B08: Stability Analysis for Mathematical Modeling of Dengue Fever Transmission and Control

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Abstract
Dengue fever is one of the greatest health challenges in the present world. In this work, mathematical modeling of dengue fever transmission and control was formulated. The model considered the human population N_h and the vector population N_m which are further subdivided into six classes, susceptible human S, infected human I, temporary recovered human class R_1, permanently recovered human class R_2, susceptible mosquito M_1, and infected mosquito class M_2. The Disease Free Equilibrium (DFE) point was obtained and the basic Reproduction number R_0 was computed. The Disease Free Equilibrium (DFE) is locally and globally asymptotically stable when $R_0 < 1$.

Keywords: basic reproduction number, dengue fever, equilibrium.

1. Introduction

Dengue fever is a mosquito-borne viral infection caused by Dengue Virus (DENV), (WHO 2015) which occurs mostly in tropical and sub-tropical areas of the world (WHO 2019). Dengue virus is categorized into four, which are genetically related but distinct serotypes. These are, DENV1, DENV2, DENV3, and DENV4 (Gubler *et al*., 2014). This means a susceptible person can be infected up to four times (WHO 2015). A person who recovered from an infection caused by one of the virus remains immune to that type but susceptible to other types (Gubler, 1998). Dengue virus is transmitted from human to mosquito and vice versa as well as from human to human through direct blood transfusion. No evidence of transmission from mosquito to mosquito (Chye *et al*., 1997). The major vector for spreading Dengue virus is *Aedes aegypti* mosquito, which is said to originate from Africa (Gubler *et al*., 2014; Ehrenkranz *et al*., 1971 and Smith, 1956).

No peculiar antiviral medication for Dengue fever, (Phaijoo and Gurung 2017 and Simmons et al., 2012) but medical health personal recommended maintaining body fluid to avoid

dehydration. There is an approved vaccine for Dengue fever but can only be effective on those who have been previously infected (WHO 2015; 2018). Dangvaxia (CYD – TDV) is a dengue virus vaccine licensed in December 2015 and approved in about 20 countries (WHO 2019).

In this paper, the mathematical model of dengue fever transmission and control was formulated, incorporating permanent recovery class which was created as a result of introduction of vaccine into the model. The Disease Free Equilibrium (DFE) point was obtained as well as the basic Reproduction number R_0. The Disease Free Equilibrium (DFE) is locally and globally asymptotically stable when $R_0 < 1$.

2. Formulation of the Model

The transmission dynamics of Dengue fever disease is presented using a system of ordinary differential equations. The total human population at time t denoted by $N_h(t)$ is subdivided into four (4) subpopulations of Susceptible $S(t)$, Infected $I(t)$, Recovery (temporary recovery) $R_1(t)$ and permanent Recovery $R_2(t)$. Similarly, the total population for the vector (mosquito) at time t, denoted by $N_m(t)$ is subdivided into susceptible mosquito $M_1(t)$ and infected mosquito $M_2(t)$.

The Susceptible population of human $S(t)$, is generated through birth rate and migration at the rate of β_1 and it increases by the number of infected human who recover temporarily $R_1(t)$ at the rate of γ_1. The rate $(1-\varepsilon)\tau$ of human who did not comply with mosquito net usage remains in susceptible class. The compartment decreases by the number of susceptible human who died naturally at the rate of μ_h, interaction with infected mosquito at the rate of α_2 and interaction with infected human $I(t)$ through direct blood transfusion which yields the force of infection $\frac{\alpha_2 M_2}{N_h} + \frac{\alpha_2 I}{N_h}$ where α_2 is the rate of interaction between $S(t)$ and $M_2(t)$, while α_3 is the rate of interaction between a susceptible human $S(t)$ and infected human $I(t)$.

The infected human compartment $I(t)$ is generated through the means of interaction with susceptible human $S(t)$ as well as the interaction between susceptible human $S(t)$ and infected mosquito $M_2(t)$. The compartment decreases due to natural death at the rate of μ_h dengue induced death at the rate of δ_h and temporary recovered human at the rate of γ_1. The temporary recovery class $R_1(t)$ is generated by the natural recovery of infected human at the rate of γ_1 and

the effective usage of mosquito net by susceptible human with total compliance at the rate of $\varepsilon\tau$. The class is decreased by the natural death of the temporary recovered human at the rate of μ_h, temporary recovered human who are vaccinated at the rate of v and those that recovered temporarily without being vaccinated at the rate of γ_2. The permanent recovery compartment $R_2(t)$ is generated by the temporary recovered human who are vaccinated at the rate of v and decreases by the natural death of permanent recovered human at the rate of μ_h.

The susceptible Vector population $M_1(t)$ is generated through birth at the rate of β_2, and decreases through the interaction with infected human $I(t)$ at the rate of α_1, natural death at the rate of μ_m and death induced by insecticide or other unnatural ways at the rate of δ_m. The infected Mosquito class $M_2(t)$ is generated by the interaction of susceptible mosquito with infected human at the rate of α_1 and decreases through natural death at the rate of μ_m and death induced by the use of insecticide and other unnatural means at the rate of δ_m.

The schematic diagram for the dynamics is represented in figure 1 below.

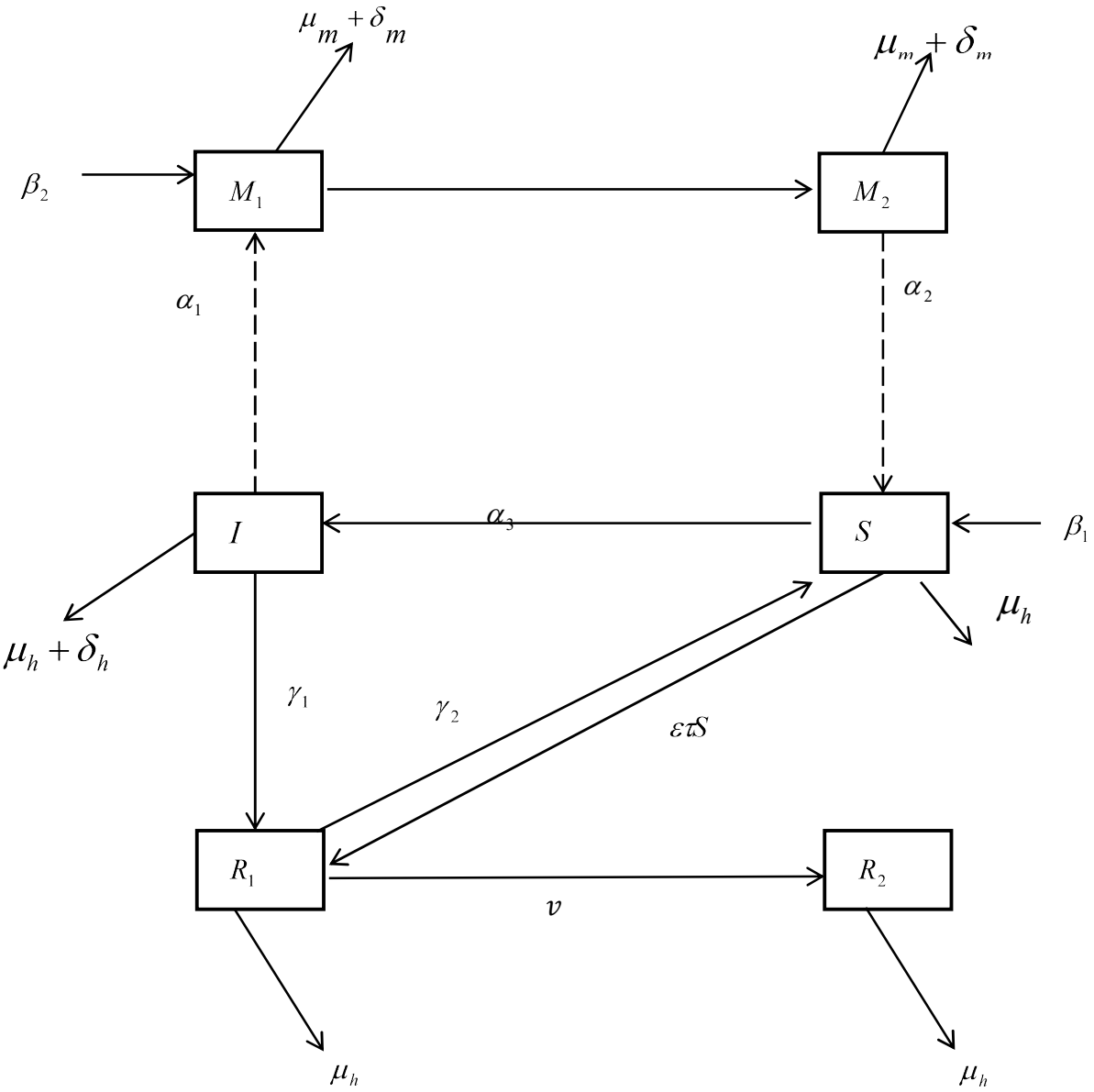

Figure 1. Schematic Diagram of the Model

3. Assumptions of the model

i. The susceptible human becomes infected through interaction with an infected vector (*Aedes aegypti* mosquito) during bite as well as interaction with an infected human by means of blood transfusion.
ii. Recruitment into the human susceptible population includes birth and those who recover temporarily.
iii. Vaccinating an infected human makes the person to have permanent immunity against Dengue virus.
iv. The control measure considered is the use of mosquito net and vaccine.
v. Dengue virus has no detrimental effect on the vector.
vi. The susceptible human who complied to the use of mosquito net will move to temporary recovery class $R_1(t)$.

4. The model equations

$$\frac{dS}{dt} = \beta_1 + \gamma_2 R_1 + (1-\varepsilon)\tau S - \left[\frac{\alpha_2 M_2}{N_h} + \frac{\alpha_3 I}{N_h} + \mu_h\right] S \tag{1}$$

$$\frac{dI}{dt} = \left[\frac{\alpha_2 M_2}{N_h} + \frac{\alpha_3 I}{N_h}\right] S - (\gamma_1 + \mu_h + \delta_h) I \tag{2}$$

$$\frac{dR_1}{dt} = \gamma_1 I - (\gamma_2 + v + \mu_h) R_1 + \varepsilon\tau S \tag{3}$$

$$\frac{dR_2}{dt} = vR_1 - \mu_h R_2 \tag{4}$$

$$\frac{dM_1}{dt} = \beta_2 - \left[\frac{\alpha_1 I}{N_m} + \mu_m + \delta_m\right] M_1 \tag{5}$$

$$\frac{dM_2}{dt} = \frac{\alpha_1 I M_1}{N_m} - (\mu_m + \delta_m) M_2 \tag{6}$$

Where,

$$N_h(t) = S(t) + I(t) + R_1(t) + R_2(t). \tag{7}$$

and

$$N_m(t) = M_1(t) + M_2(t). \tag{8}$$

With initial conditions,

$$S(0) > 0, I(0) > 0, R_1(0) > 0, R_2(0) > 0, M_1(0) > 0, M_2(0) > 0.$$

Table 4.1: Variables and Parameters used in the Model

Variable and Parameter	Definition
S	Susceptible human at time t
I	Infected human at time t
R_1	Recovered human (temporary recovery) at time t
R_2	Recovered and vaccinated human (permanent recovery) at time t
M_1	Susceptible vector (*Aedes aegypti* mosquito) at time t
M_2	Infected Vector at time t
N_h	Total number of human population at time t
N_m	Total number of mosquito population at time t
β_1	Recruitment rate for susceptible human population
β_2	Recruitment rate for Vector population
μ_h	Natural death for human
μ_m	Natural death for Mosquito
δ_h	Death induced by Dengue fever
δ_m	Death induced by insecticide or other artificial means
α_1	Rate of transmission of Dengue virus from infected human to susceptible mosquito
α_2	Rate of transmission of Dengue virus from infected mosquito to Susceptible human
α_3	Rate of transmission of Dengue virus from infected human to susceptible human
γ_1	Rate of recovery from first infection
γ_2	Rate of unvaccinated recovered human
ν	Vaccination rate
τ	Compliance of mosquito net usage
ε	Efficacy of mosquito net

5. Disease Free Equilibrium (DFE)

At the equilibrium state, the rate of change of the variables with respect to t will be equal to zero.

$$\frac{dS}{dt}=\frac{dI}{dt}=\frac{dR_1}{dt}=\frac{dR_2}{dt}=\frac{dM_1}{dt}=\frac{dM_2}{dt}=0 \tag{9}$$

Let the arbitrary DFE points be

$$(S,I,R_1,R_2,M_1,M_2)=(S^*,I^*,R_1^{`*},R_2^*,M_1^*,M_2^*) \tag{10}$$

Equation (1) – (6) can be written as

$$\beta_1+\gamma_2R_1^*+A_1S^*-\frac{\alpha_2M_2^*}{N_h}S^*-\frac{\alpha_3I^*}{N_h}S^*-\mu_hS^*=0 \tag{11}$$

$$\frac{\alpha_2M_2^*}{N_h}S^*+\frac{\alpha_3I^*}{N_h}S^*-A_2I^*=0 \tag{12}$$

$$\gamma_1I^*-A_3R_1^*+\varepsilon\tau S^*=0 \tag{13}$$

$$vR_1^*-\mu_hR_2^*=0 \tag{14}$$

$$\beta_2-\frac{\alpha_1I^*}{N_m}M_1^*-A_4M_1^*=0 \tag{15}$$

$$\frac{\alpha_1I^*}{N_m}M_1^*-A_4M_2^*=0 \tag{16}$$

Where, $A_1=(1-\varepsilon)\tau$, $A_2=(\gamma_1+\mu_h+\delta_h)$ $A_3=(\gamma_2+v+\mu_h)$

` $A_4=(\mu_m+\delta_m)$

Making M_2^* the subject from equation (16) we have

$$M_2^* = \frac{\alpha_1 I^* M_1^*}{A_4 N_m} \tag{17}$$

Substituting (17) in (12) we have,

$$\frac{\alpha_1\alpha_2 I^* M_1^*}{A_4 N_h^* N_m^*} S^* + \frac{\alpha_3 I^*}{N_h^*} S^* - A_2 I^* = 0$$

$$\alpha_1\alpha_2 I^* M_1^* S^* + \alpha_3 I^* A_4 N_m^* S^* - A_2 A_4 N_h^* N_m^* I^* = 0$$

$$I^*(\alpha_1\alpha_2 M_1^* S^* + \alpha_3 A_4 N_m^* S^* - A_2 A_4 N_h^* N_m^*) = 0$$

$$I^* = 0 \tag{18}$$

or

$$\alpha_1\alpha_2 M_1^* S^* + \alpha_3 A_4 N_m^* S^* - A_2 A_4 N_h^* N_m^* = 0 \tag{19}$$

Substituting for I^* from equation (18) in to equation (17), we have

$$I^* = M_2^* = 0 \tag{20}$$

Equation (20) gives the Disease Free Equilibrium (DFE)

$$M_1^o = \frac{\beta_2}{A_4} \tag{21}$$

$$R_1^o = \frac{\varepsilon\tau S^o}{A_3} \tag{22}$$

$$S^o = \frac{A_3\beta_1}{A_5} \tag{23}$$

Where $A_5 = \left(A_3\mu_h - \gamma_2\varepsilon\tau - A_1A_3\right)$

$$R_2^o = \frac{v\varepsilon\tau\beta_1}{A_5\mu_h} \tag{24}$$

$$E^o = \left(S^o, I^o, R_1^o, R_2^o, M_1^o, M_2^o\right) = \left(\frac{A_3\beta_1}{A_5}, 0, \frac{\varepsilon\tau\beta_1}{A_5}, \frac{v\varepsilon\tau\beta_1}{A_5\mu_h}, \frac{\beta_2}{A_4}, 0\right) \tag{25}$$

At DFE

$$N_h^o = S^o + I^o + R_1^o + R_2^o$$

$$N_h^o = \frac{\beta_1\left(\mu_h A_3 + \varepsilon\tau\mu_h + v\varepsilon\tau\right)}{\mu_h A_5} = \frac{\beta_1 A_6}{\mu_h A_5} \tag{26}$$

Where $A_6 = \left(\mu_h A_3 + \varepsilon\tau\mu_h + v\varepsilon\tau\right)$

6. Basic Reproduction Number R_0

The basic Reproduction number, denoted by R_0, is the expected number of secondary cases produced in a completely susceptible population, by a typical infective individual (Diekmann *et al*., 1990 and Driessche and Watmough 2002). If $R_0 < 1$, the infection can be wiped out at a point in time. On the other hand, if $R_0 > 1$, the disease can invade the population.

The Basic Reproduction number R_0 was calculated using the next generation matrix approach by calculating the spectral radius of the next generation matrix (Driessche and Watmough 2002). The matrices has to do with only infected classes with partial derivatives F_i and V_i. In our model, we have two infected compartments only, which are infected human I and infected mosquito M_2. F_i is the rate of appearance of new infection in infective compartment. V_i is the rate of transfer of individual out of infective compartment by other means.

Hence, looking at equations (1) – (6), we have;

$$F_i = \begin{bmatrix} \left(\frac{\alpha_2 M_2}{N_h} + \frac{\alpha_3 I}{N_h}\right) S \\ \frac{\alpha_1 I M_1}{N_m} \end{bmatrix} \tag{27}$$

And

$$V_i = \begin{bmatrix} (\gamma_1 + \mu_h + \delta_h) I \\ (\mu_m + \delta_m) M_2 \end{bmatrix} \tag{28}$$

Equation (3.79) can be written as

$$V_i = \begin{bmatrix} A_2 I \\ A_4 M_2 \end{bmatrix} \tag{29}$$

Where, $A_2 = (\gamma_1 + \mu_h + \delta_h)$ and $A_4 = (\mu_m + \delta_m)$

Differentiating F_i and V_i partially with respect to I and M_2, we obtain F and V as;

$$F = \begin{bmatrix} \frac{\alpha_3 S}{N_h} & \frac{\alpha_2 S}{N_h} \\ \frac{\alpha_1 M_1}{N_m} & 0 \end{bmatrix} \tag{30}$$

This can be written as

$$F = \begin{bmatrix} \frac{\alpha_3 S^o}{N_h^o} & \frac{\alpha_2 S^o}{N_h^o} \\ \frac{\alpha_1 M_1^o}{N_m^o} & 0 \end{bmatrix} \tag{31}$$

At DFE

$$\frac{S^o}{N_h^o}=\frac{A_3\beta_1}{A_5}\times\frac{\mu_h A_5}{\beta_1 A_6}=\frac{\mu_h A_3}{A_6},\qquad M_2^o=0,\Rightarrow M_1^o=N_m^o \qquad \text{Since} \quad N_m(t)=M_1(t)+M_2(t)$$

So, (3.82) becomes

$$F=\begin{bmatrix}\frac{\alpha_3\mu_h A_3}{A_6} & \frac{\alpha_2\mu_h A_3}{A_6}\\ \alpha_1 & 0\end{bmatrix} \tag{32}$$

And

$$V=\begin{bmatrix}A_2 & 0\\ 0 & A_4\end{bmatrix} \tag{33}$$

So,

$$FV^{-1}=\begin{bmatrix}\frac{\alpha_3\mu_h A_3}{A_2A_6} & \frac{\alpha_2\mu_h A_3}{A_4A_6}\\ \frac{\alpha_1}{A_2} & 0\end{bmatrix} \tag{34}$$

To get the spectral radius $\rho\left(FV^{-1}\right)$, we evaluate $\left|FV^{-1}-\lambda I\right|=0$

$$\left|FV^{-1}-\lambda I\right|=\begin{vmatrix}\frac{\alpha_3\mu_h A_3}{A_2A_6}-\lambda & \frac{\alpha_2\mu_h A_3}{A_4A_6}\\ \frac{\alpha_1}{A_2} & -\lambda\end{vmatrix}=0 \tag{35}$$

$$\Rightarrow \qquad \lambda^2-\lambda\frac{\alpha_3\mu_h A_3}{A_2A_6}-\frac{\alpha_1\alpha_2\mu_h A_3}{A_2A_4A_6}=0 \tag{36}$$

Solving (3.86) for λ, using the general formula for solving quadratic equation, we have;

$$\lambda = \frac{\frac{\alpha_3 \mu_h A_3}{A_2 A_6} \pm \sqrt{\left(\frac{\alpha_3 \mu_h A_3}{A_2 A_6}\right)^2 + 4\frac{\alpha_1 \alpha_2 \mu_h A_3}{A_2 A_4 A_6}}}{2} \tag{37}$$

Therefore,

$$R_0 = \rho\left(FV^{-1}\right) = \frac{\frac{\alpha_3 \mu_h A_3}{A_2 A_6} + \sqrt{\left(\frac{\alpha_3 \mu_h A_3}{A_2 A_6}\right)^2 + 4\frac{\alpha_1 \alpha_2 \mu_h A_3}{A_2 A_4 A_6}}}{2} \tag{38}$$

7. Local Stability of the Disease Free Equilibrium E_c

Theorem 8.1

The DFE of the system of equations (11) – (16) is locally asymptotically stable if $R_0 < 1$ and unstable if $R_0 > 1$.

Proof;

The Jacobian matrix J_{E^0} of our model equations (11) – (16) at DFE is given as

$$J_{E^0} = \begin{bmatrix} A_1 - \mu_h & -\frac{\alpha_3 A_3 \mu_h}{A_6} & \gamma_2 & 0 & 0 & -\frac{\alpha_2 A_3 \mu_h}{A_6} \\ 0 & \frac{\alpha_3 A_3 \mu_h}{A_6} - A_2 & 0 & 0 & 0 & \frac{\alpha_2 A_3 \mu_h}{A_6} \\ \varepsilon\tau & \gamma_1 & -A_3 & 0 & 0 & 0 \\ 0 & 0 & \nu & -\mu_h & 0 & 0 \\ 0 & -\alpha_1 & 0 & 0 & -A_4 & 0 \\ 0 & \alpha_1 & 0 & 0 & 0 & -A_4 \end{bmatrix} \tag{39}$$

Applying elementary row operation on (28), gives the upper triangular matrix

$$J_{E^0}=\begin{bmatrix} A_1-\mu_h & -D & \gamma_2 & 0 & 0 & -\dfrac{\alpha_2 A_3\mu_h}{A_6} \\ 0 & D-A_2 & 0 & 0 & 0 & \dfrac{\alpha_2 A_3\mu_h}{A_6} \\ 0 & 0 & L & 0 & 0 & G-\dfrac{E\alpha_2 A_3\mu_h}{A_6(D-A_2)} \\ 0 & 0 & 0 & -\mu_h & 0 & \left(\dfrac{v}{L}\right)\left(G-\dfrac{E\alpha_2 A_3\mu_h}{A_6(D-A_2)}\right) \\ 0 & 0 & 0 & 0 & -A_4 & \dfrac{\alpha_1\alpha_2 A_3\mu_h}{A_6(D-A_2)} \\ 0 & 0 & 0 & 0 & 0 & -A_4-\dfrac{\alpha_1\alpha_2 A_3\mu_h}{A_6(D-A_2)} \end{bmatrix} \tag{40}$$

Now, we calculate the eigenvalues to determine the basic reproduction number, R_0 by taking the dominant eigenvalue from the characteristic equation using $\left|J_{E^0}-\lambda I\right|=0$. we get the characteristic equation as

$$\begin{vmatrix} A_1-\mu_h-\lambda & -D & \gamma_2 & 0 & 0 & -\dfrac{\alpha_2 A_3\mu_h}{A_6} \\ 0 & D-A_2-\lambda & 0 & 0 & 0 & \dfrac{\alpha_2 A_3\mu_h}{A_6} \\ 0 & 0 & L-\lambda & 0 & 0 & G-\dfrac{E\alpha_2 A_3\mu_h}{A_6(D-A_2)} \\ 0 & 0 & 0 & -\mu_h-\lambda & 0 & \left(\dfrac{v}{L}\right)\left(G-\dfrac{E\alpha_2 A_3\mu_h}{A_6(D-A_2)}\right) \\ 0 & 0 & 0 & 0 & -A_4-\lambda & \dfrac{\alpha_1\alpha_2 A_3\mu_h}{A_6(D-A_2)} \\ 0 & 0 & 0 & 0 & 0 & -A_4-H-\lambda \end{vmatrix}=0 \tag{41}$$

Where, $H = \frac{\alpha_1 \alpha_2 A_3 \mu_h}{A_6 (D - A_2)}$, $D = \frac{\alpha_3 A_3 \mu_h}{A_6}$ and $L = \left(A_3 + \frac{\varepsilon \tau \gamma_2}{(A_1 - \mu_h)} \right)$

$$\left.\begin{aligned} \lambda_1 &= -\mu_h + A_1 < 0 \\ \lambda_2 &= -A_2 + D < 0 \\ \lambda_3 &= -L < 0 \\ \lambda_4 &= -\mu_h < 0 \\ \lambda_5 &= -A_4 < 0 \\ \lambda_6 &= -(H + A_4) < 0 \end{aligned}\right\} \quad (42)$$

Hence, the Disease Free Equilibrium (DFE) is locally asymptotically stable if $A_1 < \mu_h$ and $D < A_2$ otherwise unstable.

8. Global Stability of Disease Free Equilibrium

Theorem; The disease free equilibrium of equations (11) – (16) is globally asymptotically stable provided $R_0 < 1$ and unstable if $R_0 > 1$.

Proof;

The system of equations (11) – (16) are written as

$$\frac{dx(t)}{dt} = F(x, y) \quad \text{and} \quad \frac{dy(t)}{dt} = G(x, y) \quad (43)$$

Where $x = (S, R_1, R_2, M_1) \in \Re$ denote the differential classes of uninfected human and uninfected mosquito. While $y = (I, M_2) \in \Re$ denote the differential classes of infected human and infected mosquito.

The disease free equilibrium (DFE) $= (x^0, 0)$. Where;

$$x^0 = \left(\frac{A_3 \beta_1}{A_5}, \frac{\varepsilon \tau \beta_1}{A_5}, \frac{v \varepsilon \tau \beta_1}{\mu_h A_5}, \frac{\beta_2}{A_1} \right) \quad (44)$$

Case 1; considering the uninfected subsystem, we have

$$\frac{dx(t)}{dt} = F(x,y) = \begin{bmatrix} \beta_1 + \gamma_2 R_1 + A_1 S - \frac{\alpha_2 M_2}{N_h} S - \frac{\alpha_3 I}{N_h} S - \mu_h S \\ \gamma_1 I - A_3 R_1 + \varepsilon\tau S \\ \nu R_1 - \mu_h R_2 \\ \beta_2 - \frac{\alpha_1 I M_1}{N_m} - A_4 M_1 \end{bmatrix} \tag{45}$$

At $y = 0,\ I = M_2 = 0.$ So equation (32) becomes

$$F(x,0) = \begin{bmatrix} \beta_1 + \gamma_2 R_1 + A_1 S - \mu_h S \\ \varepsilon\tau S - A_3 R_1 \\ \nu R_1 - \mu_h R_2 \\ \beta_2 - A_4 M_1 \end{bmatrix} \tag{46}$$

This gives

$$\frac{dS(t)}{dt} = \beta_1 + \gamma_2 R_1 + A_1 S - \mu_h S \tag{47}$$

$$\frac{dR_1(t)}{dt} = \varepsilon\tau S - A_3 R_1 \tag{48}$$

$$\frac{dR_2(t)}{dt} = \nu R_1 - \mu_h R_2 \tag{49}$$

$$\frac{dM_1(t)}{dt} = \beta_2 - A_4 M_1 \tag{50}$$

Solving the differential equations (32a) to (32d) using integrating factor as $t \to \infty$

$S(t) \to \frac{(\beta_1 + \gamma_2 R_1)}{(\mu_h - A_1)}$, $R_1(t) \to \frac{\varepsilon\tau S}{A_3}$, $R_2(t) \to \frac{\nu R_1}{\mu_h}$, and $M_1(t) \to \frac{\beta_2}{A_4}$ Irrespective of the values for $S(0), R_1(0), R_2(0),$ and $M_1(0)$.

Thus, $x^0 = \left(\frac{A_3\beta_1}{A_5}, \frac{\varepsilon\tau\beta_1}{A_5}, \frac{\nu\varepsilon\tau\beta_1}{A_5\mu_h}, \frac{\beta_2}{A_4}\right)$ is globally asymptotically stable.

Case 2; considering the infected subsystem

$$y = G(x, y) = \begin{bmatrix} \frac{\alpha_2 M_2 S}{N_h} + \frac{\alpha_3 IS}{N_h} - A_2 I \\ \frac{\alpha_1 I M_1}{N_m} - A_4 M_2 \end{bmatrix} \tag{51}$$

Given that;

$$\hat{G}(x, y) = cy - G(x, y) \tag{52}$$

Where $c = \frac{\partial G(x,0)}{\partial t}$

Therefore,

$$c = \begin{bmatrix} \frac{\alpha_3 S}{N_h} - A_2 & \frac{\alpha_2 S}{N_h} \\ \frac{\alpha_1 M_1}{N_m} & -A_4 \end{bmatrix} \quad \text{and} \quad y = \begin{bmatrix} I \\ M_2 \end{bmatrix} \tag{53}$$

$$cy = \begin{bmatrix} \left(\frac{\alpha_3 S}{N_h} - A_2\right) I + \frac{\alpha_2 S M_2}{N_h} \\ \frac{\alpha_1 M_1 I}{N_m} - A_4 M_2 \end{bmatrix} \tag{54}$$

Substituting in (53), we have

$$\widehat{G}(x,y)=\begin{bmatrix}\left(\frac{\alpha_3 S}{N_h}-A_2\right)I+\frac{\alpha_2 S M_2}{N_h}\\ \frac{\alpha_1 M_1 I}{N_m}-A_4 M_2\end{bmatrix}-\begin{bmatrix}\frac{\alpha_2 M_2 S}{N_h}+\frac{\alpha_3 IS}{N_h}-A_2 I\\ \frac{\alpha_1 I M_1}{N_m}-A_4 M_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix} \quad (55)$$

Thus, $\widehat{G}(x,y)=0$

Therefore, the disease free equilibrium is globally asymptotically stable.

9. Result and Discussion

Dengue fever is becoming a global health issue which can be controlled using mosquito management strategies and effective administration of vaccine as shown in the study. SIR and SI epidemic model was used in the study to observe the effect and influence of control strategy to control the spread of dengue virus.

The results in this paper shows that DFE of the model is locally as well as globally stable when $R_0<1$, unstable and endemic when $R_0>1$. Increasing the level of control measure will decrease the value of reproduction number.

10. Conclusion

The importance of mathematical model in studying the epidemiology of infectious diseases like dengue fever can never be over emphasized.

It can be concluded from the investigation that effective implementation of control measure will eventually eradicate the disease or at least reduce it to a significantly minimum level. Vaccine plays a crucial role in curtailing the menace of dengue pandemic. Vaccination strategy is the best alternative in controlling the spread of dengue fever in human population. These include awareness campaign and administration.

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