Analytical Method of Land Surface Temperature Prediction

[1]M. D. Shehu [2]R. A. Olowu

[1,2]Department of Mathematics, Federal University of Technology, Niger State
[1] m.shehu@futminna.edeu.ng
[2]olowurafiu1985@gmail.com

Abstract

In this paper, we formulated a model for the prediction of Land Surface Temperature (LST) using the Analytical Method, with the incorporation of atmospheric scale height. The following parameters were considered, meridional wind speed "v", scale height of pressure difference "H", coriolis force "f", time "t", gravitational force "g" and temperature "T". The novelty of this study centers on the attempt to predict and analyze the behaviour of LST for a particular area using shallow water equation. The result revealed that the Land Surface Temperature increases with decrease in altitude, and this re-affirms the efficiency of Shallow Water Equation in making accurate predictions of the Land Surface Temperature.

Keywords: land surface temperature, zonal wind speed, meridional wind speed, scale height, atmospheric altitude.

1.0 Introduction

Weather prediction is an essential method for meteorologists to forecast the weather and lead to more precision in forecasting. Since the weather is a complex and chaotic system, numerical weather prediction also displays a high complexity. Although most people do not know how weather prediction works, they are highly interested in the results, presented by TV weathermen or accessible through the Internet (Albrecht, *et al.,* 2016; Nycander and Doos, 2003)

Land Surface Temperature (LST) is an important variable in climate, Hydrologic, Ecological, Biophysical and Biochemical studies (Ådlandsvik, 2008). LST plays a key role in modeling the surface energy balance and has a substantial impact on analyzing the heat-related issues such as soil moisture, evapotranspiration and urban heat islands (Alexander and Arblaster, 2009; Hallberg and Rhines, 2016), Batteen and Han (1981) considered a Smoothed Particle Hydrodynamics (SPH) approaches were used to solve Shallow Water Equations (SWEs), and this is used to study practical dam-break flows at

South-Gate Gorges Reservoir. The model is first tested on two benchmark collapses of water columns with the existence of downstream obstacle. Subsequently the model is applied to forecast a prototype dam-break flood, which might occur in South-Gate Gorges Reservoir area of Qinghai Province, China. It shows that the SWE-SPH modeling approach could provide a promising simulation tool for practical dam-break flows in engineering scale.

In remote sensing land surface temperature data are usually obtained through the weather satellite, the satellite does not measure the temperature directly; it only scans through the atmosphere and measure the radiant of the sun. A retrieval algorithm is used to obtain the temperature data from the satellite. However, direct estimation of LST from the radiation emitted in the TIR spectral region is difficult to perform with correct accuracy by the satellite, since the radiances measured by the radiometers onboard satellites depend not only on surface parameters (temperature and emissivity) but also on atmospheric effects (Buhler, 1998; Nelson and Markley, 2014)

This necessitated an analytical approach for the prediction of Land Surface Temperature using shallow water equation.

2.1 Model Development

2.2 Shallow Water Equation

The two-dimension form of shallow water equation as given by (Mesinger and Arakawa, 2017) is considered

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+g\frac{\partial h}{\partial x}-fv=0 \quad (1)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+g\frac{\partial H}{\partial y}+fu=0 \quad (2)$$

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+v\frac{\partial H}{\partial y}+h\frac{\partial u}{\partial x}=0 \quad (3)$$

$$\frac{\partial H}{\partial y} = -f\frac{\overline{U}}{g} \quad (4)$$

where,

U represents velocity component in the x- direction

v represents velocity component in the y- direction

h represents depth of the fluid

f represents Coriolis force

t represents time

T represents temperature

g represents acceleration due to gravity

$\overline{U}$ represents average mean wind speed of the atmosphere.

Equation (1) is the momentum equation for u along the x-axis

Equation (2) is the momentum equation for v along the y-axis

Equation (3) is the continuity equation

3.0 Model Formulation

To obtain a mathematical model for the Land surface temperature using the set of equaion (1) to (4) is to define the fluid depth to be consistent with the boundary layer of the troposphere, so the temperature at this lower atmosphere can varies with altitude. A very common way to describe the behavior of temperature and altitude at this atmosphere layer is by its 'scale height'. Scale height is related to the temperature (T) and mean molecular mass (m) of the atmosphere given by the formula:

$$H = \frac{RT}{g} \quad (5)$$

where

H = Scale Height
T = Temperature
R = Universal gas constant
g = Gravitational force

Equation (5) is incorporated into the model formulation due to the existence of various gaseous element within the atmosphere.

By differentiating equation (5) with respect to *y*, yield

$$\frac{\partial H}{\partial y} = \frac{R}{g}\frac{\partial T}{\partial y} \tag{6}$$

substituting $\frac{R}{g}\frac{\partial T}{\partial y}$ for $\frac{\partial H}{\partial y}$ in equation (4)

$$\frac{\partial T}{\partial y} = -f\frac{\overline{U}}{R} \tag{7}$$

To represent the behavior of the atmosphere in other to derive a model for the land surface temperature there is a need to consider the temperature lapse rate "Υ"at which temperature decreases with increase in altitude. Combine equation (2) and (3) taking the shallow height of the atmosphere "*h*" to be constant; this leads to a new continuity equations.

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} - fv = 0 \tag{8}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + fu = 0 \tag{9}$$

$$\frac{\partial T}{\partial y} = -f\frac{\overline{U}}{R} - \gamma \tag{10}$$

where;

Equation (8) represent momentum equation for *u*

Equation (9) represent momentum equation for *v*-

Equation (10) represent lapse rate equation

To obtain solution for our model equation, we consider the analytical method of solution.

3.1 Analytical Method

In this section we employ an analytical method for the solution of our model equation.

Equation (8) , (9) and (10), can be written as

$$\frac{du}{dt} = fv \tag{11}$$

$$\frac{dv}{dt} = -fu \tag{12}$$

Let 0<f<<1, and suppose the solution (u, v) can be expressed in series form as: (13)

$$u(x,t) = u_0(x,t) + fu_1(x,t) + \dots$$

$c_4 = 0$

Thus

$$v_1(x,t) = -u_{o}t \qquad (24)$$

Substituting equation (21), (22), (23) and (24) into equation (13) and (14), respectively yields,

$$u(x,t) = u_o - fv_o t \qquad (25)$$

$$v(x,t) = v_o - fu_o t \qquad (26)$$

Integrating equation (10) with respect to y, yields

$$\frac{\partial T}{\partial y} = -(f\frac{U}{R} + \gamma)\phi \qquad (27)$$

Thus the analytic solution to our model equations (8), (9) and (10) is giving as;

$$u(x,t) = u_o - fv_o t \qquad (28)$$

$$v(x,t) = v_o - fu_o t \qquad (29)$$

$$T_{(\phi)} = T_{(O)} - (f\frac{\bar{u}}{R} + \gamma)\phi \qquad (30)$$

3.2 Data Consideration

The following data for the evaluation of our analytical solution where considered

u_o	=	0.8, initial value for zonal wind speed
v_o	=	5.4 initial value for meridional wind speed
t	=	3.0 time
T_0	=	37^oC
γ	=	0.0065
R	=	8.3144598

Ubar "$\bar{u}$"=product of the mean value of zonal and meridional wind speed, and is giving

$$\bar{u} = \frac{\sum_{i}^{n} u_i}{n} \cdot \frac{\sum_{i}^{n} v_i}{n} \; for\; i = 1,2...n \qquad (31)$$

4.0 Results

This section shows the graphical solution obtained from the simulation of the equations (28), (29) and (30).

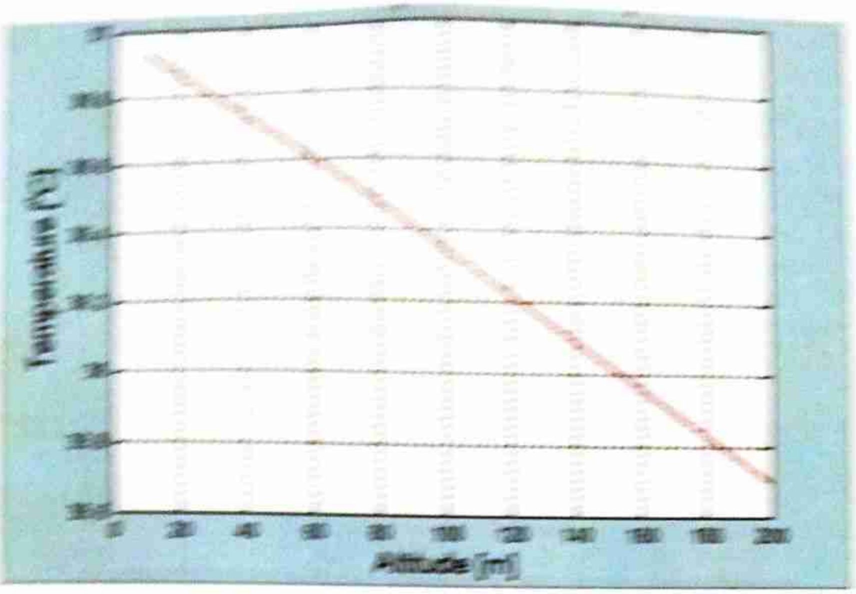

Figure 1: Graph of Temperature (°C) against the atmospheric Altitude (m).

gure 1 is the graphical solution to the simulation of the equations (28), (29) and (30) with the mean alue of the zonal and meridional wind speed, using Matlab software.

able 1: Predicted temperature extracted from figure 1

Altitude (m)	20	40	60	80	100	120	140	160	180
Temperature(°C)	36.84	36.71	36.60	36.44	36.38	36.20	36.10	35.95	35.85

t can be observed from table 1, that the land surface temperature decreases as the atmospheric altitude ncreases, also the temperature increases as the altitude decreases. It can be see how the temperature gradually decreases from 36.84°C to 35.85°C for the altitude 0m to 200m.

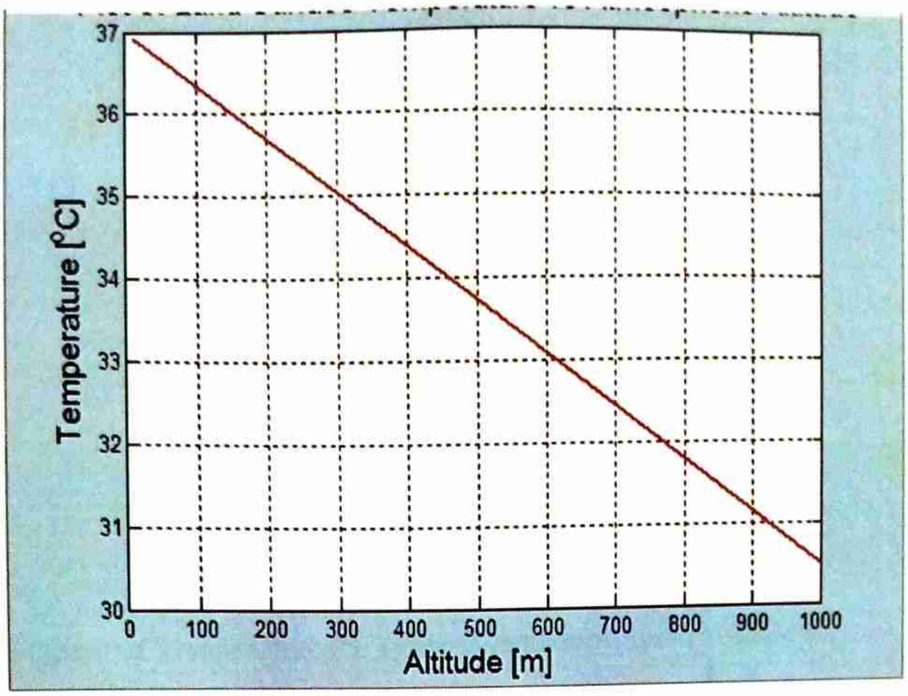

Figure 2: Graph of Temperature (^{0}C) against the atmospheric altitude (m).

Figure 2 is the graphical solution to the stimulation of the equations (28), (29) and (30) with the mean value of the zonal and meridional wind speed, using Matlab software.

Table 2: Predicted temperature extracted from figure 2

Altitude (m)	100	200	300	400	500	600	700	800	900
Temperature(^{0}C)	36.40	35.80	35.00	34.51	33.92	33.20	32.50	31.80	31.20

It can be observed from table 2, that the land surface temperature decreases as the atmospheric altitude increases, also the temperature increases as the altitude decreases. It can be see how the temperature gradually decreases from 36.40^{0}C to 31.2^{0}C for the altitude 0m to 1000m.

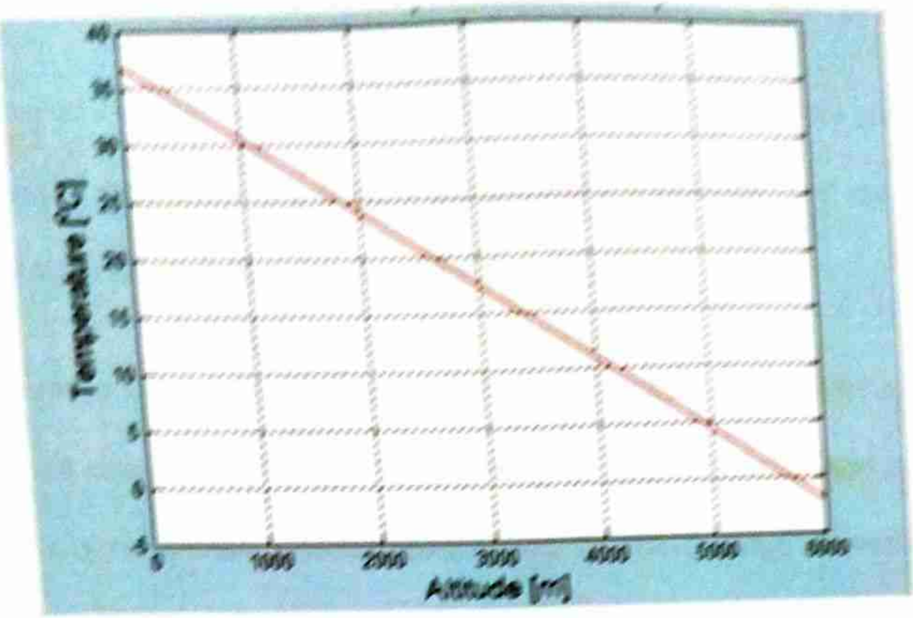

Figure 3: Graph of Temperature (°C) against the atmospheric altitude (m)

Figure 3 is the graphical solution to the simulation of the equations (28), (29) and (30) with the mean value of the zonal and meridional wind speed , using Matlab software.

Table 3: Predicted temperature extracted from figure 3

Altitude (m)	1000	2000	3000	4000	5000	6000	7000	8000	5800
Temperature(°C)	30.00	24.00	17.00	11.00	5.00	-2.00	-5.00	-7.00	-12.0

It can be observed from table 3, that the land surface temperature decreases as the atmospheric altitude increases, also the temperature increases as the altitude decreases. It can be seen how the temperature gradually decreases from 37°C to -3°C for the altitude 0m to 6000m.

5.0 Conclusion

This research work presents an analytical method for the prediction of Land Surface Temperature using the shallow water equation. The atmospheric behavior was carefully studied and most important parameters were identified and well represented accordingly as required in the prediction of Land Surface Temperature using the shallow water equation. The demographic profile of Nigeria meridional and zonal wind speed were used to study the behavior of this model. However, the behaviors exhibited by both the atmosphere altitude and the land surface temperature in this analytic result shows that at any given time on the earth surface, "the higher the location of a particular area, the lower will be the temperature the people living in that environment will experience.

Information on weather dataset should be made available, accessible and usable for all researchers who take research in the area of Land Surface Temperature and other weather related areas. The Federal Government of Nigeria should establish commission for the training of weather and climate management and development for lecturers and students in Nigeria.

References

AchutaRao, K., Covey, C. & Doutriaux, C. (2004). An Appraisal of Coupled Climate Model Simulations, Lawrence Livermore National Laboratory report UCRL-TR-202550.

Adlandsvik, B. (2008). Marine downscaling of a future climate scenario for the North Sea.Tellus, 60A, 451–458.

Albrecht, B. A., Betts, K. A. & Schubert, W. H. (2016). A model for the thermodynamic structure of the trade-wind boundary layer: I. Theoretical formulation and sensitivity tests. J. Atmos. Sci., 36, 73–89.

Alexander, L. V., & Arblaster, J. M (2009). Assessing trends in observed and modeled climate extremes over Australia in relation to future projections. Int. J. Climatol., 29,417–435.

Batteen, M. L. & Han, Y.-J. (1981). On the computational noise of finite-difference schemes used in ocean models, Tellus, 33, 387–396.

Buhler, O. (1998). A shallow-water model that prevents nonlinear steepening of gravity waves. J. Atmos. Sci., 55, 2884–2891.

Hallberg, R. & Rhines, P. B. (2016). Buoyancy-driven circulation in an ocean basin with isopycnals intersecting the sloping boundary. J. Phys. Oceanogr., 26, 913–940.

Nelson G. & Markley A. (2014). Principles of Differential Equations. Pure and Applied Mathematics, Wiley-Interscience, Hoboken, NJ, USA, 99-105.

J. & Doos, K. (2003). Open boundary conditions for barotropic waves, Journal of
physical Research: Oceans, 108.

F. & Arakawa, A. (2017). Numerical methods used in atmospheric models, GARP technical
ort 17, WMO/ICSU. Geneva, Switzerland, 1, 17, 976.