RADIATION EFFECTS ON AN UNSTEADY SQUEEZING FLOW IN THE PRESENCE OF A MAGNETIC FIELD

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Abstract

This work is focused on the examination of the effect of thermal radiation on the heat and mass transfer characteristics of an incompressible electrically conducting fluid squeezed between two parallel plates in the presence of a transverse magnetic field. Using the similarity transformation, the governing system of nonlinear partial differential equations is transformed into similarity equations which are solved numerically using the Nachtsheim and Swigert shooting iteration technique together with the Runge–Kutta sixth-order integration scheme. Numerical results are presented through graphs and tables for pertinent parameters to show interesting aspects of the solution.

1. Introduction

The squeezing flow of Newtonian and non-Newtonian fluids has received considerable attention from researchers because of numerous applications in different engineering disciplines, such as polymer processing, transient loading of mechanical components, compression, injection molding, and squeezed films in power transmission. These flows are induced by two approaching parallel surfaces in relative motion. The application of a magnetic field in these flows makes it possible to prevent the unpredictable deviation of lubrication viscosity with temperature under certain extreme operating conditions. Stefan [1] was the first to initiate pioneering works on the squeezing flow by invoking a lubrication approach. Two-dimensional and axisymmetric squeezing flows between parallel plates have been addressed by Rashidi et al. [2]. Siddiqui et al. [3] discussed the effects of a magnetic field in the squeezing flow between infinite parallel plates due to the normal motion of plates. Domairry and Aziz [4] provided an approximate analytic solution for the squeezing flow of a viscous fluid between parallel disks with suction or blowing. Hayat et al. [5] extended the work presented in [4] to analyze the squeezing flow of non-Newtonian fluids taking second grade fluids. The homotopy perturbation method (HPM) has been applied in [3, 5] for the presentation of analytic solutions of the arising nonlinear problems.

In recent years, the study of heat and mass transfer of viscous fluids in a squeezing flow has increased due to their applications in many branches of science and engineering. A few representative fields of interest in which a combined heat and mass transfer effect plays an important role are the design of chemical processing equipment, the formation and dispersion of fog, and the distribution of temperature and moisture over agricultural fields and groves of fruit trees. Heat transfer characteristics in a squeezing flow between parallel disks were studied by Duwairi et al. [6]. Khaled and Vafai [7] analyzed the hydromagnetic effects on flow and heat transfer over a horizontal surface placed in an externally squeezed free stream. Squeezing flows and heat transfer over a porous plate were investigated by Mahmood et al. [8]. Mustafa et al. [9] considered the combined effects of heat and mass transfer in a viscous fluid squeezed between two parallel plates. Chamkha [10] investigated the problem of heat and mass transfer by a steady flow of an electrically conducting fluid past a moving vertical surface in the presence of a first order chemical reaction. Kandasamy et al. [11] studied the nonlinear MHD flow with heat and mass transfer of an incompressible viscous electrically conducting fluid on a vertically stretching surface with chemical reaction and thermal stratification effects. Patil and Kulkarni [12] considered the effects of chemical reaction on the free convection flow of a polar fluid through a porous medium in the presence of internal heat generation.

However, the effect of thermal radiation on the flow and heat transfer has not been taken into account in most of the investigations. The radiation effect on an MHD flow and the heat transfer problem have become more important for industry. Cogley et al. [13] showed that, in an optically thin limit, the fluid does not absorb its own emitted radiation but the fluid does absorb radiation emitted by the boundaries. Raptis [14] investigated the steady flow of a viscous fluid through a porous medium bounded by a porous plate subject to a constant suction velocity in the presence of thermal radiation. Makinde [15] examined the transient free convection interaction with thermal radiation of an absorbing emitting fluid along a moving vertical permeable plate. Ibrahim et al. [16] discussed the case of a mixed convection flow of a micropolar fluid past a semi-infinite steady moving porous plate at varying suction velocity normal to the plate in the presence of thermal radiation and viscous dissipation. Das [17] analyzed the problem analytically to consider the effect of a first order chemical reaction and thermal radiation on a micropolar fluid in a rotating frame of reference. Recently, Das [18] has investigated the impact of thermal radiation on an MHD slip flow over a flat plate with variable fluid properties. The present study focuses on an unsteady MHD squeezing flow and heat transfer between two parallel plates in the presence of a transverse magnetic field and thermal radiation.

2. Mathematical formulation of the problem

Consider an unsteady two-dimensional squeezing flow of an incompressible viscous electrically conducting fluid between infinite parallel plates. The coordinate system is chosen such that the x-axis is along the plate and the y-axis is normal to the plate. The two plates are placed at $y = \pm h(t)$, where $h(t) = H(1 - \alpha t)^{1/2}$ and α is a characteristic parameter having dimensions of time inverse. The two plates are squeezed at a velocity of $v(t)= dh/dt$ until they touch. A uniform magnetic field of strength $B(t) = B_0(1 - \alpha t)^{-1/2}$ [19] is applied perpendicular to the plate, and the electric field is taken as zero. In addition, it is assumed that there exists a homogeneous first-order chemical reaction with time dependent reaction rate of $K_1(t) = k_1(1 - \alpha t)^{-1}$ between the diffusing species and the fluid. Here, the symmetric nature of the flow is adopted.

Under the stated assumptions, the governing conservation equations of mass, momentum, energy and mass transfer at an unsteady state can be expressed as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+v(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})-\frac{\sigma B^2(t)}{\rho}u \tag{2}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+v(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}) \tag{3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{\nu}{C_p}\left[4\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right]-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{4}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)-K_1(t)C \tag{5}$$

where *u* and *v* are velocity components along the *x*- and *y*-axis, respectively, ρ is the density of the fluid, υ is the kinematic viscosity, C_p is the specific heat at constant pressure *p*, κ is the thermal conductivity of the medium, *T* is the temperature of the fluid, *C* is the concentration of the solute, and D is the molecular diffusivity.

Following the Rosseland approximation with the radiative heat flux, q_r is modelled as follows:

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{6}$$

where σ^* is the Stefan–Boltzmann constant and k^* is the mean absorption coefficient. Assuming that the differences in temperature within the flow are such that T^4 can be expressed as a linear combination of temperature, we expand T^4 in Taylor's series about T_∞ and neglecting higher order terms, we obtain

$$T^4=4T_\infty{}^3T-3T_\infty{}^4 \tag{7}$$

Thus, we have

$$\frac{\partial q_r}{\partial y}=-\frac{16T_\infty{}^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{8}$$

Therefore, Eq. (4) reduces to

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho C_p}\left[\frac{\partial^2 T}{\partial x^2}+\left(1+\frac{16T_\infty{}^3\sigma^*}{3k^*\kappa}\right)\frac{\partial^2 T}{\partial y^2}\right]+\frac{\nu}{C_p}\left[4\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right]. \tag{9}$$

The appropriate initial and boundary conditions for the problem are

$$\left.\begin{aligned} &u=0, v=v_w=\frac{dh}{dt}, T=T_H, C=C_H \text{ at } y=h(t), \\ &v=\frac{\partial u}{\partial y}=\frac{\partial T}{\partial y}=\frac{\partial C}{\partial y}=0 \text{ at } y=0 \end{aligned}\right\}. \tag{10}$$

Using the following dimensionless quantities

$$\left.\begin{aligned} &\eta=\frac{y}{H\sqrt{1-\alpha t}}, \quad u=\frac{\alpha x}{2(1-\alpha t)} f'(\eta), \quad v=-\frac{\alpha H}{2\sqrt{1-\alpha t}} f(\eta), \\ &\theta=\frac{T}{T_H}, \quad \phi=\frac{C}{C_H}, \end{aligned}\right\} \tag{11}$$

for Eqs. (2) and (3) and then eliminating the pressure gradient from the resulting equations, we can finally obtain

$$f''''-S\left(\eta f'''+3f''+ff'-f'f'''\right)-M^2 f''=0 \tag{12}$$

Now Eqs. (9) and (5) take the following forms:

$$(1+N_r)\theta''2+\Pr S\left(f\theta'-\eta\theta'\right)+\Pr Ec\left(f''^2+4\delta^2 f'^2\right)=0, \tag{13}$$

$$\phi''+ScS(f\,\phi'-\eta\phi')-Sc\gamma\phi=0, \tag{14}$$

with the associated boundary conditions

$$\left.\begin{aligned} &f=0,\ f''=0,\ \theta'=0,\ \phi'=0 \text{ for } \eta=0 \\ &f=1,\ f'=0,\ \theta=1,\ \phi=1 \text{ for } \eta=1 \end{aligned}\right\}. \tag{15}$$

Here, $S=\frac{\alpha H^2}{2\upsilon}$ is the squeeze number where $S > 0$ corresponds to the plates moving apart, while $S < 0$ corresponds to the plates moving together (the so-called squeezing flow), $M=HB_0\sqrt{\frac{\sigma}{\mu}}$ is the Hartmann parameter, $\mathrm{Nr}=\frac{16T_\infty^3\sigma^*}{3k^*\kappa}$ is the thermal radiation parameter, $\Pr=\frac{\mu C_p}{\kappa}$ is the Prandtl number, $Ec=\frac{v^2{}_w}{H^2T_H C_p}$ is the Eckert number, $Sc=\frac{\upsilon}{D}$ is the Schmidt number, $\delta=\frac{H\sqrt{1-\alpha t}}{x}$ is the dimensionless number whose value is fixed throughout the entire study, and $\gamma=\frac{k_1H^2}{\nu}$ is the chemical reaction parameter, where $\gamma>0$ represents a destructive reaction, $\gamma<0$ represents a generative reaction, and $\gamma=0$ represents no reaction.

The parameters of interest for our problem are skin friction coefficient C_{fr}, reduced Nusselt number Nu_{r}, and reduced Sherwood number Sh_{r}. The local dimensionless skin friction coefficient is given by

$$C_{f\,r} = f''(1) \text{ , where } C_{f_r} = \frac{\sqrt{1-\alpha t}H^2 \mathrm{Re}_x}{x^2} C_f \, . \tag{16}$$

The rate of heat transfer in terms of the dimensionless Nusselt number is defined as follows:

$$\mathrm{Nu}_r = -\theta'(1) \text{ where } \mathrm{Nu}_r = \sqrt{1-\alpha t} Nu \tag{17}$$

Similarly the rate of mass transfer in terms of the dimensionless Sherwood number is given by

$$Sh_r = -\phi'(1) \text{ where } \mathrm{Sh}_r = \sqrt{1-\alpha t} Sh \tag{18}$$

3. Method of solution

The set of Eqs. (12), (13), and (14) under boundary conditions (15) was solved numerically by applying the Nachtsheim and Swigert shooting iteration technique [20] together with the Runge–Kutta sixth-order integration scheme. The unspecified initial conditions were assumed and then integrated numerically as an initial value problem to a given terminal point. A step size of $\Delta\eta = 0.001$ was selected to be satisfactory for a convergence criterion of 10^{-6} in nearly all cases. In order to ascertain the accuracy of our numerical results, the present study (in absence of a magnetic field and thermal radiation) was compared with the data of Mustafa et al. [9]. The $-f''(1)$, $-\theta'(1)$ and $-\varphi'(1)$ values were calculated for various S values. Excellent agreement was found between the results, as shown in Table 1. Thus, the use of the present numerical code for the current model was justified.

Table 1. Comparison of the values of Skin friction coefficient, Local Nusselt number, and local Sherwood number for various S values at $M = Nr = 0$

	Mustafa	et al.	[9]	Present	Results	
S	$-f''(1)$	$-\theta'(1)$	$-\varphi'(1)$	$-f''(1)$	$-\theta'(1)$	$-\varphi'(1)$
-1.0	2.170090	3.319899	0.804558	2.170091	3.319899	0.804558
-0.5	2.614038	3.129491	0.7814023	2.614038	3.129491	0.7814023
0.5	3.336449	3.026324	0.7442243	3.336449	3.026324	0.7442243
2.0	4.167389	3.118551	0.7018132	4.167389	3.118551	0.7018132

4. Numerical results and discussion

To analyze the effects of different parameters of practical importance on the behaviour of the flow, heat and mass transfer characteristics, we plot the velocity, temperature and concentration profiles against η for different values of the pertinent parameters. In the simulation, the default values of the parameters are considered as M = 5.0, Sc = 1.0, S =1.0, Pr = 0.71, γ = 0.5, δ = 0.1, Ec = 1.0, and Nr = 2.0 (Mustafa et al. [9]) unless otherwise specified. For illustrations of the results, numerical values are plotted in Figs. 1–6 and Table 2.

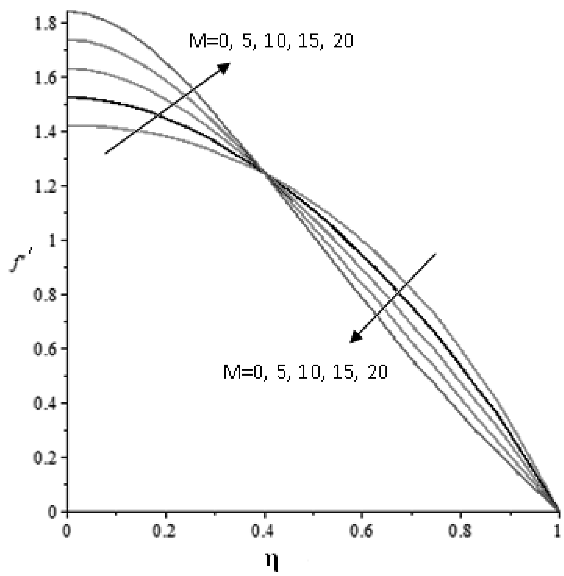

Fig. 1. Velocity profiles for various *M* values.

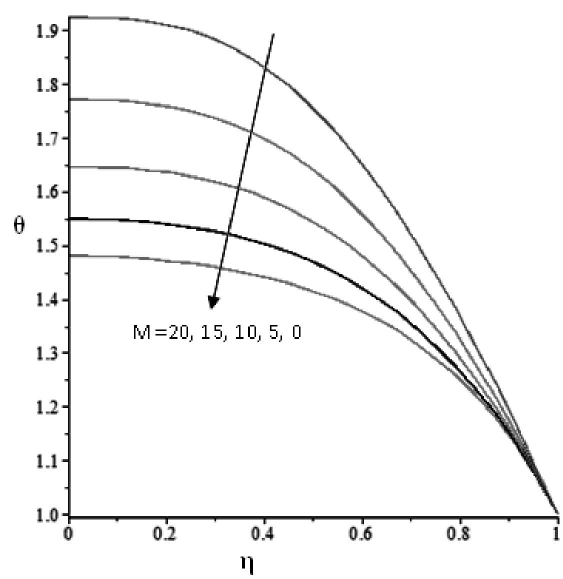

Fig. 2. Temperature profiles for various *M.* values.

The effect of Hartmann number M on the fluid velocity and temperature distribution is shown in Figs. 1 and 2, respectively. It is evident from Fig. 1 that the fluid velocity increases with increasing M values; therefore, the thickness of the momentum boundary layer for $\eta < 0.4$ (not precisely determined) decreases. However, an opposite trend is observed for $\eta \geq 0.4$ (not precisely determined). The reason behind this phenomenon is that application of a magnetic field to an electrically conducting fluid gives rise to a resistive-type force, which is referred to as the Lorentz force. This force exhibits a tendency to slow down the motion of the fluid in the boundary layer region. Figure 2 clearly indicates that the fluid temperature increases with increasing Hartmann number M in the central region and, as a consequence, the thickness of the thermal boundary layer decreases. This result qualitatively agrees with the expectations because the magnetic field exerts a retarding force on the convection flow and increases its temperature profiles. Table 2 presents the C_{fr} , Nu_r, and Sh_r values which are proportional to skin friction, rate of heat transfer, and rate of mass transfer from the surface of the plate, respectively, for the various M values. It is evident from the table that both the C_{fr} and Nu_r values decrease with increasing M. On the other hand, the effect of M value on Sherwood number Sh_r is not significant.

Table 2. Effects of various parameters on skin friction, local Nusselt number, and local Sherwood number

M	Nr	$-f''(1)$	$-\theta'(1)$	$-\varphi'(1)$
0.0	1.0	3.6384	1.7430	0.7287
0.2		3.6166	1.7410	0.7286
0.6		3.5730	1.7372	0.7283
1.0	0.0	3.5296	0.0000	0.7281
	0.5	-	1.1034	-
	1.0	-	1.7339	-

Figure 3 demonstrates the effects of thermal radiation parameter Nr on fluid temperature in the presence of a magnetic field. It is observed from the figure that temperature increases with increasing Nr in the middle region of the parallel plates and is minimal near the surface of the plates. Physically, this can be explained as follows: an increase in the radiation parameter means the release of heat energy from the flow region; therefore, the fluid temperature decreases as the thermal boundary layer becomes thinner. It is evident from Table 2 that with an increase in thermal radiation parameter Nr, the Nusselt number increases.

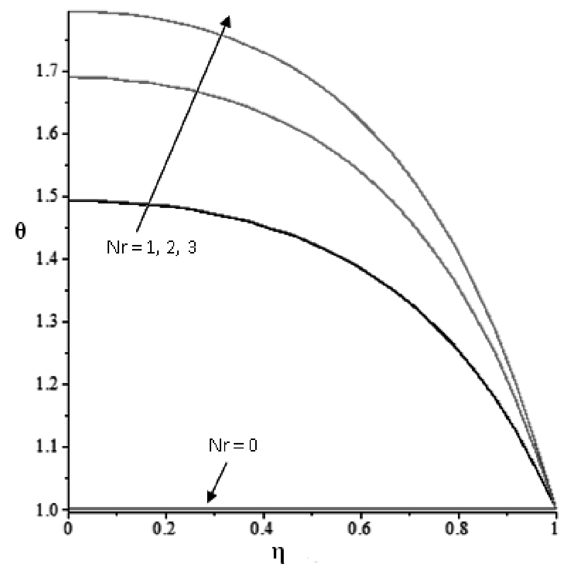

Fig. 3. Temperature profiles for various *Nr* values.

Effects of varying the value of squeeze number *S* on the velocity, temperature, and concentration distribution are illustrated in Figs. 4–6, respectively. With the increase in squeeze number *S*, the fluid velocity decreases first for $\eta < 0.45$ (not precisely determined); then it starts increasing, as shown in Fig. 4. It is evident from Fig. 5 that the fluid temperature decreases as *S* increases. It is worth mentioning that an increase in *S* can be associated with a decrease in the kinematic viscosity, an increase in the distance between the plates, and an increase in the velocities at which the plates move. The concentration distribution increases with increasing *S* values, as depicted in Fig. 6.

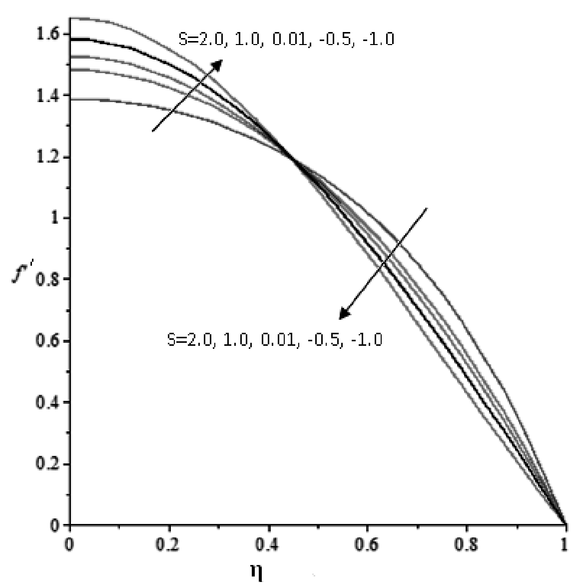

Fig. 4. Velocity profiles for various *S* values.

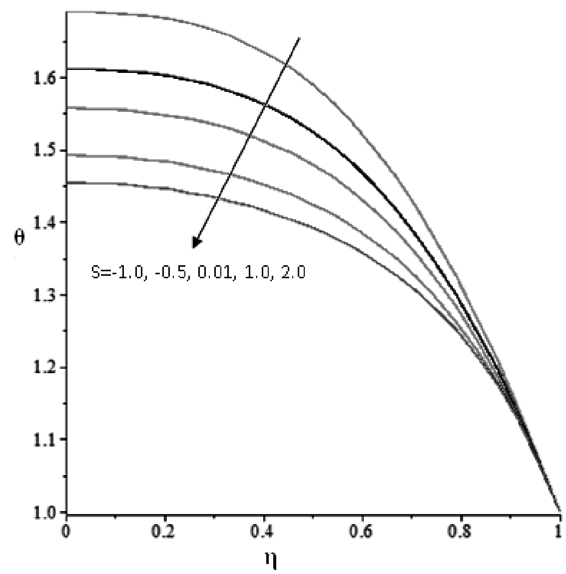

Fig. 5. Temperature profiles for various S values.

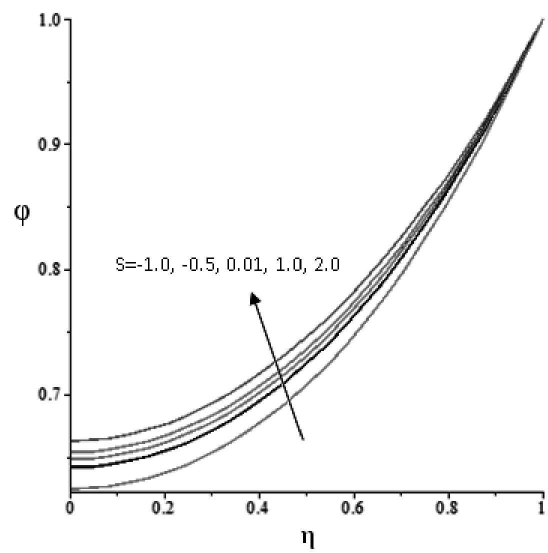

Fig. 6. Concentration profiles for various φ values.

5. Conclusions

In the present paper, the effect of a magnetic field and thermal radiation on the unsteady squeezing flow in a viscous incompressible electrically conducting fluid between two parallel plates has been analyzed using a numerical technique. Numerical results are presented through graphs and tables to illustrate the details of the flow characteristics and their dependence on

material parameters. Based upon the above-described results, the following conclusions can be drawn:
(i) The results indicate a decrease in the skin friction coefficient with an increase in the magnetic field strength. From an industrial point of view, this outcome is desirable since the drag force in the fluid squeezing between the plates decreases with increasing magnetic field strength.
(ii) With an increase in the thermal radiation parameter, the dimensionless heat transfer rate increases.
(iii) For an enhanced squeeze number, both the mass transfer coefficient and the heat transfer coefficient tend to decrease, while the skin friction coefficient tends to increase.
(iv) An increase in the relative strength of the magnetic field leads to a decrease in both the wall shear stress and the rate of heat transfer.

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