# Application of Differential Equation to Economics 

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#### Abstract

In recent years, many Business analyst and Economist discovered the abnormality in price of products or commodities, rather than a fixed price or a little increase, the price increases drastically affecting the demand or supply of goods with respect to time, which in turn inflates the price of such commodities, and affects the various policy alternatives. These problems were modeled into differential equations and the solutions were obtained. With a closer view at the Evans Price Adjustment Model, which is a determinant factor of Price as regards quantity of Demand $D(\mathrm{t})$ and supply $S(\mathrm{t})$ with respect to time $t$. Variation of time clearly shows that as time increases, price increases, supply increases and demand decreases.


Keywords: Revenue function, Marginal revenue, profile function, Evans Price model.

### 1.0 INTRODUCTION

The relationship between the financial sector, business and economic growth is an important issue that has been examined in a wide range of research papers, both theoretical and empirical. Many of them have focused on the impact of the financial sector through derivative on economic growth. Pioneering studies that highlight the role of the financial sector in the dynamism of the economy include Wicksell (1934), Goldsmith (1969), which found that the financial system serves as an engine driving the economic activity.
On the other hand, Levine (1991) points out that stock markets facilitate long-term investments, helping to reduce risk and simultaneously offering liquidity to savers and funding to companies. The author concludes that stock markets do contribute to economic growth. Moreover, Levine and Zervos (1998) highlight that a significant number of empirical studies support the existence of a relationship between capital markets and economic growth in the long term.
Derivatives markets have experienced robust growth in recent decades. In December 2008, the volume of derivatives worldwide was approximately USD 592 trillion, much higher than the gross domestic product (GDP) of the United States (the world's largest economy), which was just over 13.8 trillion in 2007. In 2003, $92 \%$ of the 500 largest firms in the world used derivatives to manage risk in several ways, especially interest rate risk, according to information provided by the BIS (Bank for International Settlements). The derivatives market is not only an enormous market, but also one that is growing dramatically. Derivative contracts increased more than sevenfold in the period 1998-2014. The role played by derivatives markets in boosting economic growth has been analyzed by authors such Sundaram (2013), Sipko (2011), Prabhaet al. (2014), among many others. Most have found a positive relationship between the development of the derivatives market and economic growth; however, a
worldwide analysis of such a relationship has yet to be carried out. This research paper examines the impact of derivatives markets on economic growth in four major world economies. Specifically, we assess the impact of variables such as the volume of the derivatives market in US dollars and the volume of the derivatives market as a proportion of GDP on economic growth over the period 20022014 and it is a new development in the literature

### 2.0 MATHEMATICAL FORMULATIONS

Consider the differential equation of the form
$D(t)=\alpha_{0}+\alpha_{1} p(t)+\alpha_{2} p^{\prime}(t), \alpha_{0}>0, \alpha_{1}>0, \alpha_{2}>0$.
$S(t)=\beta_{0}+\beta_{1} p(t)+\beta_{2} p^{\prime}(t), \beta_{0}>0, \beta_{1}>0, \beta_{2}<0$.
If $\alpha_{2}=\beta_{2}=0$ we have the Evans price Adjustment Model in which $\alpha_{1}<0$ since when price increases, demand decreases and $\beta_{1}>0$ since when price increases, supply increases. In Allen's Model, co-efficients $\alpha_{2, \beta_{2}}$ accounts for effect of speculation.

The Evans price adjustment model, assumes that the rate of change of price $p$ with respect to time $t$ is proportional to the shortage $D-S$ so that

$$
\begin{align*}
& \frac{d p}{d t}=K(D-S)  \tag{2.2}\\
& D(t)=\alpha_{0}+\alpha_{1} p(t)  \tag{2.3}\\
& S(t)=\beta_{0}+\beta_{1} p(t) \tag{2.4}
\end{align*}
$$

From the above
$D(t)=\alpha_{0}+\alpha_{1} p(t)+\alpha_{2} p^{\prime}(t)$
$S(t)=\beta_{0}+\beta_{1} p(t)+\beta_{2} p^{\prime}(t)$
$D(t)-S(t)=\left(\alpha_{0}-\beta_{0}\right)+\left(\alpha_{1}-\beta_{1}\right) p(t)+\left(\alpha_{2}-\beta_{2}\right) p^{\prime}(t)$
At dynamic equilibrium $\alpha_{2}=0=\beta_{2}$
$\mathrm{D}(\mathrm{t})-\mathrm{S}(\mathrm{t})=0$
$\Rightarrow D(t)=S(t)$
$\alpha_{0}+\alpha_{1} p(t)=\beta_{0}+\beta_{1} p(t)$
Let

$$
\begin{align*}
& D(t)=S(t)=p \\
& \frac{d p}{d t}=0  \tag{2.8}\\
& \left(\alpha_{0}-\beta_{0}\right)+\left(\alpha_{1}-\beta_{1}\right) p+\left(\alpha_{2}-\beta_{2}\right) \frac{d p}{d t}=0  \tag{2.9}\\
& \left(\alpha_{2}-\beta_{2}\right) \frac{d p}{d t}+\left(\alpha_{1}-\beta_{1}\right) p=-\left(\alpha_{0}-\beta_{0}\right)  \tag{2.10}\\
& \left(\alpha_{2}-\beta_{2}\right) \frac{d p}{d t}+\left(\alpha_{1}-\beta_{1}\right) p=\beta_{0}-\alpha_{0}
\end{align*}
$$

Divide through by $\alpha_{2}-\beta_{2}$ we have

$$
\begin{equation*}
\frac{d p}{d t}+\frac{\left(\alpha_{1}-\beta_{1}\right)}{\alpha_{2}-\beta_{2}} p=\frac{\beta_{0}-\alpha_{0}}{\alpha_{2}-\beta_{2}} \tag{2.11}
\end{equation*}
$$

Let
$\frac{\alpha_{1}-\beta_{1}}{\alpha_{2}-\beta_{2}}=\gamma$
and

$$
\begin{equation*}
\frac{\beta_{0}-\alpha_{0}}{\alpha_{2}-\beta_{2}}=\xi \tag{2.12}
\end{equation*}
$$

By substitution we have

$$
\begin{equation*}
\frac{d p}{d t}+\gamma p=\xi \tag{2.13}
\end{equation*}
$$

By using application of linear differential equation of the form

$$
\begin{equation*}
\frac{d y}{d x}+p(x) y=Q(x) \tag{2.14}
\end{equation*}
$$

Using integrating factor

$$
\begin{equation*}
v(t)=\ell^{\int y d t}=\ell^{\gamma t} \tag{2.15}
\end{equation*}
$$

The solution becomes

$$
\begin{equation*}
p(t)=\frac{\xi}{\gamma}+\left(p_{0}-\frac{\xi}{\gamma}\right) \ell^{-\gamma t} \tag{2.16}
\end{equation*}
$$

Let $\frac{\xi}{\gamma}=p_{e}$
$p(t)=p_{e}+\left(p_{0}-p_{e}\right) \ell^{-\gamma t}$
$p(t)=\frac{\alpha_{0}-\beta_{0}}{\beta_{1}-\alpha_{1}}$
Where $D(t)$ represents demand and $S(t)$ represents supply, both with respect to time, and $\alpha, \beta$ are both co-efficients of demand and supply, while $P(t)$ is price with respect to time.

Case 1:
Assume a commodity is introduced with an initial price of 5 naira per unit and $t$ months later, the price is $P(t)$ naira per unit. A case study indicates that at a time $t$, the demand for the commodity will be $D(t)=3+10 e^{-0.01 t}$ thousand units and that $S(t)=2+p$ thousand units will be supplied. Suppose that at each time $t$, the price is changing at the rate of $2 \%$ of the shortage $D(t)-S(t)$.
a) According to the given information, the unit price $P(t)$ satisfies

$$
\begin{equation*}
\frac{d p}{d t}=0.02(D(t)-S(t))=0.02\left(\left(3+10 \ell^{-0.01 t}\right)-(2+p)\right)=0.02+0.2 e^{-0.01 t}-0.02 p \tag{2.19}
\end{equation*}
$$

Or equivalently,
$\frac{d p}{d t}+0.02 p=0.02+0.2 e^{-0.01 t}$
Which we recognize as a first order linear differential equation with $P(t)=0.02$ and $Q(t)=0.02+0.2 e^{-0.01 t}$.

The integrating factor is
$I(t)=e^{\int 0.02 t}=e^{0.02 t}$
So the general solution is

$$
\begin{aligned}
& P(t)=\frac{1}{e^{0.02 t}}\left(\int \ell^{0.02 t}\left(0.02+0.2 \ell^{-0.01 t} d t+c\right)=e^{-0.02 t}\left(\frac{0.02 \ell^{0.02 t}}{0.02}+\frac{0.2 \ell^{-0.01 t}}{0.01}+c\right)\right. \\
& =1+20 e^{-0.01 t}+c e^{-0.02 t}
\end{aligned}
$$

Since the initial price is $P(0)=5$, we have

$$
\begin{equation*}
P(0)=5=1+20 e^{-0.01 t}+c e^{0}=1+20+c \tag{2.23}
\end{equation*}
$$

So that $c=5-21=-16$
And
$P(t)=1+20 e^{-0.01 t}-16 e^{-0.02 t}$
b) After 6 months $(t=6)$ the price is

$$
\begin{equation*}
P(6)=1+20 e^{-0.01(6)}-16 e^{-0.02(6)} \mid 5.6446 \tag{2.25}
\end{equation*}
$$

That is, approximately 5.64 naira per unit
c) To maximize the price, we first compute the derivatives

$$
\begin{equation*}
P^{\prime}(t)=20\left(-0.01 e^{-0.01(6)}\right)-16\left(-0.02 e^{-0.02 t}\right)=-0.2 e^{-0.01 t}+0.32 e^{-0.02 t} \tag{2.26}
\end{equation*}
$$

And then note that $P^{\prime}(t)=0$ when
$P^{\prime}(t)=-0.2 e^{-0.01 t}+0.32 e^{-0.02 t}=0$
$0.32 e^{-0.02 t}=0.2 e^{-0.01 t}$
Divide both sides by 0.2
$\frac{0.32 e^{-0.02 t}}{0.2}=\frac{0.2 e^{-0.01 t}}{0.2}$
Multiply both sides by $\frac{1}{e^{-0.02 t}}$
$\frac{e^{-0.01 t}}{e^{-0.02 t}}=\frac{0.32}{0.2}$
$e^{-0.01 t(-0.02 t)}=1.6$
$e^{-0.01+0.02 t}=1.6$
Taking $\ln$ of both sides
$0.01 t=\ln 1.6$
Taking logarithms of both sides
$t=\ln 1.6$
$t=47$
The critical number corresponds to a maximum, the largest price occur after 47 months Since

$$
\begin{equation*}
P(47)=1+20 \ell^{-0.01(47)}-16 \ell^{-0.02(47)}=7.25 \tag{2.31}
\end{equation*}
$$

The largest unit price is approximately 7.25
Since

$$
D(47)=3+10 \ell^{-0.01(47)}=9.25
$$

and

$$
S(47)=2+P(47)=2+7.25=9.25
$$

It follows that approximately 9.250 units will be both demanded and supplied at the maximum price of 7.25 naira per unit
d) Since

$$
\begin{equation*}
\lim _{t \rightarrow \infty} P(t)=\lim \left(1+20 \ell^{-0.01 t}-16 \ell^{-0.02 t}\right)=1+20(0)-16(0)=1 \tag{2.33}
\end{equation*}
$$

The Price tends towards 1 naira per unit in the "Long run".
Case 2:- SupplyS $(t)$ and demand $D(t)$ functions for a commodity are given in terms of the unit price $P(t)$ at time t . Assume that price changes at a rate proportional to the shortage $D(t)-S(t)$ with indicated constant of proportionality K and initial price $P_{0}$ in each problem:
a. Set up and solve a differential equation for $P(t)$
b. Find the unit price of the commodity when $t=4$
c. Determine what happens to the price at $t \rightarrow \infty$.
$S(t)=2+3 p ; D(t)=10-p(t) ; k=0.02 ; p_{0}=1$
From equation (3.1)
$\frac{d p}{d t}=K(D-S)$
$\frac{d p}{d t}=k(D(\mathrm{t})-\mathrm{S}(\mathrm{t}))=k((10-p)-(2+3 p))=k(D(\mathrm{t})-\mathrm{S}(\mathrm{t}))=k(10-2-(p+3 p))$
$\frac{d p}{d t}=k(8-4 p)$
Or equivalently,

$$
\begin{align*}
& \frac{d p}{d t}=0.02(8-4 p)=0.16-0.08 p \\
& \frac{d p}{d t}+0.08 p=0.16 \tag{2.36}
\end{align*}
$$

a) Implying it's a first order linear differential equation with $P(t)=0.08$ and $Q(t)=0.16$
b) The integrating factor is $I(t)=e^{\int 0.08 d t}=e^{0.08 t}$

So the general solution is
$P(t)=\frac{1}{e^{0.08 t}}\left(\int \ell^{0.08 t}(0.16) d t+c\right)=e^{-0.08 t}\left(\frac{0.16 \ell^{0.08 t}}{0.08}+c\right)=\ell^{-0.08 t}\left(2 \ell^{0.08 t}+c\right)=2+c \ell^{-0.08 t}$
Since the initial price is $P(0)=1 ; \Rightarrow P(0)=5=2+c e^{-0.08 t}=2+\ell^{-0.08(0)}=2+c$
So that $c=1-2=-1$
And
$P(t)=2-e^{-0.08 t}$
b) After 4 months $(t=4)$ the price is
$P(4)=2-e^{-0.08(4)}=2-e^{-0.032} \square 1.2739$
That is, approximately 1.27 naira per unit
c) Price $P(t)$ as $t \rightarrow \infty$ will become
$\lim _{t \rightarrow \infty} P(t)=\lim \left(2-\ell^{-0.08 t}\right)=2-0=2$
The Price tends towards 2 naira per unit in the "Long run".
Case 3:
Assume
$S(t)=1+4 p(t) ; D(t)=15-3 p(t) ; k=0.015 ; p_{0}=3$

$$
\begin{aligned}
& \frac{d p}{d t}=K(D-S) \\
& \frac{d p}{d t}=k(D(t)-S(t))=k((15-3 p)-(1+4 p))=k(15-3 p-1-4 p)=k(15-1-(3 p+4 p)) \\
& \frac{d p}{d t}=k(14-7 p)
\end{aligned}
$$

Or equivalently,
$\Rightarrow \frac{d p}{d t}=0.015(14-7 p)=0.21-0.105 p$
$\frac{d p}{d t}+0.105 p=0.21$

Implying it's a first order linear differential equation with $P(t)=0.105$ and $Q(t)=0.21$ The integrating factor is $I(t)=e^{\int 0.105 d t}=e^{0.105 t}$

So the general solution is
$P(t)=\frac{1}{e^{0.105 t}}\left(\int \ell^{0.105 t}(0.21) \mathrm{dt}+\mathrm{c}\right)=e^{-0.105 t}\left(\frac{0.21 \ell^{0.105 t}}{0.105}+c\right)=\ell^{-0.105 t}\left(2 \ell^{0.105 t}+c\right)$
$=2+c \ell^{-0.105 t}$
Since the initial price is $P(0)=1$;
$\Rightarrow P(0)=3=2+c e^{-0.105 t}=2+\ell^{-0.105(0)}=2+c \ell^{0}=2+c$
So that $c=3-2=1$
And
$P(t)=2+e^{-0.105 t}$
b) After 4 months $(t=4)$ the price is
$P(4)=2+e^{-0.105(4)}=2+e^{-0.042} \square 2.6570$
That is, approximately 2.6570 naira per unit
c) Price $P(t)$ as $t \rightarrow \infty$ will become
$\lim _{t \rightarrow \infty} P(t)=\lim \left(2+\ell^{-0.105 t}\right)=2+0=2$

The Price tends towards 2 naira per unit in the "Long run".
Case 4:
Assume
$S(t)=2+p(t) ; D(t)=3+7 \ell^{-t} ; k=0.02 ; p_{0}=4$
From equation (3.1)
$\frac{d p}{d t}={ }^{`} K(D-S)$
$\frac{d p}{d t}=k(D(t)-S(\mathrm{t}))=k\left(\left(3+7 \ell^{-t}\right)-(2+p)\right)=k\left(3-2-7 \ell^{-t}-p\right)=k\left(1+7 \ell^{-t}-p\right)$
$\frac{d p}{d t}=k\left(1+7 \ell^{-t}-p\right)$
Or equivalently,
$\Rightarrow \frac{d p}{d t}=0.02\left(1+7 \ell^{-t}-p\right)=0.02+0.14 \ell^{-t}-0.02 p$
$\frac{d p}{d t}+0.02 p=0.02+0.14 \ell^{-t}$

Implying it's a first order linear differential equation with $P(t)=0.02$ and $Q(t)=0.02+0.14 \ell^{-t}$
The integrating factor is $\quad I(t)=e^{\int 0.02 d t}=e^{0.02 t}$
So the general solution is
$P(t)=\frac{1}{e^{0.02 t}}\left(\int \ell^{c .02 t}\left(0.02+0.14 \ell^{-t}\right) d t+c\right)=e^{-0.02 t}\left(\frac{0.02 \ell^{0.02 t}}{0.02}+\frac{0.14 \ell^{-t}}{-1}+c\right)$
$=\ell^{-0.02 t}\left(\ell^{0.02 t}-0.14 \ell^{-t}+c\right)$
$P(t)=1-0.14 \ell^{-1.02 t}+c \ell^{-0.02 t}$
Since the initial price is $P(0)=4$;
$\Rightarrow P(0)=4$
$=1-0.14 \ell^{-1.02}+c e^{-0.02 t}=1-0.14 \ell^{-0.02(0)}+c \ell^{-0.02(0)}=1-0.14 \ell^{0}+c \ell^{0}=1-0.14+c$
So that $c=4-1+0.14=3.14$
And

$$
P(t)=1-0.14 \ell^{-0.02 t}+3.14 e^{-0.02 t}
$$

Or

$$
\begin{equation*}
P(t)=1-\frac{1}{7} \ell^{-0.02 t}+\frac{22}{7} e^{-0.02 t} \tag{2.51}
\end{equation*}
$$

b) After 4 months $(t=4)$ the price is

$$
\begin{equation*}
P(4)=1-0.14 \ell^{-0.02(4)}+3.14 e^{-0.02(4)}=1-0.14 \ell^{-0.08}+3.14 e^{-0.08}=3.7693 \tag{2.52}
\end{equation*}
$$

That is, approximately 3.7693 dollars per unit
c) Price $P(t)$ as $t \rightarrow \infty$ will become

$$
\begin{equation*}
\lim _{t \rightarrow \infty} P(t)=\lim \left(1-0.14 \ell^{-0.02 t}+3.14 \ell^{-0.02 t}\right)=1-0+0=1 \tag{2.53}
\end{equation*}
$$

The Price tends towards 1 naira per unit in the "Long run".

### 3.0 RESULTS AND DISCUSSION

With a closer view at the Evans Price Adjustment Model in chapter three, it's a determinant factor of Price as regards quantity of Demand $D(\mathrm{t})$ and supply $S(\mathrm{t})$ with respect to time ${ }^{t}$.variation of time clearly shows that as time increases, price increases, supply increases and demand decreases. This is the more reason we have to use MATLAB.

## CASE 1

Table 3.1: Values of Price and Quantity functions ( $\mathrm{D}(\mathrm{t})$ and $\mathrm{S}(\mathrm{t})$ ) with respect to time t

| $\mathbf{t}$ | $\mathbf{0}$ | $\mathbf{6}$ | $\mathbf{1 2}$ | $\mathbf{1 8}$ | $\mathbf{2 4}$ | $\mathbf{3 0}$ | $\mathbf{3 6}$ | $\mathbf{4 2}$ | $\mathbf{4 8}$ | $\mathbf{5 4}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| p1 | 5.0000 | 5.6446 | 6.1524 | 6.5426 | 6.8320 | 7.0354 | 7.1655 | 7.2336 | 7.2494 | 7.2214 |  |
| d1 13.0000 | 12.4176 | 11.8692 | 11.3527 | 10.8663 | 10.4082 | 9.9768 | 9.5705 | 9.1878 | 8.8275 |  |  |
| s1 | 7.0000 | 7.6446 | 8.1524 | 8.5426 | 8.8320 | 9.0354 | 9.1655 | 9.2336 | 9.2494 | 9.2214 |  |



Figure 3.1: Graph of Price and Quantity functions (d(t) and $s(t))$ against time for case 1

$$
p(t)=1+20 \ell^{-0.01 t}-16 \ell^{-0.02 t}, d(t)=3+10 \ell^{-0.01 t}, s(t)=2+p
$$

The above shows that within the period of 6 years as time increases there is an increase in price, and supply, while demand decreases drastically, at the point where demand equals supply, it said to be point of Equilibrium.

## CASE 2

Table 3.2: Values of Price and Quantity functions $(d(t)$ and $s(t))$ with respect to time $t$

| $\mathbf{t}$ | $\mathbf{0}$ | $\mathbf{6}$ | $\mathbf{1 2}$ | $\mathbf{1 8}$ | $\mathbf{2 4}$ | $\mathbf{3 0}$ | $\mathbf{3 6}$ | $\mathbf{4 2}$ | $\mathbf{4 8}$ | $\mathbf{5 4}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| p2 | 1.0000 | 1.3812 | 1.6171 | 1.7631 | 1.8534 | 1.9093 | 1.9439 | 1.9653 | 1.9785 | 1.9867 |
| d2 | 9.0000 | 8.6188 | 8.3829 | 8.2369 | 8.1466 | 8.0907 | 8.0561 | 8.0347 | 8.0215 | 8.0133 |
| s2 | 5.0000 | 6.1436 | 6.8513 | 7.2892 | 7.5602 | 7.7278 | 7.8316 | 7.8958 | 7.9355 | 7.9601 |



Figure 3.2: Graph of Price and Quantity functions ( $d(t)$ and $s(t)$ ) against time for case 2

$$
p(t)=2-\ell^{-0.08 t}, \mathrm{~d}(t)=10-2 p, \mathrm{~s}(t)=2+3 p
$$

The above shows that within the period of 6 years as time increases there is an increase in price, and drastic increase in supply while demand decreases, at the point where demand equals supply, it said to be point of Equilibrium.

## CASE 3

Table 3.3: Values of Price and Quantity functions ( $d(t)$ and $s(t)$ ) with respect to time $t$

| t | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| p3 | 3.0000 | 2.5326 | 2.2837 | 2.1511 | 2.0805 | 2.0429 | 2.0228 | 2.0122 | 2.0065 | 2.0034 |
| d3 | 6.0000 | 7.4022 | 8.1490 | 8.5468 | 8.7586 | 8.8714 | 8.9315 | 8.9635 | 8.9806 | 8.9897 |
| s3 13.0000 | 11.1304 | 10.1346 | 9.6043 | 9.3218 | 9.1714 | 9.0913 | 9.0486 | 9.0259 | 9.0138 |  |


Figure
3.3:
Graph
of
Price
and
Quantity functions $(d(t)$ and $s(t))$ against time for case 3 $p(t)=2+\ell^{-0.105 t}, d(t)=15-3 p, s(t)=1+4 p$

The above shows that within the period of 6 years as time increases there is a decrease in price, and supply decreases drastically while demand increases significantly, at the point where demand equals supply it said to be point of Equilibrium.

CASE 4

Table 3.4: Values of Price and Quantity functions $(d(t)$ and $s(t))$ with respect to time $t$

| t | 0 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| p 4 | 4.0000 | 3.6608 | 3.3599 | 3.0930 | 2.8564 | 2.6464 | 2.4603 | 2.2951 | 2.1487 | 2.0188 |
| d 4 | 10.0000 | 3.0174 | 3.0000 | 3.0000 | 3.0000 | 3.0000 | 3.0000 | 3.0000 | 3.0000 | 3.0000 |
| s4 | 6.0000 | 5.6608 | 5.3599 | 5.0930 | 4.8564 | 4.6464 | 4.4603 | 4.2951 | 4.1487 | 4.0188 |



Figure 3.4: Graph of Price and Quantity functions ( $d(t)$ and $s(t)$ ) against time $t$ case 4 $p(t)=1-\frac{1}{7} \ell^{-0.02 t}+\frac{22}{7} \ell^{-0.02 t}, \mathrm{~d}(t)=3+7 \ell^{-t}, \mathrm{~s}(t)=2+p$

The above shows that within the period of 6 years as time increases there is an decrease in price, and supply, while demand decreases significantly, at the point where demand equals supply it said to be point of Equilibrium.

### 4.0 CONCLUSION

The problems formulated in this work were solved analytically using differential calculus through the application of Evans Price Adjustment Model, as the Price increases with respect to time, it gets to a point where it stops increasing and then begins to decrease until it gets to a fixed point which is the point of Equilibrium and also this in turn also affects the demand for the products and also supply which with respect to time, providing the Graphical summaries of the system responses.

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# Panel Data Regression Method for Evaluating Financial Performance of Commercial Banks in Nigerian 

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#### Abstract

Evaluation of financial performance of the banking sector is an effective measure and indicator to check the soundness of economic activities of a nation because thesector's performance is perceived as the nation's replica of economic activities. The key indicators of banks' financial performance are their return on assets (ROA), which indicates the proportion of profit a company makes in relation to its assets and return on equity (ROE), whichmeasures a corporation's profitability by revealing how much profit a company generates with the money shareholders have invested. Panel data are data on two or more entities for multiple time periods.Therefore, this study sought to model the overall performance of some sampled commercial banks (in terms of ROA and ROE) in Nigeria using panel data regression methods. This performance is modeled in relation to the factors that affect it, which include capital adequacy ratio(CAR), credit risk(CRISK), management, liquidity ratio(LIQ.RAT.) and bank size. The results revealed that capital adequacy ratio (CAR), credit risk (CRISK), and liquidity ratio (LIQ.RAT) have highly significant effects on the estimated ROA model at both $1 \%$ and $5 \%$ significance levels with the given p-values. This model accounted for over $82 \%$ of the total variability in the data. However, for the fitted ROE model, only credit risk (CRISK) and liquidity ratio (LIQ.RAT)


