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Application of Assignment Model in Optimizing Cost of a Building Project

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Abstract

The study sought to minimize the total cost of building a three bed-room Detached House in Minna. An Assignment problem was formulated and using Hungarian solution method, a realistic result was obtained. The sighting study with unpopular scenario in building recommended the sum of N3,885,000 which can be used to build a house of the type and specifications as provided in the study.

1. Introduction

To optimize means to enhance effectiveness of something. To make something to function at its best or to use something to its best advantage. Optimization is the process of enhancing the effectiveness of something. In most cases, optimization seeks to minimize cost or maximize profit. Assignment on the other hand is characterized by a need to pair items in one group with items in another group in a one-for-one matching. For example, a manager may be faced with the task of assigning three jobs to three machines, one job to one machine (Aderinbigbe, 1998).

This empirical research work was carried out based on a hypothetical conditions of one who intended to build a three bedroom Detached House in Minna. As can be

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seen in the sub-section titled 'the problem situation', the conditions/specifications are uncommon in the general practice of building and construction. However, an assignment model was employed in order to achieve the task.

Aim of the Study

The aim of the study was to obtain the minimum cost of building a three bed-room Detached House in Minna.

2. The Problem Situation

A hypothetical case of one who intended to build a three bed-room Detached House in Minna is considered. For the reasons best known to him, he wanted the work to be done in phases in the order: Foundation, Block-work (Super-Structure), Roofing/Ceiling, Plumbing, Electrical and Finishes. Instead of negotiating for the cost of building the whole house with a contractor, he decided that different contractors should handle different phases. In other words, he did not want any one contractor to handle any two jobs (phases). The researchers then carried out a study that would determine the minimum cost of the building.

2.1. **The Model.** The model which was used for the study is Assignment Model. According to Taha (1997), "the best person for the best job" is an apt description of what the assignment model seeks to accomplish. The situation can be illustrated by the assignment of workers to jobs, where any worker may undertake any job, even though with varying degrees of skill. The Assignment Model describes exactly the kind of problem we are faced with, where we need to assign 6 Job Areas to 6 contractors who can handle any of the jobs.

2.2. **Applications of Assignment Model.** Assignment model has several applications such as:

i. Assigning employees to machines
ii. Assigning sales persons to sales districts
iii. Assigning scientists to various research projects
iv. Assigning jobs to machines
v. Assigning airplanes to schedule routes
vi. Assigning buses to cities
vii. Assigning contracts to bidders through systematic evaluation of bids from competing suppliers. This is the kind of assignment problem we are faced with in this study (Olokoyo, 2001).

3. The Methodology

Our method of solution for this research is the Hungarian algorithm. Other methods available for solving Assignment problems include Enumeration, Simplex and Transportation methods. Our choice of Hungarian algorithm is based on the fact as argued by Nikhat (2011) that, "Hungarian method has been used with

a good deal of success on various assignment problems". The other three methods have their related limitations which are handled successfully by the Hungarian algorithm. For example, the Enumeration method becomes unsuitable for manual calculations if the number of assignments is large. Also, the degeneracy problem of solution makes the Transportation and Simplex methods computationally inefficient for particular assignment problems with high degree of degeneracy.
According to Ozigbo (2000), the Hungarian method of Assignment provides us with an efficient means of finding the optimal solution without having to make a direct comparison of every option. It operates on a principle of matrix reduction. This means that, by subtracting and adding appropriate numbers in the cost table or matrix, we can reduce the problem to a matrix of opportunity costs. In this case, the opportunity costs show the relative penalties associated with assigning any person to a project as opposed to making the best or least cost assignment.

Furthermore, if we can reduce the matrix to the point where there is one zero element in each row and column, it will then be possible to make optimal assignment. That is, assignment in which all of the opportunity costs are zero.

3.1. **Mathematical Formulation of Assignment Problem.** According to Kalavathy (2001), the Assignment problem can be stated in the form of n×n cost matrix [c_{ij}] of real numbers as given below:

Matrix 1: n×n cost matrix $[c_{ij}]$ of real numbers

$$\begin{array}{cc} & \text{Job} \\ & \begin{array}{cccccc} 1 & 2 & 3\ldots & j\ldots & n \end{array} \\ \text{Person} \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ i \\ \vdots \\ n \end{array} & \left[\begin{array}{ccccc} c_{11} & c_{12} & c_{13}\cdots & c_{1j} & c_{1n} \\ c_{21} & c_{22} & c_{23}\cdots & c_{2j}\cdots & c_{2n} \\ c_{31} & c_{32} & c_{33}\cdots & c_{3j}\cdots & c_{3n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{i1} & c_{i2} & c_{i3}\cdots & c_{ij}\cdots & c_{in} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ c_{n1} & c_{n2} & c_{n3}\ldots & c_{nj}\ldots & c_{nn} \end{array}\right] \end{array}$$

Mathematically, the assignment problem can be stated as Minimize

$$Z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} \; i = 1, 2, \ldots, n; J = 1, 2, \ldots, n.$$

Subject to the restrictions

$$x_{ij} = \begin{cases} 1 \text{ if } i^{th}\text{person is assigned } j^{th}\text{job} \\ 0 \text{ if not} \end{cases}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ (job is done by } j^{th} \text{ person)}$$

$$\sum_{i=1}^{n} x_{ij} = 1 \text{ (only one person should be assigned the } j^{th} \text{ job)}$$

Where x_{ij} denotes that the j^{th} job be given to the i^{th} person.

3.2. **Hungarian Method Procedure.** Due to the highly degeneracy nature of the Assignment Model, a specially designed algorithm, widely known as Hungarian method proposed by H.W. kuhn is mostly used for its solution (Kadhirvel and Balamurugan, 2012). The steps involved in the Hungarian method of solving Assignment problem are as follows:
Step 1 Prepare a cost matrix. If the cost matrix is not a square matrix then add dummy row (column) with zero cost element;
Step 2 subtract the minimum element in each row from all the elements of the respective rows;
Step 3 Further modify the resulting matrix by subtracting the minimum element of each column from all the elements of the respective columns. Thus, obtain the modified matrix;
Step 4 Draw minimum number of horizontal and vertical lines to cover all zeros in the resulting matrix. Let the minimum number of lines be N. now there are two possible cases:
Case i If N=n, where n is the order of matrix, then an optimal assignment can be made. So make the assignment to get the required solution;
Case ii If N<n then proceed to step 5;
Step 5 Determine the smallest uncovered element in the matrix (element not covered by N lines). Subtract this minimum element from all uncovered elements and add the same element at the intersection of horizontal and vertical lines. Thus, the second modified matrix is obtained;
Step 6 Repeat steps 3 and 4 until we get the case I of step 3;
Step 7 To make zero assignment, examine the rows successively until a row-wise exactly single zero is found. Circle this zero to make the assignment. Then mark a cross (×) over all zeros if lying in the column of the circled zero, showing that they can't be considered for future assignment. Continue in the manner until all the zeros have been examined. Repeat the same procedure for column also;
Step 8 Repeat the step 6 successively until one of the following situation arises:

(1) If no unmarked zero is left, then process ends, or
(2) If there lies more than one of the unmarked zero in any column or row then circle one of the unmarked zero arbitrarily and mark as cross in the

cells of remaining zeros in its row or column. Repeat the process until no unmarked zero is left in the matrix;

Step 9 Thus, exactly one marked circled zero in each row and each column of the matrix is obtained. The assignment corresponding to these marked circled zeros will give the optimal assignment (Vinay, 2009).

4. Research Design

4.1. **Stages in Building a House.** Any house being built goes through the following stages, some of which may be progressing simultaneously:
i. Home plans purchased: getting the design ready;
ii. Builder/contractor hired;
iii. Loan process completed;
iv. Building permits issued;
v. Ground preparation: site clearing, stump removal and simple grading;
vi. Well or Borehole installation (if necessary) or tie-in to water system;
vii. Septic field installation (if necessary) or tie-in to sewage system;
viii. Basement excavation (if necessary) or footings poured;
ix. Temporary utilities established;
x. Basement floor and foundation walls or monolithic slab poured;
xi. Framing: House begins to take shape, skeleton erected;
xii. Sheathing and roofing takes place;
xiii. Rough plumbing: Plumbers rough-in pipes;
xiv. Rough electrical: Electricians begin wiring;
xv. Insulation installed;
xvi. Drywall installed: reinforced gypsum plaster sandwiched between two layers of strong paper in large sheets, used chiefly for interior walls;
xvii. Trim, subfloors, hardwood floors and cabinetry installed;
xviii. Electrical finish work- ceiling fixtures, switch-plates, etc;
xix. Finish plumbing: Set toilets, plumbing fixtures, disposal and sinks;
xx. Interior design- paint, wallpaper, tiles
xxi. Exterior finish: allowing for proper water drainage away from your home. Septic or sewer lines are installed and inspected. The driveway, walkways and concrete yard are poured, then the landscape features are planted - sod, seed & straw, shrubs, plants, trees and sometimes irrigation systems;
xxii. Moving Day!!! (www.standardhouseoffices, www.vhdesign.com/building.htm).

4.2. **Categorization of Stages to Building a House into Phases.** The following tables show the categorization of stages to building a house into Foundation (Sub-Structure), Block work (Super-Structure), Roofing/Ceiling, Plumbing, Electrical and Finishes (Painting and Tiling); in accordance to the requirement and specification of the client.

Phase I. Foundation: This includes Building permit, ground preparation, basement excavation,

sub-structure.

Phase II. Block work (Super-structure): This is framing of the super-structure to lintel level.

Phase III. Roofing/Ceiling

Phase IV. Plumbing: This includes rough and finish plumbing, septic field installation.

Phase V. Electrical: This includes rough electrical and finish electrical

Phase VI. Finishes: This includes painting, tiling, doors, widows and exterior finish

Note: The categorization defines the boundary of each phase. It also specifies the work load for each phase.

Contractors

The following Six Contractors (One Quantity Surveyor, two building contractors, and three Architects) were contacted. All of them practice in Minna.

Table 1: Names of contractors A, B, C, D, E and F.

Contractor	**Name**
A	Arc.ola.
B	Arc. Iorlamen
C	Arc. Nathaniel
D	Mr. Koko
E	Mr. Johnson O
F	Cret construct Ltd

Independently, the contractors were asked to submit Bills of quantities according to the categorization of phases (Foundation, Super-Structure, Roofing/Ceiling, Plumbing, Electrical and Finishes). Their Bills of quantities were based on the following House Brief:

House Brief

1 Living Room
1 Dining Room
3 Bedrooms ensuit
1 Kitchen/Store
1 Study Room
1 Entrance Porch

The Specification of materials was also discussed and made clear to the contractors. Generally, corporate grade was given to contractors as a specification for all the building materials to be used among Monumental, Federal, Corporate, Commercial quality levels in the order of grading, which are found in the market.

4.3. **Data Collection.** The Bills were discussed as each contractor submitted his bill of quantities. The table below shows the minimum cost per phase that each contractor accepted to carry out the contract if awarded to him. The data were collected in July 2011.

Table 2: Contractors' Costs per Phases and Totals

╲ Phase ╲ Contractor	I Foundation N(000)	II Super-Structure N(000)	III Roofing/Ceiling N(000)	IV Plumbing N(000)	V Electrical N(000)	VI Finishes N(000)	Total N(000)
A	950	780	670	300	320	888	3905
B	970	790	700	450	350	900	4160
C	1000	800	690	400	370	950	4210
D	965	750	750	350	390	1000	4205
E	985	765	675	430	360	895	4110
F	1000	740	600	300	380	1000	4020

Source: Primary Data Collected from 6 Contractors

4.4. **Data Analysis.** From Table 2, we shall form the original cost matrix (6x6) for the assignment problem. Using the cost matrix, we shall determine

(1) The optimal job assignment
(2) The cost of assignments.

Matrix 2: Original matrix

$$\text{Contractor}\quad \begin{array}{c} \\ A\\ B\\ C\\ D\\ E\\ F \end{array} \overset{\displaystyle\text{Phase}}{\begin{array}{cccccc} I & II & III & IV & V & VI \end{array}} \atop \begin{bmatrix} 950 & 780 & 670 & 300 & 320 & 888\\ 970 & 790 & 700 & 450 & 350 & 900\\ 1000 & 800 & 690 & 400 & 370 & 950\\ 965 & 750 & 750 & 350 & 390 & 1000\\ 985 & 765 & 675 & 430 & 360 & 895\\ 1000 & 740 & 600 & 300 & 380 & 1000 \end{bmatrix}$$

Matrix 2 is showing the costs (in thousands) of phases I, II, III, IV, V, VI as given by contractors A, B, C, D, E and F.

Matrix 3: Row Reduced Matrix

$$\text{Contractor}\quad \begin{array}{c} \\ A\\ B\\ C\\ D\\ E\\ F \end{array} \overset{\displaystyle\text{Phase}}{\begin{array}{cccccc} I & II & III & IV & V & VI \end{array}} \atop \begin{bmatrix} 650 & 480 & 370 & 0 & 20 & 588\\ 620 & 440 & 350 & 100 & 0 & 550\\ 630 & 430 & 320 & 30 & 0 & 580\\ 615 & 400 & 405 & 0 & 40 & 650\\ 625 & 405 & 315 & 70 & 0 & 535\\ 700 & 440 & 300 & 0 & 80 & 700 \end{bmatrix}$$

Matrix 3 is obtained by selecting the smallest element in each row of Matrix 2 and subtracting this smallest element from all the elements in the row.

Matrix 4: First Modified Matrix

Phase

Contractor	I	II	III	IV	V	VI
A	35	80	70	0	20	53
B	5	40	50	100	0	15
C	15	30	20	30	0	45
D	0	0	100	0	40	115
E	10	5	15	70	0	0
F	85	40	0	0	80	165

Matrix 4 is a Column Reduced Matrix which is obtained by selecting the smallest element in each column of Matrix 3 and subtracting this smallest element from all the elements in the column.

Matrix 5: Solved First Modified Matrix

Phase

Contractor	I	II	III	IV	V	VI
A	35	80	70	~~0~~	~~20~~	~~53~~
B	5	40	50	~~100~~	~~0~~	~~15~~
C	15	30	20	~~30~~	~~0~~	~~45~~
D	~~0~~	~~0~~	~~100~~	~~0~~	~~40~~	~~115~~
E	~~10~~	~~5~~	~~15~~	~~70~~	~~0~~	~~0~~
F	~~85~~	~~40~~	~~0~~	~~0~~	~~80~~	~~165~~

Matrix 5 is done by drawing the minimum number of lines to cover all zeros (horizontal and vertical).

Observe in matrix 5 that,

Number of lines drawn to cover all zeros is N = 5

The order of matrix is n = 6

Hence, $N < n$.

Now, we get the second modified matrix by subtracting the smallest uncovered element from the remaining uncovered elements and adding it to the element at the point of intersection of lines.

Matrix 6: Second Modified Matrix

Phase

Contractor	I	II	III	IV	V	VI
A	30	75	65	0	20	53
B	0	35	45	100	0	15
C	10	25	15	30	0	45
D	0	0	100	5	45	120
E	5	0	10	70	0	0
F	80	40	0	5	85	170

Matrix 7: Solved Second Modified Matrix

Phase

Contractor	*I*	*II*	*III*	*IV*	*V*	*VI*
A	~~30~~	~~75~~	~~65~~	~~0~~	~~20~~	~~53~~
B	~~0~~	~~35~~	~~45~~	~~100~~	~~0~~	~~15~~
C	~~10~~	~~25~~	~~15~~	~~30~~	~~0~~	~~45~~
D	~~0~~	~~0~~	~~100~~	~~5~~	~~45~~	~~120~~
E	~~5~~	~~0~~	~~10~~	~~70~~	~~0~~	~~0~~
F	~~80~~	~~40~~	~~0~~	~~5~~	~~8~~5	~~170~~

This is done by drawing the minimum number of lines to cover all zeros
Observe in Matrix 7 that,
Number of lines drawn to cover all zeros is $N = 6$
The order of matrix is $n = 6$
Hence, $N = n$. Now, the optimum assignment can be determined

Matrix 8: Optimal Assignment Matrix

Phase

Contractor	*I*	*II*	*III*	*IV*	*V*	*VI*
A	30	75	65	0✓	20	53
B	0✓	35	45	100	0×	15
C	10	25	15	30	0✓	45
D	0×	0✓	100	5	45	30
E	5	0×	10	70	0×	0✓
F	80	40	0✓	5	85	170

Row A has a single zero in column IV, that is cell (A, IV). Make an assignment in this cell as marked($\surd$). Mark (X) for all other zeros in that column showing that they can't be used for making other assignments. Note that there is no other zero in that column.
Row B contains two zeros, so proceed to row C.
Row C has a single zero corresponding to cell (C, V). Make an assignment here by marking it ($\surd$) and mark every other zeros in its column with ($\times$).Note that there is no other zero in that column.
Row D has two zeros, but the zero in cell (D, I) has already been marked (X) so no other assignment can be made here. The next zero corresponds to cell (D, II), an assignment is made here and every other zero in its column is marked (X).
Row E contains three zeros, but the first two zeros are already marked (X). Since no assignment can be made to the zeros marked (X), the assignment in row E is made to column VI.
Row F has exactly one zero in cell (F, III). An assignment is made to this cell.
Row B which was initially skipped is now considered. It has two zeros but the zero corresponding to cell (B, V) has been marked (X). Obviously, the assignment in row B is to cell (B, I).

Optimal Assignment and Optimal cost of Assignment

Table 3 shows the 0ptimal assignment of Jobs (phases) to contractors, with their corresponding costs.

Table 3: Optimal Assignment

contractor	**Phase**	**Cost(# '000)**
A	IV	300
B	I	970
C	V	370
D	II	750
E	VI	895
F	III	600
Total		3,885

5. Discussion of Result

The optimal assignment of contracts in phases to different contractors result to a total amount of N3,885,000.00 to complete a three bedroom Detached House in Minna. This amount of money is less than each total amount of money per contractor which is N3,905,000; N4,160,000; N4,210,000; N4,205,000; N4,110,000; and N4,020,000 (see Table 2). This means that, any one contractor that handles the job from start to finish will execute the contract at a cost higher than the cost of executing the contracts by six different contractors. In other words, it is more cost effective to award the contracts to different contractors.

5.1. **Recommendations.** Based on the study, it is recommended that, the Client should award the contracts as shown in Table 4.

Table 4: Recommended Assignments of Contracts

contractor	**Phase**	**Cost (# '000)**
A (Arc. Ola and sons Ltd)	IV (Foundation)	300
B (Arc. Iorlamen)	I (Super-Structure	970
C (Arc. Nathaniel)	V (Roofing/Ceiling)	370
D (Mr. koko)	II (Plumbing)	750
E (Mr. Johnson O.)	VI (Electrical)	895
F (Cret construct Ltd)	III (Finishes)	600
Total		3,885

Note that the cost is in thousands. That is, the total of N3,885 means N3,885,000.

6. Conclusion

It is obvious from this study that, in Minna, one can build a three bedroom Detached House of the kind described in the study with N 3, 885, 000.00. However, a generalization cannot be made given the peculiar conditions on which the model worked. Thus, this amount of money should serve only as a clue to any other person who might be willing to build such house in Minna, Nigeria.

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