MHD MIXED CONVECTION FLOW IN MELTING FROM A HEATED VERTICAL PLATE EMBEDDED IN POROUS MEDIUM

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Abstract

The MHD mixed convection flow in melting from a heated vertical plate embedded in porous medium is investigated. The similarity equations are derived and solved numerically using Nachtsheim–Swigert shooting iteration technique along with the sixth order Runge–Kutta integration scheme. Comparisons with previously published work are performed, and the results are found to be in excellent agreement. Results for the non-dimensional velocity and temperature are displayed graphically. During the investigation, it was found that the melting phenomenon decreases the local Nusselt number at the solid-liquid interface.

Keywords: Liquid phase,mixed convection, Magnetic effect, Buoyancy forces

Nomenclature

B_0 Magnetic flux density [T]

C_ρ Liquid specific heat capacity [J/kg K]

C_s Solid specific heat capacity [J/kg K]

Da Darcy number, $\frac{K}{x^2}$

Λ Inertia parameter $\mathrm{Re}\, F \sqrt{Da}$

f Dimensionless stream function

G_r Grashof number ν^2

h Heat transfer coefficient [W/m^2 K]

Ha Hartman number, $\sqrt{\ \ }\ \rho\nu$

Ec Eckert Number $C_p(T_\infty - T_m)$

K Permeability of porous media [m^2]

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κ Thermal conductivity [W/m K]

L Plate length [m]

M Melting parameter, $1+C_s(T_m - T_s)$

Nu Nusselt number k

q_w Heat flux [W/m^2]

Re Reynolds number ν

T Temperature [K]

T_m Melting Temperature [K]

T_s Solid temperature [K]

T_∞ Liquid temperature [K]

u_∞ External flow velocity [m/s]

U, V Velocity in x and y direction [m/s]

x, y Coordinate axes along and perpendicular to plate [m]

Greeks

α Thermal diffusivity [m^2/s]

β Coefficient of kinematic viscosity [m^2/s]

η Dimensionless similarity variable

ρ Liquid density [kg/m^3]

ν Kinematic viscosity [m^2/s]

σ Electrical conductivity of fluid [mho m^{-1}]

θ Dimensionless Temperature, $\dfrac{T - T_m}{T_\infty - T_m}$

ψ Dimensionless stream function [m^2/s]

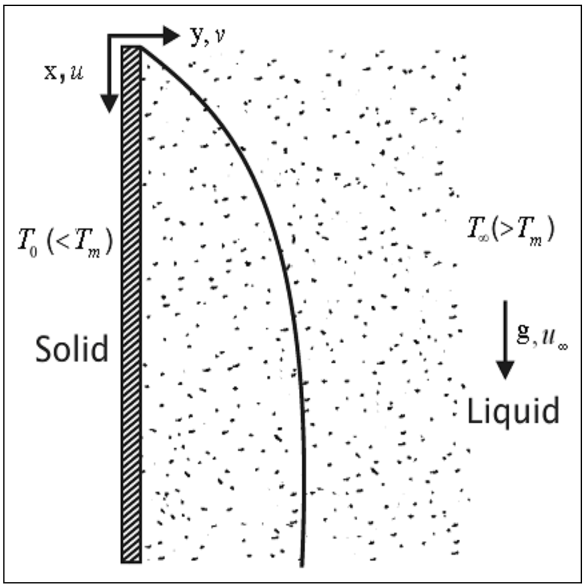
λ constant defined in equation (4)

1. Introduction

The Problem of MHD laminar flow through a porous medium has become very important in recent years because of its possible applications in many branches of Science and Technology, particularly in the field of Agricultural Engineering to study the underground water resources, seepage of water in river beds; in Chemical Engineering for filtration and purification process; in Petroleum Technology to study the movement of natural gas, oil and water through the oil reservoirs. There has been a renewed interest in MHD flow and heat transfer in porous and clear domains due to the important effect of magnetic field on the boundary layer flow control and on the performance of many systems using electrically conducting fluid such as MHD power generators, the cooling of nuclear reactors, plasma studies, purification of molten metal’s from non-metallic inclusion, geothermal energy extractions etc. Many problems of MHD Darcian and non- Darcian flow of Newtonian fluid in porous media have been analyzed and reported in the literature. Chamkha (1997) analyzed the hydromagnetic natural convection flow from an isothermal inclined surface adjacent to thermally stratified porous medium. Bian *et al.* (1996) have studied the natural convection in an inclined porous medium with the effect of electromagnetic field. Aldoss *et* al. (1995) has considered the magneto hydrodynamic mixed convection flow from a vertical plate embedded in a porous medium. Tashtoush (2005) analyzed the effects of magnetic and buoyancy on melting from a vertical plate embedded in saturated porous media. The unsteady MHD combined convection over a moving vertical sheet in a fluid saturated porous medium with uniform surface heat flux was studied by El-Kabeira *et al.* (2007). Afify (2007) analyzed the effects of variable viscosity on non-Darcy MHD free convection along a non-isothermal vertical surface in a thermally stratified porous medium.

Effect of melting and thermo diffusion on natural convection heat and mass transfer of a non-Newtonian fluid in a saturated non-Darcy porous medium was studied by Kairi and Murthy (2009). It is noted that the velocity, temperature and concentration profiles as well as the heat and mass transfer coefficients are significantly affected by the melting phenomena and thermal-diffusion in the medium. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effect of thermal radiation and mass diffusion. Bakier *et al.* (2009) studied Group method analysis of melting effect on MHD mixed convection flow from a radiative vertical plate embedded in saturated porous medium for Newtonian fluids. He developed linear transformation group approach to simulate problem of hydro magnetic heat transfer by mixed convection along vertical plate in a liquid saturated porous medium in the presence of melting and thermal radiation effects for opposing external flow. He studied the effects of the pertinent parameters on the rate of the heat transfer in terms of the local Nusselt number at the solid-liquid interface. More recently melting and radiation effects on mixed convection from a vertical surface embedded in a non-Newtonian fluid saturated non-Darcy porous medium for aiding and opposing external flows is analyzed by Chamkha *et al*. (2010). They obtained representative flow and heat transfer results for various combinations of physical parameters.

Of interest, among the reviewed articles in this work is Tashtoush (2005), in which the combined effect of magnetic and buoyancy influences is considered on melting with constant temperature from a vertical plate. However, several fluids of interest exist with variable temperature for which results in (2005) are not applicable. The present article therefore is aimed at addressing the limitation and extends its results to fluids with variable properties in presence of viscous dissipation.



2. Mathematical Formulation

Let us consider the problem of magnetohydrodynamic combined heat convective steady incompressible laminar boundary layer flow of a gray optically thick electrically conducting viscous Newtonian fluid in a porous medium adjacent to vertical flat plate with opposing external flow U_∞ in the presence of a transverse magnetic field. It is assumed that this plate constitutes the interface between and solid phases during melting inside the porous matrix as shown in Fig. 1. The uniform transverse magnetic field to the plate surface is applied. The temperature of the plate, T_m is the melting temperature of the material occupying the porous matrix. The liquid phase temperature is T_∞. The temperature of the solid far from the interface is T_0. The governing equations for the problem can be written as Bakier (1997) and Tashtoush (2005)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$\left(1+\frac{\sigma B_0^2}{\rho v}+\frac{2F\sqrt{K}}{\upsilon}u\right)\frac{\partial u}{\partial y}=\frac{-Kg\beta}{v}\frac{\partial T}{\partial y} \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\mu\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0^2u^2 \quad (3)$$

The boundary conditions for the present problem are as follows:

$$y=0,\quad T=T_m(=T_\infty+Ax^\lambda),\ k\frac{\partial T}{\partial y}=\rho\left[1+C_s\left(T_m-T_s\right)\right]v(x,0)$$ **(4)**

$$y\to\infty,\ T\to T_\infty,\ u\to U_\infty$$ **(5)**

Where u and v are Darcy's velocity in the x and y directions, also $K, \alpha, \rho, \sigma, \nu$, β, g and T are the inertia coefficient, the permeability, liquid thermal diffusivity , liquid density, electric conductivity, kinematic viscosity, thermal expansion coefficient, acceleration due to gravity and temperature respectively. The first boundary conditions on the melting surface simply stated that the temperature of the interface equals the melting temperature of the material saturating the porousmatrix. The second condition at y = 0 is a direct result of a heat balance. It states that the heat conducted to the melting surfaceis equal to the heat of melting plus the sensible heat required to raise the temperature of the solid T_0 to is melting temperature T_m, (Baker (997), Epstein and Cho (1976)])..

To seek similarity solution to equation (2) and (3) with boundary condition (4) and (5), we introduce the following dimensionless variables

$$\eta=\sqrt{\frac{u_\infty}{\alpha}}\ y\ x^{\frac{\lambda-1}{2}}\qquad \psi=\sqrt{\alpha u_\infty}\,x^{\frac{\lambda+1}{2}}\,f(\eta),\quad \theta(\eta)=\frac{T-T_m}{T_\infty-T_m}$$ (6)

Substituting equation (6) into equation (2) and (3), we obtain the following transformed governing equations:

$$\left(1+Ha^2+2\Lambda f'\right)f''+\frac{Gr}{\mathrm{Re}}\theta'=0$$ (7)

$$\theta''+\frac{1}{2}(1+\lambda)f\theta'-\lambda f'\theta+Ec\left(f''\right)^2+\frac{Ha^2Ec}{Da}f'^2=0$$ (8)

The boundary conditions are

$$\eta=0,\theta=0,\ f(0)+2M\theta'(0)=0$$ (9)

$$\eta\to\infty,\theta=1,\ f'=1$$ (10)

Where $M=\dfrac{C_p\left(T_\infty-T_m\right)}{1+C_s\left(T_m-T_s\right)}$ is the melting parameter, $\mathrm{Re}=\dfrac{U_\infty x}{\nu}$ is the Reynolds number,

$\Lambda=\mathrm{Re}\,F\sqrt{Da}$ is the inertia parameter, $Gr=\dfrac{Kg\beta_T\left(T_\infty-T_m\right)x}{\nu^2}$ is the Grashof number,

$Ec = \frac{u_0^2}{C_p(T_\infty - T_m)}$ is the Eckert number, $Da = \frac{K}{x^2}$ is the Darcy number and $Ha = \sqrt{\frac{\sigma B_0^2}{\rho \nu}}$ is the Hartman number.

The ratio $\frac{Gr}{\mathrm{Re}}$ in equation (7) is a measure of the relative importance of free and force convection and is the controlling parameter for the present problem.

The heat transfer rate along the surface of the plate q_w can be computed from the Fourier heat conduction law.

$$q_w = -k \left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{11}$$

The heat transfer results can be represented by the local Nusselt number Nu, which is defined as

$$Nu = \frac{hx}{k} = \frac{q_w x}{k(T_m - T_\infty)} \tag{12}$$

Where h denotes the local heat transfer coefficient and k represent the liquid phase thermal conductivity. Substituting equation (6) and (11) into equation (12) we obtain

$$\frac{Nu}{Ra^{\frac{1}{2}}} = \theta'(0). \tag{13}$$

3. Numerical Methods

The dimensionless equations (7)-(8) together with the boundary condition (9)-(10) are solved numerically by means of sixth order Runge-kutta methods coupled with shooting technique. The solution thus obtained is matched with the given values of $f'(\infty)$ *and* $\theta(0)$. In addition, the boundary condition $\eta \to \infty$ is approximated by $\eta_{\max} = 4$ which is found sufficiently large for the velocity and temperature to approach the relevant free stream properties.

Table I. : Values of $\theta'(0)$ and $f(0)$ for different values of buoyancy parameter and melting parameters, $Ha = 0, \lambda = 0, \Lambda = 0$

		Tashtoush (2005)		**Present work**	
M	Gr/Re	$\theta'(0)$	$f(0)$	$\theta'(0)$	$f(0)$
0.4	0.0	0.4571	-0.3657	0.4570	-0.3656
	1.4	0.6278	-0.5023	0.6278	-0.5022
	20	1.6866	-1.3493	1.6866	-1.3493
2.0	0.0	0.2743	-1.097	0.2743	-1.097
	1.4	0.3807	-1.5231	0.3808	-1.5232
	3.0	0.4747	-1.8988	0.4747	-1.8988
	8.0	0.6902	-2.7607	0.6902	-2.7607
	10.0	0.7587	-3.0290	0.7593	-3.0375
	20.0	1.0382	-4.1529	1.0382	-4.1529

4. Result and discussion

In order to test the accuracy of our results, we have compared our result with those of Tashtoush (2005) without the constant λ. The comparison is found to be in excellent agreement, as presented in table1. Numerical results are presented graphically for the mixed convection (Gr/Re) ranging from 0.0 to 30.0, melting parameter ranging from 0.0 to 10, magnetic field ranging from 0.0 to 0.4, constant λ ranging from 0.0 to 0.4 and Eckert number ranging from 0.0 to 2.0. Figures 2 and 3 depict the effect of mixed convection parameter (Gr/Re) in velocity and temperature distribution for $\lambda=1/5$ respectively. It should be mentioned that increases in the values of Gr/Re have a tendency to increase the buoyancy effect due to temperature differences and this leads to increase in slip velocity on the plate in Fig. 2. However, the thermal boundary layer thickness decreases with increase in Gr/Re and it results to increase in the fluid temperature as shown in Figure 3.

Figures 4 and 5 depict the influence of melting parameter (M) on velocity and Temperature profiles respectively. It is obvious that increasing the melting parameter causes higher acceleration to the fluid flow which in turn, increases its motion and causes decrease in temperature. This is established by respective increases in the boundary layer thickness of velocity and temperature..

The effect of flow inertia (Λ) on the velocity profiles is shown in figure 6. It can be seen that as Λ increases the velocity of the slip on the plate decreases, this decrease in slip velocity is observed to cause an increase in the thermal boundary layer thickness which consequently lead to a decrease in the temperature as shown in figure 7.

Figures 8, 9 depict the effect of magnetic parameter (Ha) on the velocity and temperature distribution, respectively. It can be seen that application of magnetic field normal to the flow of an electrically conducting fluid gives rise to a resistive force that acts in the direction opposite to that of flow. Thus, thermal boundary layer thickness is significantly increased. These behaviors are depicted in the respective decrease in velocity as well as an increase in the temperature as the magnetic parameter Ha is increased.

Fig 10, 11 depict the influence of increasing the constant parameter (λ) on the behavior of the velocity and temperature profile, respectively. It can be seen that increasing the value of constant λ increases the temperature of the plate and has a tendency to accelerate the flow. This in turn increases the velocity and decreases the temperature profile. Moreover, as λ increases, hydrodynamic and thermal boundary layers increase.

Figures 12, 13 show the effect of Eckert number (*Ec)* on velocity and temperature profiles in the mixed convection flow. It is seen from the figures that the velocity curve decreases with the increase of Eckert number. However, with the increase of Eckert number, there is significant increase in heat generation due to fluid motion and this translates to temperature increase as shown in figure 13.

FIGURES

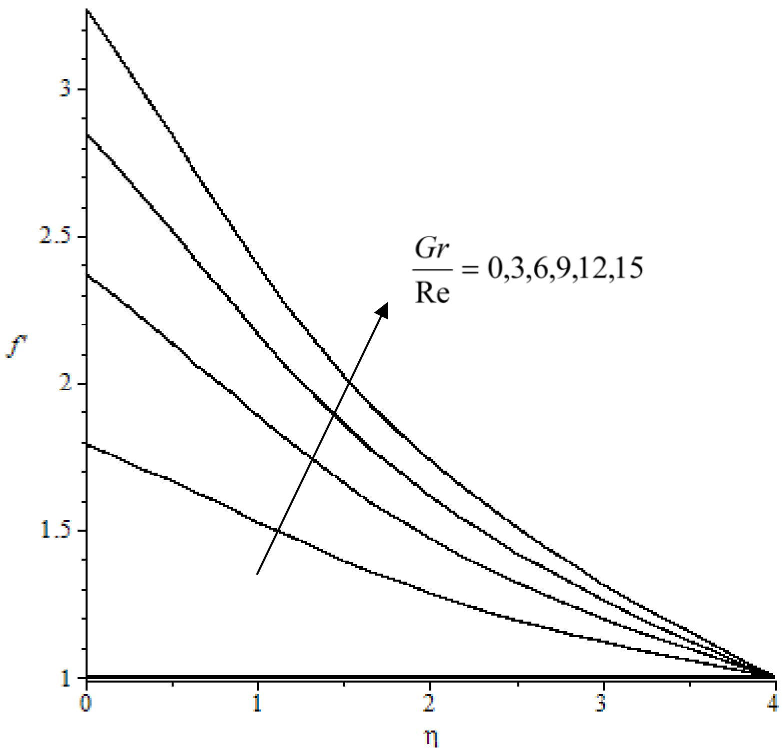


Fig. 2. Velocity profile for different values of Gr/Re, λ=0.3,M=1.0,$\Lambda = 1.0$,Ec=0.1, Ha=0.1 and Da=0.1

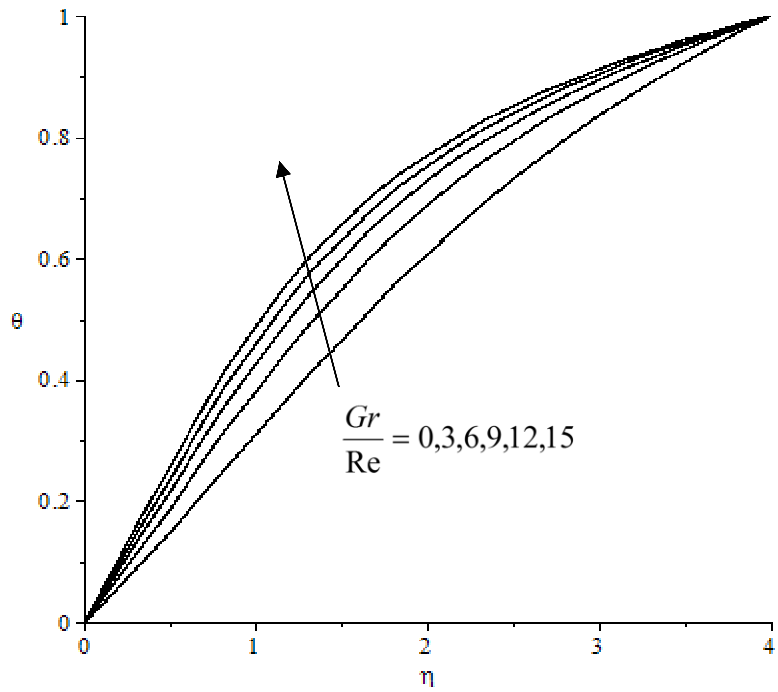


Fig. 3. Temperature profile for different values of Gr/Re, λ=0.3,M=1.0, $\Lambda = 1.0$,Ec=0.1, Ha=0.1 and Da=0.1

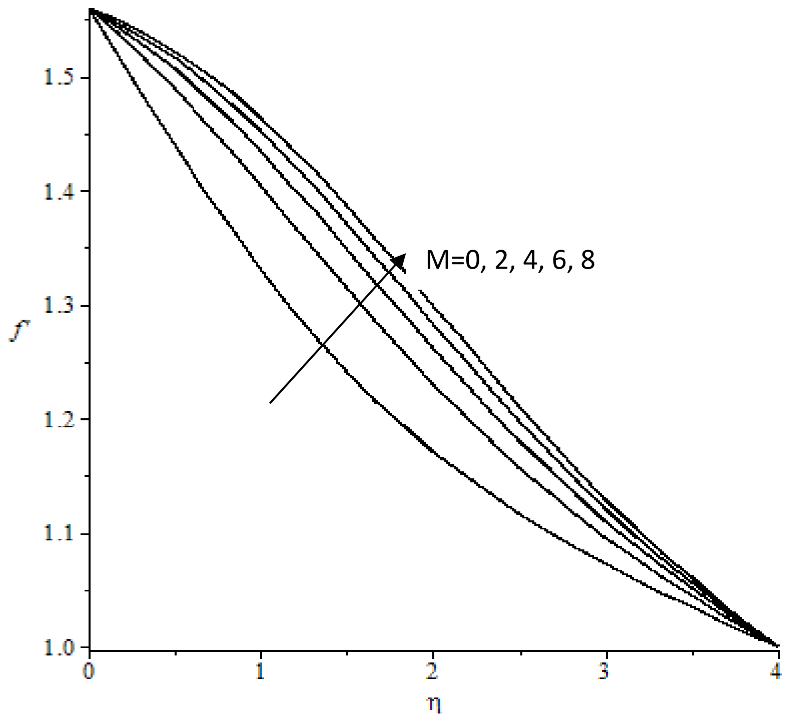


Fig. 4. Velocity profile for different values of M,Gr/Re=2.0, λ=0.3, $\Lambda = 1.0$,Ec=0.1, Ha=0.1 and Da=0.1

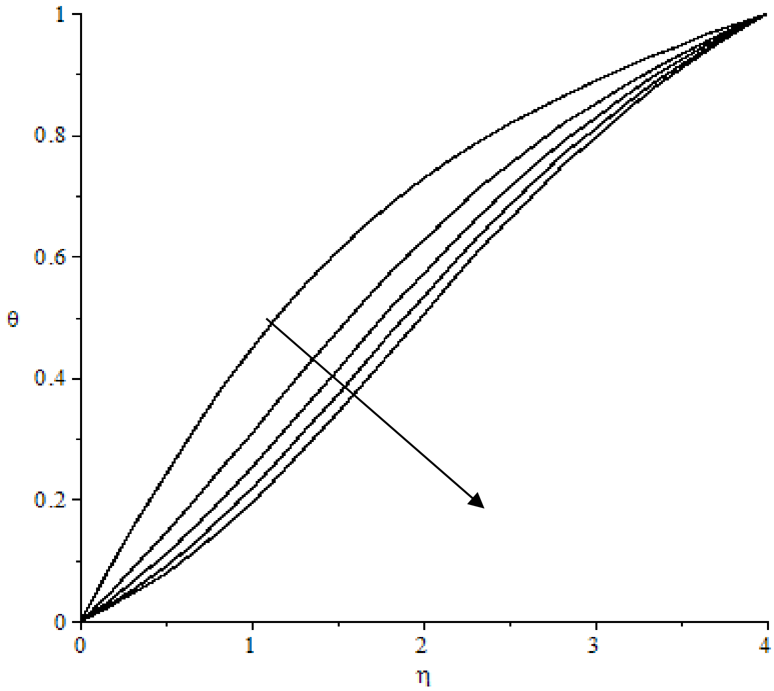


Fig. 5. Temperature profile for different values of M,Gr/Re=2.0, λ=0.3,$\Lambda = 1.0$,Ec=0.1, Ha=0.1 and Da=0.1

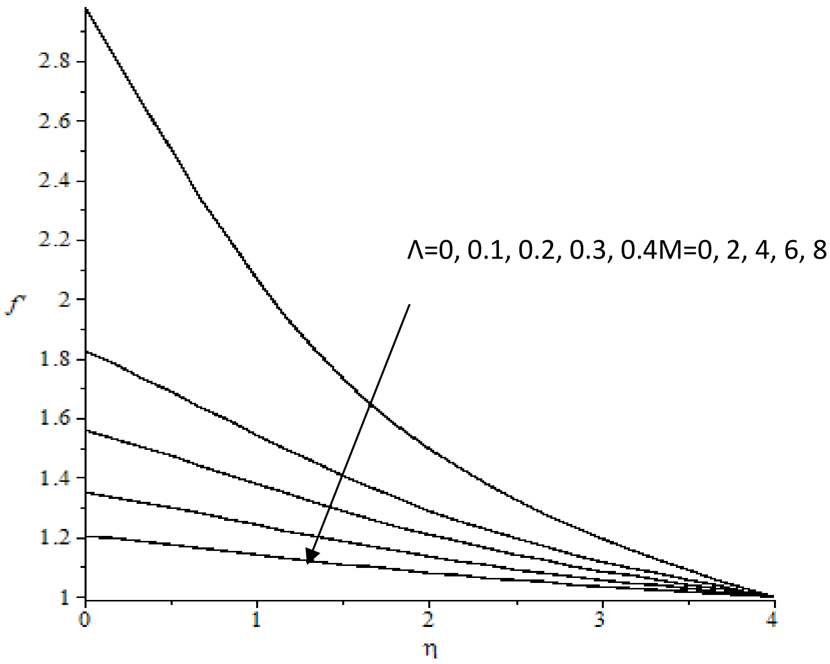


Fig. 6. Velocity profile for different values of Λ M=0.5,Gr/Re=3.0, λ=0.2,Ec=0.1, Ha=0.5 and Da=0.1

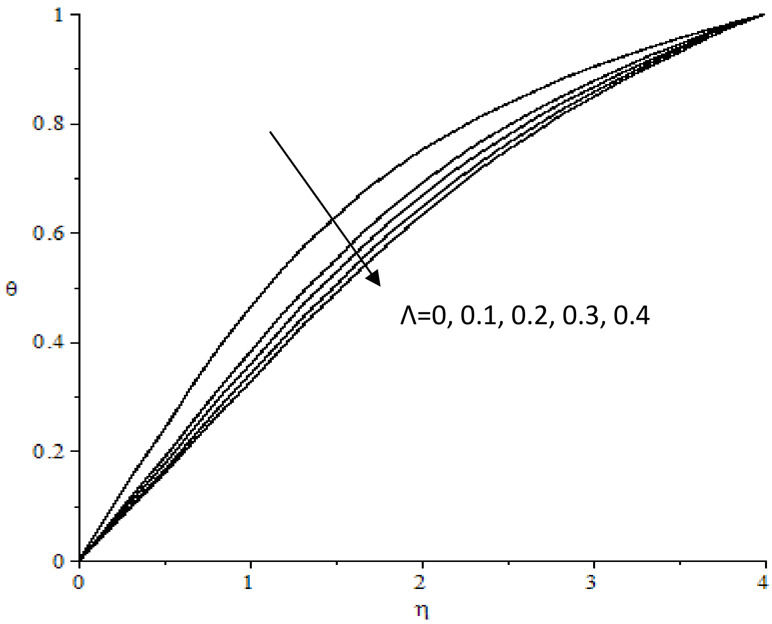


Fig. 7. Temperature profile for different values of Λ M=1.0,Gr/Re=2.0, λ=0.3,Ec=0.1, Ha=0.1 and Da=0.1

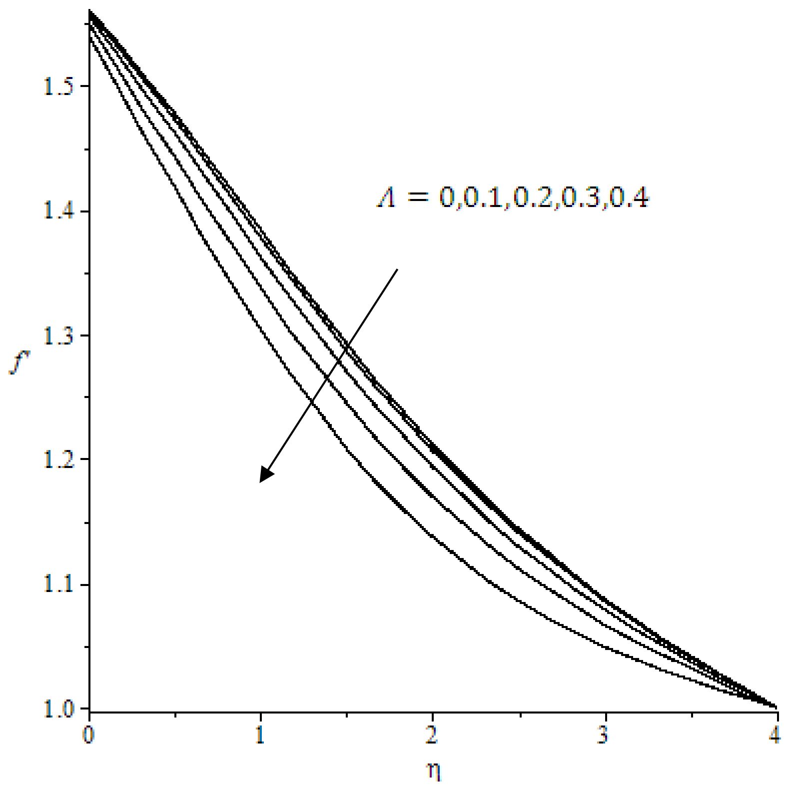


Fig. 8. Velocity profile for different values of Ha, M=1.0, Gr/Re=2.0, λ=0.3, $\Lambda = 1.0$, Ec=0.1, Ha=0.1 and Da=0.1

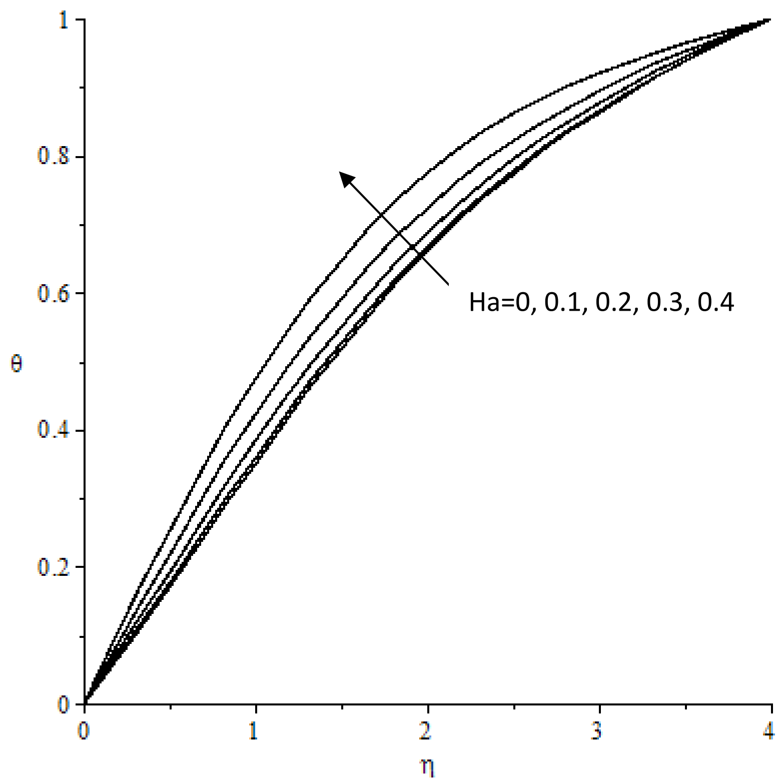


Fig.9. Temperature profile for different values of Ha, M=1.0,Gr/Re=2.0, λ=0.3, $\Lambda = 1.0$, Ec=0.1, Ha=0.1 and Da=0.1

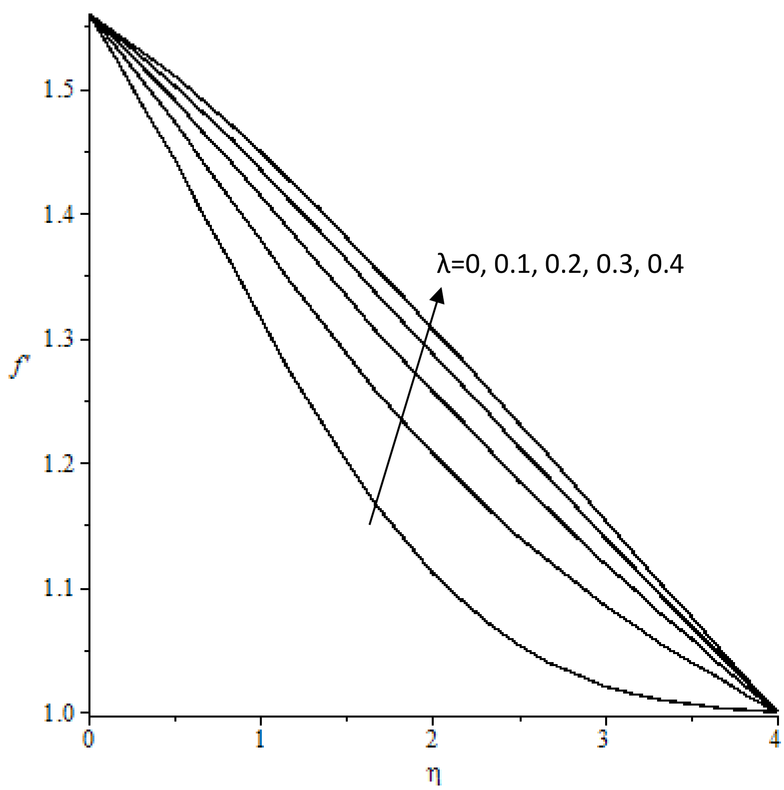


Fig. 10. Velocity profile for different values of λ , M=1.0,Gr/Re=2.0, $\Lambda = 1.0$,Ec=0.1, Ha=0.1 and Da=0.1

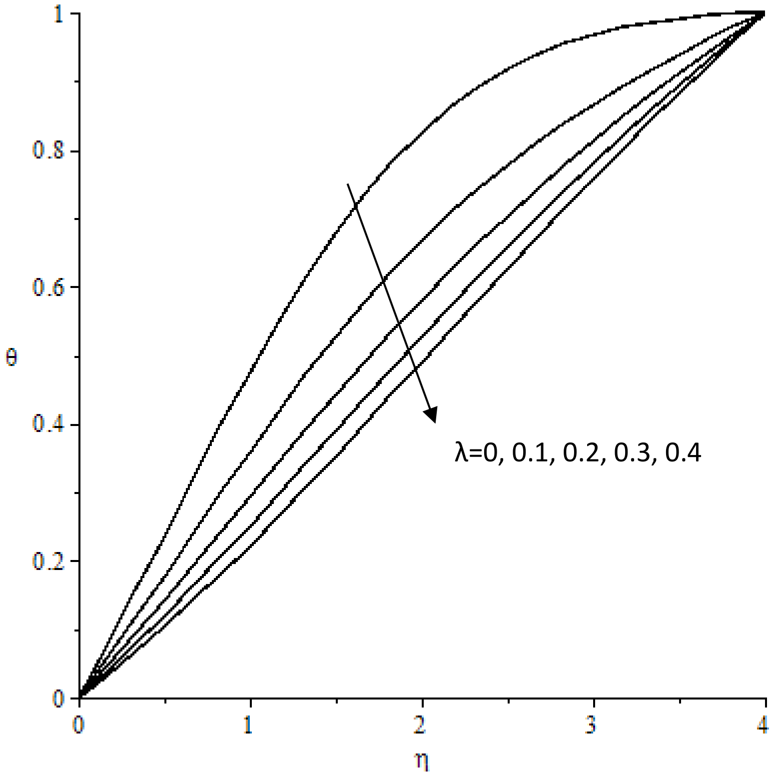


Fig. 11. Temperature profile for different values of λ=0.3 , M=1.0, Gr/Re=2.0, $\Lambda = 1.0$,Ec=0.1, Ha=0.1 and Da=0.1

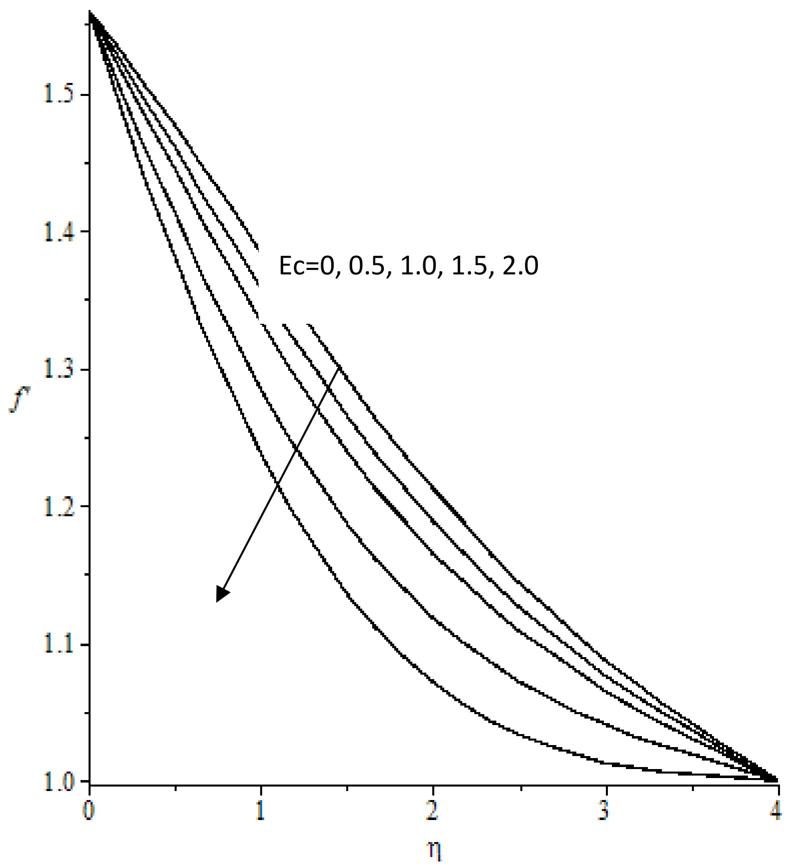


Fig. 12. Velocity profile for different values of Ec,λ =0.3, M=1.0,Gr/Re=2.0, $\Lambda = 1.0$, Ha=0.1 and Da=0.1

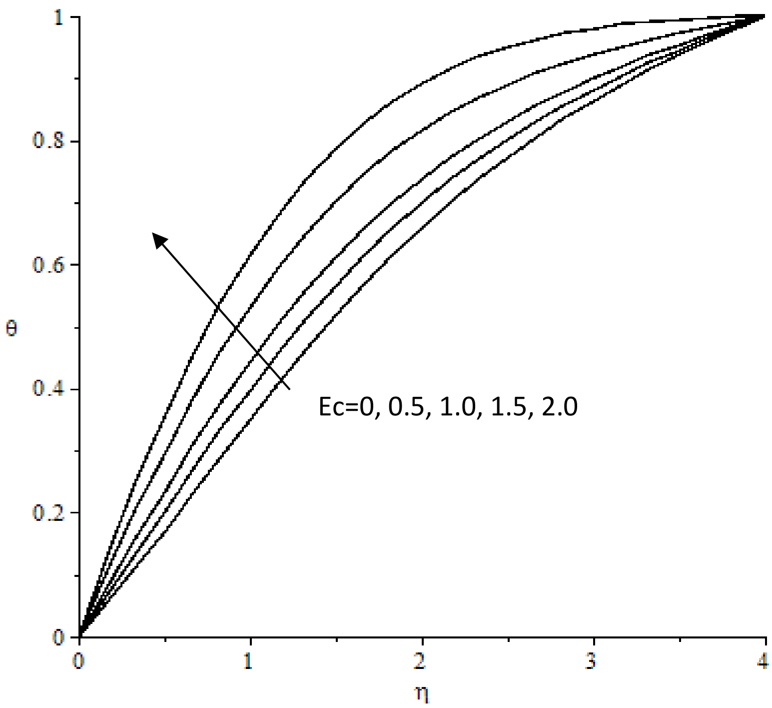


Fig. 13. Temperature profile for different values of Ec,λ =0.3, M=1.0,Gr/Re=2.0, $\Lambda = 1.0$, Ha=0.1 and Da=0.1

5. Conclusion

In this study The MHD mixed convection flow in melting from a heated vertical plate having variable temperature embedded in porous medium is analyzed. The heat transfer coefficients are obtained for various values of flow influencing parameters. It is noted that the velocity and temperature profiles as well as the heat transfer coefficients are significantly affected by the melting in the medium. The major conclusion is that the heat transfer coefficients are reduced with increasing melting parameter and grows with increasing Gr/Re. The results obtained in the present work have been validated by works in existing literature and an excellent agreement is found.

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