Performance Criteria Design Of Mixture Proportions For Laterite Cement Bricks Using The Schéffé Mixture Approach

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Abstract

A guideline to select variable proportions produce laterite-cement bricks meeting a user defined requirements was developed using the Scheffe's theory. Using this Mixture experimental design approach, five blends of a three component mixture using water, cement and laterite with percentage sand replacement to produce building bricks was carried out with cement content ranging between 8-20 percent by weight of laterite. The physical, and geotechnical properties of the laterite samples were determined. The machine mixing, compaction using Hydraform Twin-M7 machine and curing were carried out in a laboratory environment. The compressive strength at the specified ages of 7 and 28 days were measured using Testometric FS300CT Universal Testing Machine and responses were modeled as a second order quadratic polynomial. Guidelines for development of constraint formulation were carried out. An inverse relationship for strength was obtained and compressive strength achievable ranges between 7.46 - 18.85N/mm^2. Two analytical methods using the Genetic Algorithm stochastic search technique, and an approximate method are presented with examples and were found adaptable computationally, to obtain response prediction, satisfying the constraints of strength, cost, component proportions and durability. This method is intended to replace the trial method of mixture proportioning which is incapable of developing specifications writing procedure to meet user defined requirements.

Keywords

Performance, criteria, latrite, cement, Sheffe mixture approach

1. Introduction

The production of a high performance laterite cement bricks demands a higher complexity of the mixture design. To achieve this, a number of imposed criteria that the mixture must satisfy need to be clearly stated. In order to achieve this high performance laterite bricks, there is perhaps a need to employ useful numerical and optimization tools to aid the process of satisfying specified objectives. User defined criteria are usually presented as user-specified requirements satisfying some imposed constraints. Typical performance criteria could include mechanical properties such as strength, young modulus of elasticity, creep and shrinkage. It could also include durability properties such as abrasion resistance, capillary movement, chloride penetration etc.

In selecting laterite samples for brick production, certain properties of laterite material are required. Gidigasu, (1976) described laterite samples as a light to dark homogeneous, vesicular, unstratified and clinker like soil material consisting mainly of oxides and hydroxides of aluminium, iron, manganese and silica which hardens on extraction and exposure. These laterites samples are similarly described a class of pedogenics where the cementing materials are the sesquioxides content and should normally constitute not less than 50 percent of the mineralogical composition by this definition.

Laterite brick confers technical advantage largely because of the primary requirement of strength which is often three (3) times higher than the minimum strength requirements for the conventional sandcrete building blocks available in the Nigerian market. In addition, laterite bricks have a very good thermal property, shock and earthquake resistance (Hydraform, 2014) and particularly impact resistance. Several published research output (Osunade and Fajobi, 2000; Madu, 1984; Awoyera and Akinwumi, 2014; Hydraform, 2014; GIZ et al, 2013) have tried to confirm the acceptability of its properties for a series of acceptance criteria. These properties include compressive strength, absorption characteristics, reduction in the number of structural frames required in a building up to two-storey high and resistance to abrasion. These research reports have also reported its durability properties under exposure to weather and other climatic conditions. The laterite cement mixture with sand which would produce a durable, yet a cheap and affordable bricks is reported in this research work with the aim of meeting user specified requirements

Attempts have been made to improve laterite cement material as a building material for sustainable housing construction. These include development and manufacturing of compression machines for mechanical stabilization (Adeyemi, 1987 and 2004; Cinva Ram, 1999; NBRRI, 2013; Hydraform, 2014). Stabilization of soil with cement otherwise called soil-cement was also investigated (Madu, 1984; Aguwa, 2009; Osunade and Fajobi, 2000; Hydraform, 2014). Stabilization with pozzolanic material such as Corn Cob Ash (Ogunbode and Apeh, 2012). Stabilization with Locust Waste Bean Ash (Osinubi and Oyelakin, 2013), Stabilization with Coir (Aguwa, 2013). Bentonite Treatment (Amadi et al, 2011), Stabilisation with lime (Singh, 2006; Hydraform, 2014).

2. Literature Review: Mixture Experimental Design

In the design and use of laterites for road bases, there exists starting set of mixtures for the trial batch process; such is not available for laterite brick production. Historical information on trial mixture is often used. The results of the responses are evaluated and mix proportions are adjusted until certain specified requirements are met. This type of trial method of mix proportioning practice procedure obviously does not yield an optimized mixture.

In using this design procedure, experimental design points would have to be followed strictly and thus this method is different from any other renowned trial mix procedure. Empirical models are fitted for each of the responses to be measured and it also incorporates further refinements for modifying the response equations after detecting insignificant terms in a model. The final refined equations after removing all insignificant terms now form the response prediction equation called fitness functions which forms the basis for optimization subject to imposed constraints.

One of the importance of statistical experimental design procedures is that the responses can be characterized by an uncertainty (variability) which has an important implication for specification writing (FHWA, 1999; Simons et al, 1999). The mixture proportions are designed to yield responses to obtain target or mean strength which implies that at least 95 percent of the result are expected to fall within the normal distribution curve or more precisely, with probability $p \leq 0.05$.

2.1 The Concept of the Mixture Approach

This method uses essentially, the Scheffe mixture polynomial and the scheme is implementable using Design Expert software (Design Expert, 2000). However, the simplest solution procedure for the mixture approach which can be solved manually using the graphical approach involves experimental variables which are two or three. When the number of component variables is three, it can easily be described by the three-component Simplex, where the vertices of the triangle represent numerically, the pure components. For this mixture experiment involving laterite-cement mixes, it is impracticable to have pure component mixtures as we cannot have only water, only cement, or only laterite. It is upon the construction of constraints that can yield a workable or a feasible region and therefore the experimental region can be defined naturally.

To satisfy the requirement of this mixture approach, the constituent proportions are estimated in absolute volume which is fixed and constrained to be summed equal to unity. This is a pre condition of this method of solution procedure where one of the constraint equations must be equality (Simons et al, 1999; Montgomery, 2001). In this particular case, the components are water, cement and laterite. Then to reduce the mixture to a three-component variable, a natural choice could be to assume sand as a percentage replacement of laterite (Simons et al, 1999). The constraint equation therefore is:

$$\sum_{i=1}^{n} x_i = 1 \tag{1}$$

and $x_i \geq 0$

for this three-component mixture experiment, where $x_1, x_2,\ x_3$ represents water, cement and laterite respectively, the expression in equation (1) can be re-written more precisely as:

$$x_1 + x_2 + x_3 = 1 \tag{2}$$

There exist standard forms of mixture models for linear, quadratic, full cubic and special cubic models. All responses of interest in this mixture experiment are measured for each mixture factor in the experiment and modeled as a second-order quadratic polynomial shown in equation (3). The general form for the quadratic polynomial (Montgomery, 2001), can be expressed as:

$$E(y) = \sum_{i=1}^{p} \beta_i x_i + \sum \sum_{i<j}^{p} \beta_{ij} x_i x_j \quad (3)$$

or more precisely, for a three-component mixture:

$$y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + e \quad (4)$$

$where$: $x_1 = water, x_2 = cement, x_3 = laterite$
and b_i are the constant terms. The error e is the random error representing the combined effect of all the variables not included in the model. This form of polynomial is often called Scheffé mixture quadratic polynomial (Meyers, 1995; Derringer and Suich, 1980). This quadratic polynomial equation are re-parameterized in the form:

$$y_1 = b_1x_1 + b_2x_2 + b_3x_3 + b_{12}x_1x_2 + b_{13}x_1x_3 + b_{23}x_2x_3 \quad (5)$$

The expressions $x_1x_2,\ x_1x_3,\ x_2x_3$ are the interaction terms and $b_{12},\ b_{13},\ b_{23}$ are coefficients of the interaction terms of water, cement and laterite.In estimating the component proportions for all the design points in the Simplex lattice design, Pseudo and actual components in the factor space are used and transformation is made easy because of the inverse relationship that exists between them, (Onuamah, 2015; Onwuka et al, 2011; Anya and Osadebe, 2015).

2.2 The Scheffe's [3, 2] augmented Simplex lattice design
In order to make predictions about the full properties within the Simplex, an augmented [3, 2] lattice design can be used by fitting mixtures at the vertices of the Simplex in a manner as to yield an optimum mixture. More runs in the interior of the Simplex are included using both axial runs and the entire centroid (Montgomery, 2001; Mama and Osadebe, 2011; Mbadike and Osadebe; 2013). An augmented [3,2] Simplex lattice shown in Figure 1 consists of ten runs which include (1,0,0), (0,1,0), (0,0,1) pure blends, (½, ½, 0), (½, 0, ½), (0, ½, ½) binary blends, (⅔, ⅙, ⅙), (⅙, ⅔, ⅙), (⅙, ⅙, ⅔) axial blends and (⅓, ⅓, ⅓), the centroid.

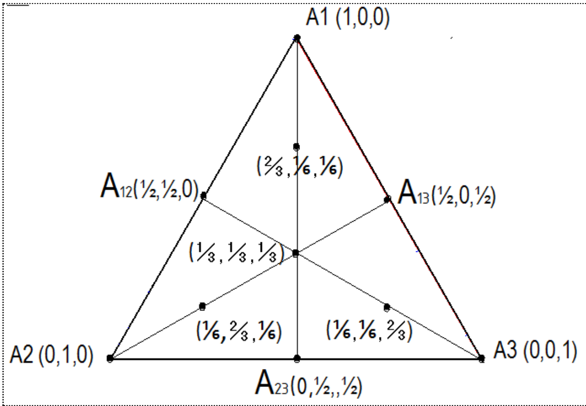

Figure 1: An augmented [3,2] Simplex lattice points

However, pure and binary blends are not practicable in its natural form, because the three components have to be mixed together, then a D-optimal design can be used, a procedure also implementable using Design Expert Software (Design Expert, 2000).

2.3 Pseudo and actual component construction and transformation of variable components

In an attempt to keep within a practicable compositional boundary, the method of transformation can be used (Mama and Osadebe, 2011; Mbadike and Osadebe, 2013; Onwuka et al, 2011; Scheffe, 1958). Coded variables called pseudo components at the vertices of the Simplex are used which are further transformed into actual variable components within the factor space. In practice, mixtures are specified in volumetric ratios called mix ratios at a given water cement ratio.

A transformation T is possible between coded and an actual component in the factor space because the vectors P_i, and P_{ij} in the factor space of real variables corresponds to the points A_i, and A_{ij} in the factor space of coded variables. By this, the procedure is to assign points to these vectors within the design domain considered. The actual component and coded or pseudo component coordinates have inverse relationship (Scheffe, 1958; Mama and Osadebe, 2011) as:

$$Q = TA \tag{6}$$

where $\boldsymbol{T}$ represents a linear transformation at any given point within the factor space between actual and pseudo component vector of variables, and $\boldsymbol{Q}$ is an identity matrix of the pseudo/coded component variables in the factor space. Multiplying both sides of equation (6) by the inverse $\mathbf{A}^{-1}$ gives:

$$QA^{-1} = TAA^{-1} \tag{7}$$

which yields:

$$A^{-1} = T \tag{7a}$$

and the transformation $\boldsymbol{T}$ is therefore the inverse of the matrix **A**. This methodology can be used to estimate proportions of all other design points within the [3, 2] augmented lattice points.

3.0 Materials and Methodology

The laterite sample was sourced in Wara within Ilorin environs, Kwara State (KW-31, Elevation 317, and Coordinates **663093**, **935109**). The laterite sample was sourced from the site of an existing burrow pit used for constructing the 500 housing units using Hydraform compressed bricks and using the method of disturbed sampling at a depth 0.5m – 1.5m depth for the collection. Two grading zones, namely zones 2 and 3 sand otherwise called coarse (C) and fine (F) sands were used as percentage replacement for the laterite. The physical and geotechnical properties of the sample tested are in accordance with BS 1377 (1975) is shown in Table 1.

Using 0% sand replacement as a control, two percentage sand replacements with proportions 10% and 20% silica sand were carried out. A starting set of mixture proportions was carried out using the absolute volume method within a domain of 8-percent and 20-percent cement content with a starting water cement ratio of 0.5 which was later revised to produce a mix that would produce one cubic meter of the mixture at maximum dry density. The specimen samples were mixed with pan mixer and ompaction using a hydraulically compressed M7-Twin Hydraform brick moulding machine was used. The brick samples was cured and tested at 7 and 28 days to obtain compressive strengths and other mechanical properties using a Testometric Universal Testing Machine Model FS300CT. The ASTM C 170-90 test plan was used.

Table 1: Properties of the laterite sample measured

	Physical and Geotechnical Properties	Value
i)	Liquid limit (%)	49
ii)	Plastic limt (%)	30.6
iii)	Plasticity Index (%)	18.4
iv)	Specific gravity	2.64
v)	Linear Shrinkage (mm)	10.1
vi)	Maximum Dry Density (kg/m^3)	1821
vii)	Optimum Moisture Content (%)	14.1
viii)	Colour	Reddish Brown
ix)	Condition of Sample	Air Dry
x)	Soil Classification	A-2-7
Mineralogical Propoerties		
i)	Iron Oxide Content (Fe_2O_3) (%)	18.01
ii)	Sesquioxide Content (%)	42.21

3.1 Example of estimation of constituent proportions using the Mixture method

The mix proportion for the various component mixes was calculated for the selected workable design domain and fitted to the design points in Figure 1. In this Scheffe's method of mixture experimental design, the constituent proportions estimated in absolute volume is fixed and constrained to be equal to one. This is an important characteristic property and a pre condition of the Simplex method (Montgomery, 2001; Simons et al, 1999). The practical interpretation of this constraint equal to unity in equation (1) can now be expressed in the estimation of the absolute volumes of each of the mixture factors as:

$$\frac{cement}{\rho_{cement} \times 1000} + \frac{water}{\rho_{water} \times 1000} + \frac{laterite}{\rho_{laterite} \times 1000} \quad (8) = 1$$

where: ρ = specific gravity

Using an example of cement content of 10% of the dry weight of laterite, the mix ratio can be expressed as 1:10. Here, a starting water/cement ratio can be adopted as 0.5, which represents assumed starting water required for the hydration of cement to produce a maximum dry density of the laterite cement mix. The mixture proportions will be adjusted further after determining the optimum moisture content corresponding to the maximum dry density. These steps are:

(i) The ratio 1:10 represents one (1) part of cement and ten (10) parts of laterite and water represents 0.5 by weight of cement. This ratio can be expressed as water:cement:laterte ratio 0.5:1:10. The laterite content can be expressed as Laterite, L=10*C. Subsequently, the water required based on the adopted initial water/cement ratio can similarly be expressed as Water, W=0.5*C

(ii) The equation which satisfies the equality constraint condition of equation (7) can therefore be re-written as:

$$\frac{\mathbf{0.5C}}{\mathbf{1000}} + \frac{\mathbf{C}}{\mathbf{3.15 \times 1000}} + \frac{\mathbf{10C}}{\mathbf{2.64 \times 1000}} = \mathbf{1}$$

And collecting the like term and solving for the unknown Cement C, the solution can be obtained as: Cement, C = 217.16kg/m^3 ; Water, W = 0.5*C = 108.58kg/m^3 and Laterite, L = 10*C = 2171.60kg/m^3

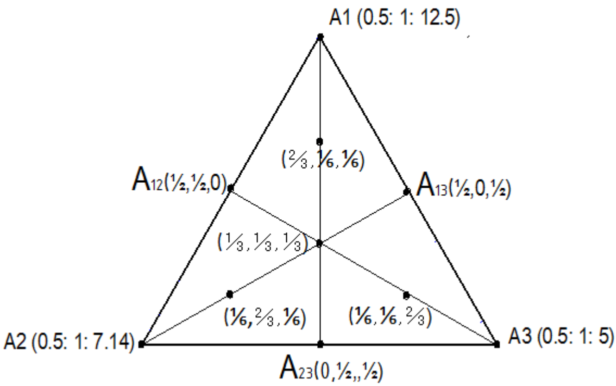

Figure 2 An augmented [3,2] Simplex lattice points
for the pure, binary, control and center points

Similarly, the remaining constituent proportions for other ratios of cement to laterite corresponding to the points in the factor space can be calculated. Sand is not included in the estimation of the constituent proportions because it is treated as a percentage replacement of laterite. A simple excel relative referencing address can be used to implement all the quantities as designated within the augmented [3,2] lattice points in Figure 2.

In an array form, the pure components are A_i = [0.5, 1, 12.5; 0.5, 1, 7.14; 0.5, 1, 5]^T is fitted to the vertices representing the 8 percent, 14 percent and 20 percent cement to laterite as shown in Figure 2. This is re-written in matrix form as:

$$A_i = \begin{bmatrix} 0.5000 & 0.5000 & 0.5000 \\ 1.0000 & 1.0000 & 1.0000 \\ 12.5000 & 7.1400 & 5.0000 \end{bmatrix} \quad (8)$$

The binary components, representing the A_{ij}'s is also written out as B_{Binary}=[.5,.5,0; .5,0,.5; 0,.5,.5]^T which is written in matrix array as:

$$B_{Binary} = \begin{bmatrix} 0.5000 & 0.5000 & 0 \\ 0.5000 & 0 & 0.5000 \\ 0 & 0.5000 & 0.5000 \end{bmatrix} \quad (9)$$

The transformation of the binary point factor variables from pseudo components into real component variables can now be carried out using equations (8) and (9) as:

$$A_{ij} = A_i * B_{Binary} = \begin{bmatrix} 0.5000 & 0.5000 & 0.5000 \\ 1.0000 & 1.0000 & 1.0000 \\ 9.8200 & 8.7500 & 6.0700 \end{bmatrix}$$

In like manner, the transformation of the factor variables of the control and centre points into real component variables can similarly be carried out.

The domain of 8-20 percent cement and 0-20 percent sand replacement were used (Hydraform, 2014; Aguwa, 2010; Osunade, 1995) because:
(i) it represents a cement content percent of laterite where curvature can be detected

(ii) maximum of 20 percent sand replacement would enable extrusion from the hydraform machine mould with minimum friction on the wearing plate.
(iii) the limits would maintain plastic bonds of the laterite.

3.2 Optimum Moisture Content (OMC) determination and methodology for revised mixing water determination

Initially, an assumed starting mixing water was adopted. The procedure as described in BS 1377 (1975) was employed for OMC determination using the 4.5kg rammer heavy compaction because; the machine compactive effort is $10MN/m^2$ The resulting revised design is shown in Table 2.

again, using a stastistical significance with probability $p \leq 0.05$, re-calculate a new revised response prediction for water (response), in column (3) of Table 4 against the ratio of cement to laterite (as the variable) which is the ratio of column (4):column (5) of Table 4 to obtain a revised linear relationship which reflects the equality constraint of equation (1). This revised response prediction for water is shown in equation (10). The Letters C1, F1 and C2, F2 immediately after the hyphen represents Coarse (C) and Fine (F) sand. The figures 0, 1 and 2 represents zero(0), ten(10) and twenty(20) percent sand replacement respectively.

$$MX-0; \qquad Y = 269.5 - 36.93 * \left(\frac{Cement}{Laterite}\right) \qquad (10a)$$

$$MX-C1; \qquad Y = 259.8 - 39.40 * \left(\frac{Cement}{Laterite}\right) \qquad (10b)$$

$$MX-F1; \qquad Y = 258.5 - 18.42 * \left(\frac{Cement}{Laterite}\right) \qquad (10c)$$

$$MX-C2; \qquad Y = 269.4 - 29.77 * \left(\frac{Cement}{Laterite}\right) \qquad (10d)$$

$$MX-F2; \qquad Y = 276.6 - 99.04 * \left(\frac{Cement}{Laterite}\right) \qquad (10e)$$

Table 2: Design matrix at Optimum Moisture Content using an augmented [3, 2] Simplex lattice

S/no.	Coordinate Points		Pseudo component ratios x1=water, x2=cement, x3=laterite			Actual components ratios			Actual component mixes, kg/m3 (0% sand replacement)			
			X1	X2	X3	x1 water	x2 Cement	x3 Laterite	water	cement	sand	laterite
(1)	(2)		(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1		A1	1	0	0	1.83	1.00	12.50	265.75	145.33		1816.63
2	PURE	A2	0	1	0	1.09	1.00	7.14	264.69	243.32		1737.29
3		A3	0	0	1	0.78	1.00	5.00	261.26	334.06		1670.30
4		A12	½	½	0	1.46	1.00	9.82	265.66	181.90		1786.22
5	BINARY	A13	½	0	½	1.31	1.00	8.75	265.45	202.25		1769.70
6		A23	0	½	½	0.94	1.00	6.07	263.55	281.44		1708.35
7		C1	⅙	⅔	⅙	1.16	1.00	7.68	265.03	227.79		1749.40
8	CONTROL	C2	⅔	⅙	⅙	1.53	1.00	10.36	265.71	173.11		1793.44
9		C3	⅙	⅙	⅔	1.01	1.00	6.61	264.22	260.80		1723.88
10	CENTRE	0	⅓	⅓	⅓	1.24	1.00	8.21	265.28	214.37		1760.00

*The highlighted are the upper and the lower limits on the domains of constituent proportions

*The domains of other blends are constructed in like manner

*The quantities in columns 9,10,11 are divided by the respective unit weights of 1000, 3150 and 2640kg/m^3 for water, cement and laterites respectively

(v) Similarly, using a statistical significance $p \leq 0.05$, a perfect linear relationship for response prediction for laterite (the response) in columns (5) plus (6) against cement (the variable) in column (4) of Table 4 can be carried out. The resulting predictive responses for all the blends are shown in equation (11). This response satisfies the condition of equation (1).

$$MX-0; \qquad L = 1927 - 0.7767 * Cement \qquad (11a)$$
$$MX-C1; \qquad L = 1956 - 0.9058 * Cement \qquad (11b)$$
$$MX-F1; \qquad L = 1959 - 0.8697 * Cement \qquad (11c)$$
$$MX-C2; \qquad L = 1928 - 0.7886 * Cement \qquad (11d)$$
$$MX-F2; \qquad L = 1907 - 0.6749 * Cement \qquad (11e)$$

The summary of the design table is shown in Table 3.

Table 3: Revised mixing water to produce 1m^3 maximum dry density for the [3,2] lattice design

Revised components mixes by weight at OMC (kg/m^3)					
S/no.	100% LATERITE				
(1)	(2) CODE COL 1	(3) WATER	(4) CEMENT	(5)	(6) 100% LAT
1	P1 - 0	**265.75**	145.33		1816.63
2	P2 - 0	**264.69**	243.32		1737.29
3	P3 - 0	**261.26**	334.06		1670.30
4	B12 - 0	**265.66**	181.90		1786.22
5	B13 - 0	**265.45**	202.25		1769.70
6	B23 - 0	**263.55**	281.44		1708.35
7	C1 - 0	**265.03**	227.79		1749.40
8	C2 - 0	**265.71**	173.11		1793.44
9	C3 - 0	**264.22**	260.80		1723.88
10	X0 - 0	**265.28**	214.37		1760.00
S/no.	10% COARSE SAND AND 90% LATERITE				
(1)	(2) CODE COL 2	(3) WATER	(4) CEMENT	(5) 10% SAND	(6) 90% LAT
11	P1 - C1	**262.36**	146.00	182.50	1642.51
12	P2 - C1	**265.65**	243.00	173.50	1561.53
13	P3 - C1	**267.05**	331.44	165.72	1491.49
14	B12 - C1	**263.79**	182.36	179.08	1611.68
15	B13 - C1	**264.49**	202.52	177.20	1594.83
16	B23 - C1	**266.44**	280.34	170.17	1531.49
17	C1 - C1	**265.25**	227.72	174.89	1574.01
18	C2 - C1	**263.47**	173.64	179.89	1619.02
19	C3 - C1	**266.04**	260.15	171.96	1547.65
20	X0 - C1	**264.86**	214.49	176.10	1584.90
S/no.	10% FINE SAND AND 90% LATERITE				
(1)	(2) CODE COL 2	(3) WATER	(4) CEMENT	(5) 10% SAND	(6) 90% LAT
21	P1 - F1	**259.29**	146.61	183.26	1649.34
22	P2 - F1	**261.37**	244.42	174.51	1570.62
23	P3 - F1	**261.46**	333.97	166.98	1502.86
24	B12 - F1	**260.30**	183.22	179.93	1619.33
25	B13 - F1	**260.74**	203.55	178.11	1602.95
26	B23 - F1	**261.63**	282.17	171.28	1541.52
27	C1 - F1	**261.17**	228.98	175.86	1582.73
28	C2 - F1	**260.08**	174.44	180.72	1626.47
29	C3 - F1	**261.53**	261.75	173.02	1557.17
30	X0 - F1	**260.96**	215.63	177.03	1593.30

Revised components mixes by weight at OMC (kg/m^3)					
S/no.	20% COARSE SAND AND 90% LATERITE				
(1)	(2) CODE COL 4	(3) WATER	(4) CEMENT	(5) 20% SAND	(6) 80% LATERITE
31	P1 - C2	**266.28**	145.23	363.06	1452.26
32	P2 - C2	**265.63**	243.01	347.02	1388.07
33	P3 - C2	**262.65**	333.43	333.43	1333.73
34	B12 - C2	**266.33**	181.73	356.92	1427.67
35	B13 - C2	**266.21**	202.04	353.58	1414.30
36	B23 - C2	**264.67**	281.02	341.15	1364.61
37	C1 - C2	**265.90**	227.52	349.47	1397.87
38	C2 - C2	**266.35**	172.96	358.38	1433.51
39	C3 - C2	**265.24**	260.44	344.30	1377.20
40	X0 - C2	**266.08**	214.14	351.61	1406.46
S/no.	20% FINE SAND AND 90% LATERITE				
(1)	(2) CODE COL 5	(3) WATER	(4) CEMENT	(5) 20% SAND	(6) 80% LATERITE
41	P1 - F2	**267.77**	144.93	362.32	1449.29
42	P2 - F2	**263.22**	243.81	348.15	1392.62
43	P3 - F2	**255.81**	336.52	336.52	1346.09
44	B12 - F2	**266.46**	181.70	356.85	1427.40
45	B13 - F2	**265.54**	202.23	353.90	1415.59
46	B23 - F2	**260.50**	282.61	343.08	1372.34
47	C1 - F2	**264.17**	228.06	350.29	1401.17
48	C2 - F2	**266.82**	172.85	358.15	1432.58
49	C3 - F2	**262.04**	261.57	345.80	1383.19
50	X0 - F2	**264.92**	214.48	352.17	1408.69

*Columns(5) plus(6) represent 100 percent laterite. The corresponding % replacement of sand can then be calculated to obtain the value in column (5). *P1, P2, P3 represent pure blends, B12, B13, B23 represent binary blends, C1, C2, C3 represent control points and X0 represents centre point fitted in the factor space. * The Letters C1, F1 and C2, F2 immediately after the hyphen represents Coarse and Fine sand, 10 percent and 20 percent blends respectively.

3.3 Development of constraint for constituent proportions

The domain of the constituent proportions in Table 3 can used for building constraints on the bounds for the propotions to yield 1m^3 of compacted volume. Using 0% sand replacement as an example, The vertices P1 – 0 and P3 – 0 represent the lower and upper limits respectively on water, cement and laterite. This is shown in row (1) and row (3), columns (3), (4) and (5) plus (6). From Table 3, for water, dividing the proportions by the respective unit weight of water (1000kg/m^3) gives 0.261 and 0.266. For cement, dividing the proportions by unit weight of cement (3150kg/m^3) gives 0.046 and 0.106. Similarly for laterite, dividing the proportions by the unit weight of laterite (2640kg/m^3) gives 0.633 and 0.688. The domains for other blends are summarized in equations 12(a) – (e).

$$\left.\begin{aligned} 0.261 \le\ & x_1 \le 0.266 \\ 0.046 \le\ & x_2 \le 0.106 \\ 0.633 \le\ & x_3 \le 0.688 \end{aligned}\right\} \quad MX-0; \quad 12(a)$$

$$\left.\begin{aligned} 0.262 \le\ & x_1 \le 0.267 \\ 0.046 \le\ & x_2 \le 0.105 \\ 0.628 \le\ & x_3 \le 0.691 \end{aligned}\right\} \quad MX-C1; \; 12(b)$$

$$\left.\begin{aligned} 0.259 \le\ & x_1 \le 0.262 \\ 0.046 \le\ & x_2 \le 0.106 \\ 0.633 \le\ & x_3 \le 0.694 \end{aligned}\right\} \quad MX-F1; \; 12(c)$$

$$\left.\begin{aligned} 0.263 \le\ & x_1 \le 0.266 \\ 0.046 \le\ & x_2 \le 0.106 \\ 0.631 \le\ & x_3 \le 0.688 \end{aligned}\right\} \quad MX-C2; \; 12(d)$$

$$\left.\begin{aligned} 0.256 \le\ & x_1 \le 0.268 \\ 0.046 \le\ & x_2 \le 0.107 \\ 0.637 \le\ & x_3 \le 0.687 \end{aligned}\right\} \quad MX-F2; \; 12(e)$$

4. Results and discussion

The modeling of response predictions for laterite cement mixes for strength at 7, 28 days and cost was carried out here using the second order quadratic polynomial in equation (3). The results have shown that strength still remains the primary response prediction for describing all other measured properties. For example the bricks with higher strength yield higher Young's modulus of elasticity. Similarly, the brick with higher strength corresponds with higher cost.

4.2 Description of the Mixture model selected

The models that adequately explain the fitted data are shown in Tables 4(a) - (c). The responses from input data were analyzed using Design Expert where the runs are randomized so as to avoid extraneous variables in the experiment (Simon et al, 1999; Montgomery, 2001). Replicate mixes are also required and carried out in this approach to provide an estimate of repeatability or statistical significance of the fitted coefficients.

A low value of $p \leq 0.05$ statistical significance shows that a model, coefficient or intercept is significant and should be included in the model. Contour plots produced can then be used to identify the conditions that give the extremum visually which shows only two (2) components at a time. The response prediction equations obtained reflected the form of the statistical method. By default, the Mixture method does not include the intercept because in the Scheffe quadratic polynomial expression, the polynomial equation has been re-parameterized and the constant term eliminated. The interaction terms that are not included in the model shows that they are not significant because probability $p \geq 0.05$. The response prediction for three of the selected responses are as shown in Tables 5(a) – (c). The contour plot is presented in the Appendix.

Table 4(a) Response prediction for 28-day strength: Mixture method

MX-0;	$1/(fc,_{28})$ = -3.54724 * Water + 0.10341 * Cement + 1.53865 * Laterite
MX-C1;	$1/(fc,_{28})$ = -1.1069895 * Water -0.0184988 * Cement + 0.568564 * (Lat + 10% CS)
MX-F1;	$1/(fc,_{28})$ = -0.170573 * Water -0.564135 * Cement + 0.271676 * (Lat + 10% FS)
MX-C2;	$1/(fc,_{28})$ = -5.92898 * Water + 1.30799 * Cement + 2.36679 * (Lat + 20% CS)
MX-F2;	$1/(fc,_{28})$ = -0.94989 * Water -0.69887 * Cement + 0.57971 * (Lat + 20% FS)

Table 4(b) Response prediction for 7-day strength: Mixture method

MX-0;	$1/(fc,_{7})$ = -4.13545 * Water + 0.21151 * Cement + 1.79349 * Laterite
MX-C1;	$1/(fc,_{7})$ = -1.80853 * Water - 0.10833 * Cement – 0.90936 * (Lat + 10% CS)
MX-F1;	$1/(fc,_{7})$ = -0.45955 * Water - 0.34136* Cement + 0.399116 * (Lat + 10% FS)
MX-C2;	$1/(fc,_{7})$ = -8.2352 * Water + 1.54417 * Cement + 3.30771 * (Lat + 20% CS)
MX-F2;	$1/(fc,_{7})$ = -1.66045 * Water -0.9286 * Cement + 0.9332 * (Lat + 20% FS)

Table 4(c) Response prediction for Cost: Mixture method

MX-0;	Cost = -9.48243 * Water + 236.04554 * Cement + 24.41443 * Laterite
MX-C1;	Cost = -42.28416 * Water + 252.64347 * Cement + 35.74273 * (Lat + 10% CS)
MX-F1;	Cost = 3.24307 * Water + 231.6168 * Cement + 19.85138 * (Lat + 10% FS)
MX-C2;	Cost = 9.08176 * Water + 231.02906 * Cement + 17.51829 * (Lat + 20% CS)

4.3 Optimization formulation

Minimize f(x) = MX(x) strength

Subject to inequalities:

$x_{il} \leq x_i \leq x_{iu}$ upper and lower levels on the variables

$\sum_{i=1}^{3} a_i x_i \leq c_i$ cost

Equalities:

$x_1 + x_2 + x_3 = 1$ absolute volumes must be equal to 1

$-a_i x_{2i} + x_{3i} = 0$ ratio of cement: laterite

$x_i = y_i$ water requirement

4.4 Example of optimization of component using the GA method (Mixture Approach)

Problem statement: To obtain mix proportions to achieve prescribed 28days strength for laterite cement brick (100% laterite with no sand replacement, which is coded as MX-0. The data input for this requirement are as stated:

i) Use cement content of 8% representing ratio 1:12.5 of cement to laterite
ii) The equality constraint of the sum of all the absolute volumes must be equal to 1
iii) The total cost should not exceed ₦30:00 per brick

The objective function for strength at 28 days from Table 4(a) is:

$$MX - 0\%;\ \frac{1.0}{fc_{28}} = -3.54724 * water + 0.10341 * cement + 1.53865 * laterite$$

The response prediction for cost of producing one brick for MX-0 from Table 4(c) is:

$$\mathrm{MX} - 0\%;\ \ \mathrm{Cost} = -9.48243 * \mathrm{water} + 236.04554 * \mathrm{cement} + 24.41443 * \mathrm{laterite}$$

The constraint on the ratio of cement to laterite which is to be 1:12.5 can be constructed as

:

$\frac{x_3}{x_2} = 12.5;$ at $x_2 =$ cement and $x_3 =$ laterite. This can be re-written as:

$x_3 = 12.5x_2$ and re-arranging gives the linear relationship $-12.5\,x_2 + x_3 = 0$, and multiplying by their respective unit weights per cubic metre, 3150kg/m^3 and 2640kg/m^3 for cement and laterite respectively gives : $-39375x_2 + 2640x_3 = 0$. Input GA Solver is shown in Appendix.

4.5 Example 2. Optimization of component mixes using approximate method for Mixture design

This method starts as an iterative process by initially selecting an absolute volume on cement which can be obtained from the limits for the component mixes. Absolute volumes of each constituent proportion are estimated and must sum equal to unity. The actual weights are obtained by multiplying the absolute volumes by each respective unit weight per meter cube

4.6 Comparative compressive strength results using the Scheffe Mixture Design approach

A comparative results of compressive strength using the GA example in 4.4 is shown in Table 5. It reveals that the measured properties of bricks produced are largely dependent on the quantity of cement and compactive effort (Hydraform, 2014; Osunade and Fajobi, 2000; Aguwa, 2009; Awoyera and Akinwumi, 2014). Similarly, production of bricks within 8 - 20 percent cement content design domain has shown reasonable results that would guide on quality brick production that would be durable and this can be adopted as a useful guide in specification writing for mass housing production. The compressive strength values are well above the minimum requirement of 2.8N/mm^2 in accordance with NIS (2004) requirements.

5.0 Conclusions and Recommendations

In using the Mixture method, it has been shown that statistically designed composite bricks satisfying user specified requirements is practicable. Similarly, in using this constrained method of mixture proportioning, responses capable of achieving target mean strengths can be developed and thus specification writing for site production is possible. The GA stochastic method and the approximate procedures presented are implementable computationally.

Table 5 Comparative compressive strength results using Central Composite Design

			Hydraform (2014)	Aguwa (2009)	Guettala et al (2005)	Awoyera & Akinwumi (2014)	MX - 0	MX - C1	MX - F1	MX - C2	MX - F2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
S/No.	Cement Content(%)	Compactive effort MN/m^2	10	4	15	2	10	10	10	10	10
1	4	COMPRESSIVE STRENGTH	-	1.9	-	-	-	-	-	-	-
2	5		3	-	15.4	-	-	-	-	-	-
3	6		-	3.5	-	-	-	-	-	-	-
4	7		5	-	-	-	-	-	-	-	-
5	8		-	5.1	-	2.3	8.4	10.3	10.1	9.08	9
6	10		8	6.1	18.4	3.49	9.6	11.6	12	10.05	10.25
7	12		-	6.5	-	3.86	11.17	12.5	13.7	11.19	11.55
8	14		-	7.1	-	-	12.79	14.4	15.6	12.54	13.07
9	15		10	-	-	-	-	-	-	-	-
10	16		-	8.3	-	-	14.55	15.9	18.2	14.22	14.98
11	18		-	9.2	-	-	18.32	17.9	21.9	15.74	17.41
12	20		12	9.6	-	-	-	-	-	-	-
13	25		14	-	-	-	-	-	-	-	-

* The highlighted header row represents the compactive effort in MN/m^2

* The serial numbers 5 through 11; columns (8) through (12) are estimated using the example in Section 4.15

* MX represents Mixture Approach

* The Letters C1, F1 and C2, F2 immediately after the hyphen represents Coarse and Fine sand, 10 percent and 20 percent blends respectively

Fine sand within grading zone 3 or coarse sand within grading zone 2 can be used, the blends are suitable and yielding nearly same results within the domain of cement:laterite considered. .Compressive strength and compactive effort still represent major factors in predicting the properties of the bricks moulded.

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Appendix I

Input for the GA solver
The inputs for the optimization process using Genetic Algorithm Solver are:
Function file name extension: function z=@mx0
The Fitness function:
function z = ((x(1).*(-3.54724))+(x(2).*(0.10341))+(x(3).*(1.53865)))
the linear inequality for cost per brick which is not to exceed N30 is:
((-9.48243*x(1))+(236.04554*x(2))+(24.41443*x(3))) <= 30
First linear equality for all the absolute volumes of the materials equal to one is:
((x(1))+(x(2))+(x(3))=1
Second linear equality for cement 8% cement content representing a ratio 1:12.5 is:
((-39375*(x(2))+(2640*(x(3)))=0

Number of variables: 3
Constraints:
Linear inequalities: A: -9.48243, 236.04554, 24.41443 b: 30
Linear equalities: Aeq: 1, 1, 1 ; 0, -39375, 2640 beq: 1;0
-equation 2.1 and ratio of laterite cement constraint
Bounds: Lower: 0.261; 0.046; 0.633 Upper: 0.266; 0.106;0.688
- the limits on minimum and maximum absolute volumes on constituent materials of water, cement and laterite in equation 12(a)
Population type: Double vector
Creation function: Constraint dependent
The solution satisfying all the constraints is: x1 = 0.266, x2 = 0.046, x3 = 0.688
Or by multiplying by the respective unit weights per m3: x1 = 266, x2 = 145, x3 = 1816.32kg/m3 representing water, cement and laterite respectively, with functional evaluation 1/f(x) = 0.11956 and the inverse is 8.364N/mm2

Appendix II

The procedure is stated thus:

Start by selecting an absolute volume on cement from within the limits suggested

Calculate the corresponding absolute volume of laterite from the equation relating the cement quantity and laterite

Calculate the absolute volume of water from the equation relating water to cement/laterite ratio

Substitute the absolute volumes of the respective quantities in the equation relating strength at 28days in Table 4a to obtain the compressive strength at 28 days

Calculate the inverse or reciprocal of the value obtained in (iv)

Calculate the cement laterite ratio and cement percentage per m3 of mix

Now calculate the cost per brick or per m2

Substitute the values in the problem statement:

Using a value of cement within the suggested limit (absolute volume = 0.057) represents 179.55kg of cement, that is (0.057 x 3150 = 179.55kg), where unit weight of cement is 3150kg/m3

The corresponding absolute volume of laterite from equation 11(a) relating the calculated cement quantity is: laterite=(1927-0.7767*cement) which gives (1927-(0.7767*179.55))/2640 = 0.6771 and the weight of laterite is 0.6771*2640 = 1787.54352kg/m3

The corresponding quantity of water from equation 10(a) relating the calculated cement/laterite ratio is: water=269.5-36.93*cement/laterite. this substitution gives = (269.5-(36.93*(179.55/1787.54352))) = 265.791kg/m3. The absolute volume of water is 266.55/1000 = 0.265791 ≈ 0.266

Substituting the absolute volumes of all the constituent materials in equation 5(a)

$1/fc_{28}$ =-3.54724*water+0.10341*cement+1.53865*laterite=

$1/fc_{28}$ =-3.54724*0.266+0.10341*0.057+1.53865*0.6771=0.104891

The inverse is 9.5337N/mm2

The cement laterite ratio is179.55/1787.54352 which represents ratio 1:10

The cost function in equation 5(c) for MX - 0 is:

Cost=-9.48243*water+236.04554*cement+24.41443*laterite which can be substituted to yield: cost = (-9.48243*0.266 + (236.04554*0.057 + (24.41443*0.0.6771))) = N27.46 per brick < N30.00