International Journal of Mathematics and
Computer Science, **11**(2016), no. 2, 187–198

An Airline Crew Scheduling for Optimality

K. Rauf[1], **N. Nyor**[2], **R. U. Kanu**[3],**J. O. Omolehin**[4]

[1]Department of Mathematics
University of Ilorin
Ilorin, Nigeria

[2]Department of Mathematics
Federal University of Technology
Minna, Nigeria

[3]Department of Basic Sciences
Babcock University
Ilishan-Remo, Ogun State, Nigeria

[4]Department of Mathematics
Federal University
Lokojo, Nigeria

email: krauf@unilorin.edu.ng, ngutornyor@yahoo.com,
richmondkanu2004@yahoo.com, omolehin_joseph@yahoo.com

(Received June 25, 2016, Accepted September 24, 2016)

Abstract

In this work, air crew scheduling problem is formulated using a practical case of IRS Airline, Nigeria. The formulated Integer Program was solved using TORA Optimization software and a recommended solution was obtained with the optimal value of 3183 minutes and 13 $x_{variables}$ as pairings.

Key words and phrases: Linear programming, Optimality, Airline Crew Formulation, TORA.
AMS (MOS) Subject Classifications: 47H10, 54E40.
ISSN 1814-0432, 2016, http://ijmcs.future-in-tech.net

1 Introduction

One major problem for airlines is the scheduling of their flight crews. The problem of crew scheduling involves the optimal allocation of crews to flights. It is obvious that the problem is much more important today since costs for flying personnel of organizations or companies or major government parastatals have so much grown and are second largest cost (next to fuel) of the total operating costs for airlines. As a result of this, even small percentage savings amount to substantial amounts (Balaji and Ellis, 2005; Michael, 1996; Karla and Manfred, 1999). Therefore, the aim of the Study is to formulate Crew Scheduling Model for IRS Airline that minimizes crew cost.

2 Methodology

The step-by-step approach to obtaining the result is as follows:
i. IRS website was studied and its routine local routes were gotten.
ii. Crew pairings were established and flight duration were calculated for the schedules.
iii. Integer Programming problem was formulated
iv. Using TORA software, the result was obtained and interpreted.

3 Literature Review

According to Da Lu and Fatma (2014), Michael and David (2013) and Balaji and Ellis (2005); the airline industry is characterized by some of the largest scheduling problems of any industry. The problem of crew scheduling involves the optimal allocation of crews to flights. Balaji and Ellis (2005) argued that, over the last two decades the magnitude and complexity of crew scheduling problems have grown enormously and airlines are depending more and more on automated mathematical procedures as a practical necessity. Michael A.T. (1996) also reiterated that, One major problem for airlines is the scheduling of their flight crews.

The airline industry is severely unionized and there are stringent limitations on how to use a crew. For example, there are rules on how many hours a crew must be in the air in a day; and there are restrictions on the number of hours a crew can be away from their home base before they must stop over in a hotel. But crew Overheads are the second largest operating expense an airline has (after gasoline). Therefore, there is an opening to work with a hard problem influenced by enormous potential cost savings (Michael, 1996).

According to Karla and Manfred (1999), the air scheduling problem is one that has been studied almost continually for the past 40 years. It is obvious that, the problem is much more important today since costs for flying personnel of organizations or companies or major government parastatals have so much grown and are second largest cost (next to fuel) of the total operating costs for airlines. As a result of this, even small percentage savings amount to substantial amounts

3.1 Case Study: IRS Airlines Limited

IRS Airlines is an airline that operates from Nnamdi Azikiwe International Airport Abuja. The airline provides scheduled domestic services. IRS Airlines was established in 2002 and commenced operations in March 2002. IRS Airlines has it's main operating base at Nnamdi Azikiwe International Airport Abuja (www.nigeria.to/airlines/IRS)

3.1.1 IRS Airlines Destinations

IRS Airlines operates regular scheduled flights between these domestic destinations:
i. Abuja (Nnamdi Azikiwe International Airport)
ii. Gombe (Gombe Lawanti International Airport)
iii. Kaduna (Kaduna Airport)
iv. Lagos (Murtala Muhammed International Airport)
v. Maiduguri (Maiduguri International Airport)
vi. Port Harcourt (Port Harcourt International Airport)
vii. Yola (Yola Airport)

4 Definition of Terms

1. Crew category: This comprises of Cockpit (Pilots and Co-Pilots) and Cabin (Flight director and attendants)

2. Crew Base: An airport in a town where crew resides. Crews are assigned to few crew bases

3. A pairing is a sequence of duties intermingled with rest periods; starting and ending at same crew base

4. A duty is a sequence of flights, deadheads and connections forming a working day

5. A deadhead is a flight on which the crew travel as passengers

6. Feasibility: A pairing is feasible if all the safety and collective agreement rules are satisfied. Such safety and collective agreement rules are:

i. maximum number of calendar days in a pairing
ii. maximum number of duties in a pairing
iii. minimum rest time between two consecutive duties
iv. maximum number of landings per duty
v. maximum span of a duty
vi. maximum flying time per duty
vii. minimum connection time between two consecutive flights

5 Flight Crew Scheduling Mathematics

We have n flights and assign m crews. One possibility is to define decision variables x_{ij}. $1 \leq i \leq n, 1 \leq j \leq m$; Where

$$x_{ij} = \begin{cases} 1 \text{ flight } j \text{ has a crew } i \\ 0 \text{ otherwise} \end{cases}$$

To cover flight j, we introduce a constraint of the form:

$$\sum_{i=1}^{n} x_{ij} \geq 1$$

for each flight j. A crew pairing problem can be visualized as:
Given:
i. A set of scheduled flight;
ii. safety and working rules;
iii. Minimum or maximum credited hours per crew base.
Find least-cost feasible crew pairings
Subject to
i. each flight is covered by an active crew
ii. the maximum or minimum credited hours per crew base is represented (Tran, 2013).

6 Problem Formulation

The IRS Crew problem formulation starts with a diagrammatic representation of flight schedules. This is done by assigning a flight from one city to another within the routes operated by IRS.

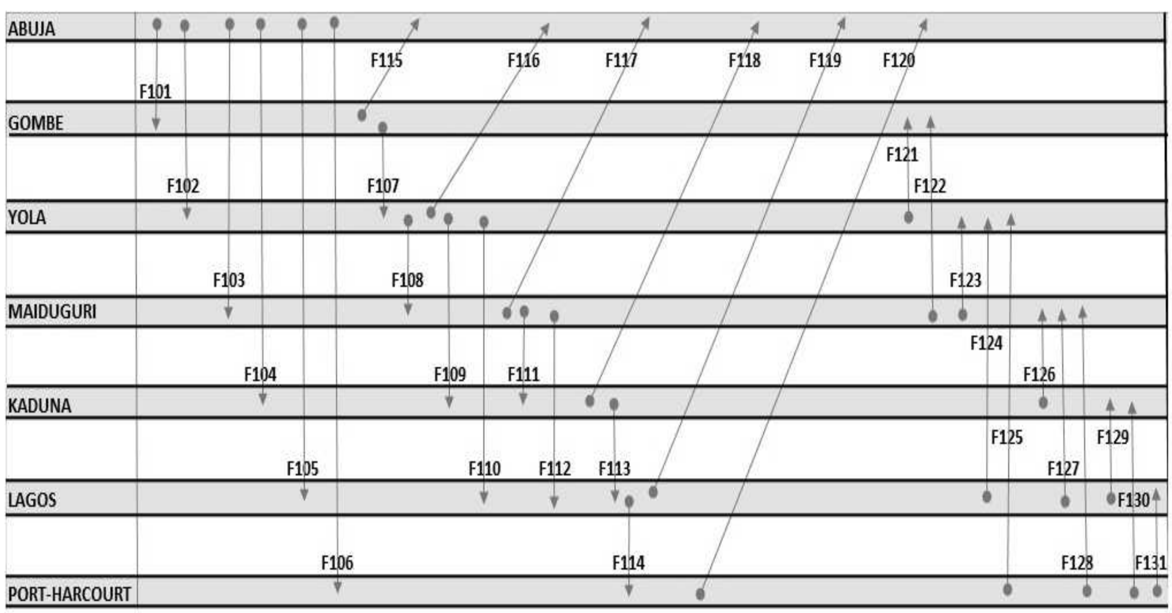
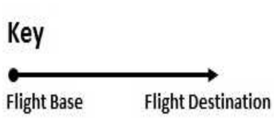


Fig 1: Diagram of Flight Schedule



F101 = Flight 101; F102 = Flight 102, and so on.

Table 2: Flying and Driving Times with Distances
The flying and driving distances and times in the table below were calculated using an online calculator on www.travelmath.com. The driving (Road) distances and times between the same flight cities were also calculated for the purpose of knowledge and information, not necessarily to be used in our formulation.

S/N	ROUTE	FLYING			DRIVING		
		Distance (miles)	**Distance** (km)	**Time** (hr,min=min)	**Distance** (miles)	**Distance** (km)	**Time** (hr,min)
1	ABJ-GMBE	283	455	1,04=64	560	901	10,50
2	ABJ-YLA	362	583	1,13=73	952	1532	18,32
3	ABJ-KD	94	152	0,41=41	115	185	02,10
4	ABJ-LAG	320	515	1,08=68	455	732	08,35
5	ABJ-MAID	446	718	1,24=84	584	940	11,00
6	ABJ-PORT	302	486	1,06=66	732	1176	13,26
7	GMBE-ABJ	283	455	1,04=64	523	842	10,04
8	GMBE-YLA	117	188	0,44=44	178	286	03,37
9	KD-ABJ	94	152	0,41=41	106	171	02,01
10	KD-LAG	393	633	1,17=77	487	784	08,55
11	KD-MAID	399	642	1,18=78	470	756	08,56
12	LAG-ABJ	320	515	1,08=68	457	735	08,40
13	LAG-KD	393	633	1,17=77	487	784	08,47
14	LAG-MAID	763	1227	2,02=122	1017	1637	18,54
15	LAG-PORT	273	439	1,03=63	376	605	06,46
16	LAG-YLA	651	1047	1,48=108	1385	2229	26,26
17	MAID-ABJ	446	718	1,24=84	546	879	10,15
18	MAID-GMBE	172	277	0,51=51	267	430	05,08
19	MAID-KD	399	462	1,18=78	470	756	08,56
20	MAID-LAG	763	1227	2,02=122	948	1526	17,43
21	MAID-YLA	188	302	0,53=53	370	595	07,33
22	PORT-ABJ	302	486	1,06=66	450	724	08,30
23	PORT-KD	395	636	1,17=77	746	1201	15,12
24	PORT-LAG	273	439	1,03=63	376	605	06,46
25	PORT-MAID	642	1034	1,47=107	1215	1955	24,02
26	PORT-YLA	483	778	1,28=88	1052	1693	20,32
27	YLA-ABJ	362	583	1,13=73	815	1473	17,47
28	YLA-GMBE	117	188	0,44=44	394	634	08,13
29	YLA-KD	356	572	1,13=73	486	782	09,00
30	YLA-LAG	651	1047	1,48=106	813	1308	17,13
31	YLA-MAID	188	302	0,53=53	370	595	07,33

Key:

ABJ = ABUJA

GMBE = GOMBE

YLA = YOLA

KD = KADUNA

MAID = MAIDUGURI
PORT = PORT-HARCOURT
LAG = LAGOS

7 IRS Pairings

Pairings are derived from the diagram of flight schedule in Figure 1. It is done randomly within the routine flight routes. Note that in flight pairings, flight crew must start and end at a crew base. That is, crew must come back to where it took off from. This is to avoid costs of hotel accommodation and other logistics since we are minimizing crew costs. In this formulation, we have assumed two crew bases - Abuja and Lagos. Table three shows the pairings of flight with their respective duration from one city to another.

Table 3: IRS Pairings with Respective Flight Duration

x_j	**Pairings**	**Interpretation**	**Time Aggregate**
1	$F101$, $F115$	$A \overset{64}{\to} G \overset{64}{\to} A$	128
2	$F101$, $F107$, $F116$	$A \overset{64}{\to} G \overset{44}{\to} Y \overset{73}{\to} A$	181
3	$F101$, $F107$, $F108$, $F117$	$A \overset{64}{\to} G \overset{44}{\to} Y \overset{53}{\to} M \overset{84}{\to} A$	245
4	$F101$, $F107$, $F108$, $F111$, $F118$	$A \overset{64}{\to} G \overset{44}{\to} Y \overset{53}{\to} M \overset{78}{\to} K \overset{41}{\to} A$	280
5	$F101$, $F107$, $F108$, $F111$, $F113$, $F119$	$A \overset{64}{\to} G \overset{44}{\to} Y \overset{53}{\to} M \overset{78}{\to} K \overset{77}{\to} L \overset{68}{\to} A$	384
6	$F101$, $F107$, $F108$, $F111$, $F113$, $F114$, $F120$	$A \overset{64}{\to} G \overset{44}{\to} Y \overset{53}{\to} M \overset{78}{\to} K \overset{77}{\to} L \overset{77}{\to} P \overset{66}{\to} A$	445
7	$F102$, $F116$	$A \overset{73}{\to} Y \overset{73}{\to} A$	146
8	$F102$, $F108$, $F117$	$A \overset{73}{\to} Y \overset{53}{\to} M \overset{84}{\to} A$	210
9	$F102$, $F108$, $F111$, $F118$	$A \overset{73}{\to} Y \overset{53}{\to} M \overset{78}{\to} K \overset{41}{\to} A$	245
10	$F102$, $F108$, $F111$, $F113$, $F119$	$A \overset{73}{\to} Y \overset{53}{\to} M \overset{78}{\to} K \overset{77}{\to} L \overset{68}{\to} A$	349
11	$F102$, $F108$, $F111$, $F113$, $F114$, $F120$	$A \overset{73}{\to} Y \overset{53}{\to} M \overset{78}{\to} K \overset{77}{\to} L \overset{63}{\to} P \overset{66}{\to} A$	410
12	$F103$, $F117$	$A \overset{84}{\to} M \overset{84}{\to} A$	168
13	$F103$, $F111$, $F118$	$A \overset{84}{\to} M \overset{78}{\to} K \overset{41}{\to} A$	203
14	$F103$, $F111$, $F113$, $F119$	$A \overset{84}{\to} M \overset{78}{\to} K \overset{77}{\to} L \overset{68}{\to} A$	307
15	$F103$, $F111$, $F113$, $F114$, $F120$	$A \overset{84}{\to} M \overset{78}{\to} K \overset{77}{\to} L \overset{63}{\to} P \overset{66}{\to} A$	368
16	$F104$, $F118$	$A \overset{41}{\to} K \overset{41}{\to} A$	82
17	$F104$, $F113$, $F119$	$A \overset{41}{\to} K \overset{77}{\to} L \overset{68}{\to} A$	186
18	$F104$, $F113$, $F114$, $F120$	$A \overset{41}{\to} K \overset{77}{\to} L \overset{63}{\to} P \overset{66}{\to} A$	247
19	$F105$, $F119$	$A \overset{68}{\to} L \overset{68}{\to} A$	136
20	$F105$, $F114$, $F120$	$A \overset{68}{\to} L \overset{63}{\to} P \overset{66}{\to} A$	197
21	$F106$, $F120$	$A \overset{66}{\to} P \overset{66}{\to} A$	132
22	$F101$, $F107$, $F121$, $F115$	$A \overset{64}{\to} G \overset{44}{\to} Y \overset{44}{\to} G \overset{64}{\to} A$	216
23	$F102$, $F108$, $F122$, $F115$	$A \overset{73}{\to} Y \overset{53}{\to} M \overset{51}{\to} G \overset{64}{\to} A$	241
24	$F103$, $F123$, $F116$	$A \overset{84}{\to} M \overset{53}{\to} Y \overset{73}{\to} A$	210
25	$F114$, $F120$, $F105$	$L \overset{63}{\to} P \overset{66}{\to} A \overset{68}{\to} L$	197
26	$F114$, $F131$	$L \overset{63}{\to} P \overset{63}{\to} L$	126
27	$F114$, $F130$, $F113$	$L \overset{63}{\to} P \overset{77}{\to} K \overset{77}{\to} L$	217
28	$F124$, $F116$, $F105$	$L \overset{108}{\to} Y \overset{73}{\to} A \overset{68}{\to} L$	249
29	$F129$, $F118$, $F105$	$L \overset{77}{\to} K \overset{41}{\to} A \overset{68}{\to} L$	186
30	$F127$, $F123$, $F110$	$L \overset{122}{\to} M \overset{53}{\to} Y \overset{106}{\to} L$	281
31	$F119$, $F104$, $F113$	$L \overset{68}{\to} A \overset{41}{\to} K \overset{77}{\to} L$	186
32	$F114$, $F125$, $F108$, $F112$	$L \overset{63}{\to} P \overset{88}{\to} Y \overset{53}{\to} M \overset{122}{\to} L$	326
33	$F114$, $F125$, $F109$, $F113$	$L \overset{63}{\to} P \overset{88}{\to} Y \overset{73}{\to} K \overset{77}{\to} L$	301
34	$F114$, $F128$, $F112$	$L \overset{63}{\to} P \overset{107}{\to} M \overset{122}{\to} L$	292
35	$F104$, $F126$, $F123$, $F121$, $F115$	$A \overset{41}{\to} K \overset{78}{\to} M \overset{53}{\to} Y \overset{44}{\to} G \overset{64}{\to} A$	280
36	$F129$, $F113$	$L \overset{77}{\to} K \overset{77}{\to} L$	154

Key:
A = Abuja; G = Gombe; Y = Yola; K = Kaduna; M = Maiduguri
P = Port-Harcourt; L - Lagos

8 IRS Integer Programming (IP) Formulation

"An integer programming problem is a mathematical optimization or feasibility program in which some or all of the variables are restricted to be integers. In many settings the term refers to Integer Linear Programming (ILP) in which the objective function and the constraints (other than the integer constraints) are linear" (Borndrer, 2012). The IRS Crew problem is formulated from the pairings thus:

Minimize Z = $128x_1 + 181x_2 + 245x_3 + 280x_4 + 384x_5 + 445x_6 + 146x_7 + 210x_8 + 245x_9 + 349x_{10} + 410x_{11} + 168x_{12} + 203x_{13} + 307x_{14} + 368x_{15} + 82x_{16} + 186x_{17} + 247x_{18} + 136x_{19} + 197x_{20} + 132x_{21} + 216x_{22} + 241x_{23} + 210x_{24} + 197x_{25} + 126x_{26} + 217x_{27} + 249x_{28} + 186x_{29} + 281x_{30} + 186x_{31} + 326x_{32} + 301x_{33} + 292x_{34} + 280x_{35} + 154x_{36}$
Subject to:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_{22} \geq 1 \quad \text{(F101)}$$

$$x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{23} \geq 1 \quad \text{(F102)}$$

$$x_{12} + x_{13} + x_{14} + x_{15} + x_{24} \geq 1 \quad \text{(F103)}$$

$$x_{16} + x_{17} + x_{18} + x_{31} + x_{35} \geq 1 \quad \text{(F104)}$$

$$x_{19} + x_{20} + x_{25} + x_{28} + x_{29} \geq 1 \quad \text{(F105)}$$

$$x_{21} \geq 1 \quad \text{(F106)}$$

$$x_2 + x_3 + x_4 + x_5 + x_6 + x_{22} \geq 1 \quad \text{(F107)}$$

$$x_3 + x_4 + x_5 + x_6 + x_8 + x_9 + x_{10} + x_{11} + x_{23} + x_{32} \geq 1 \quad \text{(F108)}$$

$$x_{33} \geq 1 \quad \text{(F109)}$$

$$x_{30} \geq 1 \quad \text{(F110)}$$

$$x_4 + x_5 + x_6 + x_9 + x_{10} + x_{11} + x_{13} + x_{14} + x_{15} \geq 1 \quad \text{(F111)}$$

$$x_{32} \geq 1 \quad \text{(F112)}$$

$$x_5 + x_6 + x_{10} + x_{11} + x_{14} + x_{15} + x_{17} + x_{18} + x_{31} + x_{36} \geq 1 \quad \text{(F113)}$$

$$x_6 + x_{11} + x_{15} + x_{18} + x_{20} + x_{25} + x_{26} + x_{27} + x_{32} + x_{33} + x_{34} \geq 1 \quad \text{(F114)}$$

$$x_1 + x_{22} + x_{23} + x_{35} \geq 1 \quad \text{(F115)}$$

$$x_2 + x_7 + x_{24} + x_{28} \geq 1 \quad \text{(F116)}$$

$$x_3 + x_8 + x_{12} \geq 1 \quad \text{(F117)}$$

$$x_4 + x_9 + x_{13} + x_{16} + x_{29} \geq 1 \quad \text{(F118)}$$

$$x_5 + x_{10} + x_{14} + x_{17} + x_{19} + x_{21} \geq 1 \quad \text{(F119)}$$

$$x_7 + x_{11} + x_{15} + x_{18} + x_{20} + x_{21} + x_{25} \geq 1 \quad \text{(F120)}$$

$$x_{22} + x_{35} \geq 1 \quad \text{(F121)}$$

$$x_{23} \geq 1 \quad \text{(F122)}$$

$$x_2 + x_{30} + x_{35} \geq 1 \quad \text{(F123)}$$

$$x_{28} \geq 1 \quad \text{(F124)}$$

$$x_{32} + x_{33} \geq 1 \quad \text{(F125)}$$

$$x_{35} \geq 1 \quad \text{(F126)}$$

$$x_{30} \geq 1 \quad \text{(F127)}$$

$$x_{34} \geq 1 \quad \text{(F128)}$$

$$x_{29} + x_{36} \geq 1 \quad \text{(F129)}$$

$$x_{27} \geq 1 \quad \text{(F130)}$$

$$x_{26} \geq 1 \quad \text{(F131)}$$

$$x_j = 0 or 1 (j = 1, ...31) \quad \text{(Integer Condition)}$$

9 IP Result

Using TORA Software, the IP formulation in section 9 above gave the following optimal result:
Objective Value = 3183

$$x_{Variables} = \begin{cases} 1 \text{ for } x_5, x_{12}, x_{21}, x_{23}, x_{26}, x_{27}, x_{28}, x_{29}, x_{30}, x_{32}, x_{33}, x_{34}, x_{35} \\ 0 \text{ otherwise} \end{cases}$$

10 Discussion of Result

The objective value of 3183 minutes is the optimal (minimum) duration of flight that IRS crew can spend on air to reduce crew cost, given the $x_{variables}$ that have solution values as 1. These variables form the recommendation of the model and can be identified from the IRS Pairings in table 3 as in table 4 below. In practical terms, the model recommends the following pairings for optimal crew cost reduction by IRS Airline.

11 Recommendation

The pairings in Table 4 were recommended by the model in order to minimize crew cost. Their interpretation and time aggregates are also stated.

Table 4: Model Recommendation of IRS Pairings

x_j	**Pairings**	**Interpretation**	**Time Aggregate in Min.**
5	$F101, F107, F108, F111, F113, F119$	$A \overset{64}{\to} G \overset{44}{\to} Y \overset{53}{\to} M \overset{78}{\to} K \overset{77}{\to} L \overset{68}{\to} A$	384
12	$F103, F117$	$A \overset{84}{\to} M \overset{84}{\to} A$	168
21	$F106, F120$	$A \overset{66}{\to} P \overset{66}{\to} A$	132
23	$F102, F108, F122, F115$	$A \overset{73}{\to} Y \overset{53}{\to} M \overset{51}{\to} G \overset{64}{\to} A$	241
26	$F114, F131$	$L \overset{63}{\to} P \overset{63}{\to} L$	126
27	$F114, F130, F113$	$L \overset{63}{\to} P \overset{77}{\to} K \overset{77}{\to} L$	217
28	$F124, F116, F105$	$L \overset{108}{\to} Y \overset{73}{\to} A \overset{68}{\to} L$	249
29	$F129, F118, F105$	$L \overset{77}{\to} K \overset{41}{\to} A \overset{68}{\to} L$	186
30	$F127, F123, F110$	$L \overset{122}{\to} M \overset{53}{\to} Y \overset{106}{\to} L$	281
32	$F114, F125, F108, F112$	$L \overset{63}{\to} P \overset{88}{\to} Y \overset{53}{\to} M \overset{122}{\to} L$	326
33	$F114, F125, F109, F113$	$L \overset{63}{\to} P \overset{88}{\to} Y \overset{73}{\to} K \overset{77}{\to} L$	301
34	$F114, F128, F112$	$L \overset{63}{\to} P \overset{107}{\to} M \overset{122}{\to} L$	292
35	$F104, F126, F123, F121, F115$	$A \overset{41}{\to} K \overset{78}{\to} M \overset{53}{\to} Y \overset{44}{\to} G \overset{64}{\to} A$	280
	Total		**3183**

12 Conclusion

The crew problem was formulated using time instead of money as cost. This is because, time can easily be converted to money once it can be established how much money is spent in a particular time. Again, the difficulty in getting financial data makes one to think of another standard option.

References

[1] G. Balaji, L. J. Ellis, Airline crew scheduling: state-of-the-art, Annals of Operations Research, **140,** no. 1, (2005), 305-337.

[2] R. M. Borndrer, Designing telecommunication networks by integer programming, 2012,
http://www.zib.de/groetscel/teaching/ss2012/120503vorlsung-designing telcomnetworks-reduced.pdf.

[3] Da Lu, Fatma Gzara, The robust crew pairing problem: model and solution methodology, 2014,
http://link.springer.com/article/10.1007/$s10898-014-0222$-y?no-access=true.

[4] http://www.nigeria.to/airlines/IRS/IRS.php5

[5] L. H. Karla, P. Manfred, Solving airline crew scheduling problems by branch-and-cut, Management Science, **39,** no. 6, (1999), 657-682.

[6] A. T. Michael , Airline Crew Scheduling,
http://mat.gsia.cmu.edu/classes/mstc/decomp/node5.html

[7] D. D. Michael, M. Clarke, David Ryan, 2013.

[8] Airline Industry Operations Research,
http://link.springer.com/referenceworkentry/10.1007/$978-1-4419-1153-7_24$.

[9] J. V. Robert, Linear Programming: Duality, 2007,
http://www.princeton.edu/ rvdb.

[10] Saravanan Thirumuruganathan (2011), Detailed Tutorial on Linear Programming, (2011),
https://saravananthirumuruganathan.wordpress.com/2011/03/26/detailed-tutorial-on-linear-programming/

[11] S. Srinivasan, Introduction to Operations Research, Prentice-Hall Inc., London, 2010.

[12] V. H. Tran, Airline crew scheduling problem 2013,
http://www.cse.hcmut.edu.vn/hoai

[13] www.travelmath.com.