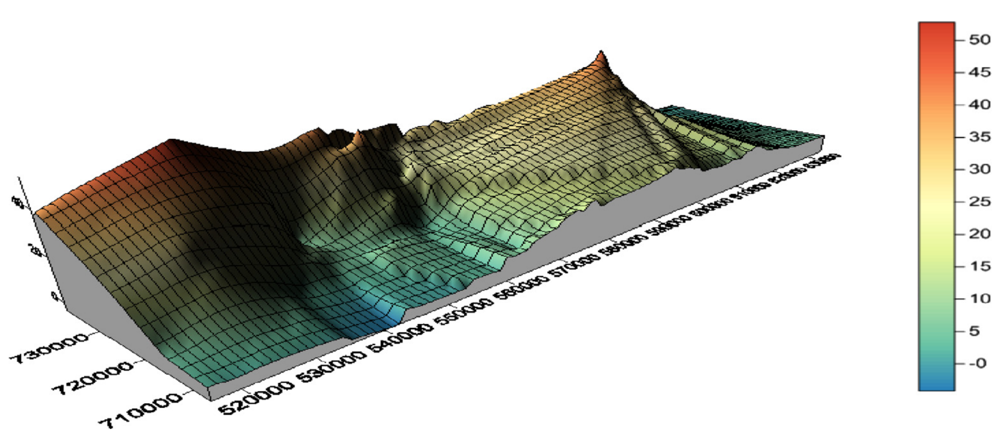

Figure 4(b) 3D terrain model of study area from spirit levelling elevation differences.

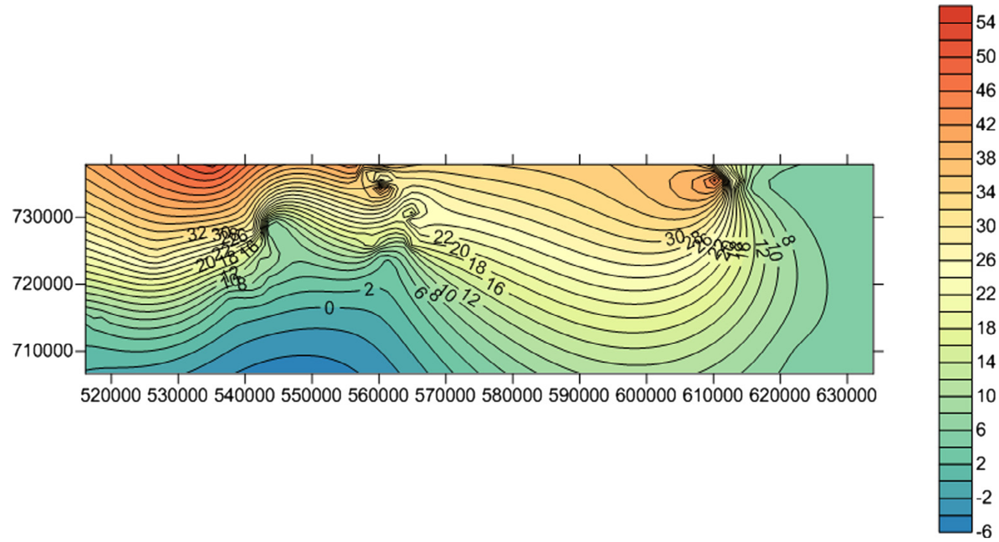

Figure 5(a) Contour plot of study area plotted from Normal Orthometric heights.

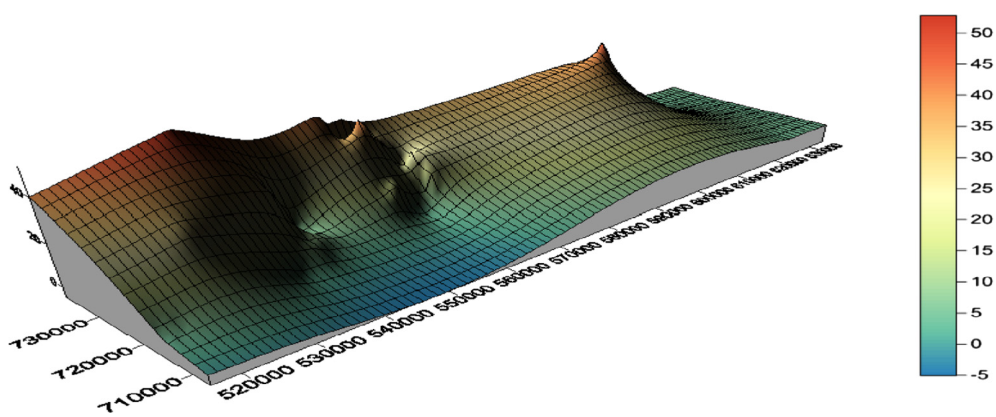

Figure 5(b) 3D terrain model of study area from Normal Orthometric heights.

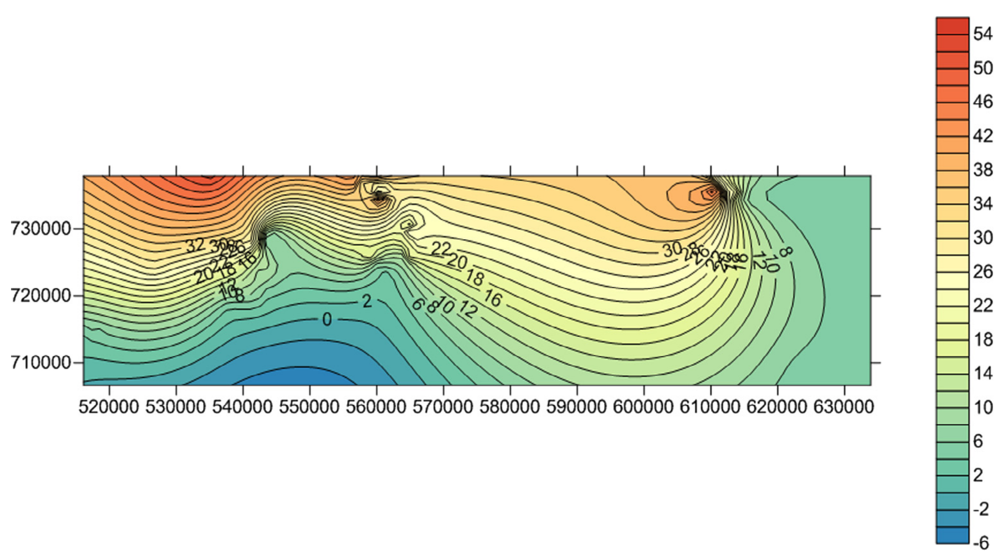

Figure 6(a) Contour plot of study area plotted from Helmert Orthometric Heights.

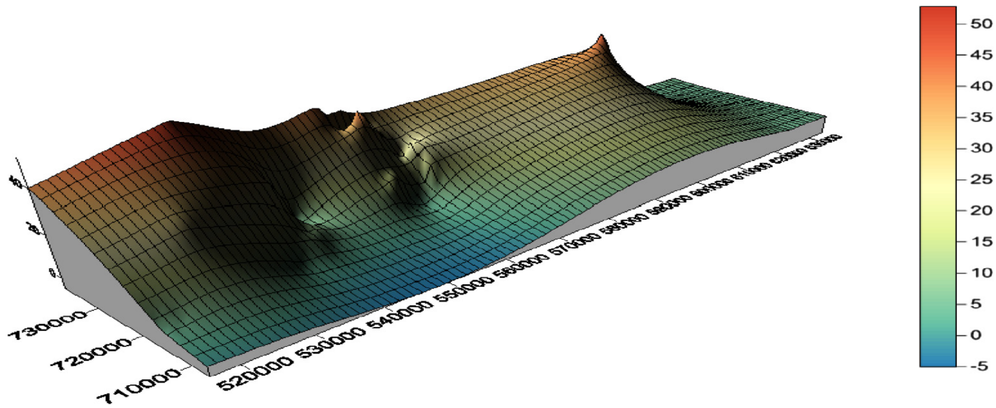

Figure 6(b) 3D terrain model of study area from Helmert Orthometric Heights.

Table 2 Result of one-way ANOVA test.

Source	SS	DF	MS	F	P (> F)
Between TR	0.00	2	0.00	0.00	1.00
Within TR	119116.43	597	199.53		
Total	119116.43	599			

DO NOT REJECT H_0 AT ALPHA = 0.05 (95% Confidence Interval).

(Null Hypothesis) H_0: No differences between the means of the 3 Groups.
(Alternative Hypothesis) HA: At least one of the means is not the same as other means.

This shows that there is no statistically significant difference between all three data sets (i.e. the Spirit levelled elevation differences, Normal Orthometric Heights and Helmert Orthometric Heights) therefore any of the three height systems can be used instead of another within the study area.

4. Conclusion

Although the statistical test allows the use of the spirit levelled elevation differences, the contour plot from the three data sets reveals that the use of "*mere elevation differences*" is not suitable for geodetic purposes as a sharp difference is evident in the nature of contour lines obtained from it (Fig. 4a) when compared with the contour lines (Figs. 5a and 6a) obtained from the other two height systems discussed (Normal Orthometric and Helmert Orthometric).

However, the Contour and Surface Plots (Figs. 5(a & b) and 6(a & b)) in conformity with statistical analysis shows that within the Study Area, the use of Normal Orthometric Heights instead of Helmert Orthometric Heights is suitable and acceptable within 3rd Order Geodetic Accuracy Levels.

Furthermore, investigations into differences between the Normal and Helmert Orthometric heights within the study area reveal a maximum of 1 mm discrepancy between the two height systems.

Though, it is not the task here to further investigate reasons for such close similarity between the two (2) height systems, a major reason might be due to the low-lying nature of the study area; hence in more rugged and undulating topographies (especially mountainous regions) with high mass surplus or deficits that can influence high variation between normal and actual gravity, such close range discrepancy between both systems *may not be obtainable*.

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The effort of the Office of the Surveyor General of Lagos State in the provision "Spirit Levelled Elevation Differences" of the used control Points is herein acknowledged. The data are greatly appreciated as this research effort would have been completely impossible without the availability of these data. The gravity data obtained from Bereau Gravimetric International (BGI) is also greatly appreciated.

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