Mathematical Modeling of Anaerobic Co-digestion of Corn Stover Hydrochar and Food Waste for Control Design Application

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Abstract
Energy crisis and lignocellulose biomass waste management are the greatest issues that the world is facing today**.** This problem can be overcome by anaerobic digestion (AD) of lignocellulose biomass waste to produced methane and carbon dioxide. To optimize and control this process accurately, mathematical model of the process is needed. Unfortunately, modeling of AD process often results in high order differential equations with many unknown parameters, a fact that complicate controller design. In this study, lignocellulose biomass (corn stover hydrochar) obtained from hydrothermal carbonization at a temperature, residential time, and biomass/water ratio of 215 ºC, 45 min and 0.115 respectively, was added to the bioreactor as a substrate inoculated with food waste and cow dung to generate biogas. Three batch processes were performed with organic loading of 0.2 m^3, two part was utilized for parameter estimation and model construction while the other part for model validation. The estimation of the model parameters was based on dynamical experiments. It is satisfied employing two techniques: 4th order Runge Kutta numerical techniques and deterministic sequential quadratic programming (SQP) algorithm. Simulation of the developed mathematical model is conducted based on the different data set of the process. The biogas developed model predict the set of the experimental data for all considered process variables with high precision. Future work will entail the application of controller design in AD system.

Keywords: *Anaerobic Digestion, Hydrothermal Carbonization, Mathematical modeling, Controller Design, State Transition Matrix, Deterministic algorithm.*

1. Introduction

The demand of fossil fuels consumption is increasing globally, which is one of the major contributors to carbon dioxide concentration in the atmosphere, leading to global warming (Mohammed *et al.,* 2020). Consideration is recently paid to various renewable energy sources. The utilization and development of lignocellulose biomass as a source of renewable energy is still very scanty equated to its potential. Biogas is a renewable gas that can be utilized to produce electricity and heat. Biogas can also be utilized directly as a vehicle's fuel after an upgrade to biomethane. Corn stover is consider a promising substrate for biogas production. However, due to its complex and rigid structure, its biodegradability during AD is usually low. hydrothermal pretreatment was studied to explore the feasibility of improving the biogas yield of corn stover hydrochar in AD. The mathematical modelling of AD processes is a complex problem, and the selection of a proper method for optimization is fundamental for the accurate estimation of model parameters. There are a few techniques that are appropriate in the considered case problem. Among them are conventional optimization methods, such as SQP (Gill and Wong, 2010). The aim of this study is to develop a mathematical model of anaerobic co-digestion of corn stover hydrochar and food waste for control design application, this aim is achieved through the following objectives: (1) the model parameters were obtained by integrating the AD differential equation with 4th order Runge–Kutta technique followed by SQP. (2) the biogas prediction model was developed by obtaining the state transition matrix of the AD system.

2. Materials and Methods

The biogas experimental data utilized in this study for model development and validation is detail in (Shaba *et al.,* 2022).

2.1. Mathematical model of AD

The mathematical model of the AD system was constructed by concatenating the two differential equations and one algebraic equation, as shown in Eq. (1).

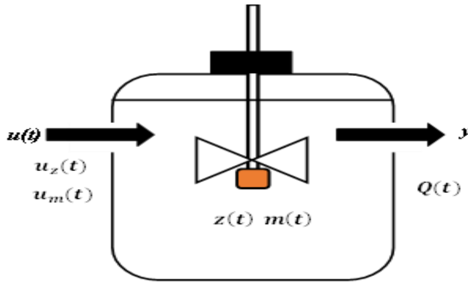

Figure. 1 Graphical representations of a batch-type bioreactor used for anaerobic digestion ($u_z(t)$ = bacterial input (kg/ (m^3 h)), $u_m(t)$ = substrate input (kg/ (m^3 h)), $z(t)$ = bacteria concentration (kg/m^3), $m(t)$ = substrate concentration (kg/m^3), and $Q(t)$ = bi-gas flow rate (m^3/h)).

one for bacterial growth and the other for substrate disintegration. The output equation was used to describe the biogas flow rate caused by substrate decay and bacterial growth. The growth rates used in these equations were given by a modified Monod equation.

$$\begin{cases} \frac{dz(t)}{dt}(\mu(m)-a)z(t)\left(1-\frac{z(t)}{z_{max}}\right)+u_z(t) \\ \frac{dm(t)}{dt}=-\frac{1}{w}\mu(m)z(t)+u_m(t) \\ y(t)=\left(Lq_1\frac{1}{w}\mu(m)+Lq_2a\right)vz(t) \end{cases} \quad \text{Eq. (1)}$$

$$\mu(m)=\mu_{max}\frac{m(t)}{K_s+m(t)+bm^2(t)}$$

Perturbation theory was applied near the point at which equilibrium was reached using the nonlinear model by ignoring the second order and higher-order terms after Taylor expansion of the two-variable functions, to give the linear-time state–space model shown in Eq. (2).

$$\frac{dX(t)}{dt} = A_q X(t) + B_q U(t) \qquad Eq.(2)$$

$X(t)$ therefore, represents vectors of the state variables and $U(t)$ represents the vectors of the manipulated variables that are partial derivative matrices for the equilibrium point of the Jacobian matrix.

2.2. Parameter identification

The set of differential equations were integrated using fourth order Runge Kutta technique follow by SQP which was resolve into mathematical modeling problem: To find a set of design parameters, $x = \{x_1, x_2, \dots x_n\}$ .The discrepancy between the model and real data is configured as an optimization criterion:

$$J = \sum_{i=1}^{k}\sum_{j=1}^{m}\left(Y_{ij} - \hat{Y}_{ij}\right)^2 \qquad Eq(3)$$

where k and m are the number of process variables and data points, Y_{ij} and $\hat{Y}_{ij}$ represents the experimental data and model predicted values.

Fourth order Range-Kutta technique was also used to determine the equilibrium point of the state variable equation by equating the input value to be $u_z = 0, u_m = 0.01$.

The coefficient of system matrix (A_q) was determined by the equilibrium point obtained from the state variable equations.

$$A_q = \begin{bmatrix} 0.0000001853 & 0.009938 \\ -0.00333 & -0.000683 \end{bmatrix} \qquad Eq(4)$$

The response of the system is obtained by solving the state transition matrix which given by $Eq(5)$

$$X(t) = e^{A_q t}X(0) + \int_0^t e^{A_q(t-\tau)}B_q U(\tau)d\tau \qquad Eq(5)$$

The developed state transition matrix of the biogas system is given by $Eq(6)$

$$\begin{bmatrix} e^{-at}\cos\omega t - \frac{a_{22}+a}{\omega}e^{-at}\sin\omega t & \frac{a_{12}}{\omega}e^{-at}\sin\omega t \\ \frac{a_{21}}{\omega}e^{-at}\sin\omega t & e^{-at}\cos\omega t - \frac{a_{11}+a}{\omega}e^{-at}\sin\omega t \end{bmatrix}$$

Eq (6)

From $Eq(6)$, the biogas prediction model was developed given in $Eq(7)$

$$u_z(t) = \begin{pmatrix} z_0(e^{-0.000341t}\cos 0.000576t + 0.0594e^{0.000341}\sin 0.00576t) + \\ m_0(1.7253e^{-0.000341t}\sin 0.00576t) \end{pmatrix}$$

$$u_m(t) = \begin{pmatrix} z_0(-0.5781e^{-0.000341t}\sin 0.00576t) + \cdots \\ m_0(e^{-0.000341t}\cos 0.00576t + 0.0594e^{-0.000341t}\sin 0.00576t) \end{pmatrix}$$

$$\mu(m) = \mu_{max}\frac{u_m(t)}{K_d + u_m(t) + bu_m^2(t)}$$

$$y(t) = \left(L_{q1}\cdot\frac{1}{W}\cdot\mu(m) + L_{q2}\cdot a\right)\cdot u_z(t).V \qquad Eq\ (7)$$

3. Results and Discussion

Table 1, present the results of the parameter estimation of the AD model by SQP algorithm.

Table 1. Model parameter Identification

parameters	Deterministic Algorithm
μ_{max}	0.0859
K_s	4.8712
a	0.0331
W	6.3511
b	0.0879
L_{g1}	0.4233
L_{g2}	0.0723

The model describes the experimental data well, where the lower and upper constrain are set on the SQP algorithm.

Figure 2 show the comparison of the experimental data and model prediction of the biogas by SQP.

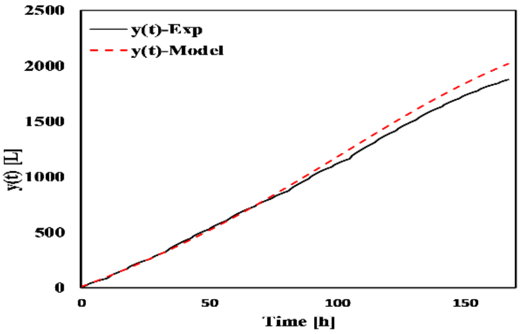

Figure 2. Comparison of the experimental data and model predictions for biogas by SQP models—identification results

The developed biogas model was simulated setting the initial state variable at $X(0) = \begin{bmatrix}10\\0\end{bmatrix}$ and $U(0) = \begin{bmatrix}0\\9\end{bmatrix}$

Figure 3 show the comparison of the experimental data and biogas model prediction which illustrate high precision.

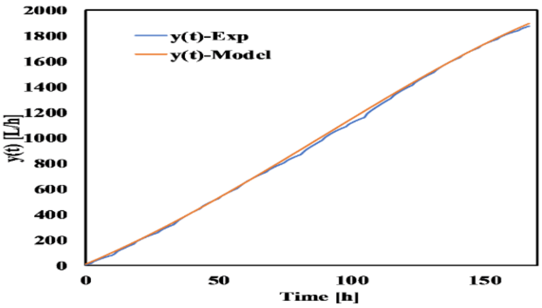

Figure 3 experimental data Vs biogas model prediction

4. Conclusions

In this study, a new model of the anaerobic co-digestion of corn stover hydrochar and food waste is developed which show high precision. Parameter identification is made using two combine methods deterministic SQP algorithm and 4th Runge Kutta method.

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