Mathematical Modeling for Cost Minimization for the Control of Air Pollutants

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Abstract. In this paper, a linear programming model for controlling the generation of industrial pollution in a given system is presented. For each system, the model incorporates the key factors of cost, abatement potentials and implementation to determine a least cost for emission reduction for each pollutant within a specified time frame. Emission sources the system considered are induction furnaces, reheating furnaces and Open-heart furnaces. Simplex method and computer application 'Solver' were applied to solve the model. Sensitivity analysis was then conducted to explore the effect of making possible adjustments in the air standards Sensitive parameters were identified and the parameters that should be estimated more closely are determined. Results from both approaches were compared,analyzeand discussed.

Keywords: Emission reduction, pollutants, constraints, time period, emission.

Introduction

Iron and steel industry is regarded as one of the basic industries for the development of any country. On the other hand, iron and steel industries are one of the major sources of air pollution, Air pollution worldwide is a growing threat to human health and the natural environment. Air pollution is described as contamination of the atmosphere by gaseous, liquid, or solid wastes or byproducts that can endanger human health, welfare of plants and animals, attack materials, reduce visibility, and or produce undesirable odors [1]

Although some pollutants are released by natural sources like volcanoes, coniferous forests, and hot springs, the effect of this pollution is very small when compared to that caused by emissions from industrial sources, power and heat generation, waste disposal, and the operation of internal combustion engines [8]. Fuel combustion is the largest contributor to air pollutant emissions, caused by man, with stationary and mobile sources equally responsible. Industries are a major source of pollution when proper controls of the emissions are not in place [2].

Linear programming is a suitable modeling approach for selecting the optimal solution of abatement actions to achieve specified air management objectives. Linear programming model incorporates the interactions between abatement actions to solve a multi-pollutant problem [2]. A linear programming problem has three key components: an objective, activities and constraints. The objective specifies something to be maximized or minimized, such as, maximize profit or minimize cost. The activities are the options available for use by the decision maker, such as, abatement methods available for application. The constraints are the restrictions on the selection of activities. These restrictions can be specified as minimum, maximum or exact level of the activities to be used in the solution [7]. A constraint is said to be 'binding' when all available units of an activity are used. A solution to a linear programming problem must satisfy all the constraints specified [3].

A mixed integer programming model for the selection of optimal air pollution reduction strategies for refineries to achieve a given pollution level was developedby [4], the model considered both costs and environmental impacts and was numerically tested on an industrial scale. An air pollution control model for multi-pollutants dispersion for the mining sector was developed by [5]; the model is based on a linear algorithm to achieve a single objective by determining the least cost option among various treatment technologies. Various simulation runs were conducted for each pollutant and every treatment option. They applied general Gaussian air dispersion model to determine the concentration after considering the treatment effect. In conclusion, the model can be used as a decision tool for planners to select the sustainable and cost- effective technology to control air pollution.

A model to minimize carbon emission from road transportation with focus on Lagos was considered by [6]. a number of control strategies were used. The situation is modeled as a

Linear programming problem and simplex method adopted to solve the problem. They came up with optimum solutions. Their findings revealed that greater attention be paid to using quality fuel for transportation without compromise.

In this paper, a linear programming model is considered for effective control of the generation of industrial pollution using the approach of cost minimization.

2. Model Development

There are three key components of linear programming methodology in the context of air quality management; they are; Control activities, objective function and emission reduction constraints.

2.1 Control Activities

These are the control options, or actions available for use across emission sources. The model diagram below describes the pollution control activities of Dana steel works.

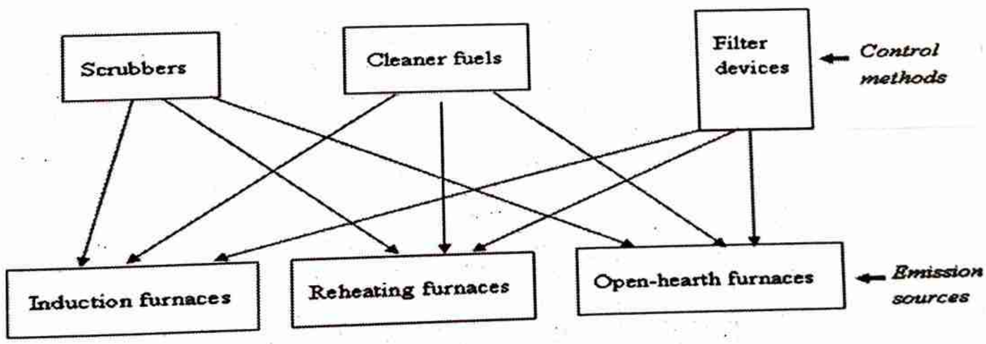

Figure 1. Model Diagram

A number of emission sources such as Induction furnaces, Reheating furnaces and Open-heart furnaces are been considered, each emitting a number of pollutants such as particulate matters, oxides of nitrogen, oxides of sulfur and hydrocarbon. For a given emission source, there are available number of control measures. These pollution control measures include scrubbers, filter devices and cleaner fuel. Each of these measures has associated with it a certain cost and emission reduction capability.

The annual cost estimates (in millions of naira per annum) of the nine control activitiesused in their full capacity are given in table 1

Table 1. Total Annual Cost from the Maximum Feasible use of a Control Method

Emission Source	Induction furnaces			Reheating furnaces		Open-hearth furnaces			
Control Activities, $ctrl_j$	$ctrl_1$	$ctrl_2$	$ctrl_3$	$ctrl_4$	$ctrl_5$	$ctrl_6$	$ctrl_7$	$ctrl_8$	$ctrl_9$
Costs (in millions of naira, ₦ per annum) ,c_j	2	3	4.5	3	2.5	5	3	4	2.5

. Source:Dana Steel Mill

2.3 Emission Reduction Constraints and Capacity of each Control Measure on a Source

Key components of an emission reduction constraint are considered, these are-

(i) Emission reduction target for each pollutant (given in table 2)

(ii) Time period when emission reduction target must be met (these targets must be met annually)

(iii) Annual emission reduction potential of each abatement action (given in table 3)

The emission reduction constraints are given by:

$$\sum_{j=1}^{9} a_{ij}ctrl_j \qquad (3)$$

Technological limit and Non-negativity Constraint

$$1 \geq ctrl_j \geq 0 \qquad \text{for } j = 1,2,3,\ldots,9 \qquad (4)$$

Where, a_{ij}represents reduction potential in annual emission rates of control j for pollutant i.$ctrl_j$represents the fraction of control activity j across emission sources, b_i represents emission reduction target (air quality standard) of pollutant i given in table 2 $i$represent the number of pollutant $j$represent the number of control activities

$$a_{11}ctrl_1+a_{12}ctrl_2+a_{13}ctrl_3+a_{14}ctrl_4+a_{15}ctrl_5+a_{16}ctrl_6+a_{17}ctrl_7+a_{18}ctrl_8+a_{19}ctrl_9 \geq b_1$$

$$a_{21}ctrl_1+a_{22}ctrl_2+a_{23}ctrl_3+a_{24}ctrl_4+a_{25}ctrl_5+a_{26}ctrl_6+a_{27}ctrl_7+a_{28}ctrl_8+a_{29}ctrl_9 \geq b_2 \quad (5)$$

$$a_{31}ctrl_1+a_{32}ctrl_2+a_{33}ctrl_3+a_{34}ctrl_4+a_{35}ctrl_5+a_{36}ctrl_6+a_{37}ctrl_7+a_{38}ctrl_8+a_{39}ctrl_9 \geq b_3$$

$$a_{41}ctrl_1+a_{42}ctrl_2+a_{43}ctrl_3+a_{44}ctrl_4+a_{45}ctrl_5+a_{46}ctrl_6+a_{47}ctrl_7+a_{48}ctrl_8+a_{49}ctrl_9 \geq b_4$$

The new air quality standards require the company to reduce its emission of oxides of sulfur, oxides of nitrogen, hydrocarbons and particulate matters to that given in table 2.

Table 2. Clean air standards for the Dana Steelworks.

I	Pollutants	Emission reduction target/annum
1	Oxidesof nitrogen	110 million pounds
2	Oxides of sulfur	140 million pounds
3	Hydrocarbons	120 million pounds
4	Particulate matters	80 million pounds

Source: Dana Steel Mill

The values on table 2 represent the limits to which the emission of the pollutants listed above must be reduced to in order to achieve the required air quality.

The emission reduction potentials a_{ij} in annual emission rate when control *j* is applied in its full capacity on an emission source to control the emission of pollutant *i*, (in millions of pound per annum) are given in table 3.

Table 3. Reduction Capability in Emission Rate from Maximum Feasible Use of a Control

S/N	Pollutants	Emission reduction potential of control methods on a source								
		Induction furnaces			Reheating furnaces			Open heart furnaces		
		$ctrl_1$	$ctrl_2$	$ctrl_3$	$ctrl_4$	gtrl5	$ctrl_6$	$ctrl_7$	$ctrl_8$	$ctrl_9$

1	Nitrogen oxides	a_{11} 24	a_{12} 16	a_{13} 36	a_{14} 22	a_{15} 18	a_{16} 22	a_{17} 17	a_{18} 23	a_{19} 24
2	Sulfur oxides	a_{21} 38	a_{21} 35	a_{21} 18	a_{21} 20	a_{21} 32	a_{21} 28	a_{21} 26	a_{21} 32	a_{21} 24
3	Hydrocarbons	a_{31} 28	a_{31} 16	a_{31} 26	a_{31} 31	a_{31} 24	a_{31} 20	a_{31} 18	a_{31} 24	a_{31} 18
4	particulate mat ters	a_{41} 14	a_{41} 12	a_{41} 22	a_{41} 9	a_{41} 18	a_{41} 13	a_{41} 18	a_{41} 16	a_{41} 14

Source of the table: Dana Steel Mill:

3. Model Formulation

The model is formulated as a linear programming problem and on a Spreadsheet. Simplex algorithm and excel solver were applied to solve the problem.

Minimize (cost)

$$Z = 2ctrl_1 + 3ctrl_2 + 4.5ctrl_3 + 3ctrl_4 + 2.5ctrl_5 + 5ctrl_6 + 3ctrl_7 + 4ctrl_8 + 2.5ctrl_9 \quad (6)$$

Subject to *Emission reduction of*

(1) Oxides of sulfur

$$24ctrl_1+16ctrl_2+36ctrl_3+18ctrl_4+22ctrl_5+17ctrl_6 104ctrl_7+23ctrl_8+24ctrl_9 \geq 110 \quad (7)$$

(2) Oxides of nitrogen

$$38ctrl_1+ 35ctrl_2+ 18ctrl_3+ 32ctrl_4 + 20ctrl_5+28ctrl_6+26ctrl_7+32ctrl_8+24ctrl_9 \geq 140 \quad (8)$$

(3) Hydrocarbons

$$28ctrl_1+16ctrl_2+26ctrl_3+31ctrl_4+ 24ctrl_5 + 20ctrl_6+ 18ctrl_7+ 24ctrl_8+ 18ctrl_9 \geq 120 \quad (9)$$

(4) pParticulate matters

$$14ctrl_1+12ctrl_2+22ctrl_3+9ctrl_4+18ctrl_5+13ctrl_6+18ctrl_7+16ctrl_8+14ctrl_9 \geq 80 \quad (10)$$

Technological limit and non negative constraint given as,

$$ctrl_j \leq 1 \qquad j = 1, 2, 3, ...,9$$

$$ctrl_j \geq 0 \qquad j = 1, 2, 3, ... ,9$$

(11)

3.1 Model Formulation on Excel Worksheet

The linear programming model in equations (1), (2) and (3) is further formulated on an excel worksheet as shown on the figure 2

	A	B	C	D	E	F	G	H	I	J	K	L	M
1		DANA STEEL MILL KATSINA											
2		EMISSION CONTROL MODEL											
3	**Control Activities**	*Ctrl1*	*Ctrl2*	*Ctrl3*	*Ctrl4*	*Ctrl5*	*Ctrl6*	*Ctrl7*	*Ctrl8*	*Ctrl9*			**Cost (minimum)**
4	**Cost**	2	3	4.5	3	2.5	5	3	4	2.5			0
5	**Pollutants**	Reduction in emission rate from the maximum feasible use of a control method									*Total Reduction achieved*	*Relation*	*Required Reduction in emission rate*
6	*Nitrogen oxides*	24	16	36	18	22	17	10	23	24	0	≥	110
7	*Sulfur oxides*	38	35	18	32	20	28	26	32	24	0	≥	140
8	*Hydrocarbon*	28	16	26	31	24	20	18	24	18	0	≥	120
9	*Particulate matters*	14	12	22	9	18	13	18	16	14	0	≥	80
10	**Technological limit**	*Ctrl1*	*Ctrl2*	*Ctrl3*	*Ctrl4*	*Ctrl5*	*Ctrl6*	*Ctrl7*	*Ctrl8*	*Ctrl9*		≤	1
11	*Fractional use of control method*	0	0	0	0	0	0	0	0	0			

Figure 2. excel worksheet formulation

4. Model Solution

4.1 Simplex Method Solution of the Model

The model in equations (6), (7), (8), (9), (10) and (11) is a minimization problem, to solve this; we formed the coefficient matrix and take its transpose. Then, interpret the transpose as maximization problem in standard form given as:-

Maximize,

$$w = 110a + 140b + 120c + 80d - e - f - g - h - i - j - k - l - m \qquad (12)$$

Subject to

$$
\begin{array}{llllll}
24a & +38b & +28c & +14d & -e & \leq 3 \\
16a & +35b & +16c & +12d & -f & \leq 4 \\
36a & +18b & +26c & +22d & -g & \leq 5.5 \\
22a & +20b & +24c & +18d & -h & \leq 4 \\
18a & +32b & +31c & +9d & -i & \leq 3.5 \\
28a & +17b & +20c & +13d & -j & \leq 6 \\
24a & +24b & +16c & +14d & -k & \leq 4 \\
23a & +32b & +24c & +16d & -l & \leq 3.5 \\
10a & +26b & +18c & +18d & -m & \leq 5
\end{array}
\qquad (13)
$$

Slack variables $ctrl_1$, $ctrl_2$, $ctrl_3$, $ctrl_4$, $ctrl_5$, $ctrl_6$, $ctrl_7$, $ctrl_8$, and $ctrl_9$ are introduced. The following tables give initial system for the dual model and subsequent iterations. Optimal solution was reached after nine (9) successive iterations

4.2 Solvers Solution of the Model

Solver found it optimal solution after thirteen (13) successive iterations

1		DANA STEEL MILL KATSINA											
2		EMISSION CONTROL MODEL											
3	Control Activities	Ctrl1	Ctrl2	Ctrl3	Ctrl4	Ctrl5	Ctrl6	Ctrl7	Ctrl8	Ctrl9			Cost (minimum)
4	Cost	2	3	4.5	3	2.5	5	3	4	2.5			14.18507853
5	Pollutants	Reduction in emission rate from the maximum feasible use of a control method									Total Reduction achieved	Relation	Required Reduction in emission rate
6	Nitrogen oxides	24	16	36	18	22	17	10	23	24	110	≥	110
7	Sulfur oxides	38	35	18	32	20	28	26	32	24	140	≥	140
8	Hydrocarbon	28	16	26	31	24	20	18	24	18	120	≥	120
9	Particulate matters	14	12	22	9	18	13	18	16	14	80	≥	80
10	Technological limit	Ctrl1	Ctrl2	Ctrl3	Ctrl4	Ctrl5	Ctrl6	Ctrl7	Ctrl8	Ctrl9		≤	1
11	Fractional use of control method	1	0.225	0.471	0.601	1	0	0.863	0	1			

Figure 3. Solvers Solution

4.3 Results from both Approaches

Simplex algorithm and Microsoft excel solver were used to obtain optimal solution to meet the objectives of minimizing the cost of control plans across the emission sources and the air quality standards.

4.3.1 Results from Simplex Method

The optimal solution is reached after nine (9) successive iterations. The maximum (minimum) cost (Total annual cost),

$$w = \frac{4752}{335} = ₦\ 14.19 \text{ million}$$

Optimal solution of control activities used in their fractional capacities are as follows;

$x_1 = ctrl_1 = 1$ $\quad x_2 = ctrl_2 = \frac{459}{2044} = 0.225$ $\quad x_3 = ctrl_3 = \frac{418}{887} = 0.471$

$x_4 = ctrl_4 = \frac{2051}{3414} = 0.601$ $\quad x_5 = ctrl_5 = 1$ $\quad x_6 = ctrl_6 = 0$

$x_7 = ctrl_7 = \frac{824}{955} = 0.863$ $\quad x_8 = ctrl_8 = 0$ $\quad x_9 = ctrl_9 = 1$

The Optimal Objective Value Z, (Total annual cost) is achieved at ₦14,1850,785.3

4.4 Sensitivity Analysis:

Sensitivity analysis were then conducted to explore the effect of making possible adjustments in the air quality standards, as well as to check on the effect of any inaccuracies in the cost data..

4.4.1 Adjustable Cells Section

(i) Reduced Cost: The amount an objective function coefficient must change before we would increase any of the control activity. Alternatively, the amount the cost would change if you were to increase the use of a control method by one unit or percent.

Currently the control activities $ctrl_6$ and $ctrl_8$ are at zero percent, (which indicate that there are no control activities taking place there), with costs of ₦5 million and ₦4 million respectively. These costs are too high to make use of the methods. The costs must decreased to ₦(5 - 1.996269634) million = ₦3.003730366 million and ₦(4 - 0.280235602) million = ₦3.719764398 million respectively, if the use methods are to be used.

While control activity $ctrl_1$, $ctrl_4$ and $ctrl_7$ are heavily used at 100% with costs of ₦2 million and ₦3 million respectively. Their objective coefficients (cost) may be increased to achieve some fractional use of the control capacity. The costs must increased to ₦ (2 + 1.828141361) million = ₦3.828141361 million, ₦ (3 + 1.245557436) million = ₦4.245557436 million and (3 + 0.788461538) million = ₦ 3.788461538 million respectively. $ctrl_2$, $ctrl_3$ and $ctrl_5$ are already at a fraction of their technological limit. They have 0 reduced cost.

(ii) Allowable Increase and Decrease: This is the amount the objective coefficient

(cost) for a decision variable (control activity) can be increased or decreased without changing the optimal solution.

$Ctrl_2$ currently have a cost of ₦3 million (objective coefficient).Its allowable Increase and decrease are ₦ 0.544811321million and ₦ 0.235382177million respectively.

4.4.2 Interpretation

As long as the unit cost for the control activity $ctrl_2$ is between ₦ 2.764617823 million and ₦ 3.544811321 million, the optimal solution (of the control activity) will remain the same.

Note: The optimal solution (control activities) stays the same within this range, but the optimal objective value (cost) changes since the unit cost is changing. So, if the cost on the control activity $ctrl_2$ goes up to ₦ 3.4 million/pound, optimal cost will increase by (₦3.4 - ₦3 million) x final value = 0.4 x 0.224559733 = ₦ 0.0898238932 million.

If it goes down to ₦2.8 million the cost will decrease by (₦3 – 2.8 million) x (0.224559733)

= ₦ 0.0449119466 million.

5. Conclusion

The control measures must be used at the following fractions on an emission source

Table 4. Fraction of Control Activities on Emission Sources

Control methods	Induction furnaces	Reheating furnaces	Open–hearth furnaces

Scrubbers	$ctrl_1$	1 or 100%	$ctrl_2$	0.863 or 86.3%	$ctrl_3$	0 or 0%
Cleaner fuels	$ctrl_4$	0.601 or 60.1%	$ctrl_5$	1 or 100%	$ctrl_6$	1 or 100%
Filter devices	$ctrl_7$	0.225 or 22.5%	$ctrl_8$	0 or 0%	$ctrl_9$	0.471 or 47.1%

Control activities $ctrl_4$, $ctrl_5$, $ctrl_6$ represent the use of cleaner fuels in the three furnaces. Cleaner fuels are preferred over other fuels amount because of its ability to reduce emissions and improve the furnace performance.

The total cost of control plan per annum is therefore, minimized at ₦ 14.18507853. Oxide of nitrogen, Oxide of sulfur, Hydrocarbons and Particulate matters has required targets in annual emission reduction rate of 110 million pounds, 140 million pound, 120 million pounds, 80 million pounds respectively, the reduction targets are all achieved.

The framework presented in this work is a cost-effective portfolio of control measures, which meet multi-pollutant emission reduction targets within specific timeframe. Findings from study show that seven out of the nine control activities should actually be in place. These seven control activities; $ctrl_1$, $ctrl_2$, $ctrl_3$, $ctrl_4$, $ctrl_5$, $ctrl_7$,and $ctrl_9$ are very significant in the process of the minimization. For emission of these air pollutants to be reduced to the required air quality standards from Dana Steel furnaces, control activities $ctrl_4$, $ctrl_5$, $ctrl_6$ that represent the use of cleaner fuels must be used in the three furnaces. The three methods must be used in the Induction Furnaces; because of the high volume of emission from the source especially during changing of materials to furnace.

The required air quality standards specified in table 2 was satisfied and binding at a minimized total cost of control per annum of ₦ 14,185,078.53.

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