STOCHASTIC MODELLING OF HYDROLOGICAL DROUGHT SEVERITY IN THE SOKOTO-RIMA RIVER BASIN (SRRB), NIGERIA

Otache, M. Y.., O. D. Jimoh.,** J. J. Musa., * and Danmagaji, A****
** Department of Agricultural & Bioresources Engineering, Federal University of Technology, Minna*
*** Department of Civil Engineering, Federal University of Technology, Minna*
**** Sokoto-Rima River Basin Development Authority, Sokoto*
martynso_pm@futminna.edu.ng; drotachemartyns@gmail.com

Abstract
Hydrological drought is affected by a complex relation between intra-and inter annual climate variability and stores in the catchment, though it has a crucial link with drought impacts. Thus in this study,attempt was made at modelling the severity characteristic of hydrological drought. To do this, historical streamflow data for some gauging stations of the Sokoto-Rima River Basin, spanning period of years was obtained. The modelling exercise was done by employing Autoregressive Integrated Moving Average (ARIMA) and Composite Stochastic models.The results indicate that the stochastic component accounts for 92.52 %, 91.77 %, 93.18 %, and 92.71 % of the variance of the entire hydrological drought severity series in the stations under discourse. However, adoption of stochastic models (ARIMA)and its variants in the forecast of hydrological drought severity time series may not be effective since their forecast functions failed to reproduce the standard deviations of the series in a long-term. Instead of general ARIMA Models, ARFIMA and Fuzzy Logic Systems as well as Artificial Neural Network should be considered; at least for short-term forecasting and effective definition of the drought domain.
Keywords: *Hydrological drought, severity, ARIMA, Streamflow*

Introduction

As reported by Van Lanen *et al.* (2013), drought, desertification and other forms of water shortage are anticipated to affect as many as one-third of the World's population. For instance, according to ISDR (2009) and WWDR (2009) reports, in West Africa, an average of 30% of the people is exposed to drought every year and in Nigeria Five billion dollars is lost to drought annually. Therefore, considering the dynamics of drought occurrence and its appurtenant characteristics, it should not be confused with low flow, aridity, water scarcity or desertification or with related hazards such as heat waves and forest fires. Thus, drought according to Amin *et al.* (2011), can be looked at as a stochastic natural hazard that is instigated by intense and persistent shortage of precipitation, which impacts greatly on agriculture and hydrology. In this context, it suffices to note that drought in the submission of Amin (2011), varies by multiple dynamic dimensions; that is, including severity and duration which by the very nature of its subjective network of impacts makes them difficult to characterise.

According to Van Loon (2015), more types of drought impacts are related to hydrological drought than to meteorological drought. This is simply due to the fact that the areal extent of a drought, although very useful for meteorological drought, according to Nalbantis (2008), is not of interest for hydrological droughts since water managers are interested on streamflow only at a small number of points in space. According to Nalbantis (2008), hydrological drought is characterised by four attributes: (i) severity which is expressed by an index, (ii) time of its onset and duration, (iii) extent of area covered and (iv), the frequency of occurrence. Hydrological drought can cover extensive areas and can last for months to years, with devastating impacts on the ecological system and many economic sectors (Tallaksen and Van Lanen, 2004). It is marked by characteristics such as the magnitude which is a measure of the amount of deficiency and represents the ratio between drought deficit volume and drought duration. Other important characteristics are the severity which deals with the cumulative amount of deficiency, i.e., how the available level of water has deviated from the normal desired level, the duration which marks when the streamflow, groundwater or lake level falls below the threshold value and when the level rises above the threshold.

Hydrological drought is determined accordingly by Van Loon and Laaha (2015) in terms of the propagation of meteorological drought through the terrestrial hydrological cycle. This means that it is the significant decrease in the availability of water in all its forms appearing in the land phase of the hydrological cycle (Nalbantis, 2008); that is, a hydrological drought episode is related to stream flow deficit with respect to normal condition (Nalbantis, 2008). Since the definition of drought is dependent on the objective of a particular study, it is imperative to look at hydrological drought as a broad term related to negative anomalies in surface and sub-surface water. However, how hydrological drought duration and deficit relate to climate and catchment characteristics and which is dominant (Van Loon and Lanen, 2012) is still very unclear, especially as noted by Van Lanen *et al.* (2013), the relative importance of climate versus physical catchment structure on the development of hydrological drought still remains poorly understood. According to Mishra and Singh (2010), Pozzi *et al.* (2013), hydrological drought deserves more attention due to its crucial link with drought impacts. According to IPCC report as in Sceneviratne *et al.* (2012), there is need to pay more attention to the space-time development of hydrological drought. Against this backdrop therefore, this study is primarily designed to model and simulate hydrological drought regime of the Sokoto-Rima River Basin, Nigeria.

Methodology

i. Study location Data mobilisation

The Sokoto-Rima Basin is located within Latitude 10°N and 14° N and Longitude 4° E and 9°.5'E, respectively. The entire Basin is classified as belonging to Hydrological Area One (HA.1) according to Nigerian Hydrological Service Agency (NIHSA). The average annual rainfall is between 364- 970 mm. The River Basin is majorly serviced by four dams (i.e., Goronyo, Bakolori, Zobe, and Jibia); largely for water supply, irrigation and domestic use by the riparian communities. Figure 1 shows the gauging stations that were considered for the study; this includes the network of the stations and the dam sites. For the study, monthly time series data of streamflow for the Sokoto-Rima Basin were obtained for Sokoto, Goronyo, Gusau, Bakolori, Katsina, Zobe, Jibia gauging stations from NIMET and Sokoto-Rima River Basin Development Authority (SRRBDA) Zonal offices across the four States (Katsina, Zamfara, Sokoto and Kebbi) within the Basin.

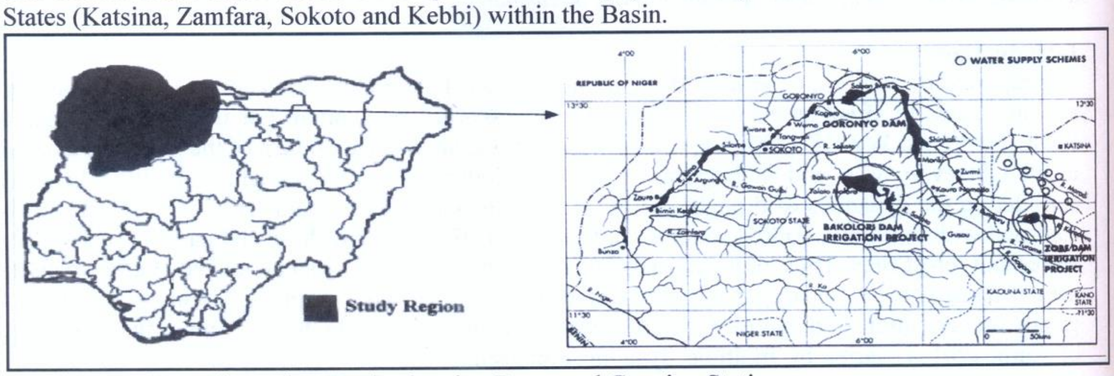


Figure 1: Sokoto-Rima River Basin showing Dams and Gauging Stations
Source: SRRBDA (2017)

ii. Extraction of drought severity series

According to the Threshold Level Method (TLM), a drought is observed once the variable of interest X (e.g. Streamflow) is equal to or drops below a pre-defined threshold. For this study, this threshold was defined from the streamflow percentile statistics. Based on the recommendation of Huijgevoort *et al.* (2012), the 20th percentile, also known as the 80th exceedance percentile was adopted. To do this, the entire streamflow series of monthly series was arranged in ascending order; that is, from the smallest to the largest. The monthly data points were converted into their respective percentiles: $\overline{X}_{P,T}$, where T = 1, 2, ...,12 or simply P_T. The percentile (P_i) of the data set was completed (Robertson, 2004) as in equation (1).

$$P_i = 100\left(\frac{i-0.5}{n}\right) \qquad (1)$$

where n = total number of observation, xi = the pith percentage of the data set. Due to the possibility of seasonal variability in the streamflow series, thresholds were defined for each month; i.e., $P_{20,T}$, T= 1, 2, 3,... 12. For this scheme, a drought was assumed to occur if the calculated percentile value is equal to or smaller than the defined threshold (i.e., $P_T \leq$ Thresholds, T). Since hydrological drought is considered as the period during which streamflow are below the expected amount (threshold) for a certain period of time, the ***cumulative volume of deficit*** was taken as the temporal extent of severity.

This line was adopted based on the fact that the truncation or threshold levels which in this case vary seasonally, divides the series into deficit and surplus sections. In addition, due to the randomness of the factors responsible for the occurrence and severity of drought, it is considered as a stochastic process.

Modelling drought severity

For the simulation exercise, since hydrologic phenomenon usually depicts cyclic and stochastic process, Fourier analysis was carried out to explain probable variance of the periodic component. This was done by employing the traditional classical harmonic analysis (Kottegoda, 1980). Based on this, the time series was expressed as

$$x_{\alpha,t} = \lambda_t + \eta_t \qquad (2)$$

where$x_{\alpha,t}$ is the severity of hydrological drought for the month t and year α whereas λ_t is the periodic component reflected in the monthly mean value t = 1 for April and t = 12 for March and $\eta_t$is the stochastic component with mean zero and variance. The periodic component was expressed as:

$$\lambda_t = A_0 + \sum_{j=1}^{6}\left(A_j cos\left(\frac{\Pi jt}{6}\right) + B_j sin\left(\frac{\Pi jt}{6}\right)\right) \qquad (3)$$

while the harmonic coefficients A_j and B_j were computed as

$$\begin{cases} A_0 = \frac{1}{12}\sum_{t=1}^{12}\lambda t \\ A_j = \frac{1}{6}\sum_{t=1}^{6}\lambda t cos\left(\pi i \frac{t}{6}\right) \\ B_j = \frac{1}{6}\sum_{t=1}^{6}\lambda t sin\left(\pi i \frac{t}{6}\right) \end{cases} \qquad (4)$$

The total series may then be written as a combination of the size of the periodic component depending on the adjudged number of significant harmonics and the adopted stochastic model. In this study, the stochastic component η_t of the series was modelled based on the Box and Jenkins (1976) methodology. Based on this approach, different seasonal ARIMA models of order (p, d, q)) x P, D, Q) were respectively fitted to the streamflow series. The seasonal ARIMA model is expressed as

$$(1 - B)^d(1 - B^s)^D \emptyset(B)\Phi(B)x_t = \theta(B)\Theta(B)\varepsilon_t \qquad (5)$$

where B is the backward shift operator, s is the period of the season. Here, $\phi(B)$, $\theta(B)$ are the autoregressive and moving average operators of order p and q of regular series while, $\Phi(B)$ and $\Theta(B)$ are autoregressive and moving average operators of order P and Q of the seasonal series and ε_t is the residual. To obtain the necessary orders for the model, both seasonal and non-seasonal differencing were employed, this was followed by analysis of the autocorrelation and partial autocorrelation functions with the view to determining the presence of notable spikes and randomness in the residuals of the fitted respective models (See Appendix).

Discussion

(a) ***Modelling of the periodic component***

Table 1 illustrates explained variance and percentage contributions of each harmonic to the overall periodic behaviour. Since the harmonics are uncorrelated, no two harmonics can explain the same part of the variance of the time series. This implies that the fractions can be added up to determine how many harmonics that may be needed to explain most of the variations in the time series. The first harmonic explains about 94.21 %, 68.05 %, 70.68 %, and 44.53 % for Bakolori, Goronyo (pre dam), Goronyo (post dam) and Zobe, respectively of the variance in the periodic means and in a similar manner accounts for a paltry 10.48 %, 8.23 %, 6.82 %, and 7.29 %, respectively. On the other hand, for the other harmonics in total on the average explained only 15.47 %, 13.99 %, 18.41 %, and 15.79 % of the variations in the periodic means for the respective stations whereas 89.52 %, 91.77 %, 93.18 %, and 92.71 % of the variance of the total hydrological drought series was accounted for by the stochastic component; this shows that it has a significant influence on the series. Based on this, the total hydrological drought severity series could be expressed for Bakolori, Goronyo (pre dam), Goronyo (post dam) and Zobe stations as follows:

$$\beta\alpha, t = 1.78021 + 1.5679Cos(\pi t / 6) - 2.7192Sin(\pi t / (6) + \eta_t \qquad (6)$$

$$\beta\alpha, t = 1.48898 + 1.5679Cos(\pi t \ / \ 6) - 2.7192Sin(\pi t \ / \ (6) + \eta_t \qquad (7)$$

$$\beta\alpha, t = 3.14833 + 1.5742Cos(\pi t/6) - 3439Sin(\pi t \ / \ (6) + \eta_t \qquad (8)$$

$$\beta\alpha, t = 0.1684075 + 0.2244Cos(\pi t \ / \ 6) - 0.3022Sin(\pi t \ / \ (6) + \eta_t \qquad (9)$$

where, η_t in the respective models stands for the stochastic component

Table 1: Harmonics and associated explained variance based on total drought severity series

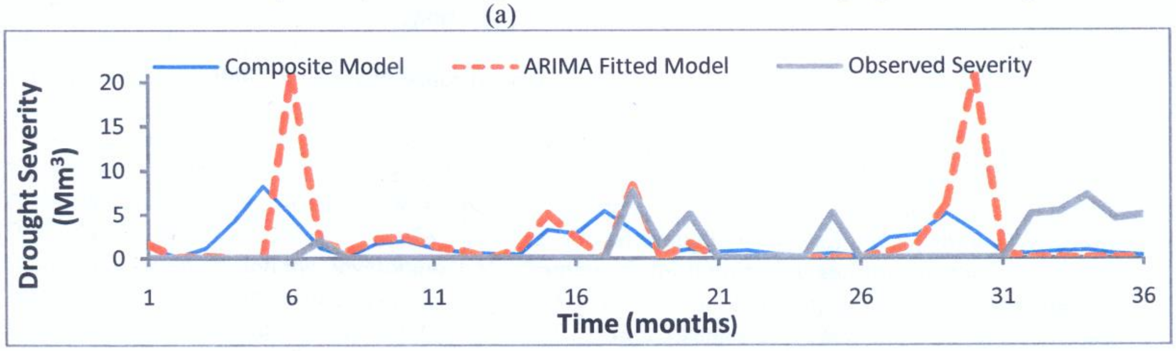
Station	Harmonics	Explained Variance	%
Bakolori			
	1	9.18961	10.48
	2	4.63038	5.25
	3	1.70922	1.95
	4	0.94243	1.07
	5	0.26317	0.30
	6	0.00000	0.00
Goronyo (pre dam)			
	1	5.60555	8.23
	2	3.07520	4.52
	3	1.16571	1.71
	4	0.75817	1.11
	5	0.76607	1.12
	6	0.00000	0.00
Goronyo (post dam)			
	1	17.51195	6.82
	2	12.37531	4.82
	3	6.015593	2.34
	4	2.667126	1.04
	5	1.756686	0.68
	6	0.000000	0.00
Zobe			
	1	0.093182	7.29
	2	0.093226	7.29
	3	0.056886	4.45
	4	0.015016	1.17
	5	0.000136	0.01
	6	0.000000	0.00

(b) Modelling of the stochastic component

Figures 2 shows the performances of the respective models for the Goronyo station as an example. From the figure, it is glaringly apparent that in all instances the models though replicated the behavioural response pattern of the drought severity but failed to capture the necessary feature vectors of the process; for instance, the peak drought severity in all the stations though the flows were replicated much better but to a varying degree. This trend is clearly manifested in Table 3 as all the models failed to reproduce the periodic means in the drought severity series.

The failure of these models brings a lot of issues to the fore for consideration though probable reasons could be advanced for this development. Baring data quality problems, stationarity issues, for an ideal forecast function, forecasts in the distant future for a trend-free series should be the unconditional estimates of the means. In this regard, the failure of both the composite and ARIMA models to account for the seasonal standard deviations is a major limitation of the models. Taking this further, it is normally hoped that seasonal differencing removes the periodic contribution but on the other hand it does distorts the covariance of the stationary part. As noted here, in tandem with the findings of Ahaneku and Otache (2014), the multiplicative ARIMA model assumes there is a serial correlation structure within the months of the year but it does not preserve the monthly standard deviations. This adversely transmits the negative effect to the model fitted to the periodic component in the coupling strategy too. In this regard, considering the non-practical nature of model structure and given the number of physical variables that could be related as potentially relevant by influence on drought, it is apparent that a very large number of different combinations of such variables and mathematical relationships that link them together be available when developing a forecasting model

(a)



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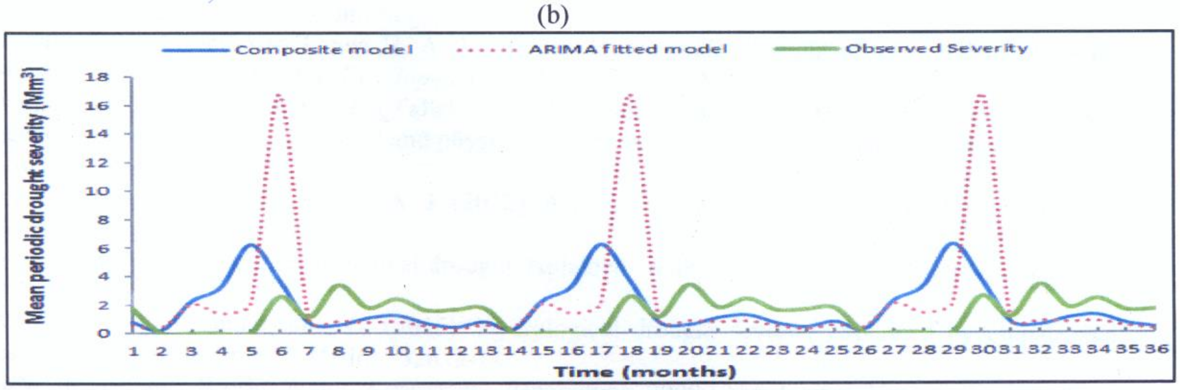

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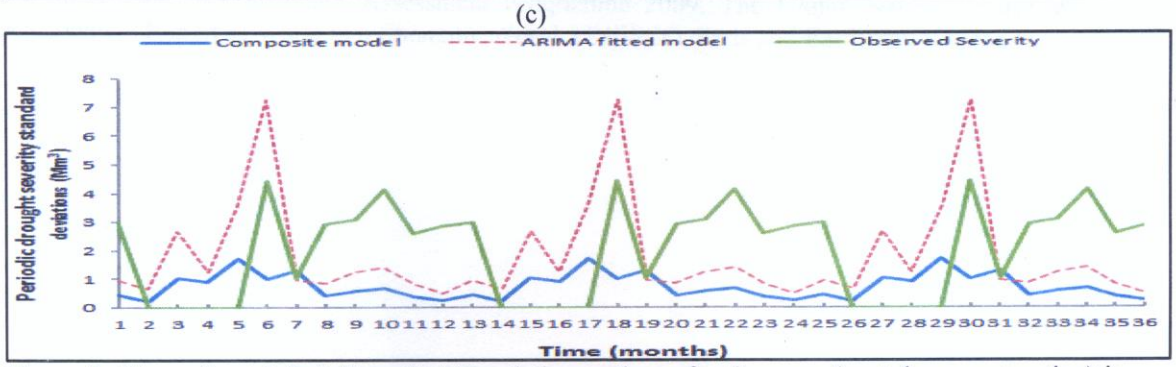

Figure 3: Streamflow deficit (Severity) Simulation analyses for Goronyo (post dam construction) in terms of (a) comparative model predictive ability, (b) reproduction of observed streamflow first moments, and (c) long-term forecast errors.

Conclusions

Based on the findings of this study, the following conclusions were drawn:

The stochastic component accounts for 92.52 %, 91.77 %, 93.18 %, and 92.71 % of the variance of the entire hydrological drought severity. This shows that it has a significant influence on the series and thus connotes high heteroscedasticity in the hydrological drought phenomenon perhaps attributable to climate change dynamics.

ii. Adoption of stochastic models in the forecast of hydrological drought severity time series is not effective even though the process has some degrees of intrinsic randomness since their forecast functions do not often reproduce the standard deviations of the series in a long-term. This seeming shortcoming is usually manifested in their failure to reproduce the feature vectors of the drought severity accurately. This derives directly from the fact that the multiplicative ARIMA model assumes there is a serial correlation structure within the months of the year; however, does not preserve the monthly standard deviations.

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