Electronic circuit realization and Adaptive Control of a Chaotic Permanent Magnet Synchronous Motor

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ABSTRACT
In this paper, the electronic circuit realization of the mathematical model of a normalized chaotic permanent magnet synchronous motor (PMSM) is presented. The mathematical model of a permanent magnet synchronous motor is remodeled so that it conforms to nonlinear chaotic-algebraic structure. The chaotic PMSM is thus, diffeomorphic to the canonical Lorenz chaotic system in it simplified form which is a three-dimensional coupled system consisting of the torque, q-axis current and d-axis currents as state vectors. Robust adaptive control laws are formulated to drive the state variables of the chaotic PMSM master and slave systems into states of synchrony in finite time while simultaneously estimating the unknown state parameters of the slave system. The results of the numerical simulations of the mathematical model of the chaotic PMSM with MATLAB matched those realized through the design of autonomous electronic circuit using NI Multisim simulation software. The dynamics generated by the chaotic PMSM is suitable for cross-discipline applications such as secure communications and biometric security amongst others.

Keywords: *Adaptive control, permanent magnet synchronous motor, synchronization*

1 INTRODUCTION

Chaos is a phenomenon that occurs in nonlinear dynamic systems that are highly sensitive to disturbances in their system's structures or unexcited states (Kellert, 1993). Chaos phenomena have been observed widely in natural and man-made systems, which has inspired engineers and scientists to utilize them for modelling of real life systems and management of of the behaviours of sysems. In the medical fields, chaotic dynamics has been used to study and understand electroencephalography (Albert, 1992)(Kumar & Hegde, 2012). In finances and economics, it has been used to study and model prices and stock market fluctuations (Guegan, 2009). In signal analysis, chaos has utilized extensively in the design of secure communication systems and multimedia security systems (Carroll & Pecora, 1991). In power systems and machines, chaos has been observed in the dynamics of machines and electric power cycles (Harb, Batarseh, Mili, & Zohdy, 2012). Chaos is generally undesirable in systems. However, in recent studies, chaos has been found to be useful in the study of power outages (Harb & Smadi, 2004), resulting in the possibility of anti-synchronizing chaotic dynamics to counteract power outages (Abbasi, Gholami, Rostami, & Abbasi, 2011). With particular reference to electric motors, the presence of chaos can lead to undesirable performances (Pennacchi, 2009). Several approaches have been proposed and applied to control chaos in various dynamic systems (Moaddy, Radwan, Salama, Momani, & Hashim, 2013; E.A. Umoh, 2013, 2014; Edwin A. Umoh, 2014a). Synchronization is a regulating strategy that is used to drive the trajectories of two or more chaotic systems to achieve synchrony in finite time (Pecora, 2007). Several synchronization and antisynchronization techniques have been reported and used to regulate system dynamics in the literature (Emadzadeh & Haeri, 2005; E.A. Umoh, 2014; Edwin A. Umoh, 2014b; Edwin Albert Umoh, 2014). In recent years, studies on the existence of chaos and bifurcation in motors has increased in the literature (Gao & Chau, 2002; Jing, Yu, & Chen, 2004; Li, Park, Joo, Zang, & Chen, 2002). Chaotic dynamics are exhibited by PMSM in the presence of disturbances and can lead to unpredictable behaviours, with dire consequences during operation. The complex behaviours of motors adds to the difficult of controlling and synchronizung chaos in them. However, several works on controlling and synchronizing the dynamics of PMSM have been reported in recent years (Ge & Lin, 2007; Zribi, Oteafy, & Smaoui, 2009). In this paper, the adaptive control of the mormalized model of a smooth air-gap permanent magnet synchronous motor (Choi, 2012) with unknown parameters is presented. The bifurcation diagram of these parameters are generated and an electronic circuit of the model is realized using discrete electronic components, simulated in the virtual environment of the NI Multisim software is also presented.

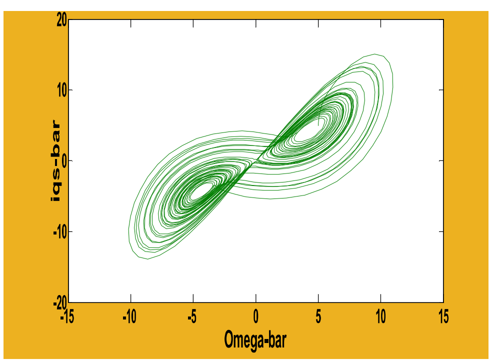
GENERAL FORM OF THE NORMALIZED CHAOTIC PMSM

The basic form of the chaotic PMSM (Choi, 2012)is given as

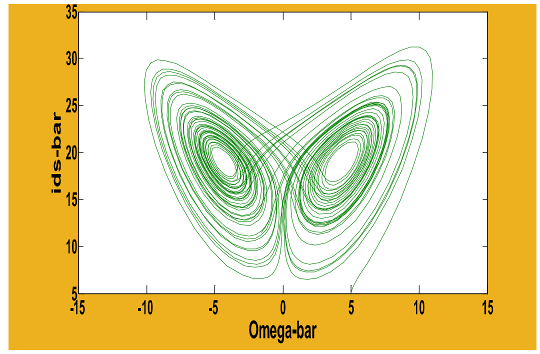
$$\begin{cases} \dot{\bar{\omega}} = \sigma(\bar{i}_{qs} - \bar{\omega}) - \bar{T}_L \\ \dot{\bar{i}}_{qs} = -\bar{i}_{qs} - \bar{\omega}\bar{i}_{ds} + \gamma\bar{\omega} + \bar{V}_{qs} + \bar{d}_q \\ \dot{\bar{i}}_{ds} = -\bar{i}_{ds} + \bar{\omega}\bar{i}_{qs} + \bar{V}_{ds} + \bar{d}_d \end{cases} \quad (1)$$

Where $\bar{\omega}$ is the electrical rotor angle speed, $\bar{i}_{qs}$ = q-axis current, $\bar{i}_{ds}$ = d-axis current, $\bar{T}_L$ = load torque

σ and γ are parameters of the motor, $\bar{V}_{qs}$ = q-axis voltage, $\bar{V}_{ds}$ = d-axis voltage, $\bar{d}_q$ and $\bar{d}_d$ are disturbances applied the q and d axes respectively. The MATLAB-based simulated results of the open loop dynamics of the chaotic PMSM when the load torque, axes voltages and disturbances are negligible, i.e. $\bar{T}_L = \bar{V}_{qs} = \bar{V}_{ds} = \bar{d}_q = \bar{d}_d = 0$ and the initial conditions $\bar{\omega} = \bar{i}_{qs} = \bar{i}_{ds} = 0.01$ are depicted in the following figures.



(a)



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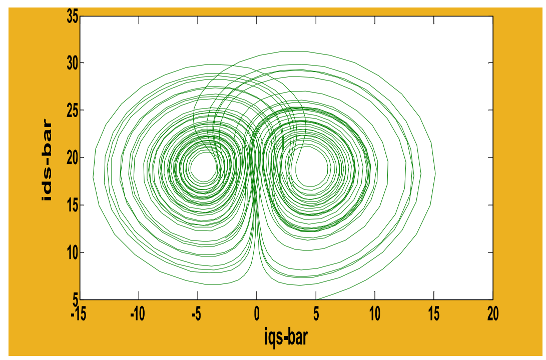
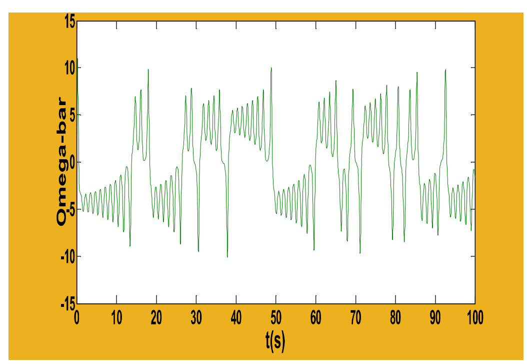


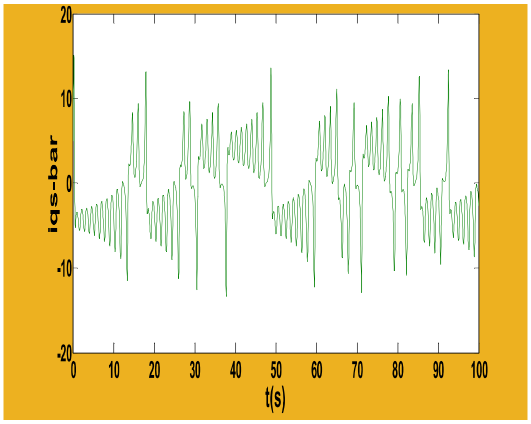
(c)

Figure.1. 2D Phase portraits of the chaotic PMSM

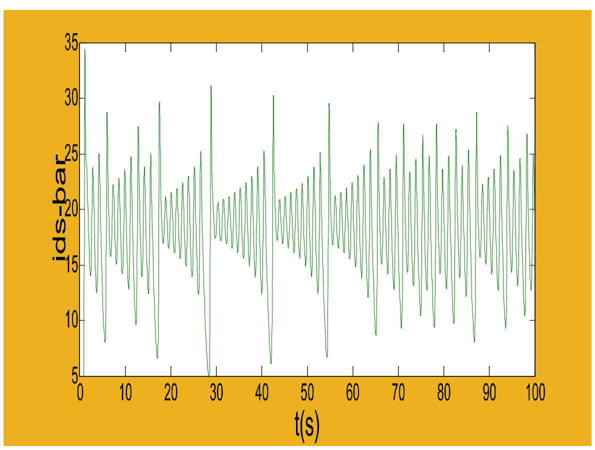
(a) $\bar{i}_{qs}$ *vs* $\bar{\omega}$ (b) $\bar{i}_{ds}$ *vs* $\bar{\omega}$ (c) $\bar{i}_{ds}$ *vs* $\bar{i}_{qs}$



(a)



(b)



(c)

Figure 2. Dynamics of the State trajectories in time space

(a) $\overline{\varpi}\ vs\ \ t(s)$ (b) $\overline{i}_{qs}\ vs\ t(s)$ (c) $\overline{i}_{ds}\ vs\ t(s)$

The bifurcation plots of parameters σ and γ are given in Figure 3 and Figure 4 respectively.

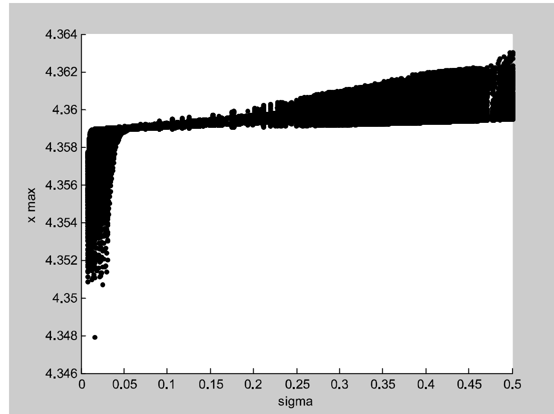


Figure 3. Bifurcation diagram of parameter σ

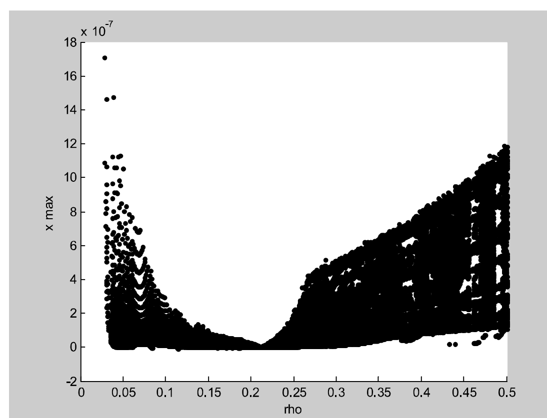


Figure 4. Bifurcation diagram of parameter γ

THEORETICAL ANALYSIS OF THE ADAPTIVE CONTROLLER

In this section, the theoretical framework of the adaptive controller and parameter estimate law are summarized. In practice, owing to real time uncertainties in dynamic systems, not all of system's parameter are always available during operations. As a result, an adaptive controller which acts as an observer is essential to infer the parameters from existing variables. Several methods of designing adaptive controllers have been utilized to control and estimate unknown parameters of systems in the literature (Naseh & Haeri, 2005; Vaidyanathan, Volos, & Pham, 2016). In this paper, an elegant approach proposed in (Wang & Liao, 2005) is used to stabilize and synchronize the aperiodic dynamics of the two identical chaotic PMSMs.

Given a chaotic PMSM known as the Master system, of the form

$$\dot{x} = f(x) \tag{2}$$

Where $x \in \Re^n$ is the state vector of the PMSM. $f(x)$ is a vector field. Eq. (2) can be decomposed into the following form

$$\dot{x} = P_m(x) + S_m(x)\alpha \tag{3}$$

$P_m \in \Re^n, S_m \in \Re^{n\times m}$ are nonlinear and linear functions of the parameters of the PMSM system, $\alpha \in \Re^1$ is the vector of the system parameters σ and γ and $x = (\overline{\varpi}^m, \overline{i}_{qs}^m, \overline{i}_{ds}^m)^T \in \Re^1$ are vectors of the state variables of the master system. Let the controlled Slave PMSM system which has the same definition as the master system be of the form

$$\dot{y} = g(y) \tag{4}$$

be decomposed into the following form

$$\dot{y} = P_s(y) + S_s(y)\overline{\alpha} + C_i(x,y) \tag{5}$$

Where $\overline{\alpha}$ is an estimate of the vector α. $C_i(x,y) \in \Re^1 (i = 1,2,3)$ is the adaptive controllers to be designed and $y = (\overline{\varpi}^s, \overline{i}_{qs}^s, \overline{i}_{ds}^s)^T \in \Re^1$. By defining the state error as

$$\begin{aligned} e &= (e_1, e_2, e_3) = y(t) - x(t) \\ &= (\overline{\varpi}^s - \overline{\varpi}^m, \overline{i}_{qs}^s - \overline{i}_{qs}^m, \overline{i}_{ds}^s - \overline{i}_{ds}^m) \end{aligned} \tag{6}$$

And the parameter estimation error as

$$\phi(t) = \overline{\alpha}(t) - \alpha \tag{7}$$

Then, it can be inferred that based on the Lyapunov stability principle, if an adaptive controller can be designed such that

$$\lim_{x\to\infty} \|e\| = \lim_{x\to\infty} \|y(t) - x(t)\| = 0 \tag{8}$$

Then the master and slave chaotic PMSM can completely synchronized in finite time and the unknown parameters can be estimated. From (6), the error dynamic system can thus be written in the form

$$\begin{aligned} \dot{e} = \dot{y} - \dot{x} = P_s(y) - P_m(x) + (S_s(y) - S_m(x))\overline{\alpha} \\ + S_m(x)\phi(t) + C_i(x,y) \end{aligned} \tag{9}$$

From (8), the adaptive controller to be designed is structured in the form

$$C_i(x,y) = P_x(x) - P_s(y) - (S_s(y) - S_x(x))\overline{\alpha} - \xi_i \tag{10}$$

Where $\xi_i \in \Re^{n\times m}, \Re^1$ is a positive definite matrix of the form

$$\begin{bmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \end{bmatrix} = \Delta \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \tag{11}$$

Where Δ is a diagonal matrix is whose elements $\Delta_{11}, \Delta_{22}, \Delta_{33}$ constitutes the positive coefficients of the adaptive controllers to be designed. By substituting inserting (10) into (9), the compact form of the error system and parameter estimation error system are determined as

$$\dot{e} = S_s(y)\phi(t) - \xi_i$$
$$\dot{\alpha} = -S_s^T(y)e \quad (12)$$

APPLICATION TO THE NORMALIZED CHAOTIC PMSM SYSTEMS

The theory outlined in the previous section is now applied to the synchronized the state trajectories and estimate the unknown system parameters of the chaotic PMSMs. The nonlinear part of the master system of (1) is

$$P_m(x) = \begin{bmatrix} 0 \\ -\bar{\varpi}^m i_{ds}^m \\ \bar{\varpi}^m i_{qs}^m \end{bmatrix} \quad (13)$$

The nonlinear part is given as

$$S_m(x) = \begin{bmatrix} i_{qs}^m - \bar{\varpi}^m & 0 & 0 \\ 0 & \bar{\varpi}^m & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (14)$$

The slave system has the following structures.

The Nonlinear part is given as

$$P_s(y) = \begin{bmatrix} 0 \\ -\bar{\varpi}^s i_{ds}^s \\ \bar{\varpi}^s i_{qs}^s \end{bmatrix} \quad (15)$$

The nonlinear part is given as

$$S_s(y) = \begin{bmatrix} i_{qs}^s - \bar{\varpi}^s & 0 & 0 \\ 0 & \bar{\varpi}^s & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (16)$$

The transposition of (16) gives

$$S_s^T(y) = \begin{bmatrix} i_{qs}^s - \bar{\varpi}^s & 0 & 0 \\ 0 & \bar{\varpi}^s & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (17)$$

Based on (12), the parameter estimate error system is given as

$$\dot{\alpha} = -S_s^T(y)e = \begin{bmatrix} \dot{\bar{\sigma}} \\ \dot{\bar{\gamma}} \\ 0 \end{bmatrix} = -\begin{bmatrix} i_{qs}^s - \bar{\varpi}^s & 0 & 0 \\ 0 & \bar{\varpi}^s & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} (\bar{\varpi}^s - i_{qs}^s)e_1 \\ -\bar{\varpi}^s e_2 \\ 0 \end{bmatrix} \quad (18)$$

The adaptive controllers are derived from (10) and (11) as

$$C_i = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} \begin{bmatrix} \bar{\sigma}(e_2 - e_1) - \Delta_{11} e_1 \\ -x_1 x_3 + y_1 y_3 - \bar{\sigma} e_1 - \Delta_{22} e_2 \\ x_1 x_2 - y_1 y_2 - \Delta_{33} e_1 \end{bmatrix} \quad (19)$$

2 RESULTS AND DISCUSSION

The results obtained from numerical simulation of the master and slave system and the controller and error systems for the following initial conditions using MATLAB software, are depicted in the following figures.

Master PMSM system, $\bar{\varpi}^m = \bar{i}_{qs}^m = \bar{i}_{ds}^m = 0.01$

Slave PMSM system, $\bar{\varpi}^s = \bar{i}_{qs}^s = 1,\ \bar{i}_{ds}^s = 10$

Parameter estimate error system, $\dot{\bar{\sigma}} = \dot{\bar{\gamma}} = 1$

The controller coefficients were selected as $\Delta_{11} = \Delta_{22} = \Delta_{33} = 10$.

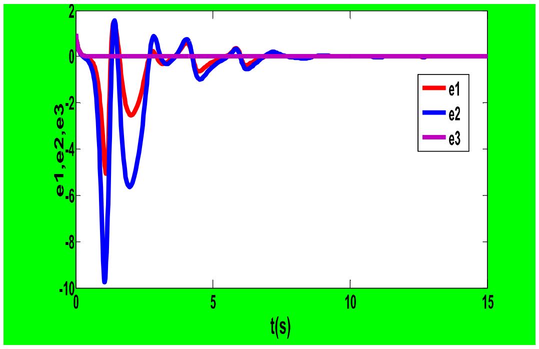


Figure 5. Dynamics of the synchronization error states e_1, e_2, e_3

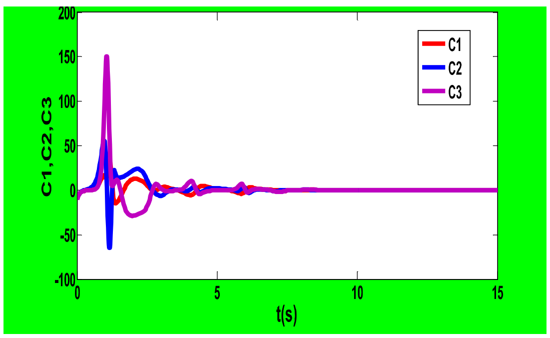


Figure 6. Asymptotically stabilized dynamics of the adaptive controllers C_1, C_2, C_3

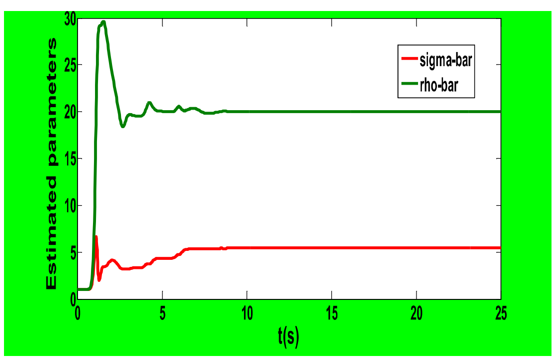


Figure 7. Dynamics of the estimated parameters $\overline{\sigma}, \overline{\gamma}$

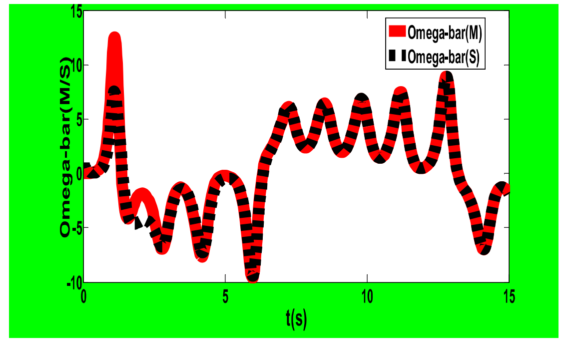


Figure 8. Synchronized dynamics of state variables $\overline{\omega}^s$ *and* $\overline{\omega}^m$

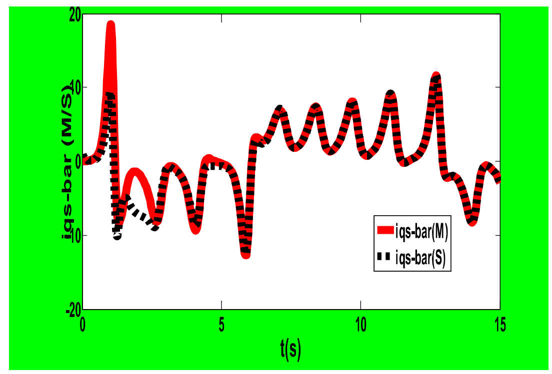


Figure 9. Synchronized dynamics of state variables $\overline{i}_{qs}^s$ *and* $\overline{i}_{qs}^m$

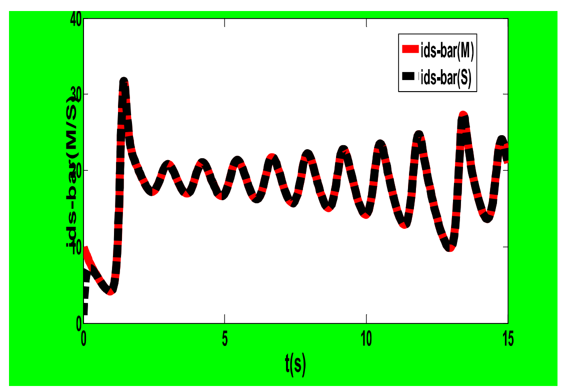


Figure 10. Synchronized dynamics of state variables $\overline{i}_{ds}^s$ *and* $\overline{i}_{ds}^m$

ELECTRONIC CIRCUIT REALIZATION OF THE NORMALIZED CHAOTIC PMSM

Autonomous electronic circuits comprising basic circuit components such as resistors, capacitor and analogue integrated circuits such as operational amplifiers can be built to mimic the behaviours and dynamics of a nonlinear chaotic system. The behaviours of the chaotic system – quasi-periodic, periodic and transient behaviours can be captures as the parameters of the circuits are varied accordingly. The essence of constructing an electronic chaotic system is to authenticate the chaoticity of a proposed system because stochastic noise often have the same dynamic as a deterministic chaotic system (Kenedy, 1995). In the literature, almost all chaotic systems are provides with their electronic circuit equivalent. In this section, the electronic circuit is realized and simulated in the virtual environment of NI MultiSim software. The time constant of the ideal simulation environment and the ideal components are not compatible and are therefore rescaled. In this section, the mathematical model of chaotic PMSM (1) is transformed into electronic circuit through appropriately selected parameters of the components. Basically, the operational amplifier provides the means to perform addition, subtraction, differentiation and integration of the coupled differential equations that constitutes the mathematical model of the system. The multiplier produces the nonlinear combination of two variables which introduces nonlinearities into the algebraic structure of the model. The model circuit equation is given as

$$\begin{cases} \dot{\bar{\omega}} = \dfrac{R_4}{R_5 C_1}\left(\dfrac{\bar{i}_{qs}}{R_2} - \dfrac{\bar{\omega}}{R_1}\right) \\ \dot{\bar{i}}_{qs} = \dfrac{R_{11}}{R_{12} C_2}\left(-\dfrac{\bar{i}_{qs}}{R_8} - \dfrac{10\bar{\omega}\bar{i}_{ds}}{R_9} + \dfrac{\bar{\omega}}{R_{10}}\right) \\ \dot{\bar{i}}_{ds} = \dfrac{R_{16}}{R_{17} C_3}\left(-\dfrac{\bar{i}_{ds}}{R_{18}} + \dfrac{10}{R_{15}}\bar{\omega}\bar{i}_{qs}\right) \end{cases} \quad (20)$$

The circuit diagram of equation of (1) is of the form

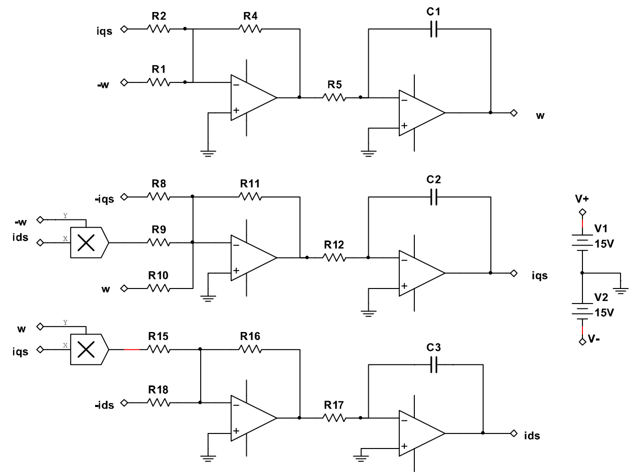
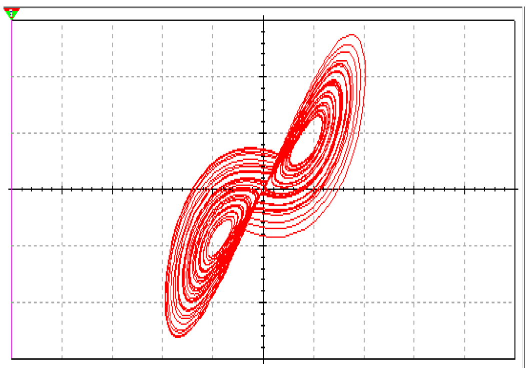


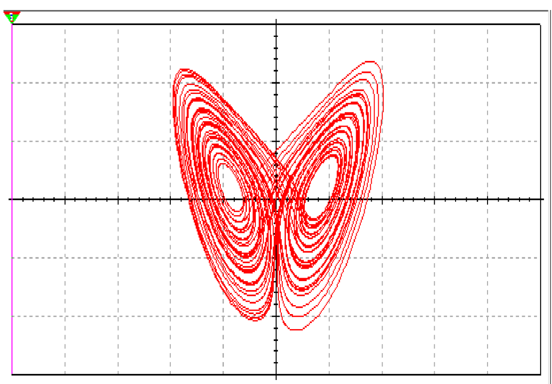
Figure 11. Chaotic Electronic PMSM

The time constant $\tau = \tau_0 t$, where $\tau_0 = 0.01$. The multiplier voltage gain is fixed at $10V/V$. Therefore, using operational amplifier circuit theory, the following parameter values were selected.

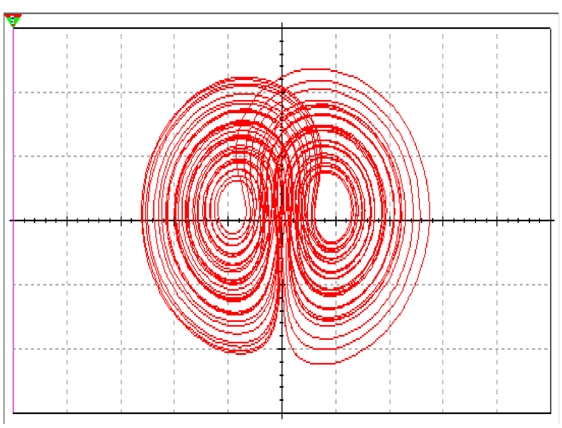
For the value of $\sigma = 5.46$, $R_1 = R_2 = 18.3K$, $R_4 = R_8 = R_9 = R_{11} = R_{15} = R_{16} = R_{18} = 100K$, $R_5 = R_{17} = 10K$, $R_{10} = R_{12} = 5K$. $C_1 = C_2 = C_3 = 100nF$. Using Multisim software, the following captures were observed.



$\overline{i}_{qs}$ vs $\overline{\omega}$

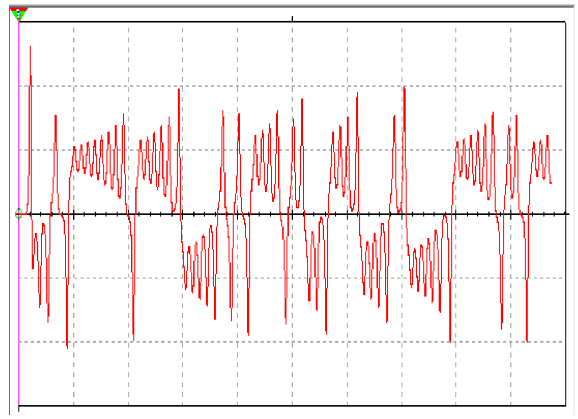


$\overline{i}_{ds}$ vs $\overline{\omega}$

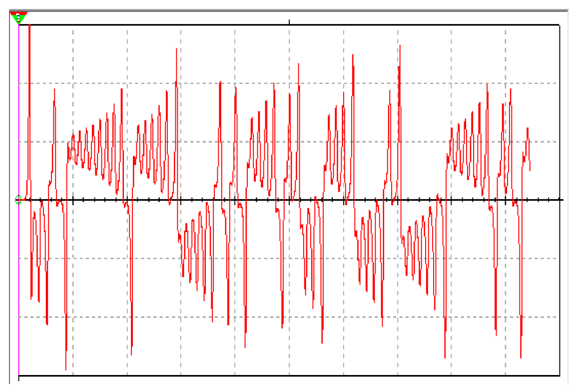


$\overline{i}_{ds}$ vs $\overline{i}_{qs}$

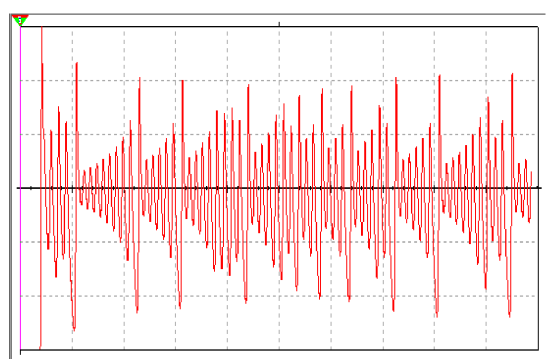
Figure 12. 2D phase portraits of the electronic chaotic PMSM circuit



$\overline{\omega}$ vs $t(s)$



$\overline{i}_{qs}$ vs $t(s)$



$\overline{i}_{ds}$ vs $t(s)$

Figure 13. Dynamics of the state trajectories in time space

3 CONCLUSION

This paper has demonstrated how the dynamics of a permanent magnet synchronous motor can be modelled with electronic circuits. This approach makes the unwanted dynamics of the motor to be suitable for application in such areas as secure communication, cryptography and related fields where the dynamics can be used to hide information on transmission channels. It can also be seen that the designed adaptive controller robustly synchronized the chaotic orbits and estimated the unknown parameters of the slave system. The electronic circuit of the chaotic PMSM mimics the chaotic dynamics of the mathematical model, thus validating the existence of chaos in the normalized chaotic PMSM.

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