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ANALYTICAL COMPARISON OF SQUARE CIRCLE PACKING AND TRIANGULAR CIRCLE PACKING BASED ALGORITHMS FOR MAXIMUM COVERAGE AREA DENSITY OF MULTIPLE DEPLOYED UAV-ABS

Suraju A. Fadare [1,3], Nathaniel Salawu[1] , Henry Ohize [2]

[1]Department of Telecommunication Engineering, Federal University of Technology, Minna, Nigeria
[2]Department of Electrical & Electronic Engineering, Federal University of Technology Minna, Nigeria
[3]Department of Electrical & Electronic Engineering, Federal Polytechnic, Ilaro, Nigeria
suraju.pg207326@st.futminna.edu.ng, nathsalawu@futmina.edu.ng, henryohize@futminna.edu.ng

ABSTRACT

Circle packing theory (**CPT**) proffers square circle packing (**SqCP**) and triangular circle packing (**TriCP**) as two basic tools for closed-form packing or placement optimizations of 2D circular geometrical objects. Placement optimization of multiple deployed unmanned aerial vehicles with mounted aerial base stations (**UAV-ABS**) had been classified as a circle placement problem that **CPT** as a tool could resolve. However, the optimization constraints utilized by most researchers are more favourable to **SqCP** than **TriCP**. Furthermore, the non-linear constraints need to be simplified to linear forms. A proper analysis of the basic geometry of both **SqCP** and **TriCP** was used to derive linear mathematical models for both. The mathematical models were used to develop **SqCP**-based and **TriCP**-based algorithms for the placement of multiple deployed **UAV-ABS** in target rectangular border regions and a given optimal wireless network (**WN**) coverage radius. The algorithms equate the 2D position of the UAV-ABS on the horizontal plane as the centroid of the circle for the respective circular **WN** coverage regions. Both algorithms were implemented using MATLAB® 2023a and simulated using a rectangular border region of 3 km by 3 km and coverage radii ranging from 50 m to 1000 m at an interval of 10 m. Coverage area density (**CAD**) was used as the performance metric. The **TriCP-**based algorithm was observed to perform better with an average of 13.97% higher **CAD**. Its minimum obtainable CAD was 44.57% but SqCP-based alogorithm recorded 19.90%. The outcome of the research indicated that the TriCP is more appropriate for optimal deployment or placement of UAV-ABS and any other circular objects on a 2D plane when coverage area density is the utmost optimization objective.

Keywords: UAV, ABS, Coverage Area Density, Circle Packing, Optimization

INTRODUCTION

Unmanned Aerial Vehicles – Aerial Base Station (**UAV-ABS**) can be described as a small aircraft or drone that is improvised with telecommunication base station capability and Command and Control (**CNC**) systems can be remotely controlled or preprogrammed (Mozaffari et al., 2019a; Sivalingam et al., 2020). It is used as a cost-effective solution for the provision of telecommunication services (Arabi et al., 2018; Romero et al., 2022; Viet & Romero, 2022). Its adaptable functions, agility, mobility, ability to be located at different different altitudes and flexible programmable control systems are prominent attributes for its applications in telecomunication engineering (Azari et al., 2019; Bulut & Guevenc, 2018; Khamidehi & Sousa, 2019; Mozaffari et al., 2019a; Nunns et al., 2019; Viet & Romero, 2022). In telecommunication engineering, **UAV-ABSs** are utilized as mobile or flying base stations (**BS**), wireless network relays and access points (**AP**) to enhance and optimize the capacity and coverage density of wireless networks (**WN**) (Banagar & Dhillon, 2022; Mayor et al., 2022; Mozaffari et al., 2019; Nunns et al., 2019; Sawalmeh et al., 2022).

Most researchers affirmed the placement of multiple **ABS**s is equivalent to the circle packing optimization problem to achieve their cooperative optimal performance, maximum **WN** coverage area density (**CAD**), minimum or no inter-channel interference, and minimal power consumption (Mozaffari et al., 2016; Sawalmeh et al., 2018; Shakhatreh et al., 2021). Base on circle packing theory (**CPT**), there are two basic categories of circle packing for homogenous non-overlapping circle based on pattern of connecting the circles without overlapping. The two categories of the closed form circle packing are – square circle packing (**SqCP**) and hexagonal or triangular circle packing (**TriCP**). However, majority formulate their optimization constraints to favour square packing than rectangular packing pattern for target rectangular region. Visual examination of Figure 1 and Figure 2 with the basic knowledge of geometry, it could be affirmed that the **TriCP** has greater tendency to have lower areas of blind zones and higher coverage density than the **SqCP** between all the three and four connected adjacent circles that form the respective triangle or square (*Circle Packing -- from Wolfram MathWorld*, n.d.; Fukshansky, 2011). Using **CAD** as performance the performance metric, the aim of this paper is to analytically compare both CPT based circular object packing techniques in reference to placement optimization of multiple **UAV-ABSs**. The significance of the work is to encourage formulation of optimization constraints that cater for **TriCP** approach instead of the common presented constraint in relevants literatures that is more favourable to **SqCP**. The seconadary benefits of the paper is the possibility of utilizing a linear mathematical model algorithms to simulate packing of closed form circles at given radius inside a given target rectangular region to achieve maximum CADs for both SqCP and TriCP approaches. The centroids of the circles depict the 2D Cartesian postions of the UAV-ABS inside the target border region.

LITERATURE REVIEW

The placement of UAV-ABSs/APs for optimal coverage had been an identified a thought-provoking optimization research puzzle that can be realated (Kalantari et al., 2016; Li & Cai, 2017; Mozaffari et al., 2019; Pourbaba et al., 2019; Sun & Masouros, 2018). Pourbaba et al. (2019) proposed a convex optimization based algorithm to find a position for a full-duplex UAV relay in a vehicular network. Li & Cai (2017) proferred an optimization algorithm for UAV based floating relay cell deployment inside the existing macrocell in order to achieve dynamic and adaptive coverage. Kalantari et al. (2016) developed a heuristic based algorithm for the 3D placement of the UAV-ABSs in various geographical areas with different user densities. Sun & Masouros (2018) proffered two methods for drone positioning for user coverage maximization. The first method is the successive deployment of aerial BSs, and the second method is the utilization of K-means clustering for simultaneous deployment of multiple aerial BSs. (Al-hourani et al., 2016) affirmed the requirement for optimization of response period of UAV-ABSs for emergency services by minimizing their deployment period while maintaining optimal QoS and servicing coverage radius. The authors worked on maximizing coverage probability within a given coverage radius at specific SNR threshold. Sun & Masouros (2018) modelled the UAV-ABS placement problem as a circle-packing problem with no coverage overlap so that ICI avoided. Furthermore, a placement or deployment optimization technique based on linear approximation was introduced as a solution to multiple and simultaneous deployment technique with the help of K-means clustering to solve the placement problem (Sun & Masouros, 2018, 2019).

Therefore, the UAV-ABS/AP placement problem boils down to a circle placement problem that requires solution that multiple circles in the horizontal plane such that the number of enclosed feasible user service points are maximized (Mozaffari et al., 2016; Sawalmeh et al., 2018; Sun & Masouros, 2019). However the optimization constraints provided by most researchers favoured **SqCP** compare to **TriCP** (Alzenad et al., 2017, 2018; Hu et al., 2020; Mozaffari et al., 2016; Sawalmeh et al., 2018, 2022; Sun & Masouros, 2018, 2019; Zhao et al., 2020; Zhou et al., 2020). Equation (1) and its other variants in equation (2) were commonly used as the optimization constraints that usually indicate square packing pattern. Here, the $\boldsymbol{m}$x$\boldsymbol{n}$ number of centroids of the **SqCP** based closely packed non overlaping circles were described with the 2D coordinates for both ***x***- and ***y***- Cartesian directions as (x_{ca}, y_{cb}). These centroids were assumed to be the **2D** positioned in horizontal plane for placement of the **UAV-ABS** as shown in Figure 1.

$$(\boldsymbol{x}_{ca+1} - \boldsymbol{x}_{ca})^2 + (\boldsymbol{y}_{cb+1} - \boldsymbol{y}_{cb})^2 \geq 4\boldsymbol{R}^2 \qquad a=1,2\ldots m \;; \; b=1,2,\ldots,n \qquad (1)$$

$$(\boldsymbol{x}_{ca+1} - \boldsymbol{x}_{ca}) \geq 2\boldsymbol{R} \,, (\boldsymbol{y}_{cb+1} - \boldsymbol{y}_{cb}) \geq 2\boldsymbol{R} \qquad a=1,2\ldots m \;; \; b=1,2,\ldots,n \qquad (2)$$

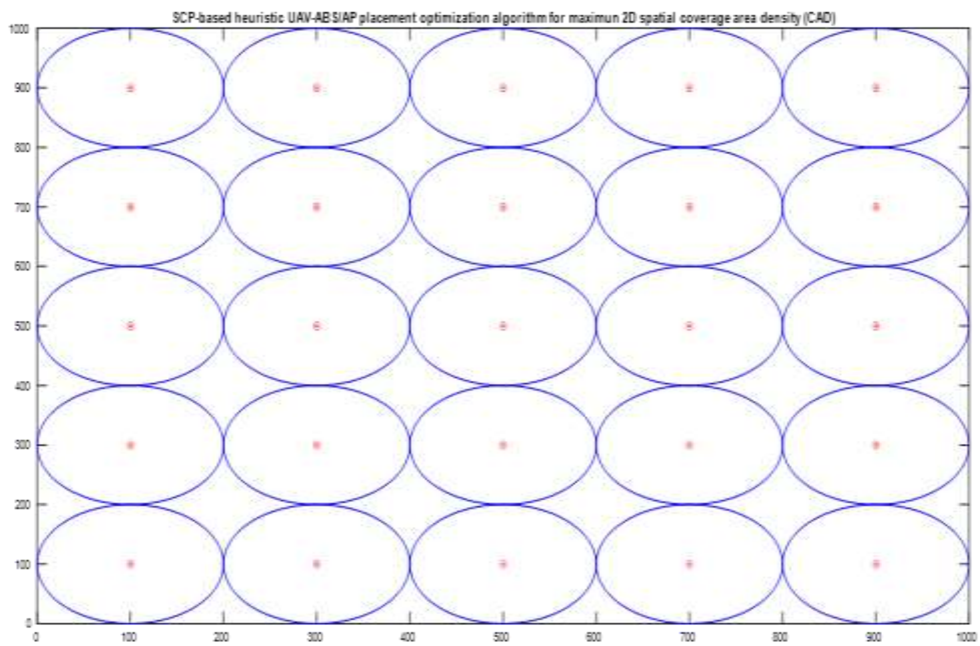

Figure 1: Illustration **SqCP** based UAV-ABS placement with homogenous radius R = 100 m

However, for **TriCP**-based packing, equation (**2**) was modified to equation (**3**) and illustrated in Figure 2. Furthermore, successful shift operation was performed on the horizotal layer or row to avoid overlapping of the circle while mainteining the first pat of the equation 3 as illustrated in Figure 3. Equation one is maintained as the euclidean distance between the centroid is double value of the homogenous radius (**2R**).

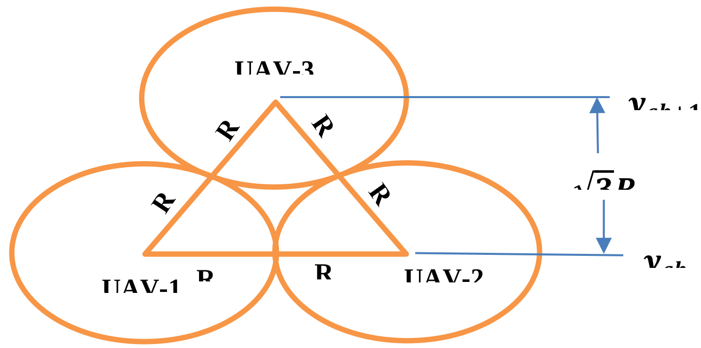

Figure 2: A basic analytical layout for **TRICP**

$$(x_{ca+1} - x_{ca}) \geq 2R \, ; \, (y_{cb+1} - y_{cb}) \geq \sqrt{3}R \qquad \begin{matrix} a=1,2\ldots.m \\ b=1,2,\ldots,n \end{matrix} \; ; \qquad (3)$$

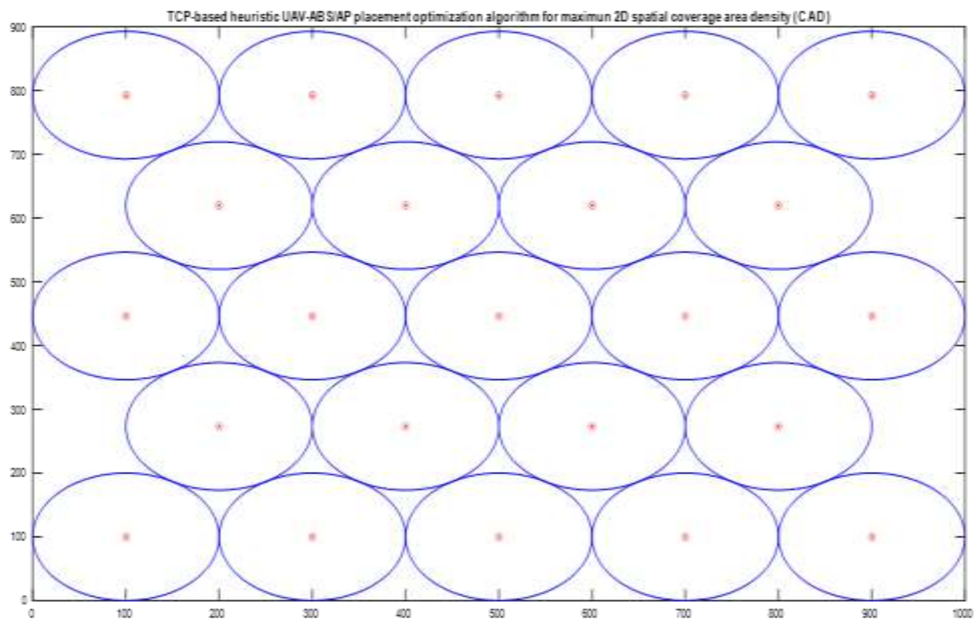

Figure 3: Illustration **TriCP** based **UAV-ABS** placement with homogenous radius R = 100 m

THE RESEARCH METHODOLOGY

The aim of the research is to analytically compared **SqCP** and **TriCP** techniques in optimal placement of UAV-ABS to achieve maximum **CAD**. The **CAD** was choise as the performance metric parameter for the research exercise as an emphaze for the target aim. Utilizing Figure 1 and Equation 1 and Equation 2, two mathematucal models were formed as shown in Equation 4 and Equation 5. A linear algorithm for **SqCP** was developed to place feasible maximum number of 2D circular **WN** on a given coverage radius (covRad) (without overlaping for avoidance of **ICI**) within given rectangualar boundary points of a target region (minX, maxX, minY and maxY). The algorithm generates **M** number of points along ***x***- axis at interval that is twice the size of the given coverage radius. Also, the algorithm generates **N** number of points along ***y***- axis at interval that is twice the size of the given coverage radius. Therefore, the total number of centroid points or feasible positions to place the **UAV-ABS**s is product of **M** and **N**. The 2D locations of the UAV-ABS are described in accordance with the result of the equation as $(x_{acm}, y_{ac.n})$.

$$\boldsymbol{x_{ac,m} = R + (m-1)(2R)\ ;\ x_{ac,m} = 2mR - R} \qquad m = 1, 2, .., N \qquad (4)$$

$$\boldsymbol{y_{ac,m} = R + (n-1)(2R)\ ;\ y_{ac,m} = 2nR - R} \qquad n = 1, 2, .., N \qquad (5)$$

Similarly, utilizing Figure 2 and Equation 1 and Equation 3, two mathematucal models were formed as shown in Equation 5 and Equation 6. Therefore, another linear algorithm was develoiped for the **TriCP** to place feasible maximum number of 2D circular **WN** on a given coverage radius (covRad) (without overlaping for avoidance of **ICI**) within given rectangualar boundary points of a target region (minX, maxX, minY and maxY). Conversely, the **TriCP**- based algorithm generates **M** number of points along ***x***- axis at interval that is equal to the given coverage radius ($\boldsymbol{R}$). In addition, the algorithm generates **N** number of points along ***y***- axis at interval that is equal to the product of square root of 3 and the given coverage radius ($\boldsymbol{\sqrt{3}R}$) . Therefore, the total number of centroid points or feasible positions to place the UAV-ABS is product of **M** and **N**. Also, the 2D locations of the **UAV-ABS**s are described in accordance with the result of the equation as $(x_{ac,m}, y_{ac.n})$.

$$\boldsymbol{x_{ac,m} = mR} \qquad m = 1, 2, .., N \qquad (6)$$

$$\boldsymbol{y_{ac,m} = y_{ac,n} = R + (n-1)\sqrt{3}R} \qquad n = 1, 2, .., N \qquad (7)$$

The use of non-linear contsraint was further circumvent by dividing the horizontal layers or rows of centroids into even and odd layers using even and odd separation of elements of an array using their indices. Various combinatorial forms of each even number index element of ***x***- coordinate array derived from equation (**6**) with even number index element of ***y***- coordinate array derived from equation (**7**) form the centroids to be located at the apex vertex of the equilteral triangle for **TriCP**. Similarly, various combinatorial forms of each odd number index element of ***x***- coordinate array derived from equation (**6**) with odd number index elements of ***y***- coordinate array derived from equation (**7**) form the centroids to be located at the base vertices of the equilteral triangle for **TriCP**.

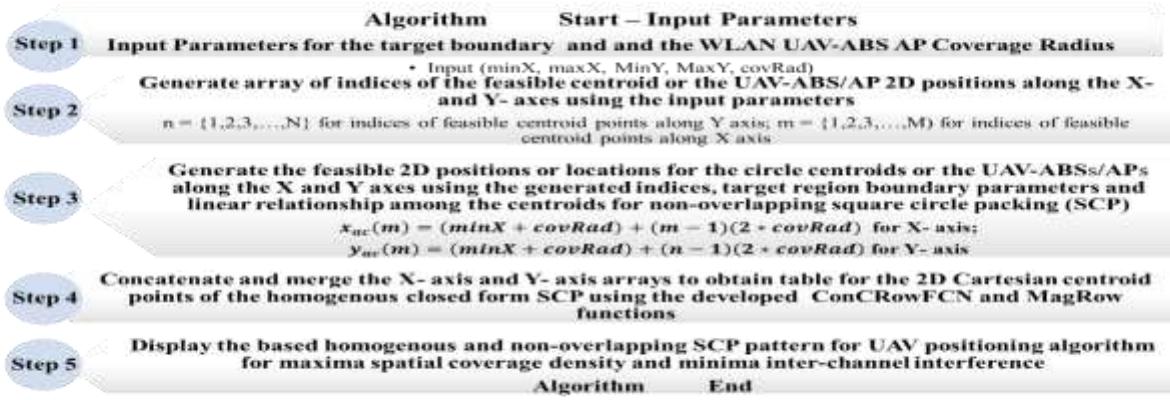

Figure 4: Heuristic linear SqCP-based UAV-ABS/AP placement optimization algorithm

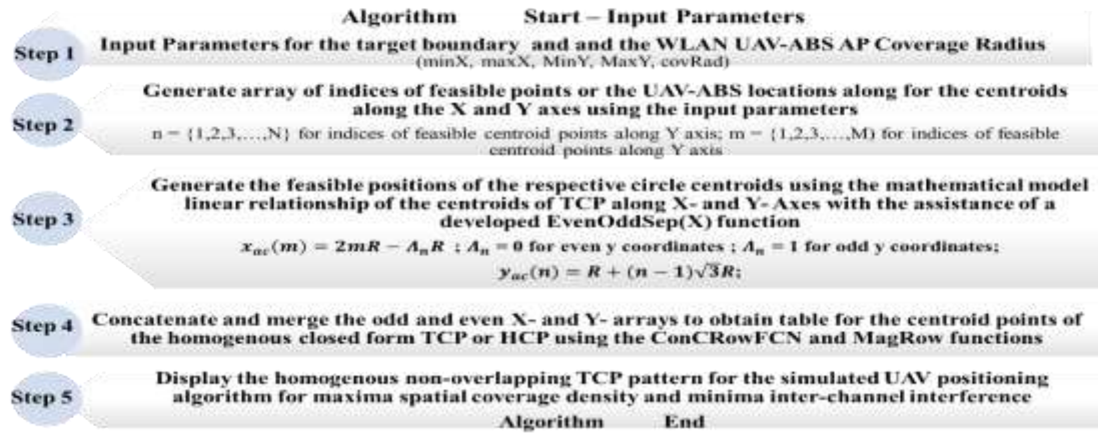

Figure 5: Heuristic linear TriCP-based UAV-ABS placement optimization algorithm

To ease the analytical comparison of **SqCP-** based and **TriCP-** based algorithms for optimal placement of **UAV-ABS**, the developed algorithm in Figure 4 and Figure 5, were generated and simulated using Matlab® 2023a successfully.

SIMULATIONS, RESULTS AND DISCUSSIONS

Matlab® R2023 was utilized to implement and simulate both algorithms. Both algorithms were written to accept the same input parameters – border parameters (minX= 0 m, maxX=3000 m, minY= 0 m and maxY= 3000 m) and the given common optimal **WN** coverage radius ranges from 50 m to 1000m at ainterval of 10 m. The **SqCP-** based algorithm output **nSCent** (maximum number of non-overlapping circles or centroids within the target border region), **SqCAD** (obtined **CAD** based on **SqCP**). Similarly, the **TriCP-** based algorithm output **nTCent** (maximum number of non-overlapping circles or centroids within the target border region), **TriCAD** (obtained **CAD** based on **TriCP**). The results of the simulations for the overall **CAD** for both algorithms (**SqCP-** based and **TriCP-** based algorithms) were presented as shown in Figure 6 to Figure 8.

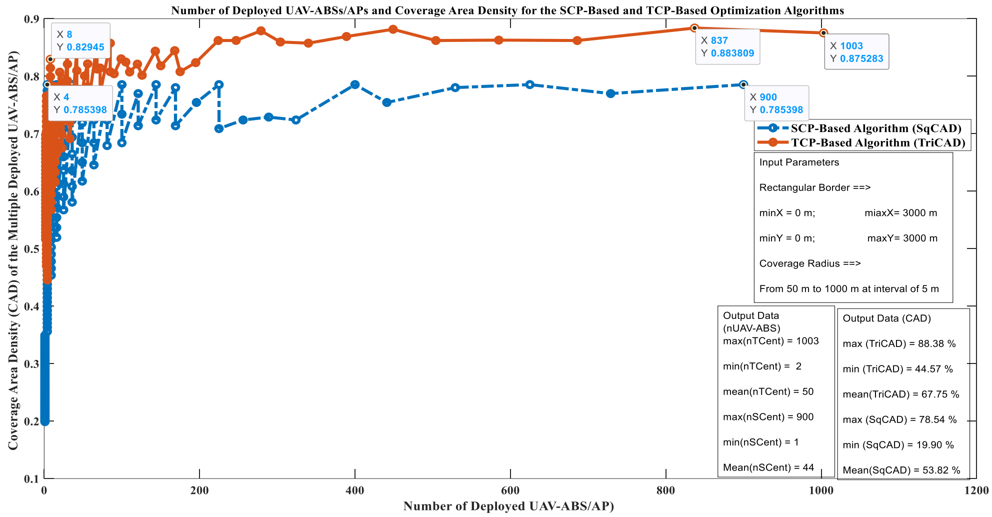

Figure 6: Coverage area density and number of deployed UAV-ABS

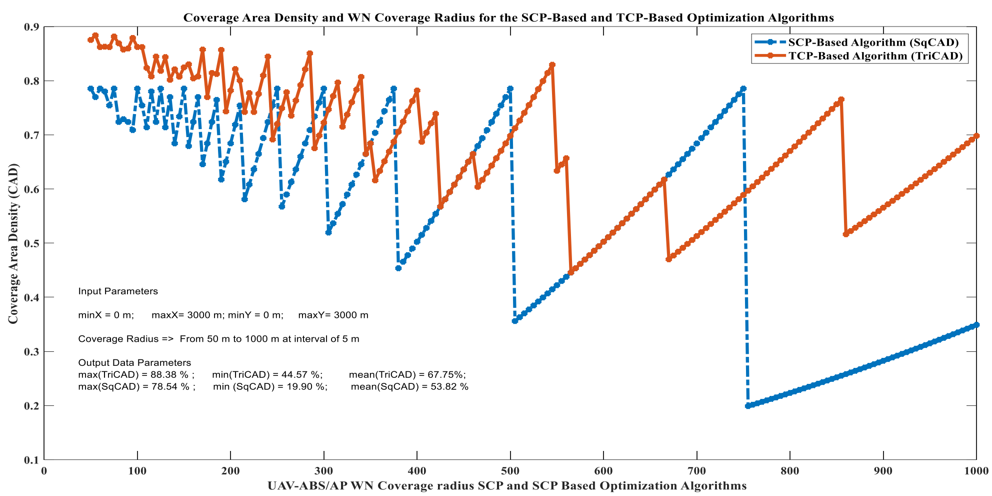

Figure 7: Coverage Area Density and UAV-ABS WN coverage radii

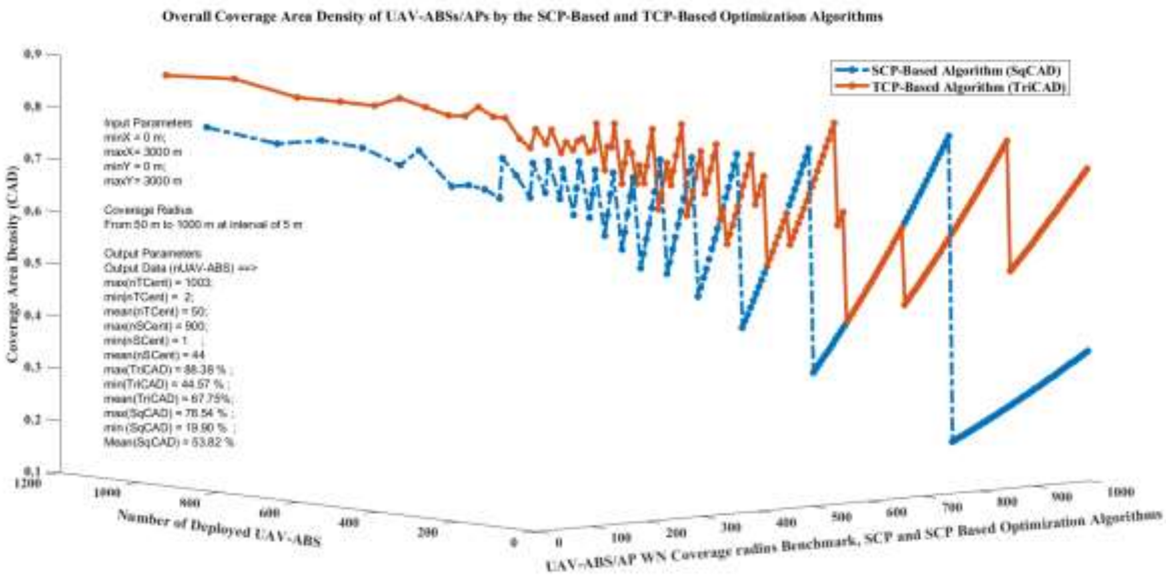

Figure 8: 3D Plot of CAD, number of deployed UAV and UAV-ABS WN Coverage Radii

The developed **TriCP**-based algorithm displayed better performance over the commonly used **SqCP** in reference with the performance metric – coverage area density (**CAD**). Considering the overall coverage area density (**CAD**) as the performace metric, the **TriCP**- based algorithm provided higher **CAD** compare to **SqCP**-based algorithm over larger distribution of coverage radii as shown in Figure 6 to Figure 8. The mean value of the coverage area density of **TriCP-** based algorithm (**TriCAD**) is 67.75 % is 13.93% higher than the 53.82 % recorded for the mean value of **CAD** of **SqCP**-based placement algorithm (**SqCAD**). Furthermore, the developed placement optimization algorithm can reach maximum of 88.38 % **CAD** which more than 10% higher the maximum **SqCAD** of 78.54 % for the given 9000000 m^2 coverage area. The minimum **CAD** attained by the TriCP-based was 44.57 % but the minimum **SqCAD** for the input simulation parameters was 19.90%.

CONCLUSION

Based on the obtained results, the **TriCP-**based algorithm could produce higher coverage area density that the **SqCP**- based algorithm for placement of **UAV-ABS**. Quality of service (**QoS**) of **WN CAD**. As shown by the results, **TriCP-**based placement algorithm will improve **QoS** of **WN** because its aggregate coverage is denser (with smaller dead zone area within the circular **WN** coverage region). Furthermore, the possibilty of using linear constraints in maximizing CAD is a great impetus in the application of **CPT** in placement optimization of **UAV-ABS**. This reseach work had analytically shown that the two basic CPT based algorithms can be implemeted using linear mathematical model and the **TriCP**-based is more benficial when high **CAD** is desired like in a target serviced area with densely and evenly distributed **WN** users.

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