Introduction to Mechanics

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ISBN: 978-978-59863-2-7

Printed and Published by
Ahmadu Bello University Press Limited, Zaria

Kaduna State, Nigeria.

Tel.: 08065949711

E-mail: abupress2020@yahoo.com

abupress2013@gmail.com

Website: www.abupress.com.ng

DEDICATION

This book is dedicated to the Almighty Allah and to the fond memories of our late fathers, **Alh. Suleiman Olas and Alhaji Mohammed Lemu**. May Allah forgive their shortcomings.

EPIGRAPH

"If I remained silent and you remained silent, then who will teach the ignorant?" ***— Ibn Taymiyyah.***

Foreword

It is indeed an honour to me when two great scholars joined heads together and produced a Book titled "**Introduction to Mechanics**" and requested me to introduce the book.

The Book, no doubt is timely, has wider coverage and useful to all students of Physics both at secondary schools and tertiary levels. I am particularly happy with the simplicity of language used and numerous examples given in the Book. This will facilitate better understanding and good comprehension.

I therefore commend the contribution of the Authors and the Book is hereby recommended for all science students.

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PREFACE

Mechanics is a compulsory course for students offering Physics as a field of study in the University, Polytechnic, Colleges of Education. Apart from students offering Physics, there are also few other field (Science Laboratory Technology, Survey and Geo-informatics, Industrial Education, etc) offering Mechanics as a borrowing course, hence, the book was initiated and compiled by the Authors to serve as a guide to these categories of students.

There are fourteen chapters in the book. Each Author contributed seven chapters each. The idea was to have a book written in simple English and provided examples in simple steps for better understanding. Each chapter is followed by an exercise to assist students to test their comprehension and to adequately prepare them in answering questions related to the topics.

Chapter One presents Units/Quantities, measurements in Physics and Dimension. These are essential components in Physics as most values are with S.I. Units. In Chapter two, motion, different types of motion, speed, velocity and acceleration were presented. The Chapter also present equation of uniform acceleration motion and motion under gravity. Chapter three presents scalar and vector quantities, how vectors are represented, adding and subtracting vectors and also resolution of vectors. Chapter four is exclusively simple harmonic motion, types of oscillation, potential and kinetic energy in oscillation. Newton's laws of motion, collision and momentum of a body were also discussed in chapter five.

Chapter six explains circular motion, relationship between linear and angular velocity, centripetal and centrifugal acceleration, motion in a vertical circle and relative velocity. Chapter seven and eight look at work, energy and power and mechanical energy respectively. While chapter nine dwells into projectile, time of flight, maximum height and range, chapter ten discusses frictional forces, types advantages, disadvantages, laws and methods of reducing friction.

Elasticity, stress, strain and young modulus were all captured in chapter eleven. Chapter twelve looks at structure of matter, where J.J. Thomson, Rutherford and Bohr's models of an atom were discussed. Chapter 13 briefly talks about fluid motion, Bernoulli's theorem and Poiseuille's equation discussed. Finally, chapter 14 centres on Gas law, Boyle's law, Charle's law, pressure law, Dalton's law and Avogadro's law/

ACKNOWLEDGEMENT

Verily, with every difficulty there is relief. Appreciation to the Almighty Allah, the Most Beneficent and the Most Merciful. May the peace and blessings of Allah be upon our Noble Prophet Muhammad (S.A.W). We thank Allah (SWT) for His guidance, mercies, protections on us and our families, for the initiative in starting this book and capability to compile and complete it.

May Allah forgive the shortcomings of our parents Alhaji Suleiman Olas, Alhaji Mohammed Lemu and make Aljannah Firdaus their final abode. Their parental caring and advise can never be compared to any and we will forever be grateful for the confidence, proper upbringing and the inculcation of societal values. To our uncles too numerous to mention, words alone cannot express our gratitude to you all but we pray to Allah to make you strong and healthy so that you can continue to enjoy the fruits of your labour.

Special thanks to our friends and colleagues, for the support and understanding at all the time. We are very grateful and may Allah continue to strengthen our relationships.

Finally, we will like to appreciate the authors whose works were consulted, their contributions have actually added colour to this Book. To the students, who are the target audience, it is our hope that a step by step reading and understanding of this book will assist you greatly.

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CHAPTER ONE: UNITS/QUANTITIES, MEASUREMENTS & DIMENSIONS

BY
DR. S. TAUFIQ

1.0 Units and Quantities

The words **Unit** and **Quantity** are both used in referring to an amount of thing or number. Though, they may possibly have similar functions, they are however, used in different instances and in diverse and distinct contexts.

The term **Unit** is a noun which is used to describe a definite and unified number of things. Even though they are more than one, they are considered as one entity. It is usually used when a whole body of things or people are subdivided into smaller groups which can have only a few or many countable things within it. Each group is referred to as a unit which is distinct from the others. The term **Quantity** on the other hand, is used to describe the number or amount of something. The quantity of things can be divided into a multitude and a magnitude as it usually refers to a huge amount or number of things and is used in a plural form. So, when one is talking about a large, uncountable number or amount of items, the term **quantity** is used.

Physicists, like other scientists, observed things around them and ask fundamental questions. For instance, how big is an object? How much of mass does it has? How far can it travel? To answer all of these questions, they make measurements with various instruments (e.g., balance, meter stick, stopwatch, etc.). The measurements of physical quantities are expressed in terms of units, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in metres or kilometres. Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way.

System International Unit

The System International Units (S.I. Unit) which is an abbreviation from the French, *Système international d'unités* is the modern form of the metric system and is the most widely used system of measurement based on the International System of Quantities. They are adopted to facilitate measurement of diverse quantities. It is a system of measurement based on 7 base units: the metre (length), kilogram (mass), second (time), ampere (electric current), Kelvin (temperature), mole (quantity), and candela (brightness).

1.1 Fundamental Quantities/Units

Fundamental quantities are those quantities that are independent of other quantities. They are standard quantities and have standard unit of measurement. There are three important fundamental quantities in physics, these are **length, mass,** and **time**. The units of these quantities form the fundamental or basic units upon which most (though not all) other units depend. Fundamental quantities and they respective units are shown in table 1

Table 1: Fundamental Quantities and Units

QUANTITY	UNIT	UNIT ABBREVIATION
Length	Metre	M
Mass	Kilogram	Kg
Time	Second	S
Temperature	Kelvin	K
Electric current	Ampere	A
Amount of substance	Mole	Mole
Luminious intensity	Candela	Cd

1.2 Derived Quantities/Units

By simple combination of SI basic unit, we can obtain other useful units. They are called derived units because they are mostly obtained by combining two or more units to form their own unit. For instance, the unit of volume is obtained by multiplying three lengths m*m*m*= m^3. Other important derived units are listed in Table 2.

Table 2: Derived Quantities

QUANTITY	DERIVATION	UNITS
Area	Length * Breadth	m^2
Volume	Length * Breadth * Height	m^3
Density	$\frac{\text{Mass}}{\text{Volume}}$	Kg/m^3
Speed or Velocity	$\frac{\text{Distance}}{\text{Time}}$	m/s
Acceleration	$\frac{\text{(Change in velocity)}}{\text{Time}}$	m/s^2

Force	Mass * acceleration	Kgm/s^2 (N)
Momentum	Mass * vel = Force*Time	Kgm/s (Ns)
Pressure	$\frac{\text{Force}}{\text{Area}}$	N/m^2
Energy or work	Force * distance	Nm or Joules

1.3 Measurements:

1.3.1 Measurement of Length

Length is a term that is used in identifying the distance from one point to another or the size of an object. It is a measure of how long an object is or the distance between two points. The length of a physical quantity or object is measured by instruments such as metre rule, micro metre screw guage, callipers, etc. The unit of length is meter.

SI unit for length is metre (m). It is a scalar quantity.

Things you need to know:

- Accuracy refers to the maximum error encountered when a particular observation is made.
- Error in measurement is normally one-half the magnitude of the smallest scale reading.
- Because one has to align one end of the rule or device to the starting point of the measurement, the appropriate error is thus twice that of the smallest scale reading.
- Error is usually expressed in at most 1 or 2 significant figures.

Tools for measuring the length

Tape

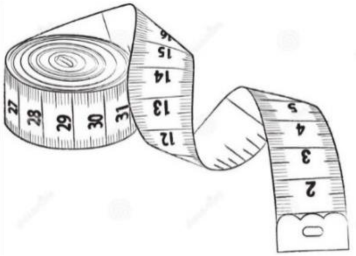


Fig. 1.1: Tape

Equipment: It is made up of a long flexible tape and can measure objects or places up to 10 – 50 m in length. It has markings similar to that of the rigid rule. The smallest marking could be as small as 0.1 cm or could be as large as 0.5 cm or even 1 cm.

How to use: The zero-mark of the measuring tape is first aligned flat to one end of the object and the tape is stretched taut to the other end, the reading is taken where the other end of the object meets the tape.

Accuracy: ± 0.1 to 1 cm

Rule

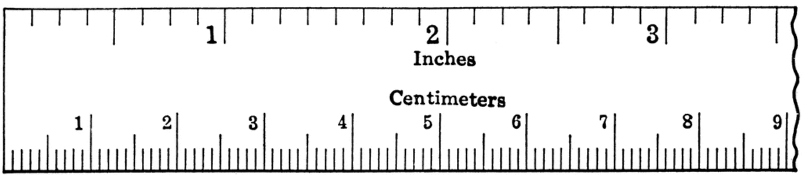


Fig. 1.2: Rule

Equipment: It is made up of a long rigid piece of wood or steel and can measure objects up to 100 cm in length. The smallest marking is usually 0.1 cm.

How to use: The zero-end of the rule is first aligned flat with one end of the object and the reading is taken where the other end of the object meets the rule.

Accuracy: ± 0.1 cm

Vernier Calliper

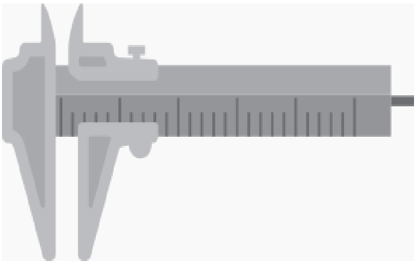


Fig. 1.3: Vernier Calliper

Equipment: It is made up of a main scale and a vernier scale and can usually measure objects up to 15 cm in length. The smallest marking is usually 0.1 cm on the main scale.
It has:

- a pair of external jaws to measure external diameters
- a pair of internal jaws to measure internal diameters
- a long rod to measure depths

How to use: The jaws are first closed to find any zero errors. The jaws are then opened to fit the object firmly and the reading is then taken.
Accuracy: ± 0.01 cm

Micrometre Screw Gauge

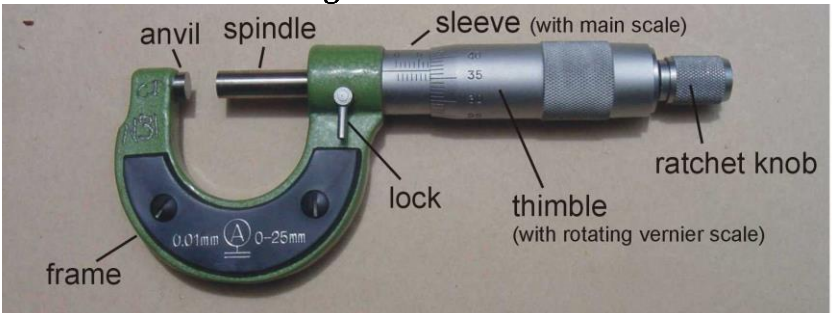


Fig. 1.4: Micrometre Screw Gauge

Equipment: It is made up of a main scale and a thimble scale and can measure objects up to 5 cm in length. The smallest marking is usually 1 mm on the main scale (sleeve) and 0.01 mm on the thimble scale (thimble). The thimble has a total of 50 markings representing 0.50 mm.

It has:

- an anvil and a spindle to hold the object
- a ratchet on the thimble for accurate tightening (prevent over-tightening)

How to use: The spindle is first closed on the anvil to find any zero errors (use the ratchet for careful tightening). The spindle is then opened to fit the object firmly (use the ratchet for careful tightening) and the reading is then taken.

Accuracy: ± 0.01 mm

Different Units of Length

The standard unit of length based on the metric system is the metre (m). However, other units are used in defining length depending on how short or long the distance is. These units are therefore converted into the S.I. Units. For examples, millimetres, centimetre or kilometres can all be converted into metre.

One hundred centimeters is converted into metre by simply divide by 100. That is:

$$100\ cm = 1m$$

One kilometre is converted to metre by multiplying by 1000. That is:

$$1\ km = 1000\ m$$

For conversion, the chart below can be used.

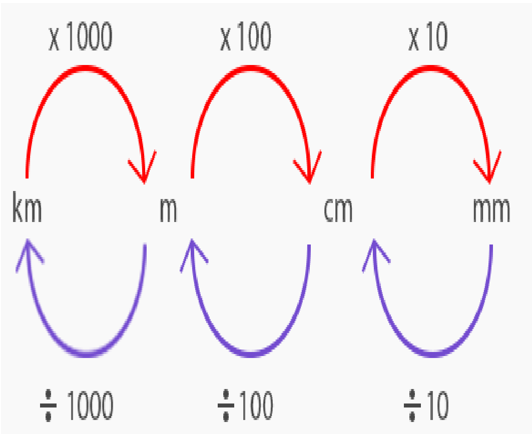


Fig. 1.5: Conversion

1.3.2 Measurement of Mass and Weight

Mass: The mass of a body is a measure of the quantity of matter it contains. It is the amount of material it contains. Mass is measured in standard metric units. Mass is usually measured by comparing it with standard masses, using a balance. There are various types e.g. beam or chemical balance, lever balance, a dial spring, direct reading balance etc

Weight is the measure of how heavy an object is. It is the force acting on the body due to the earth's gravitational pull. One instrument used for measuring weight is the spring balance. Weight is measured in Newtons. Weight is measured in standard customary units.

For everyday purposes, when you are on the surface of the earth, the difference is not important. But if you measure something on another planet, its mass will be the same as it is on earth, but its weight will be different. (Weight depends on gravity, and gravity is different on other planets). This is why a floating object in space is ***weightless***.

Metric Units

The **gram** and **kilogram** are two units used to measure mass in the metric system. From the gram, we get the rest of the metric units using the standard metric prefixes

Table 4

Milligram (mg)	0.001 gram
Centigram (cg)	0.01 gram
Decigram (dg)	0.1 gram
Dekagram (dag)	10 grams
Hectogram (hg)	100 grams

Gram (g)	1,000 milligrams
Kilogram (kg)	1,000 grams
Metric ton (t)	1,000 kilograms

Conversion

Example 1) A truck weighs 6,000 pounds. How many tons is this?

Solution:

Since, 1 ton =2,000 pounds 1 ton =2,000 pounds.

To convert pounds to tons, divide 6,000 by 2,000

$$\frac{6,000}{2000} = 3 \text{ tons}$$

Note:

(a) To convert from larger units to smaller units, multiply by the appropriate unit ratio.

(b) To convert from smaller units to larger units, multiply by the reciprocal of the appropriate unit ratio.

(c) Multiplying or dividing by a unit ratio is mathematically equivalent to using a proportion to convert between units of measure.

Differences between Mass and Weight

Mass	**Weight**
Mass is the quantity of matter present in a body	Weight occurs as a result of force of gravity
Mass is a scalar quantity	Weight is a vector quantity
The Unit of Mass is Kg	The Unit of Weight is Newton
Mass is constant	Weight varies

Relationship Between Mass & Weight

$$W = mg$$

Where W = weight (N)   m = mass (kg)
g = acceleration due to gravity (m/s^2)

1.3.3 Measurement of Time

Time is an indefinite process that does not stop. It is seen as an irreversible process irrespective of any factorial change. The role time plays in any scientific formulations cannot be neglected, this is because it plays a predominant role in determining the end factual result. Time plays an important role in human existence The SI unit of time is “Seconds”. The second (s), is the exact duration of 9,192,631,770 cycles of the radiation associated with the transition between the two hyperfine levels of the ground state of caesium-133 atom. The tool used to measure time is the clock.

Some of the more common clocks and watches can be found in the table below:

Type of clock/watch	Use and accuracy
Atomic clock	Used to measure very shorty time intervals of about 10^{-10} seconds
Digital stopwatch	Used to measure short time intervals of minutes and seconds to an accuracy of ± 0.01 s
Analogue stopwatch	Used to measure short time intervals of minutes and seconds to an accuracy of ±0.1 s
Ticker-tape timer	Used to measure short time intervals of 0.02 s

Watch	Used to measure longer time intervals of hours, minutes and seconds
Pendulum clock	Used to measure longer time intervals of hours, minutes and seconds
Radioactive decay clock	Used to measure long time intervals of years to thousands of years

Clocks
Equipment: It can be read in hours, minutes and seconds.
How to use: The clock is set to commence at a particular time or the start time is noted. The time event is then allowed to occur, and at the end of the event, the end time is noted. The difference provides the required time interval.

Accuracy: ± 1 s
Stopwatches
Equipment: It reads up to 0.01 s
How to use: As the time event occurs, the stopwatch is started at the same time. At the end of the event, the stopwatch is stopped and the end time is noted. The reading provides the required time interval. Some precise stopwatches are connected electronically to the time event and hence, more accurate.

Accuracy: ± 0.1 s. (Allowance made to human reaction time limits the accuracy of the stopwatch to 0.1 – 0.4 s for laboratory experiments. Records that show up to 2 decimal places are not appropriate.)
The following are used in time calculations:

60 seconds = 1 minute
60 minutes = 3600 seconds = 1 hr
24 hours = 1 day = 86,400 seconds

365 days = 1 year

Example 1) Calculate how many hours are in 180,000 seconds?
From the above, there are 3600 seconds in an hour, therefore, 180,000 is divided by 3600:

$$\frac{180{,}000}{3600} = 50 \text{ hours}$$

There are 50 hours in 180,000 seconds.

Example 2) Calculate how many minutes are in three hours?

3 x 60 = 180

There are 180 minutes in 3 hours

1.3.4 Measurement of Volume

The space occupied by a substance is called its volume. The SI unit for volume is cubic metre (m^3). Sub-multiples of the cubic metre, the cubic centimetre and the cubic millimetre, are used for smaller measurements.

Volume of Regular Solids

Volume of a cube = length^3
Volume of a cuboid = Length x Breadth x Height
Volume of a sphere of radius r = $\frac{4}{3}\pi r^3$
Volume of a cylinder of height h and radius r = $\pi r^2 h$

Volume of Liquids

Containers of known capacity are used to measure the volume of liquids. The 'litre' is used to measure the volume of liquids. The millilitre is used to measure smaller volumes.

1 litre = 1000 cm^3
1 millilitre (ml) = 1 cm^3

To measure the volume of liquids, it is poured into a graduated measuring cylinder and the volume of the liquid is read from the level of the liquid in the cylinder. The curved surface formed by a liquid is known as meniscus. For a concave surface, we take the lower meniscus reading. For a convex surface, we take the upper meniscus reading.

1.4 Dimensions of Physical Quantities

By the dimensions of a physical quantity, we mean the way it is related to the fundamental quantities, mass, length and time. These are usually denoted by M, L and T, respectively. The following are the dimensions of some quantities in mechanics;

1) **Area** = Length * breadth = L * L = L^2

2) **Volume** = Length * breadth * height = L*L*L = L^3

3) **Density** $= \frac{\text{Mass}}{\text{Volume}} = \frac{M}{L^3} = ML^{-3}$

4) **Velocity** $= \frac{\text{Displacement}}{\text{Time}} \ \frac{L}{T} = LT^{-1}$

5) **Acceleration** $= \frac{\text{Change in Velocity}}{T} = \frac{LT^{-1}}{T} = LT^{-2}$ Time

6) **Force** = mass * acceleration = MLT^{-2}

7) **Work** = force * distance = ML^2T^{-2}

8) **Pressure** $= \underline{\text{Force}} = \underline{\text{mass * acc}} = \underline{MLT^{-2}} = ML^{-1}T^{-2}$

Area Area LL

9) **Young Modulus** $= \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{e/L} = \frac{MLT^{-2}/L^2}{L/L} = ML^{-1}T^{-2}$

1.5 Applications of Dimension

Dimension of physical quantities have been applied to various purposes which includes:

i) Conversion from one system unit to another
ii) Deriving the relationship that exists between physical quantities
iii) Studying and modifying mathematical models

1.6 Exercise

1) Differentiate between fundamental quantity and Derived quantity
2) Itemize three (3) tools for measuring length and state their accuracies
3) Differentiate between Mass and Weight
4) Which of the following has a Unit?
   a) Strain (b) Magnification (c) Stress
5) The correct expression of thousands of joules is ……
   a) KJ (b) kJ (c) kj
6) The correct dimension of pressure is …………
   a) MLT^{-2} (b) $ML^{-1}T^{-2}$ (c) $ML^{-2}T^{2}$
7) The S.I. unit of momentum is …..
   a) gcm/s (b) kgm/s (c) MLT^{-1}
8) Convert the following to metre:
   a) 10 cm
   b) 25 mm
   c) 3 km
9) Convert the following to seconds:

a) 1 hour, 25 minutes
b) 3 hours
c) 20 hours

10) Derive the dimension of the following physical quantities
a) Momentum b) Force c) Impulse

CHAPTER TWO: MOTION
BY
DR. S. TAUFIQ

2.0 Concept of Motion
Motion is characterized by a change in position. Motion in a straight path is called **Linear** motion. Kinematics is a motion described using the concept of space and time regardless to the causes of the motions. The subject of dynamics on the other hand involves an object undergoing constant acceleration with the relationship between motion, forces and the properties of moving objects.

2.1.1 Types of Motion
1) **Linear motion**: This is the motion of an object or a body moving along a straight path or line. The physical quantities which describe this motion are displacement, velocity and acceleration or deceleration. Examples of linear motion in daily life are:

   i) Sliding a boy in a straight line
   ii) The motion of the car on the road
   iii) Motion of football

2) **Random motion**: This is the motion of an object or body in a disorderly or irregular direction. In this motion there is no specified direction or orientation, so the particles tend to collide with one another. Examples of random motion are:

i) The motion of dust or smoke particles in the air.
ii) The motion of insects and birds.
iii) The Brownian motion of a gas or liquid molecules along a zig-zag.

3) **Oscillatory motion:** This is a rapid to and fro movement of a body in a regular manner. When a body moves from one path to another and reverses itself in a repeated way continuously, that motion is said to be oscillatory or vibratory in nature. Examples of oscillatory motion include:

i) The motion of a swinging pendulum bob.
ii) The strings of a plucked guitar
iii) The movement of a mass placed on a spiral spring.

4) **Circular motion:** This is the motion of a body of mass, m, in a circular path of radius, r. The parameters that describe circular motion are, angular distance θ, angular velocity, ω, and angular acceleration, α. Examples are:

i) Wheels of a moving car
ii) The rotation of the earth about its orbit.

2.1.2 Causes of Motion

Having discussed the various types of motion with its examples, we can now begin to ponder on what causes motion. A body cannot just change its position or remain at rest without something happening to it or keeping it at rest.

A bar of magnet resting on a table will remain at rest until it is pushed by an agent. Also, a moving car will continue moving uniformly in a smooth road in a straight line except an agent pushes or brings it to a stop. The agent that changes the state of rest or uniform motion in a straight line of a body or object is called **Force**. Therefore, force is what causes motion in every body or object.

2.2 Relative Motion

Motion has already been defined as the change in position of a body. However, it may be difficult or impossible to determine the change in position without an initial frame of reference. So, every measurement is made with a certain frame of reference. Hence, every motion that is made is relative to a particular frame or point of reference when measured.

If a passenger travelling in a train looks down on the floor of the train, the train would appear to him to be static or stationary. This occurs because the passenger and the bus are moving with the same speed, hence, the motion between them is not relative. But if the passenger looks out of the window of the train, the trees outside the train will appear to be moving in the direction opposite the train motion. This scenario happens because of the relative motion between the passenger in the moving train and the stationary trees, since trees do not move or are static, they become the frame of reference for this motion.

Similarly, a body on the surface of the Earth may appear to be at rest because the observer is also on the surface of the Earth. However, the Earth itself, together with both the body and the observer, is moving in its orbit around the Sun and rotating on its own axis at all times.

2.3 Position, Distance and Displacement

Position: The position of an object is the location of that object in space. The position of body in space can be determined by usage of a frame or point of reference.

Distance: Is the change in position of a body or object in space. It is the amount or gap that exist between two objects or bodies. It is defined as the product of average speed and time:

Distance = Average speed x time

It is also defined as the length of space covered by an object when it changes from one point to another. It is important to note here that, in the above definitions, there is no specified direction, hence, distance is a scalar quantity and its S.I. unit is the metre (m).

Displacement (**s**): This is the distance travelled or covered in a specified or constant direction.

Displacement is a vector quantity and is measured in metre (m). The displacement can be along vertical direction or horizontal direction (S_x) or (S_z). It is denoted by the letter '**s**' and has both magnitude and direction.

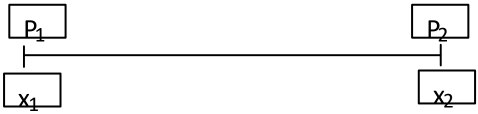
2.4 Speed, Velocity and Acceleration

Speed is the time rate of change of distance. Or simply put, distance covered over time.

Velocity (v): This is the time rate of change of displacement. Velocity is a vector quantity, and is measured in metre per second (ms^{-1}).

Note: Speed and velocity are often used interchangeably; however, this is incorrect. Speed does not specify direction while velocity specifies direction. Speed is therefore a scalar quantity while velocity is a vector quantity.

For an object moving say from point P_1 to P_2 along x-axis, it can be represented as shown in figure 2.1:



As the motion is along the x-axis the average velocity is given by:

$$\boxed{V_{av-x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}}$$

Fig. 2.1: Representative of Displacement between two points

Where t_1 is the time at x_1 and t_2 is the time at x_2. This can be represented in graphical form as follows:

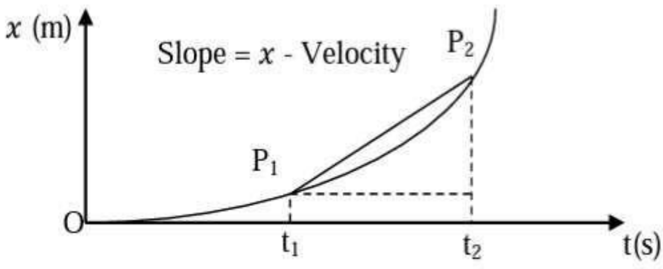


Fig. 2.2: Displacement - time graph of linear motion

The slope of the graph $= \frac{\Delta x}{\Delta t}$ = Average velocity

The average x -velocity depends only on the total displacement, that occurs during the time interval $\Delta t = t_2 - t_1$

Acceleration: When the velocity of a particle changes with time, the particle is said to be accelerating. Acceleration is defined as the time rate of change of velocity. If the velocity of a body increases from an initial value, U (V_i) to a final value V (V_f) in a time interval, t , the average acceleration is given by

$$a = \frac{(V - U)}{t} = \frac{(v_f - v_i)}{(t_f - t_i)} = \frac{\Delta V}{\Delta t}$$

Example 1) A car travelled at an average speed of 120 km/h. what distance does it covers in 6 minutes?

Speed = distance / time

:. Distance = speed x time

where speed = $\frac{120 \times 1000}{60 \times 60}$ = **33.3 m/s**

& Time = 6 x 60 = **360 secs**.

:. Distance = speed x time = 33.3 x 360 = **11988 m**

Example 2) A car travelled for 3 hrs to a distant city 250 km due East. What was its speed?

Speed = $\frac{\text{Distance}}{\text{Time}} = \frac{250 \times 1000}{3 \times 60 \times 60} = \frac{250000}{10800}$ = **23.15 m/s**

Example 3) If a bus covers a distance of 400 km in 4 hrs. cal. its average speed.

Speed = $\frac{\text{distance}}{\text{Time}} = \frac{400\text{ km}}{4}$ = 100 km/h

2.5 Average Velocity and Instantaneous Velocity

Let us try to understand and differentiate between an average velocity and an instantaneous velocity. Average velocity does not tell us exactly how fast an object is moving at every instant in time. The object can speed up, slow down, and even stop for a while, and this will not directly affect its average velocity.

For instance, it may take a student a total of 45 minutes to travel a distance of 10 km from house to school. In this regard, the average velocity here, is the total distance travelled all over time, irrespective of whether the student speed up at some point, slow down or even took a rest.

In contrast, **instantaneous velocity** is the velocity of an object at a single instant in time.

In the example cited of a student going to school from house, the student's instantaneous velocity while speeding will not be the same with when the student was slowing down or even at rest due to traffic.

Therefore, while instantaneous velocity takes cognizance of velocity at any instant of time, average velocity takes note of the entire trip, hence, average velocity cannot be zero but instantaneous velocity can be zero for a small part of the trip.

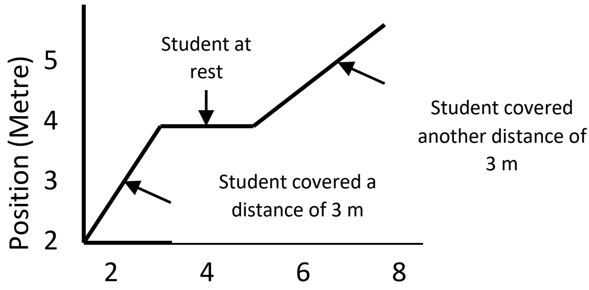
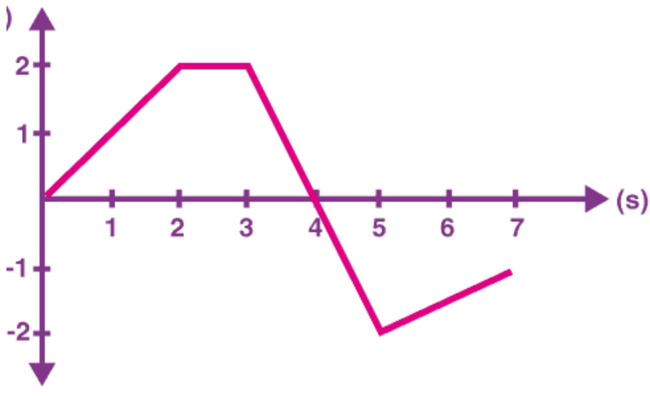


Fig. 2.3

2.6 Instantaneous Acceleration

The instantaneous acceleration is defined as the limit of the average acceleration as Δt approaches zero, that is

$$a = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt}$$



(Slope of the velocity time graph)

Fig. 2.4

Consider the velocity-time graph shown above. Here, between the time intervals of 0-2 seconds, the velocity of the particle is increasing with respect to time; hence the body is experiencing a positive acceleration as the slope of the v-t curve in this time interval is positive.

Between the time intervals of 2-3 seconds, the velocity of the object is constant with respect to time; hence the body is experiencing zero acceleration as the slope of the v-t curve in this time interval is 0.

Now, between the time intervals of 3-5 seconds, the velocity of the body is decreasing with respect to time; hence the body experiences a negative value of the rate of change of velocity as the slope of the v-t curve in this time interval is negative.

If the velocity of the body increases by equal amounts in equal time intervals, the acceleration is said to be uniform or constant, that is

$$a = \frac{(V - U)}{t} = constant$$

The unit of V is m/s, a=m/s^2.

If the velocity of a body increases uniformly from an initial velocity U to a final velocity V in a time interval, the average velocity V_{avg} will be:

$$V_{avg} = \frac{(V + U)}{2} = \frac{S}{t}$$

But from equation 1

$$\frac{S}{t} = \frac{(V + U)}{2} = \frac{((U + at) + U)}{2}$$

$$S = Ut + \frac{1}{2}at^2 \qquad (2)$$

If the body starts from rest, U=0. Equation 2 becomes

$$S = \frac{1}{2}at^2$$

Since from equation (1), V=U +at, then making t subject of the formula, we have that

$$t = \frac{(V - U)}{a}$$

Combining this equation with equation 2 eliminates t. the equation then becomes

$$S = U\left(\frac{V - U}{a}\right) + \frac{1}{2}a\left[\left(\frac{V - U}{a}\right)\right]^2$$

$$V^2 = U^2 + 2aS \qquad (3)$$

For U = 0, equation 3 becomes

$$V^2 = 2aS$$

Or

$$V = \sqrt{2aS}$$

Equations (1), (2) and (3) are the three fundamental equations of motion for linear motion under constant acceleration or uniform acceleration.

2.7 Velocity-Time Graph

Graphs of motion for bodies under constant acceleration is of two types: Displacement-time graph (fig a) and Velocity time graph. The slope of the displacement time graph of a body under constant acceleration is the velocity. That is

$$Slope = \frac{S_2 - S_1}{t_2 - t_1} = \frac{S}{t} = V$$

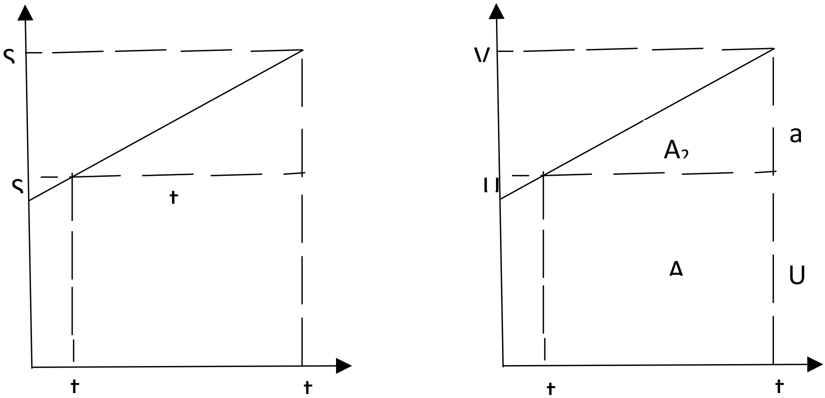
$$where\ S = displacement$$

Figure (b) is the velocity – time graph if a body under constant acceleration. From the graph we see that:

$$Slope = \frac{(V - U)}{t} = a$$

The area A_1 of the rectangle is: $A_1 = Ut$ and the area A2 of the triangle with base t is: $A_2 = (1/2)at^2$. The total area, A under the velocity -time graph is:

$$A = A_1 + A_2 = Ut + \frac{1}{2}at^2 = S(total\ displacement)$$



(a) Displacement (b) Velocity time

Fig. 2.5: Velocity-Time Graph

Examples 1)

1) A car travels 270 km in 4.5 h (a) what is its average velocity (b) how far will it go in 7.0 h at this average velocity (c) how long will it take to travel 300 km at this average velocity.

a) $V_{avg} = \dfrac{\Delta s}{t} = \dfrac{270}{4.5} = \mathbf{6.0\ km/hr}$

b) S = $V_{avg} \times t = 6.0 \times 7 =$ **420 km**

c) t = $\dfrac{s}{v} = \dfrac{300}{60} =$ **5 hrs**

Example 2a) What is the acceleration of a car that goes from 20 km to 30 km/h in 1.5 s?

(b) At the same acceleration, how long will it take the car to go from 30 to 36 km/h?

Solution:
Final velocity (V) = 30 km, Initial velocity (U) = 20 km, Time (t) = 1.5 s

Acceleration = $\dfrac{V-U}{t} = \dfrac{30-20}{1.5} = \dfrac{10}{1.5} =$ **6.7 km/hr**

b) V = 36 km, U = 30 km, a = 6.7 km/hr

$$t = \frac{V-U}{a} = \frac{36-30}{6.7} = \frac{6}{6.7} = \mathbf{0.9\ hr}$$

Example 3) The brake of a car whose initial velocity is 30 m/s are applied and the car receives an acceleration of -2 m/s^2. How will it have gone (a) when its velocity has decrease to 15 m/s? (b) when it has come to a stop?

Solution:

a) a = - 2 m/s^2, u = 30 m/s, v = 15 m/s

$$s = \frac{v^2-u^2}{2a} = \frac{15^2-30^2}{2\times -2} = \frac{-675}{-4} = \mathbf{169\ m}$$

b) When it comes to a stop, final velocity, V = 0.

$$s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 30^2}{2 \times -2} = \frac{-900}{-4} = \mathbf{225\ m}$$

Example 4) The velocity of a particle moving along the x-axis varies with time according to the expression v = (40-5t^2) m/s when t is in Sec (a) find the average acceleration in the time intervals t = 0 to t = 2s (b) determine the acceleration at t = 2 sec

a. V_i corresponds to t_i hence, $V_i = (40\text{-}5t_i^2)$; for $t_i = 0$, $V_i = 40$ m/s

V_f corresponds to t_f hence $V_f = (40\text{-}5t_i^2)$; for $t_f = 2$ sec, $V_f = 20$ m/s

b. $average\ acceleration = \frac{\Delta V}{\Delta t} = \frac{V_f - V_i}{t_f - t_i} = -10 m/s$

c. a = dV/dt at t = 2. This implies -10t m/s
hence, is equal to -20 m/s^2

or using limiting factor that is as $\Delta t \to 0$. The velocity at time t is given by V = (40-5t^2) m/s and the velocity at time t+Δt is given by

$$V = 40 - 5(t+\Delta t)^2 = 40 - 5t^2 - 10t\Delta t - 5(\Delta t)^2$$

The change in velocity over the time interval Δt is

$\Delta V = V_f - V_i = [-10t\Delta t - 5(\Delta t)^2]$ m/s

Divide through Δt and taking limit of the results as Δt approaches zero, we have acceleration at any time t as

$$a = \lim_{\Delta t \to 0}\left(\frac{\Delta V}{\Delta t}\right) = \lim_{\Delta t \to 0}(-10t - 5\Delta t) = -10t\ m/s$$

at t = 2 sec, a = - 20 m/s

2.8 Equations of Uniform Acceleration Motion

The velocity–time graph in Figure 1 shows a straight line with a non-zero intercept. This graph is a non-horizontal straight line, showing that the object is undergoing uniform, or constant, acceleration. In other words, the velocity is increasing at a uniform, or constant, rate. We know that to determine the displacement of this object from the velocity–time graph, we must determine the area under the line. For the graph in Figure 2.6, we must determine the area of a rectangle and a triangle:

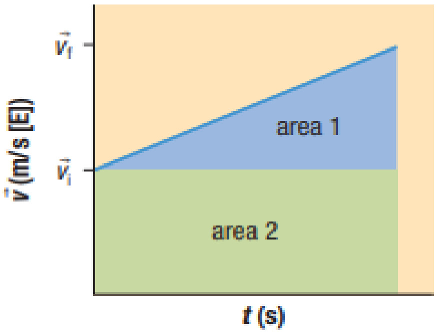


Figure 2.6: A velocity–time graph for an object undergoing uniform acceleration

$$\Delta\vec{d} = A_{triangle} + A_{rectangle}$$

$$= \frac{1}{2}bh + hw$$

$$= \frac{1}{2}\Delta t(\vec{v}_f - \vec{v}_i) + \Delta t\vec{v}_i$$

$$= \frac{1}{2}\vec{v}_f\Delta t + \frac{1}{2}\vec{v}_i\Delta t + \ \Delta t\vec{v}_i$$

$$= \frac{1}{2}\vec{v}_f\Delta t + \frac{1}{2}\vec{v}_i\Delta t$$

$$\Delta\vec{d} = \left(\frac{\vec{v}_f - \vec{v}_i}{2}\right)\Delta t \qquad Eq1.$$

We can use Equation 1 to determine the displacement of an object that is undergoing uniform acceleration. Equation 1 is very similar to an equation that we previously developed from the defining equation for average velocity: $\Delta\vec{d} = \vec{v}_{av}\Delta t$. We can relate the average velocity to the initial and final velocities by the equation

$$\vec{V}_{av} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)$$

Then substituting $\vec{v}_{av}$ in place of $\left(\frac{\vec{v}_f+\vec{v}_i}{2}\right)$ in Equation 1 gives

$\Delta\vec{d} = \vec{v}_{av}\Delta t$ directly.

As you will see, Equation 1 can help us to solve many motion problems.

2.9 Motion under Gravity

A practical example of a straight line motion with constant acceleration is the motion of an object near the surface of the Earth. We know that near the surface of the Earth, the acceleration due to gravity ‘g’ is constant. All straight line motions under this acceleration can be well understood using the kinematic equations given earlier.

Case (1): A body falling from a height h

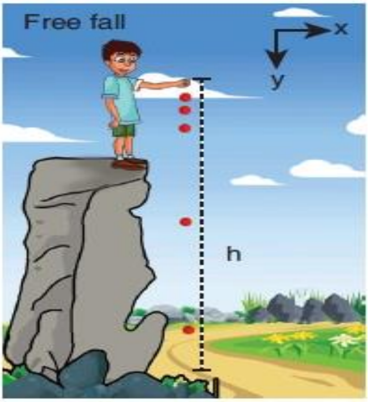


Figure 2.7: An object in free fall

Consider an object of mass m falling from a height h. Assume there is no air resistance. For convenience, let us choose the downward direction as positive y-axis as shown in the Figure 2.7. The object experiences acceleration 'g' due to gravity which is constant near the surface of the Earth. We can use kinematic equations to explain its motion. We have

The acceleration $\vec{a} = g\hat{j}$

By comparing the components, we get;

$$a_x = 0, a_z = 0, a_y = g$$

Let us take for simplicity, $a_y = a = g$

If the particle is thrown with initial velocity 'u' downward which is in negative y axis, then velocity and position at of the particle any time t is given by:

$$v = u + gt \qquad Eq1$$

$$v = ut + \frac{1}{2}gt^2 \qquad Eq2$$

The square of the speed of the particle when it is at a distance y from the hill-top, is:

$$v = u^2 + 2gy \qquad Eq3$$

Suppose the particle starts from rest.

Then u = 0

Then the velocity v, the position of the particle and v2 at any time t are given by (for a point y from the hill-top)

$$v = gt \qquad Eq4$$

$$y = \frac{1}{2}gt^2 \qquad Eq5$$

$$v^2 = 2gy \qquad Eq6$$

The time (t = T) taken by the particle to reach the ground (for which y = h), is given by using equation (5),

$$h = \frac{1}{2}gT^2 \qquad Eq7$$

$$T = \sqrt{\frac{2h}{g}} \qquad Eq8$$

The equation (8) implies that greater the height (*h*), particle takes more time(*T*) to reach the ground. For lesser height (*h*), it takes lesser time to reach the ground.

The speed of the particle when it reaches the ground (y = h) can be found using equation (6), we get:

$$V_{ground} = \sqrt{2gh} \qquad Eq9$$

The above equation implies that the body falling from greater height(h) will have higher velocity when it reaches the ground.

The motion of a body falling towards the Earth from a small altitude (h << R), purely under the force of gravity is called free fall. (Here R is radius of the Earth)

Case (ii): A body thrown vertically upwards
Consider an object of mass *m* thrown vertically upwards with an initial velocity *u*. Let us neglect the air friction. In this case we choose the vertical direction as positive y axis as shown in the Figure 2.8, then the acceleration a =-g (neglect air friction) and *g* points towards the negative *y* axis. The kinematic equations for this motion are,

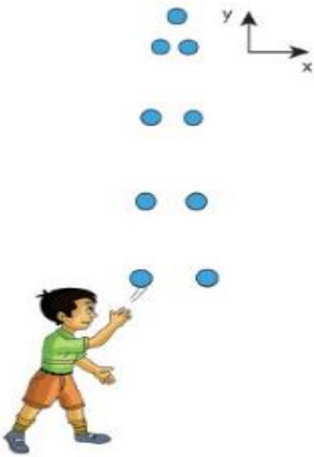


Figure 2.8: An object thrown vertically

The velocity and position of the object at any time t are,

$$v = u - gt \qquad Eq10$$

$$v = u - \frac{1}{2}gt^2 \qquad Eq11$$

The velocity of the object at any position y (from the point where the object is thrown) is:

$$v^2 = u^2 - 2gy \qquad Eq12$$

2.10 Exercises

1. Define Motion

1b) List and explain four types of motion

2. Differentiate between speed, velocity and acceleration
3. If the velocity of a particles is zero, can its acceleration ever be non zero and vice versa? Explain
4. Average velocity and instantaneous velocity are generally different quantities, can they ever be equal for a specific type of motion? Explain
5. If V is non zero for some time interval Δt, does this mean that the instantaneous velocity is never zero during the interval? Explain
6. An electron has an initial velocity of 3.0 x 10^5 m/s. If it undergoes an acceleration of 8 x 10^{14} m/s^2
   (a) how long will it take to reach a velocity of 5.4 x 10^5 m/s and (b) how far has it travelled in this time?
7. A jogger runs in a straight line with an average velocity of 5m/s for 4 min and then with an average velocity of 4 m/s for 3 min. (a) what is her total displacement (b) what is her average velocity during this time?
8. The velocity of a particles moving along the x-axis varies in time according to the relation V= (15 – 8 t) m/s. Find:

a. The acceleration of the particles
b. It's velocity at t = 3 s
c. It's average velocity in the time interval t = 0 to t = 2 s

9. The initial speed of a body is 5.2 m/s. What is its speed after 2.5 s if it (a) accelerates uniformly at 3.0 m/s^2 and (b) accelerates uniformly at -3.0 m/s^2 (-x axis)?
10. At t = 1 s, a particle moving with constant velocity is located at x = -3 m and at t = 6 s, the particle is located at x = 5 m, from this information, plot the position as a function of time. Determine the velocity of the particles from the slope of this graph

CHAPTER THREE: SCALAR AND VECTOR
BY
DR. S. TAUFIQ

3.1 Concept of Scalar and Vector

3.1.1 Scalar

This is a physical quantity that has magnitude and is completely specified by a number and a unit. Examples are: mass, volume frequency, length, time e.t.c. the rules of ordinary arithmetic are used to manipulate scalar quantities (symbols are represented in italic)

3.1.2 Vector

This is a physical quantity that has both magnitude and direction and is completely specific by a numerical number (value), a direction and a unit. Examples are: displacement, velocity, force e.t.c. A vector is represented analytically by a boldface type such as **F.** When written by hand, the designation F. When vector quantities are added, the direction must be taken into account.

3.2 Vector Representation

The following definition are fundamental

1. Two vectors **A** and **B** are equal if they have the magnitude and directions regardless of their initial point that is **A** = **B**

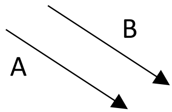


Fig. 3.1

2. A vector having direction opposite to that of vector say **A** but with the same length is denoted as –**A.**

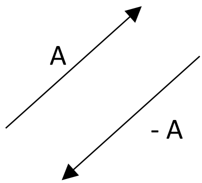


Fig. 3.2

3. The sum of resultant of vectors **A** and **B** of figure 3.3(a) below is a vector C formed by placing the initial point of **B** on the terminal point of **A** and joining the initial point of **A** to the terminal point of **B** (3.3b). We write **C**= **A**+**B**. This definition is equivalent to the parallelogram law for vector addition as indicated in fig 3.3c

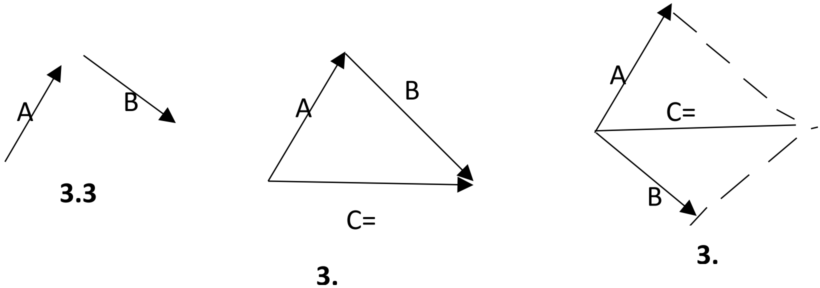


Fig. 3.3

For more than one vector: A, B, C, D = E (Resultant)

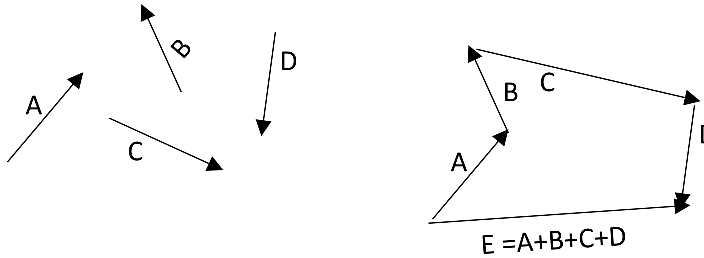


Fig. 3.4

4. The difference of vectors **A** and **B** represented by **A-B** is that vector **C** which when added to **B** gives **A**. (that is **A** +(-**B**)). If **A** = **B,** this implies that **A-B** = zero (null) vector. This has a magnitude of zero but its direction is not defined.

5. The product of a vector **A** by a scalar p is a vector p**A** or **A**p with magnitude p times the magnitude of **A** and direction the <u>same as</u> or <u>opposite</u> to that of **A** depending on whether p is +ve or -ve. If p=0 this implies that p**A**=0 (null vector)

3.3 Addition and Subtraction of Vectors

If **A**, **B** and **C** are vectors and p and q are scalars,

1. **A** + **B** = **B** + **A**
   Commutative law for addition

2. **A** + (**B** + **C**) = (**A** + **B**) + **C**
   Associative law for addition

3. p (q**A**) = pq**A** = q (p**A**)
   Associative law for multiplication

4. (p + q)**A** = p**A** + q**A**

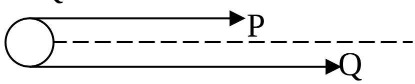
Distributive law

5. $p(\mathbf{A} + \mathbf{B}) = p\mathbf{A} + p\mathbf{B}$
   Distributive law

Example 1) Consider two vectors of magnitude P = 30 N, and Q = 40 N acting on a body O.

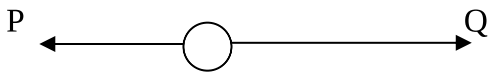
i) If the two vectors are acting in the same direction, the resultant force will be

$$R = Q + P = 30 + 40 = 70 \text{ N}$$



ii) If the two vectors are acting directly in opposite directions, the resultant force will be

$$R = Q – P = 40 – 30 = 10 \text{ N}$$



3.4 Resolution of a Vector

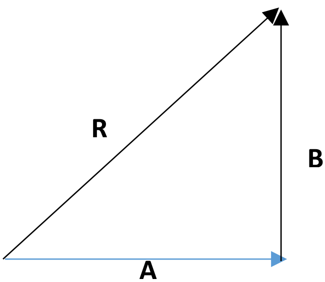
When two or more vectors are added together, those vectors must have the same unit. Thus can be done in number of ways

i. **Graphical method** – e.g fundamental 3- the order to which the vectors are added does not matter for three or more vectors. A geometric proof of this is found in associative law for additions. This method could also be called Geometric method. The rules for vector sums are conveniently described by geometric methods. To add vector **B** to vector **A**. First draw vector **A** in graph paper and then draw vector **B** with its tail starting from the tip (head) of **A**. Thus the resultant vector **R**= **A** + **B** is the vector drawn from the tail of A to the Tip to the tip of **B** . This is Triangle method of Addition.

ii. An alternative graphical procedure for adding two vectors is known as the **PARALLELOGRAM** rule of addition. In construction, the tails of the two vectors **A** and **B** are together and the resultant vector **R** is the diagonal of parallelogram law of vectors which states that: **if two vectors are represented in magnitude and direction by the adjacent side of a parallelogram, the resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from the common point (or tail) of the vectors** (figure 1c).

iii. Trigonometric Method

Although it is possible to determine the magnitude and direction of the resultant of two or more vectors of the same kind graphically with ruler and protractor, this procedure is not very exact. Thus, for accurate results, its necessary to use trigonometry. This method is easy to find the resultant **R** of two vectors **A** and **B** that are perpendicular to each other. The magnitude of the resultant is given by the Pythagorean theorem as **R** = $\sqrt{A^2 + B^2}$and the angle ϴ btw **R** and **A** may be found from tanϴ = **B**/**A**



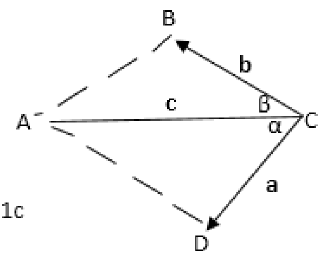


Fig 1c

Fig. 3.5

From Figure 3.5, apply cosine rule (formular), the magnitude of **C** which is the resultant or sum will be:

$$C = (a^2 + b^2 \ - 2abcosB)^{1/2} \qquad (1)$$

When the vectors **a** and **b** act in the same direction angle B will be 180^0 so that eqn (1) gives the magnitude of **C** as (**a** and **b** are parallel)

$$c = ((a + b)^2)^{1/2} = a + b \qquad \text{Cos } 180^0 = 1$$

$$c = a^2 + b^2 + 2ab$$

when **a** and **b** are in the opposite direction, angle B will be 0, then (**a** and **b** are antiparallel)

$$c = (a^2 + b^2 \ - 2abcos0)^{1/2}$$

$$c = ((a - b)^2)^{1/2} = a - b$$

If the two vectors are perpendicular, angle B = 90^0

$$c = (a^2 + b^2 \ - 2abcos\ 90^0)^{!1/2}$$

$c = (a^2 + b^2\ )^{1/2}$**a** is perpendicular to **b**

3.5 Resultant of more than two Vectors

When vectors to be added are not perpendicular 3- dimensional cases the method of addition by component is employed. The component of a vector in any given direction is the effect of that vector in the said direction.

Any vector **A** in 3- dimensions can be represented with initial point at the origin 0 of a rectangular coordinate system. Let A be the rectangular coordinates of the terminal vector **A** with initial point at O. The vectors A_1i A_2j A_3k are called the rectangular component vector (component vectors) of **A** in the x, y and z direction respectively. A_1, A_2 and A_3 are called the rectangular component (components) of A in the x, y and z direction respectively. The sum or resultant of $A_1i, A_2j\ and\ A_3$ is the vector **A** such that;

$$A = A_1i + A_2j + A_3k$$

The magnitude of A is: $A = I\boldsymbol{A}I = \sqrt{A_1^2 + A_{\mathbf{2}}^{\mathbf{2}} + A_3^2}$

in general the position vector or radius vector **r** from O to the point (x,y,z) is written as:

$$r = xi + yj + zk$$

And has magnitude r$= \boldsymbol{IrI} = \sqrt{x^2 + y^2 + z^2}$

To add two or more vectors **A**, **B** and **C** by the component method follow this procedure

1. Resolve the initial vectors into components in the x, y and z directions
2. Add the components in the x direction to give Rx likewise y-direction to give Ry and the component in the z direction to give Rz. That is, the magnitude of Rx, Ry and Rz are given below respectively as:

$$Rx = Ax + Bx + Cx$$

$$Ry = Ay + By + Cy$$

$$Rz = Az + Bz + Cz$$

Calculate the magnitude and direction of the resultant from its component Rx, Ry and Rz by using the Pythagorean theorem:

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

If the vectors being added all lies in the same plane, only two components needed to be considered.

Example 1) A particle undergoes three consecutive displacement given by $d_1 = (i + 3j\text{-}k)$ cm, $d_2 = (2i\text{-}j\text{-}3k)$ cm and $d_3 = (\text{-}i+j)$ find the resultant displacement of the particle

Solution.

$R = d_1 + d_2 + d_3 = (2i + 3j - 4k)$ cm

The resultant displacement has components Rx = 2 cm, Ry = 3 cm and Rz = -4 cm. its magnitude is

$$R = \sqrt{Rx^2 + Ry^2 + Rz^2} \;=\; R = \sqrt{2^2 + 3^2 + 4^2} \quad \mathbf{= 5.39}$$

Example 2) If a force of 9 N act at an angle of 45^0 to another force of 7 N. Calculate the resultant force of the two forces.

Soln. $R^2 = A^2 + B^2 + 2AB\,Cos\Theta$

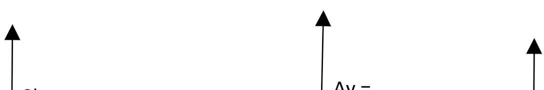
$= 9^2 + 7^2 + 2(9 \times 7)\,Cos\,45^0$

$= 81 + 49 + 126 \times 0.707$

$= 81 + 89.1 = 170.1$

$R = \sqrt{170.1}$ = **13.04 N**

Example 3) A boat headed north at a velocity of 8 km/hr. A strong wind is blowing whose pressure causes it to move sideways to the west at a velocity of 2.0 km/h.



There is also a total current that flow in a direction 30 south of east at a velocity of 5.0 km. What is the boat's velocity relative to the earth surface?

.

30

The direction ϴ can be obtained by:

$$tan\theta = \frac{Rx}{Ry} = 23^0, \qquad R = \sqrt{R_x^2 + R_y^2} = 6km$$

3.6 Exercise

1) Differentiate between Scalar and Vector quantities
2) Can a vector have a component equal to zero and still have a non zero magnitude? Explain
3) Can the magnitude of particles displacement be greater than distance travelled?
4) If A = B, what can you conclude about their component of A and B?
5) Can the magnitude of a vector have a negative value? Explain
6) Two forces 10 N and 6 N acts on a body, the direction of the forces are not known (a) what is the maximum magnitude of these forces? (b) what is the minimum magnitude?

CHAPTER FOUR: SIMPLE HARMONIC MOTION (SHM)
BY
DR.. M. I. KIMPA

4.1 Concept of Simple Harmonic Motion

In physics, simple harmonic motion is defined as a repetitive movement back and forth through an equilibrium, or central position, such that the maximum displacement on one side of this position is equal to the maximum displacement on the other side. Equally, the time interval of each complete vibration is the same and the force responsible for the motion is always directed toward the equilibrium position and is directly proportional to the distance from it.

$$F = -kx,$$

where F is the force, x is the displacement, and k is a constant.

The negative sign indicates that acceleration is in opposite direction to displacement.

This relation is called Hooke's law.

SHM can also be defined as a motion in which the acceleration, (a) is directed towards a fixed point (known as its equilibrium position) and its proportional to its displacement, (x) from that point.

:, $$a = -kx,$$

Examples of SHM are the motion of a simple pendulum, the oscillation of a clock pendulum, the vibration of a guitar string, vibration of a loaded spring, etc.

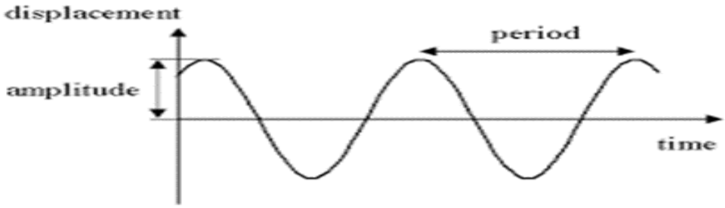


Fig. 4.1: SHM

4.1.1 Simple Pendulum

This consists of a small weight or bob suspended from a rigid support by means of a thread. If it is displaced by a small angle as shown in the diagram below, it performs simple harmonic motion. The amplitude of motion is **a**

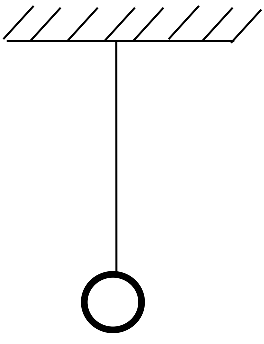


Fig. 4.2: Pendulum

It is observed that the motion of the pendulum does not continue indefinitely. This is because frictional forces and air resistance gradually reduce the motion until the amplitude a becomes zero

The period of motion T of the simple pendulum of length l can be shown to be equal to:

$$T = 2\pi\sqrt{l/g}$$

Where g is the acceleration due to gravity. This expression is valid provided that the angular displacement 0 or the amplitude of motion a is very small;

Therefore, $T \propto \sqrt{L}$

4.1.2 Period, Frequency and Amplitude of SHM

In the absence of friction, the time to complete one oscillation remains constant and is called the period (T). Its units are usually seconds.

For periodic motion, frequency (f) is the number of oscillations per unit time.

The relationship between frequency and period is:

$$f = \frac{1}{T}$$

The SI unit for frequency is the *hertz* (Hz) and is defined as one *cycle per second*:

The following equations are useful in SHM:

Period of SHM:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Frequency of SHM:

$$F = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Displacement in SHM as a function of time is given by:

$$x_{(t)} = X \cos\frac{2\pi t}{T}$$

Velocity in SHM as a function of time is given by:

$$v_{(t)} = -V_{\max} \text{Sin}\frac{2\pi t}{T}$$

$$\text{Where } V_{\max} = \sqrt{\frac{k}{m}}X$$

Acceleration in SHM as a function of time is given by:

$$a_{(t)} = -\frac{kx}{m}\text{Cos}\frac{2\pi t}{T}$$

Example 1) If shock absorbers in a car go bad, then the car will oscillate at the least provocation, such as when going over bumps in the road and after stopping. Calculate the frequency and period of these oscillations for such a car if the car's mass (including its load) is 900 kg and the force constant (*k*) of the suspension system is 6.53×10^4 N/m.

Solution

m = 900 km, k = 6.53 x 104 N/m, f = ? T = ?

$$F = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad = \frac{1}{2\times 3.142}\sqrt{\frac{6.53\times 10^4}{900}} \;=\; \mathbf{0.43\ Hz}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad = \; 2\times 3.142\sqrt{\frac{900}{6.53\times 10^4}}$$

Example 2) A 0.500 kg mass suspended from a spring oscillates with a period of 1.50 s. How much mass must be added to the object to change the period to 2.00 s?

Solution

M_1 = 0.5 kg, T_1 = 1.5 secs, T_2 = 2 secs, m = ?

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Take square of both sides

$$T^2 = 2^2\pi^2\frac{m}{k}$$

Make k subject formular

$$k = \frac{4(3.142)^2\times 0.5}{(1.5)^2} \quad = \quad \mathbf{8.8}$$

$$T^2 = 2^2\pi^2\frac{m}{k}$$

Make m subject formula

$$m = \frac{T^2 k}{4\lambda^2} \;=\; \frac{2^2\times 8.8}{4\times(3.142)^2} \;=\; \mathbf{0.89\ kg}$$

4.1.3 Important Features of Simple Harmonic Motion

1. The period of motion is independent of the amplitude. For example in a simple pendulum, the period is given by $T = 2\pi\sqrt{\frac{l}{g}}$ Where L is the length of the string and g is the acceleration due to gravity. The amplitude of the simple harmonic motion is not seen in the expression.

2. When the displacement is maximum, in either direction, the speed is zero, since the velocity must now change its direction. This implies that the body come momentarily to rest at A or B before it changes its direction of motion. The acceleration at the instant of change of direction is maximum but is directed opposite to the displacement.

3. At the equilibrium position C, the displacement is zero, the speed of the body is maximum and the acceleration is zero.
4. As the body moves from B to C, the speed increases towards C and then decreases as it moves out to the maximum displacement at A.

4.2 Types of Oscillation

There are three main types of Simple Harmonic Motion

i) **Free Oscillation:** A free oscillation possesses constant amplitude and period without any external force to set the oscillation. Ideally, free oscillation does not undergo damping.

ii) **Damped Oscillation**: An oscillation that fades with time is called damped oscillation. Due to damping, the amplitude of oscillation reduces with time. Reduction in amplitude is a result of energy loss from the system in overcoming external forces like friction or air resistance and other resistive forces. There are two types of damping; natural damping and artificial damping.

iii) Forced Oscillation: When a body oscillates by being influenced by an external periodic force, it is called forced oscillation. Here, the amplitude of oscillation, experiences damping but remains constant due to the external energy supplied to the system. For example, when you push someone on a swing, you have to keep periodically pushing them so that the swing doesn't reduce.

4.3 Potential Energy and Kinetic Energy Exchange in Oscillation

Since, P.E = ½ kx^2

K.E = ½ mv^2

Total Energy (ET) = P.E. + K.E.

Recall that: $x_{(t)} = A\cos\frac{2\pi t}{T}$

and $v_{(t)} = -A_{\max}\,\mathrm{Sin}\frac{2\pi t}{T}$

Where $w = \sqrt{\frac{k}{m}}$

:ET = $1/2KA^2C\mathrm{os}^2(wt+\Phi) + 1/2mA^2w^2\,\mathrm{Sin}^2(wt+\Phi)$

$= 1/2KA^2C\mathrm{os}^2(wt+\Phi) + 1/2mA^2\left(\frac{k}{m}\right)\mathrm{Sin}^2(wt+\Phi)$

$= 1/2KA^2C\mathrm{os}^2(wt+\Phi) + 1/2KA^2\,\mathrm{Sin}^2(wt+\Phi)$

$= 1/2KA^2(C\mathrm{os}^2(wt+\Phi) + \mathrm{Sin}^2(wt+\Phi))$

Recall that $Cos^2\Theta + Sin^2\Theta = 1$

:. $ET = \frac{1}{2} KA^2$

4.4 Relating SHM with Circular Motion

Circular Motion: This is the motion of a body of mass, **m**, in a circular path of radius, **r**. The parameters that describe circular motion are, angular distance θ, angular velocity, ω, and angular acceleration, α,

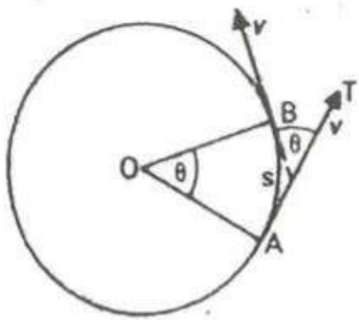


Fig. 4.3: Circular motion

Consider an object moving in a circle of radius, r, with a uniform speed, ω, round a fixed point, with O as its centre as shown in the figure above.

Angular Displacement $\boldsymbol{\theta}$:
This is the angular distance covered by the body about the centre, O, and is related to the **linear displacement, S**, which is the arc that subtends the angle, θ, at the centre, O, of the circle as:

$$\theta = \frac{S}{r} \quad \text{.................................1}$$

$$\Longrightarrow \qquad S = r\theta \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

The angle θ is measured in radians (where 2π radians = 360°).

Angular Speed, (ω):
Angular speed about a point O is defined as the time rate of change of angular displacement and is given by:

$$\omega = \frac{\theta}{t} \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

The angular speed is measured in radians per second ($rads^{-1}$).

Relationship between Linear and Angular Velocities:
Linear velocity is given by:

$$V = \frac{s}{t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

Using eqn (2), S = r into equation (4), we have:

$$V = \frac{r\,\theta}{t} \qquad = r\,\omega$$

$$\Longrightarrow \qquad V = r\,\omega \qquad 5$$

Angular Acceleration, (α):
Angular acceleration is defined as the time rate of change of angular speed and is given by:

$$\alpha = \frac{\omega}{t} \qquad 6$$

The angular acceleration is measured in radians per second per second (rad per second squared, $rads^{-2}$).

Relationship between Linear and Angular Accelerations:

Linear **acceleration** is given by:

$$a = \frac{v}{t} \qquad 7$$

Using eqn (5), V = rω into equation (7), we have:

$$a = \frac{r\omega}{t} = r\,\alpha$$

$$\Longrightarrow \quad a = r\,\alpha \qquad 8$$

4.5 Exercise

1. Define Simple Harmonic Motion
2. Differentiate between Free Oscillation, Damped Oscillation and forced Oscillation.
3. A 90.0 kg skydiver hanging from a parachute bounces up and down with a period of 1.50 s. What is the new period of oscillation when a second skydiver, whose mass is 60.0 kg, hangs from the legs of the first?
4. A car is racing on a circular track of 180 m radius and of banking angle 30^0. To avoid the chances of skidding, what should be the speed of the car?
5. A body moving with simple harmonic motion, has an amplitude of 5 cm and a frequency of 50 Hz. Calculate
   a. The period of oscillation
   b. The acceleration at the middle and end of the oscillation
   c. The velocities at the corresponding instants

CHAPTER FIVE: NEWTON'S LAWS OF MOTION

BY

DR. M. I. KIMPA

5.1 Momentum and Impulse

5.1.1 Momentum

The momentum, P of a body is defined as the product of its mass and its velocity. It is written as:

Momentum = Mass x Velocity

$$P = mv \qquad 1$$

Momentum is a vector quantity and its S.I unit is ($kgms^{-1}$) or (Ns).

Now, from Newton's second law of motion, if an object moving along x-direction has initial velocity v_{ox} and final velocity v_x then;

$$F \quad \alpha \quad \frac{\text{Change in momentum}}{\text{Time taken}}$$

$$F \quad \alpha \quad \frac{mv - mu}{t}$$

$$F \quad \alpha \quad \frac{m(v-u)}{t}$$

$$F \quad \alpha \quad ma$$

$$F = kma, \quad \text{Where k is a constant.}$$

$$F = ma$$

The S.I. unit of force is Newton, mass in kg and a in m/s^2

5.1.2 Impulse

If a force, F, acts on a body of mass, m, for a time, t, then the impulse on the body is the product of the force and the time. i.e.

$$I = Ft$$

But F, $F = \frac{m(v-u)}{t}$

cross multiply

$F_x t = m(v_x - v_{ox})$

$I = m(v_x - v_{ox})$

or

$I = mv_x - mv_{ox}$

$I = P_f - P_i =$ change in momentum

This says that, impulse is equal to the change in linear momentum.

Example 1) A ball of mass 100 g is rolling on a horizontal plane with a velocity of 20 ms^{-1} when a boy hit it from its back, it then moves with velocity of 30 ms^{-1} on its path. If the boy's toes stick to the ball for 0.5 s, calculate;

a. The impulse which the ball experienced.
b. The force applied on the ball by the boy.
c. The rate at which the ball attains its final velocity

Solution

M = 100 g = 0.1 kg, $v_i = 20\ ms^{-1}$, $v_f = 30\ ms^{-1}$, t = 0.5s,
F = ? I = ?, $a_x = ?$

Assuming the ball rolls along x-direction then,

(a) $I = m(v_x - v_{ox})$

$\Longrightarrow$ $I = 0.1(30 - 20) =$ **<u>1 Ns</u>**

(b) Using $F_x t = I$,

$F_x \times (0.5) = 1$

Divide through by 0.5

$$F_x = \frac{1}{0.5} = \mathbf{2N}$$

(c) The rate at which the ball attains its final velocity is found using:

$F = ma_x$

Make a subject formular

$$a_x = \frac{F}{m} = \frac{2}{0.1} = \mathbf{20\ ms^{-2}}.$$

Example 2) A hose ejects water at a speed of 20 cm/s through a hole of area 100 cm^{-2}. If the water strikes a wall normally, calculate the force on the wall in Newton, assuming the velocity of the water normal to the wall is zero after collision.

Solution

To calculate the force on the wall, we use:

F = mass per unit time x change in velocity

The initial speed of the water $v_i = 20\ ms^{-1}$,
the final speed $v_f = 0$,

:. change in velocity $\Delta v = 20 - 0$

$= \mathbf{20\ ms^{-1}}$

To find the mass of the water ejected per unit time we use:

Volume of the water ejected per second = $\frac{l \text{ X } b \text{ X } h}{t}$

$= l_x \text{ X } l_y \text{ X } \frac{l_z}{t}$ = area × velocity

But, Area of the nozzle = 100 cm^2.

Volume of the water ejected per second = 100×20

= 2000 cm^3/s

Since, $p = \frac{m}{v}$

Make m subject formular

$m = \rho \times V$

$= 1 \times V$ $m = V$

$\Longrightarrow$ mass of water ejected per second = Volume of the water ejected per second. Therefore, mass of water ejected per second =2000 g/s

= 2 kgs^{-1}

Therefore, the force on the wall = 2×0.2

= **0.40 N**

Example 3) A body of mass 9.0 kg moves with a velocity 15 m/s. cal. the momentum of the body.

$\rho = mv = 9 \times 15 =$ **135 kgm/s**

Example 4) A stationary ball is hit by an average force 65 N for a time 0.05 sec. What is the Impulse experienced by the body?

I = ft = 65 x 0.05 = **3.25 N-sec**

Example 5) A ball of mass 0.3 kg, moving at a velocity of 20 m/s is suddenly hit by a force of 5 N for a time of 0.03 sec. Find its new velocity of motion.

$$F = \frac{mv - mu}{t}$$

cross multiply

$$ft = mv - mu$$

Make v subject formula

$$v = \frac{ft}{m} + u$$

$$= \frac{5 \times 0.03}{0.3} + 20 = \mathbf{20.5\ m/s}$$

Example 6) A ball of mass 0.1 kg is dropped from a height. If the ball hits the ground after 2 Sec, what is the momentum of the ball just before it hits the ground?

$$V = u + at = u + gt; \quad u = 0, \quad g = 9.8m/s^2;$$

$$p = mV = 1.98kgm/s$$

5.2 Newton's Laws of Motion

5.2.1 First Law and Inertial Frame

The first law states that **"everybody continues in its state of rest or uniform motion in a straight line, unless impressed (external resultant force acts) force acts in it".** We can say that when the resultant force on a body is zero, its acceleration is zero. That is when F = 0, then a = 0 which implies that an isolated body is either at rest or moving with constant velocity This law is sometimes called **Law of Inertial**, since it applies to objects in an inertial frame of reference. An **inertial frame of reference** is one in which an object will move with constant velocity if left undisturbed. An inertial frame is one with no acceleration. In general, inertial frames are those in which Newton's law of motion are valid.

The reluctance of a stationary objects to move and the moving objects to stop is called **Inertial.** Object with big mass feels more reluctant to change its state than an object with small mass which implies that mass of an object is a measure of its inertial.

5.2.2 Second Law and Fundamental Equation

The second law states that the change in momentum per unit time is proportional to the impressed force and it takes place in the direction of the straight line along which the force acts. In another way, the law states that the time rate of change of momentum of a particle is equal to the resultant external force acting in the object /particles.

$$\sum F = \frac{dP}{dt} = \frac{d(mV)}{dt} \qquad (1)$$

$$p = mV\ (momentum\ of\ a\ particle)$$

$\sum F$ is the vector sum of all external forces. From equation (1), we have for m=constant;

$$\sum F = \frac{d(mV)}{dt} = m\frac{dV}{dt} \qquad but\ a = \frac{dV}{dt}$$

$$therefore, \quad \sum F = ma \qquad (2)$$

Equation (1) and (2) are Newton's 2nd law = Fundamental equation

Therefore, from (2) we conclude that the resultant force on a particle equal to its mass multiplied by its acceleration if the mass is constant. Equation (1) is valid only when the speed of the particles is less than the speed of light.

The resultant force is zero if a = 0, where V is constant or zero. Hence the 1st law of motion is a special case of the second law. The SI unit of force is N. A force of 1 N is defined as the force that, when acting on a 1 kg mass, produces an acceleration of 1 m/s^2. 1 N = 1 kgm/s. The table below shows different system of unit for mass, acceleration and force.

Systems of Units	**Mass**	**Acceleration**	**Force**
SI	Kg	m/s^2	N=kgm/s^2
Cgs	G	Cm/s^2	dyne = gcm/s^2
British Engineering	Slug	Ft/s^2	lb = slug ft/s^2

1 N = 105 dyne = 0.225 lb

Lb = pound: 1 pound of force when acting on a 1-slug mass produces an acceleration of 1ft/s^2

1 Lb = 1 slug ft/s^2

Deduction from Newton's Second Law

Momentum can be related to Newton's 2nd law of motion that is the time rate of change of the momentum of a particle is equal to the resultant force on the particle. that is:

$$F = ma = \frac{m(V-U)}{t} = \frac{mV - mU}{t}$$

$$F = \frac{p_f - p_i}{t} = \frac{dp}{dt}$$

$$dp = Fdt$$

$$\Delta p = \int_{t_i}^{t_f} F dt = I$$

5.2.3 Third law

If two bodies interact, the force of body 1 and 2 (F_{12}) is equal to and opposite the force of body 2 on body 1 (F_{21}) (In a direction along the line joining the particles). Then;

$$F_{21} = -F_{12}$$

The action force is equal in magnitude to the reaction force and opposite in direction. that is the action and reaction forces always act in different objects. Note that an isolated force cannot exist in nature.

Examples 1) A force of 3000 N is applied to a 1500 kg car at rest (a) what is its acceleration (b) what will its velocity be 5 s later.

(a) $a = \frac{f}{m} = \frac{3000}{1500} = \mathbf{2\ m/s^2}$

(b) v = at = 2 x 5 = **10 m/s**

Example 2) A 60 g tennis ball approaches a racket at 15 m/s in contact with the racket for 0.005 s and then rebounded at 20 m/s. Find the average force that the racket exerted on the ball.

Solution

v = -20 m/s, u = 15 m/s, t = 0.005 s

The ball acceleration, $a = \frac{v-u}{t} = \frac{-20-15}{0.005} = \frac{-35}{0.005} =$

= **7,000 m/s²**

From second law of motion,

F = ma

= 0.06 x 7,000= -420 N (Minus sign shows that the force was in opposite direction)

Example 3) An object of mass 0.3kg is places on a horizontal frictionless surface. Two forces act on the object as shown fig below. The force F1 has a magnitude of 5.0N and F2 has magnitude of 8.0N. Determine the acceleration of the object

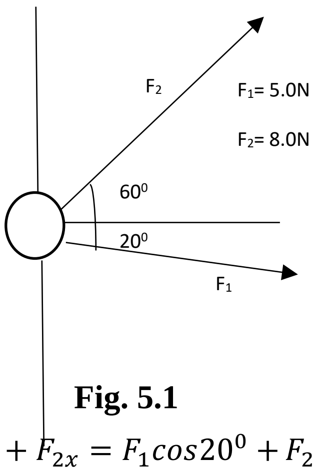


Fig. 5.1

$$\sum F_x = F_{1x} + F_{2x} = F_1 cos20^0 + F_2 cos60^0 = 8.7N$$

$$\sum F_y = F_{1y} + F_{2y} = -F_1 sin20^0 + F_2 sin60^0 =$$

If a is +ve, $m_2 sin\Theta > m_1$ (that is m_2 accelerates down the inclined plane)

If $m_1 > m_2 sin\Theta$ (acceleration of m_2 is up the incline and downward for m_1)

$$a = \frac{(\sum F)}{m}$$

If we assume that m_1 =10 kg, m_2 = 5 kg, $\Theta = 45^0$, then
a = -4.2 m/s^2

The –ve sign shows that a is against gravity.

5.3 Conservation of Linear Momentum

The law of conservation of linear momentum for two interacting particles states that if two particles from an isolated system, their total momentum is conserved regardless of the nature of the forces between them. Therefore, the total momentum at all times equal its initial total momentum. that is

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

OR when two particles collide, the total momentum of the system before collision always equals the total momentum after the collision. Or more generally

When the vector sum of the external forces that acts on a system equals zero, the total linear momentum of the system remains constant no matter what momentum changes occurs within the system

$$\Delta p_1 = \Delta p_2 \qquad (action\ of\ 1 = reactionof\ 2)$$

5.4 Collision

In collisions, the total momentum is conserved. The total kinetic energy K.E of a body of mass m moving with velocity U is given by

$$\mathrm{K.E} = ½\,\mathrm{m}U^2$$

5.4.1 Types of Collision

There are two types of collision namely

i. Elastic collision
ii. Inelastic collision

Elastic collision is the collision in which the two objects bounce back after collision, while inelastic collision is the one in which the two objects stick together and move with a common velocity, v.

When elastic collision takes place, both momentum and kinetic energy are conserved; while in inelastic collision, only momentum is conserved but kinetic energy in not conserved. Thus, for elastic collision between two objects of m_1 and m_2 with initial velocities of u_1 and u_2, and final velocities of v_1 and v_2, we have:

For conservation of momentum

$$m_1u_2 + m_2u_2 = m_1v_1 + m_2v_2$$

and for conservation of energy

$$1/2\, m_1u_{12} + 1/2\, m_2u_{22} = 1/2\, m_1v_{12} + 1/2 m_2v_{22}$$

An inelastic collision is a collision for which the mechanical energy is not conserved, but momentum is conserved .

A perfectly inelastic collision corresponds to the situation where the colliding bodies stick together after the collision. In the case of inelastic collision, part of the K.E has been converted to other forms of energy like heat and sound. Part of the K.E is not recoverable. This is the case in perfectly inelastic collision, K.E loss is maximum (momentum is conserved)

The kinetic energy of the system before collision is given by:

$$Ek_1 = \frac{1}{2}M_1U_1^2 + \frac{1}{2}M_2U_2^2$$

and after collision is given by:

$$Ek_2 = \frac{1}{2}M_1V_1^2 + \frac{1}{2}M_2V_2^2$$

For complete inelastic collision, the kinetic energy before collision is greater than the kinetic energy after collision. I.e. $Ek_1 > Ek_2$

Example 1) A bullet A of mass 0.1 kg travelling at 200 m/s embeds itself in a wooden block B of mass 0.9 kg moving in opposite direction at 20 m/s. Calculate the velocity of the block and the bullet when the bullet come to rest inside the block and state the principle you used in doing the calculation.

Solution

Given: $m_A = 0.1$ kg, $u_A = 200$ m/s, $m_B = 0.9$ kg, $u_B = 20$ m/s, $v = ?$

Using: Momentum before collision = Momentum after

$$m_Au_A + m_Bu_B = (m_A + m_B)v$$
$$0.1 \times 200 - 0.9 \times 20 = (0.1 + 0.9)v$$

$$20 - 18 = 1v$$

$$2 = 1v$$

$$\mathbf{v = 2\ m/s^2}$$

Example 2) Two bodies A and B of mass 4 kg & 2 kg move towards each other with velocities 3 m/s & 2 m/s and collide. If the collision is perfectly inelastic, find the velocity of the two bodies after collision. Find the total kinetic energy of the system before and after collision, hence cal. the loss in kinetic energy.

Solution

Momentum before collision = Momentum after

(4 x 3) – (2 x 2) = V (4+2)

8 = 6V

v = 8/6 = **1.33 m/s**

Kinetic energy before collision:

Ek_1 = ½ x 4 x 3^2 + ½ x 2 x 2^2 = **22J**

Kinetic energy after collision:

EK_2 = ½ (4 + 2) X 1.33^2 = **5.33J**

Loss in Kinetic energy = 22 – 5.33J = **16.67**

Example 3) Two particles collide in a perfectly inelastic fashion. If $m_1 = 0.5$ kg, $m_2 = 0.25$ kg, $V_{1i} = 4$ m/s and $V_{2i} = -3$ m/s.

(a) Find the velocity of the composite particles after collision.

(b) How much K.E is lost in the collision?

$$V_f = \frac{m_1 V_{1i} + m_2 V_{2i}}{m_1 + m_2}$$

$$= 1.7 m/s \ V_f \ is \ common \ to \ both \ bodies$$

$$K.E_i = \frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{1i}^2 = \mathbf{5.1J}$$

$$K.E_f = \frac{1}{2}(m_1 + m_2)V_f^2 = \mathbf{1.0J}$$

The loss n Kinetic energy is $K.E_i - K.E_f = \mathbf{4.1J}$

Example 4) A 4 kg ball moving with a velocity of 10 m/s collides with a 16 kg ball moving with a velocity of 4 m/s. (i) in the same direction (ii) in the opposite direction. Calculate the velocity of the balls in each case and the energy loss if the balls coalesce (common velocity) on impact. Hence, state whether the collision is elastic or inelastic in each case

i. $m_1V_1 + m_2V_2 = m_1U_1 + m_2U_2$

4V + 16V = 40 + 64

V= 5.2 m/s

ii. 4V + 16V= 64 – 40

V = 1.2 m/s

iii. $K.E_i = \frac{1}{2}m_1U_1^2 + \frac{1}{2}m_2U_2^2 = \mathbf{328\,J}$

iv. $K.E_f = \frac{1}{2}m_1V^2 + \frac{1}{2}m_2V^2 = \mathbf{270.4J}$

V= **5.2m/s**

Loss in kinetic energy = **57.6 J**

v. $K.E_i = \mathbf{328\,J}$

vi. $K.E_f = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = 14.4J$

where V_1=V_2

Loss in kinetic energy = **313.6 J**

Both cases are inelastic

5.5 Inertial Mass and Weight

Weight and mass are two different quantities.
The weight of a body is defined as the force acting on it due to gravitational pull (gravity). It is measured in Newton (N) while mass is the quantity of matter a body contains and is measured in kilogram (kg). Also, weight is a vector quantity while mass is a scalar quantity.

The weight and the mass of a body are related by:

Weight = mass × acceleration due to gravity, i.e.

$$W = mg$$

5.6 Exercise

1. Differentiate between momentum and impulse
2. State the three laws of motion

2b) From second law of motion, show that F = ma

3. Define Collision and differentiate between elastic and inelastic collision.
4. An object has mass of 200 g. Find its weight in dyne and in N.
5. A person weighs 120 lb. Determine (a) her weight in N (b) her mass in kg
6. A 6 kg object undergoes an acceleration of 2 m/s^2 (a) what is the magnitude of the resultant force acting on it? (b) If this same force is applied to a 4 kg object, what acceleration will it produce?
7. A 4 kg object has a velocity of 3i m/s at one instant. t seconds later, its velocity is (8i + 10j)m/s. Assuming the object was subject to a constant net force. Find (a) the component of the force (b) its magnitude

CHAPTER SIX: CIRCULAR MOTION

BY

DR. M. I. KIMPA

6.1 Circular Motion

This is the motion of a body of mass, **m**, in a circular path of radius, **r**. The parameters that describe circular motion are, angular distance θ, angular velocity, ω, and angular acceleration, α,

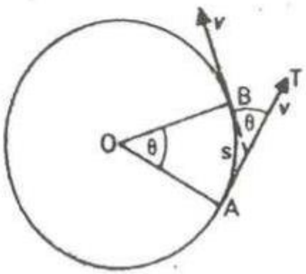


Fig. 6.1: Circular motion

6.1.1 Angular Displacement $\boldsymbol{\theta}$:

This is the angular distance covered by the body about the centre, O, and is related to the **linear displacement, S**, which is the arc that subtends the angle, θ, at the centre, O, of the circle as:

$$\Theta = \frac{s}{r}$$

$$s = r\Theta$$

The angle θ is measured in radians (where 2π radians = 360°).

6.1.2 Angular Speed (ω):

Angular speed about a point O is defined as the time rate of change of angular displacement and is given by:

$$\omega = \frac{\theta}{t}$$

The angular speed is measured in radians per second ($rads^{-1}$).

6.1.3 Relationship between Linear and Angular Velocity

Linear velocity is given by:

$$V = \frac{s}{t}$$

Substitute S = r into V, we have:

$$V = \frac{r\,\theta}{t} = r\,\omega$$

$$V = r\,\omega$$

6.2 Equations of Uniformly Accelerated Circular Motion

6.2.1 Angular Acceleration

If a body is not rotating with a uniform angular velocity, it is said to possess angular acylation. The angular acceleration of a rotating body is defined as the time rate of change of angular velocity. Thus, if a body is moving with an angular velocity w_1 and changes its velocity from w_1 to w_2 in t seconds, we have angular acceleration;

$$\frac{dw}{dt} = \frac{w_2 - w_1}{t}\,(\mathrm{rad}/s^2)$$

Now if the distance of the body from the axis of rotation is r, the linear velocity from the beginning will be rw_1 and will be changed to Θw_2 in t seconds. Then the linear acceleration of the body will be:

$$a = \frac{rw_2 - rw_1}{t}$$

$$= \frac{r(w_2 - w_1)}{t}$$

that is linear acceleration = distance from the axis of rotation (r) * angular acceleration (α)

6.2.2 Uniform Circular Motion

Consider a body of mass m moving in a horizontal circle of radius r. The acceleration of the body can be found as follows: If the body moves with a velocity V, the direction of the linear velocity at every point will be tangential to the circle at that point. Let the body move from A to B in t seconds and describe a small angle 0 during the period. The linear velocity will not change in magnitude but its direction will keep on changing as the body describes the circle at the point A. Its velocity is represented by AC and at B by the tangent (Fig 6.2). The angular velocity $w = \frac{d\theta}{dt} = \frac{\theta}{t}$

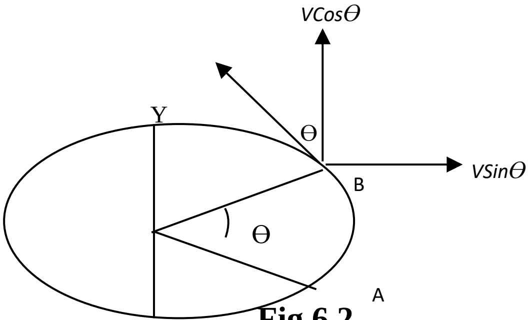


Fig 6.2

Now at point A horizontal component of the velocity is zero and the entire linear velocity is along AC. At point B, the linear velocity can be resolved into two components –

horizontal and vertical component (along AC and perpendicular to AC). The component of velocity at B parallel to AC = V$\cos\theta$ and perpendicular to AC = V$\sin\theta$ and represented by BL and BM respectively as shown in fig 1.3. Now if θ is very small, $\sin\theta$ = 0 and $\cos\theta$ = 1. Hence,
The component parallel to A0 = Vθ and the component perpendicular to A0 = V.

From the above equation, we find that the vertical component of the velocity does not change when moves near to A. While the horizontal component of the velocity changes from zero to V * 0 that is an added velocity V*0 is acquired by the body along A0 in time t, hence, the acceleration acquired by the body

$$a = \frac{\text{change in velocity}}{\text{time}} \frac{V\theta}{t}$$

Or acceleration = V. ($\frac{\theta}{t}$) v.w
Therefore

$$\text{acceleration} = \frac{\text{v2}}{r} \quad (\text{as } w = \frac{v}{r})$$

Thus, acceleration is directed such that its direction is always perpendicular to the direction of velocity at that point that is it is always directed towards the centre and acts along the radius of the circle and is called radial or centripetal **(centre seeking)** acceleration.

6.2.3 Relationship between Linear and Angular Acceleration

Linear **acceleration** is given by:

$$a = \frac{v}{t}$$

Substitute $V = r\omega$, we have:

$$a = \frac{r\omega}{t} = r\,\alpha$$

$$a = r\,\alpha$$

Example 1) A motorist speeding at 2.5 ms^{-1} decelerates uniformly to 2 ms^{-1} in 5 s as he comes across a roundabout of radius 3 m. Find:

a. His initial and final angular speeds
b. Angular and linear accelerations
c. The angle he subtends at the centre of the circle in the first 2s.

Solution

$V_1 = 2.5\ ms^{-1}$, $V_2 = 2\ ms^{-1}$, $t = 5$ s, $r = 3$ m, $\omega_1 = ?$ $\omega_2 = ?$ $\alpha = ?$ $a = ?$

a) His initial angular speed $\omega_1 = \frac{v_1}{r} = \frac{2.5}{3} =$ **.833 rads^{-1}**

b) His final angular speed $\omega_2 = \frac{v_2}{r} = \frac{2}{3} =$ **0.667 rads^{-1}**

c) For the angular acceleration

$$\alpha = \frac{\omega_2 - \omega_1}{t} = \frac{0.667 - 0.833}{5} = -\ 0.0332 rads^{-2}$$

And hence, the linear acceleration:

$a = r\omega = 3 \times (-0.0332) =$ **- 0.0996 ms^{-2}**

d) For the angle he subtends at the centre in the first 2s,

$\theta = ?$, $t = 2s$.

We have to find $\omega_{t\,=\,2s} = ?$

Using: $\alpha = \frac{\omega_{t=2s} - \omega_1}{t} = \frac{\omega_{t=2s} - 0.833}{2} = -0.0332$

$\Longrightarrow$ $-0.0332 \text{ X } 2 = \omega_{t=2s} - 0.833$

$\Longrightarrow$ $\omega_{t=2s}$

$= 0.833 - 0.0664 = 0.7666 \text{ rads}^{-1}$.

Now, using $\omega_{t=1s} = 0.7666 = \frac{\theta}{t}$

$\Longrightarrow$ $\theta = 0.7666 \text{ x } t = 0.7666 \text{ x } 2 = \mathbf{1.5332}$

$\Longrightarrow$ $\theta =$ **1.5332 rad**

$= 275.976^{o}$.

$\approx$ **276^{o}.**

Therefore, the angle he subtends at the centre of the circle in the first 2s is 276^{o}.

6.3 Period of Circular Motion, T

This is the time taking to complete one cycle.

Using $\omega = \frac{\theta}{t}$, for one complete cycle, the angular distance covered is $\theta = 2\pi$, and the time for the complete cycle is the period, T.

$$\omega = \frac{2\lambda}{T}$$

$$T = \frac{2\lambda}{\omega}$$

The S.I unit of period is second (s).

6.3.1 Frequency of Circular Motion, f

This is the number of complete circles/revolutions per unit time.

$$f = \frac{\text{No. of circles/revolutions}}{\text{time}}$$

The S.I unit of frequency is revolution/sec which is named Hertz (Hz). Therefore,

$$1 \text{ Hz} = 1 \text{ rev/s}$$

For **n** complete circles, the angular distance covered is 2π x n = 2 ,

the total time taken is therefore t = nT = n x $\frac{2\pi}{\omega} = \frac{2\pi n}{\omega}$ and the frequency is related to period of circular motion as follows:

$$f = \frac{n}{t} = \frac{n}{\frac{2\pi n}{\omega}}$$

$$\Longrightarrow \quad f = \frac{\omega}{2\pi}$$

Thus, $f = \frac{1}{T}$ and thus $T = \frac{1}{f}$

Example 1) It takes a body 10 s to complete 4 complete revolutions about a point, O, determine:

a. Period of its oscillations.
b. Its angular velocity

Solution

a. To find T, we use $f = \frac{n}{t} = \frac{4}{10}$ = **0.4 s**.

   Therefore, $T = \frac{1}{f} = \frac{1}{0.4}$= **2.5 s**

b. To find the angular velocity we use

$$T = \frac{2\lambda}{\omega}$$

$$\Rightarrow \quad \omega = \frac{2\pi}{T} = \frac{2\ X\ 3.142}{2.5} = \mathbf{2.514\ rads^{-1}}.$$

6.4 Centripetal and Centrifugal Acceleration

6.4.1 Centripetal Acceleration, (a_{rad})

Although, the magnitude of the velocity of an object undergoing circular motion remains constant, the continuous change in direction of the body is considered to be a form of change of velocity with time, hence acceleration exists. This acceleration is called Centripetal Acceleration.

For a rotating body, the centripetal acceleration is given by:

$$a_{rad} = r\,\omega^2$$

Proof that acceleration exists in a Circular Motion:

To prove that acceleration exist in a circular motion, consider an object of mass, **m**, moving around a circle of radius, **r**, as shown in the figure below:

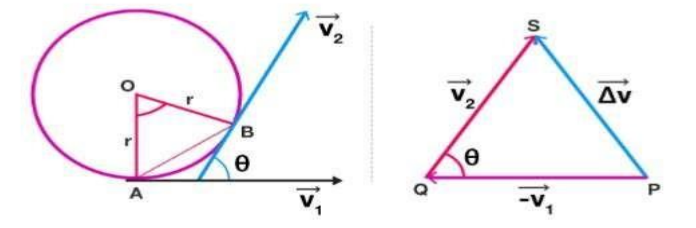


Fig. 6.3: Continuous change of direction of the constant speed in a circular motion

Let the velocities of the object at point A, and at point, B, be two vectors $\mathbf{V_1}$ and $\mathbf{V_2}$ respectively. To find the resultant velocity, $\Delta \boldsymbol{V}$, lets reverse the direction of $\mathbf{V_1}$ so that it becomes

– $\mathbf{V_1}$. Let's the resultant velocity be $\Delta \boldsymbol{V}$ as in the triangle PQS above.

As $-\mathbf{V_1} + \mathbf{V_2} = \Delta \boldsymbol{V}$

$\Longrightarrow \Delta \boldsymbol{V} = \mathbf{V_2} - \mathbf{V_1}$

The triangle PQS is similar to triangle AOB, thus,

$$\frac{\Delta \boldsymbol{V}}{\boldsymbol{AB}} = \frac{\boldsymbol{V_2}}{\boldsymbol{r}} = \frac{-\boldsymbol{V_1}}{\boldsymbol{r}} = \frac{v}{r},$$

Also, the linear velocity along arc AB is:

$$\mathbf{v} = \frac{AB}{\Delta t}$$

$\Longrightarrow$ distance along arc AB,

$$AB = \mathbf{v}\Delta t$$

Using the two equations, we have:

$$\frac{\Delta \boldsymbol{V}}{AB} = \frac{\Delta \boldsymbol{V}}{\mathbf{v}\Delta t} = \frac{v}{r}$$

$$\Longrightarrow \quad \frac{\Delta \boldsymbol{V}}{\mathbf{v}\Delta t} = \frac{v}{r}$$

And multiplying both sides by v, we have;

$$\frac{\Delta \boldsymbol{V}}{\Delta t} = \frac{v^2}{r}$$

Thus, $$a = \frac{V^2}{r}$$

$$a = \frac{(r\boldsymbol{\omega})^2}{r}$$

$$a = r\boldsymbol{\omega}^2$$

This shows the existence of acceleration in circular motion.

The difference between linear acceleration, a, and centripetal acceleration, a_{rad}, is that the linear acceleration is acting tangential to the circle while the centripetal acceleration is always directed towards the centre of the circle.

Also, **the difference between angular acceleration, α, and centripetal acceleration, a_{rad},** is that the angular acceleration is about a point, say, O, which is the centre of the circle while the centripetal acceleration is acting towards the centre, O, of the circle.

6.4.2 Centripetal and Centrifugal Forces

These are forces acting on a body moving round in a circle.

i. **Centripetal force:** is the force acting on a body moving in a circle and directed towards the centre of the circle, it is given by:

$$F_c = ma_{rad}$$

$$= m\frac{v^2}{r}$$

$$= mr\omega^2$$

ii. **Centrifugal force:** is the force acting on a body moving in a circle and directed away from the centre of the circle.

Centrifugal force has equal but opposite magnitude to that of centripetal force.

6.5 Motion in a Vertical Circle

Let us consider the motion of body tied to one end of a string whirled in a vertical circle of radius R with the other end fixed at a point say 0 as shown in fig 1.4

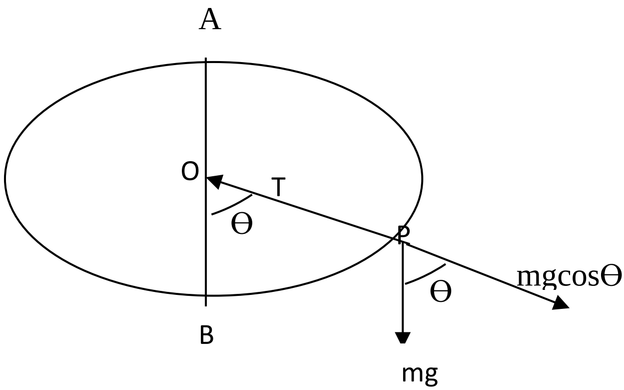


Fig 6.4

Let the body be at a point P such that the string makes an angle θ with the vertical. If T is the tension in the string and m is the mass of the body, centripetal force will be

$$\frac{MV^2}{r} = T - mgcos\theta$$

or $$T = \frac{MV^2}{r} + mgcos\theta$$

At the lowest point A, the angle θ = 0 and Cosθ = 1

Therefore $$T_A = \frac{MV^2}{r} + mg$$

At the highest point B, the angle $\theta = \pi$ and Cosθ = -1

Hence $$T_B = \frac{MV^2}{r} - mg$$

from above relation, it is clear that for $T_B < 0$ or negative, the string will become loose and the body no longer moves in circular path. While for $T_B > 0$, the string remains tight that is to keep the body in the circular path, the centripetal force must be greater than or equal to the weight of the body at the highest point. If tension is zero, it will just complete the vertical circle and the velocity of the body at the highest point called critical speed V_c will be given by:

$$0 = \frac{mV_{c^2}}{R} - mg \qquad or \qquad Vc = \sqrt{Rg}$$

The above relation given minimum velocity at the highest point in order to complete the circle shows if the velocity at the highest point $V_B < V_c$. The string will become loose and the body will no longer move in circular path and will fall downward

Velocity at the lowest point A to complete the circle making use of the law of conservation of energy for the body at A and B, we have for the total energy (KE + PE)

$$\frac{1}{2}mV_A^2 + 0 = \frac{1}{2}mV_B^2 + mg(2R)$$

Or

$$V_A^2 = V_B^2 + 4gR$$

If the body just performs the complete revolution that is if it has critical velocity at B that is $V_B = V = \sqrt{Rg}$

We have;

$$V_A^2 = Rg + 4Rg = 5Rg$$

Or

$$V_A = \sqrt{5Rg}$$

In case $V_A < \sqrt{5Rg}$, the body will either oscillate about the lowest point A or leave the circular path altogether. The tension along the chord is minimum at the top and maximum at the bottom and at no point tension should be negative otherwisebody will fall down without completing the circle.
Difference between the tensions at the highest and lowest point will be under this situation

$$T_A - T_B = 2mg + \frac{mV_A^2}{R} - \frac{mV_B^2}{R}$$

Or $$T_A - T_B = \frac{m}{R}(V_A^2 - V_B^2) + 2mg$$

Or $$T_A - T_B = \frac{m}{R}(4Rg) + 2mg$$

Or $$T_A - T_B = 6\,mg$$

This shows that the difference between the tension at the cord at the highest and lowest point is six times the weight of the body

Example 1) An object of mass 5 kg is whirled round a horizontal circle of radius 5 m with uniform speed of 5 ms^{-1} by a revolving string inclined to the vertical; calculate:

a. The tension in the string inclined to the vertical.
b. The angle of inclination of the string to the vertical and
c. The tension in the string

Solution

m = 5 kg, a = 9.8 m/s^2, r = 5 m, v = 5 m/s, T_{vert} = ?
θ = ?, T = ?

The problem is sketched as shown in Fig. 6.3:

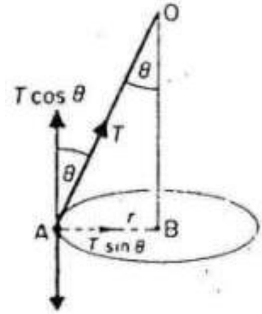


Fig. 6.5: Two components of tension in a circular motion

The tension will have vertical as well as horizontal components.

The horizontal component of the tension, T is the centripetal force and is given by:

$$F_c = T\ \text{Sin}\theta$$

$$F_c = m\frac{v^2}{r}$$

$\Rightarrow$ $$T\ \text{Sin}\theta = m\frac{v^2}{r} \quad \ldots\ldots\ldots\ldots\ldots \text{(A)}$$

a. For equilibrium, the vertical component of the tension, T_{vert}, will be equal to the weight of the object T_{vert} = mg.

$$\Rightarrow \quad T_{vert} = 5 \times 9.8 = \underline{\mathbf{49\ N}}$$

b. To find the angle of inclination to the vertical, θ, we use: $T_{vert} = T\ \text{Cos}\theta$

$\Rightarrow$ $$T\ \text{Cos} \ = mg \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(B)}$$

Dividing eq. (A) by eq. (B), we have:

$$\tan\theta = \frac{v^2}{rg}$$

$$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

$$\theta = \tan^{-1}\left(\frac{5^2}{5 \, X \, 9.8}\right)$$

$$\theta = \tan^{-1}(0.5102)$$

$$\theta = \underline{\mathbf{27.03^o.}}$$

c. From eqn. (A), the tension T in the string is:

$$T = \frac{5 \, X \, 5^2}{5 \, X \sin 27.03} = 55.05 \text{N}$$

6.5.1 Relative Velocity

The relative velocity of a body A relative to velocity of another body B moving in the same direction is simply the difference between the two velocities, $(\vec{V}_A - \vec{V}_B)$ and similarly, the relative velocity of B relative to A is $(\vec{V}_B - \vec{V}_A)$. However, if they are moving in opposite direction, we add the two velocities (V_A + V_B).

Example 1) Three cars, 1 , 2 and 3 are on the same high way, A and C are approaching the same town with speed of 100 kms^{-1} and 120 kms^{-1} respectively while B is just coming back from the town with speed of 80 kms^{-1}. Calculate the relative velocity of:

a. A with respect to B.
b. A with respect to C.
c. B with respect to C and

d. C with respect to B

Solution

$V_A = 100$ kms^{-1}, $V_B = 80$ kms^{-1}, $V_C = 120$ kms^{-1},

a. $Vr_{A \text{ wrt } B} = V_A + V_B = 100 + 80 =$ **180 kms^{-1}**.

b. $Vr_{A \text{ wrt } C} = V_A - V_C = 100 - 120 =$ **-20 kms^{-1}.**

c. $Vr_{B \text{ wrt } C} = V_B +- V_C = 80 + 120 =$ **200 kms^{-1}**

d. $Vr_{C \text{ wrt } B} = V_C +- V_{AB} = 120 +- 80 = 200$ kms^{-1}/100
= **20 kms^{-1}**

6.6 Exercise

Fill in the blank questions

1. The parameters that describe circular motion are denoted by θ, ω, and ______.
2. __________ is the arc that subtends an angle, θ, at the centre, O, of a circle.
3. Radians per second (rads^{-1}) is a unit for measuring ______________.
4. The equivalence of one Hertz (1 Hz) is ______________.
5. A body is moving along a circular path with variable speed. It has two types of accelerations namely__ and _________________.

Objective Questions

6. Average angular velocity of a body rotating at an angle of $30°$ for 5 seconds will be
   (a) 6 rad/s (b) 7 rad/s (c) 8 rad/s (d) 10 rad/s

7. In the case of uniform circular motion, which one of the following physical quantities does not remain constant?
   (a) Mass (b) Speed (c) Linear momentum (d) Kinetic energy

Essay questions

8) An object of mass 10 kg is whirled round a horizontal circle of radius 4 m by a revolving string is inclined to the vertical. If the uniform speed of the object is 5 ms^{-1}, calculate:
   a. The tension in the string.
   b. The angle of inclination of the string to the vertical.

CHAPTER SEVEN: WORK, ENERGY AND POWER

BY

DR. S. TAUFIQ

7.1 Work

When a force acts on any object, we can define the work done W, as the product of the force, F and the displacement, s in the direction of the force.

i.e. W = F x s

The definition of work as (W = Fs) does have one surprising feature: If the distance s is zero, the work is zero, even if a force is applied. Pushing on an immovable object, such as a brick wall, may tire your muscles, but there is no work done of the type we are discussing. In physics, the idea of work is intimately tied up with the idea of motion. If the object does not move, the force acting on the object does no work.

Work is a scalar quantity, and it is measured in Newton-metre (Nm) or Joule (J). The work done by a constant force F acting on a particle is defined as the product of the component of the force in the direction of the particle's displacement and the magnitude of the displacement. that is the work done by:

F is : $\boldsymbol{W = Fscos\theta}$ [from F x s = |FsCosΘ].

However, the work done by a varying force acting on an object moving along the x-axis from X_i to X_f is given by:

$$\boldsymbol{W} = \int_{xi}^{xf} Fxdx$$

If there are several forces acting on the particle, the net work done by the forces is the sum of the individual work done by each force.

7.1.2 Work done in a Gravitational field

An object in motion has kinetic energy. There are also other types of energy. For example, an object may possess energy by virtue of its position relative to the earth and is said to have gravitational potential energy.

The gravitational potential energy PE is the energy that an object of mass m has by virtue of its position relative to the surface of the earth. That position is measured by the height h of the object relative to an arbitrary zero level:

$$PE = mgh$$

Where g, is the acceleration due to gravity, which has a constant value of 9.8 or 10 m/s^2

Example 1) A footballer hits a ball of mass 300 g, and cause it to accelerate at 65 m/s^2, if the ball takes 25 secs to reach the goal post, what is the kinetic energy with which the ball moves?

Solution

Given: $m = 300\ g \quad = \quad 300/1000 \quad = 0.3\ kg$

$a = 65\ m/s^2$, $t = 25\ secs$

$$a = v\ t$$

:. $\quad v = a \times t = 65 \times 25 = \mathbf{1625\ m/s}$

Therefore,

$$K.E \quad = \quad ½\ Mv^2$$

$K.E \quad = \quad ½ \times 0.3 \times 16252 \; = \; \mathbf{396{,}093.7}\ \boldsymbol{J}$

$K.E \quad = \quad \mathbf{396.1}\ \boldsymbol{kJ}$

Example 2) A car speeding at 23 m/s was hit by another car with an energy of 185 kJ, it causes the first car to increase its speed to 28 m/s. Find the mass of the car that was hit.

Solution

Given: $vi = 23\ m/s \quad vf = 28\ m/s$

$Energy = Workdone\ by\ the\ car\ after\ it\ was\ hit = 185\ kJ$ $= 185{,}000\ J$

We make use of the work-energy theorem to find the mass,

$$W = K.E_f - K.E_i = \frac{1}{2}\,Mv_f{}^2 - \frac{1}{2}Mv_i^2$$

$$W = \frac{1}{2}Mv_f^2 - \frac{1}{2}\,Mv_i^2$$

Now, we make M subject of the formula,

$$M = \frac{2 \times W}{V^2{}_f - \mathrm{V_i}^2} = \frac{2 \times 185{,}000}{28^2 - 23^2}$$

$$\boldsymbol{M} = \mathbf{1450}\ \boldsymbol{kg}$$

Example 3) A gymnast of mass 56 kg jumps vertically upward from a trampoline, and reaches a maximum height of 4.80 m (with respect to the ground) before falling back down. Ignoring air resistance, determine his gravitational potential energy at his maximum height before he falls back. (Take g = 9.8 m/s^2)

Solution

Given: $m = 56\ kg$, $h = 4.8\ m$

$$PE = mgh$$

$$PE = 56 \times 9.8 \times 4.8 = 2634\ J$$

$$PE = \mathbf{2.6\ kJ}$$

Example 4) A loaded sack of total mass 110 kg falls down from the floor of a lorry 2.5 m high. Calculate the work done by gravity on the load.

W = mgh = 110 x 10 x 2.5 = **2750 J**

Example 5) A boy of mass 5 kg runs up a set of steps of total height 3.0 m. Find the work done against gravity.

W = mgh = 5 x 10 x 3 = **150 J**

Example 6) What is the work done when a force of 500 N pushes a car 6 m on a straight path?

W = F x s = 500 x 6 = **3000 J**

7.2 Energy

Energy is defined as capacity for doing work, thus the unit of energy is same as work (that is Joule).

7.2.1 Forms of Energy

Energy manifest in different forms, these are:

1. **Kinetic energy** such as the energy possess by a moving car or a falling stone due to its motion
2. **Chemical energy** such as the energy contain in food from which derives energy used while working

3. **Heat energy** such as the energy developed in the engine of a car which the car uses for motion
4. **Electrical energy**: energy travelling along conductors used in driving fans
5. **Light energy**: energy emitted by light bulbs
6. **Sound energy**: energy in speakers
7. **Nuclear energy**: energy derive from the nucleus of atoms, nuclear generators

7.2.2 Transformation and Conservation of Energy

Law of conservation of energy states that energy can neither be created nor destroyed, but can be changed from one form to another. Energy in one form can be transform into another form by means of appropriate mechanisms.

7.2.3 Types of Mechanical Energy

A body can possess mechanical energy or have the ability to do mechanical work either by virtue of its motion or its position. For example, a car moving with velocity **V** possess Kinetic Energy, K.E by virtue of its motion while a stone held at a height, h possess Potential Energy, P.E by virtue of its position.

Kinetic Energy

Supposed a particle has a constant mass and that at times t_1 and t_2, it is located at point p_1 and p_2 and moving with velocities $v_1 = \frac{dv_1}{dt}$ and $v_2 = \frac{dv_2}{dt}$ respectively. Therefore, the total work done in moving the particle from p_1 to p_2 is given by:

$$w = \int F.dv = \frac{1}{2}m({v_1}^2 - {v_1}^2)$$

Total work done from p_1 to p_2 → K.E at p_2 , - K.E atp_1

$$w = T_2 - T_1$$

where $T_1 = \,^1/_2\, m{v_1}^2\,; T_2 = \,^1/_2\, m{v_2}^2 \qquad (V << C)$

Kinetic energy is defined as the energy a body possess by virtue of its motion.

Potential Energy

The scalar V such that $F = -\nabla v$ is called the potential energy. Scalar potential of the particle in the conservative force field. The total work done from

$$p_1 \ to \, p_2 \ \rightarrow P.E \ at \ p_1 - P.E \ at \ p_2$$

can be written as

$$w = v_1 - v_2$$

$$v_1 = v(p_1)$$

Energy needed in some machine are stored in some components of the machine as Potential Energy. In a pure mechanical system only these two forms of mechanical energy are considered with the result that in energy conservation law, the total energy gives:

Total Energy = K.E + P.E = constant (for a pure mechanical system)

$E = T + V = constant$ (principle of conservation)

The **work-energy theorem states that the net-work done on a particle by external forces equals the change in K.E of the particle**

that is:

$$W = k_f - k_i = \,^1/_2\, m{v_f}^2 - \,^1/_2\, m{v_i}^2$$

Example 7) A mass of 25 kg moves with a velocity of 4 m/s. Find its K.E.

K.E. $= \frac{1}{2} mv^2 = \frac{1}{2} \times 25 \times 4^2 =$ **200 J**

Example 8) Find the potential of a body of mass 30 kg standing on a building floor 10 m above the ground level.

P.E. $= mgh = 30 \times 10 \times 10 =$ **3000 J**

Example 9) A ball of mass 0.1 kg is thrown vertically upward with an initial velocity of 20 m/s. Calculate the (a) P.E halfway (b) P.E at max height (c) P.E as it leaves the ground (d) K.E half way up (e) K.E as it leaves the ground (f) K.E. at the max height and use your answer to show that energy is conserved in this exercise.

a) $V^2 = U^2 - 2gh$ at h_{max}, $V = 0$

$U^2 = 2gh$

$h_{max} = 20.4$ m

half way up (1/2 h = 10.2 m)

P.E = **10 J**

b) $P.E_{max} =$ **20 J**

c) P.E at ground level

h = 0 then P.E = **0J**

d) K.E at the ground level, V = 20 m/s

K.E $= \frac{1}{2} m V^2 =$ **20 J**

e) $K.E_{1/2}h = \frac{1}{2}mv^2 =$ **10J**

f) K.E at h_{max} that is at V= 0,

K.E = **0 J**

for energy to be conserved, the total mechanical (sum of K.E and P.E) must be the same for all points or height. Thus, for the 3 heights:

P.E + K.E = (0+20) J = (10+10)J = (20+0)J = 0 J

7.3 Power

The time rate of doing work on a particle is often called the instantaneous power or power applied to the particle

$$P = \frac{dw}{dt} = \quad \text{work; P = power}$$

If **F** is the force acting on a particle and **V** is the velocity of the particle, then

$$P = F.V = \frac{work\ done\ (or\ expended)}{time} = \frac{w}{t}$$

The unit of power is Joule per second J/S or Watt (w). 1 Watt =1 J/S

(1 hp = 746 watts = 0.75 kw)

In many situations, the time it takes to do work is just as important as the amount of work that is done. The idea of power incorporates both the concepts of work and time, for power is the work done per unit time.

Average power, P is the average rate at which work W, is done, and it is obtained by dividing W, by the time t, required to perform the work:

$$Power = Work \times Time \quad = \mathrm{W} \times t$$

SI Unit of Power: joule/s = watt (W)

An alternative expression for power can be obtained from Equation 6.1, which indicates that the work W done when a constant net force of magnitude F points in the same direction as the displacement s is:

$$\boldsymbol{W} = \boldsymbol{F} \times \boldsymbol{s}$$

Divide both side by t

$$W/t = \ F \times s/t$$

Since, average power,

$\bar{P} = W\ t\,/$ and average velocity, $\bar{v} = s\ t\,/$

Then, $\bar{P} = F \times \bar{v}$

Power is a scalar quantity.

- 1 horse power (hp) = 746 W = **0.746 kW.**
- The kilowatt – hour (kW.h) is the usual commercial unit of electrical energy. 1 kWh is the total work done in 1 hour (3600 s) when the power is 1 kilowatt (103 J/s), so,

1 kW.h = (103 J/s) × (3600 s) = 3.6 × 106 J = **3.6 MJ**

Example 10) A car, starting from rest, accelerates in the +x direction. It has a mass of 1003 kg and maintains an acceleration of 4.60 m/s^2 for 5 s. Assume that a single horizontal force accelerates the vehicle. Determine the average power generated by this force.

Solution

Given: M = 1003 kg, a = 4.6 m/s^2, t = 5 s, u = 0, P = ?

We can find the power using,

$$P = F \times v$$

Recall that,

$$F = M \times a$$

$$v = u + \frac{1}{2}at$$

Therefore,

$$F = M \times a = 1003 \times 4.6 = \mathbf{4613.8}\ \boldsymbol{N}$$

$$v = u + \frac{1}{2}at = 0 + \frac{1}{2} \times 4.6 \times 5 = \mathbf{11.5}\ \boldsymbol{m/s}$$

Finally,

$$P = F \times v = 4613.8 \times 11.5 = \mathbf{53{,}058.7}\ \boldsymbol{W}$$

Example 11) A car traveled at 65 m/s produces a force of 400 N. Cal. the power of the engine in KW

Power = FV = 400 × 65 = 26000 W = **26 KW**

Example 12) A man whose mase is 70 kg walks up a flight of 10 steps each 20 cm high in 5 seconds. Calculate the power he develops (g 10m/s^2)

$$\text{Power} = \frac{F \times s}{t} = \frac{mgs}{t} = \frac{70 \times 10 \times 2}{5} = \frac{1400}{5} = \mathbf{280\ W}$$

7.4 Efficiency (ŋ)

Efficiency **ŋ** is defined as the ratio of the useful energy (or work) output of a machine (or any other energy converting system) to the energy (or work) input. that is

$$\eta = \frac{\textbf{energy output}}{\textbf{energy input}}$$

The efficiency of any real machine is less than unity (one) because the energy outpute_o is always less than the energy input e_i. The missing energy dissipates in the machine due to dissipative factors (energy) so that

$$E_i = E_o + E_d \quad or \quad W_i = W_o + W_d$$

$thus \quad \eta = \frac{E_o}{E_i} \quad = \quad \frac{W_o}{W_i} = \frac{P_o}{P_i}$ $(P_o \quad P_i \rightarrow power\ input\ /\ output)$

Example 13) A concrete slab weighing 1500 N each are to be loaded into a trailer which is 1.5 m high. The loader used an inclined plane made of plank Incline at an angle of 15^0 to the horizontal. If the coefficient of dynamic friction between the plank and the slap is 0.3, calculate the efficiency of the inclined plane.

Work input (W.I) = Work done by the loafers = Fs

F = Component of weight along the incline + dynamic frictional force

$= \quad W\,Sin\,15^0 + R_x u$

But $R_x = W\,cos\,150$, $\quad u = 0.3$, $\quad W = 1500$

F = 822.9 N

S = length of the inclined plane = 1.5/(Sin 15^0) = 5.80 m

$w_i = \quad Fs = (822.9 \text{ x } 5.8)$ J

$= \mathbf{4772.79\ N}$

Work output = WD by inclined plane = W_o

$$= mgh = wh = (1500 \text{ x } 1.5) \text{ J}$$

$$the\ efficiency = \frac{work\ output}{work\ input} = \frac{w_o}{w_i} = \frac{1500\ x\ 1.5}{822.9\ x\ 5.8}$$

$$= \mathbf{0.472} = \mathbf{47.2}\%$$

7.5 Exercise

1. Can the K.E of an object have a –ve value? Explain
2. If the speed of a pole is double, what happens to its K.E
3. What can be said about the speed of an object if the net W_{net} on that object is zero?
4. Define the following terms:
a) Work
b) Energy
c) Power
5. Itemize five forms of energy
6. Differentiate between Kinetic energy and Potential energy
7. A 15 kg block is dragged over a rough horizontal surface by a constant force of 70 N acting at an angle of 25^o to the horizontal. the lock is displaced 5 m by (a) the 70 N force (b) the force of friction (c) the normal force (d) the force of gravity (e) the net work done on the body
8. A 3 kg mass has an initial velocity of V_0 = (5i-3j) m/s a. what is its kinetic energy at this time? B. find the change in its Kinetic energy if its velocity changes to (8i+4j) m/s [hint V^2 = **V.V**]

9. A mechanic pushes a 200 kg car from rest to a speed of 3 m/s with a constant horizontal force. During this time, the car moves a distance of 30 m. neglecting friction between the car and the road, determine (a) the work done by the mechanic (b) the horizontal force exerted on the car

10. A 1500 kg Car accelerates uniformly from rest to a speed of 10 m/s in 3 secs. Find the a. work done on the car in this time (b) the average power delivered by the engine in the first 3 sec (c) the instantaneous power delivered by the engine at t = 2 sec

11. In 1970, the population of the world was about 3.5 x 10^{20} and about 2 x 10^{20} J of work was performed under human condition. Find the average power consumption per person in watts and horse power? [1yr = 3.15x10^{7}S]

12. A boy pulls a wagon with a force of 45 N by means of a rope that makes an angle of 40^{0} with the ground. How much work does he do in moving the wagon 50 m?

13. A 15 kg object initially at rest is raised to a height of 8 m by a force of 200 N. what is the velocity of the object at this height?

CHAPTER EIGHT: MECHANICAL ENERGY
BY
DR. S. TAUFIQ

8.1 Conservation of Mechanical Energy

The sum total of an object's kinetic and potential energy at any given point in time is its total mechanical energy. The law of conservation of energy says "***Energy can neither be created nor be destroyed***." So, it means, that, under a conservative force, the sum total of an object's kinetic and potential energies remains constant. Before we dwell on this subject further, let us concentrate on the nature of a conservative force.

8.1.1 Conservative Force

A conservative force has following characteristics:

- A conservative force is derived from a scalar quantity. For example, the force causing displacement or reducing the rate of displacement in a single dimension without any friction involved in the motion.
- The work done by a conservative force depends on the end points of the motion. For example, if W is the work done, $K_{(f)}$ is the kinetic energy of the object at final position and $K_{(i)}$ is the kinetic energy of the object at the initial position:

$$w = K_f - K_i$$

- Work done by a conservative force in a closed path is zero. Here, W is the work done, F is the conservative force and d is the displacement vector. In case of a closed loop, the displacement is zero. Hence, the work done by the conservative force F is zero regardless of its magnitude.

$$w = \vec{F}.\vec{d}$$

8.1.2 Proof of Conservation of Mechanical energy

Let us consider the following illustration:

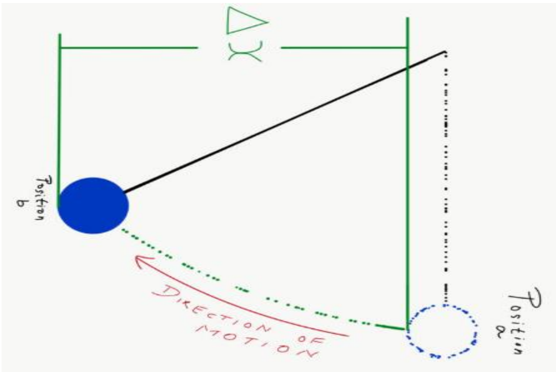


Fig. 8.1

Here, Δx is the displacement of the object under the conservative force F. By applying the work-energy theorem, we have: ΔK = F(x) Δx. Since the force is conservative, the change in potential Energy can be defined as ΔV = – F(x) Δx. Hence

$$\Delta K + \Delta V = 0 \; or \; \Delta(K + V) = 0$$

Therefore, for every displacement of Δx, the difference between the sums of an object's kinetic and potential energy is zero. In other words, the sum of an object's kinetic and potential energies are constant under a conservative force. Hence, the conservation of mechanical energy is proved.

8.2 Machines

A machine is an object, device, or system that is used to transfer energy from one place to another and allows work to be done that may not be able to be done otherwise.

8.2.1 Types of Machines

A simple machine is a device with few moving parts that allow the user to convert an applied force to some type of useful work. There are six types of simple machines: (we will also examine gear)

Lever
Wheel and axle
Pulley
Ramp (or Inclined plane)
Screw
Wedge

Simple machines allow us to:

1. Multiply force: wrench, hammer
2. Multiply speed: gears
3. Change direction: pulley, gear

A complex machine is just some type of combination of simple machines. A car, for example, is an example of a complex machine. Our human body can be considered a complex machine. There are two forces to concern ourselves with when it comes to machines: effort and resistance.

4. The effort force is the force that is applied to the machine.
5. The resistance force is the force that the machine must overcome to do work.

Example – A person applies 20 lb of force to a jack. This is the effort force. The jack produces 600 lb of force to lift the object. This is the resistance force.

8.2.2 Mechanical Advantage (M.A)

The force exerted on a machine is called the effort force F_e, the force exerted by the machine is called the resistance force F_r. The ratio of the resistance force to effort force, F_r/F_e is called the mechanical advantage (MA) of the machine.

$$MA = \frac{resistance\ force}{effort\ force}$$

MA is unit-less because it is a ratio.

The higher the mechanical advantage, the greater the output of the machine.

Many machines, such as bottle opener, have a mechanical advantage greater than one. When the mechanical advantage is greater than one, the machine increases the force you apply. In the case of a pulley system, the force F_e and F_r are equal, consequently MA is 1. The usefulness of this pulley arrangement is not that the effort force is lessened, but that the direction is changed; now the direction of the effort is in the same direction as displacement.

Example 1) If a force of 10 N level up a load of 50 N, what is the mechanical advantage?

$$\text{M.A} = \frac{load}{effort} = \frac{l}{e} = \frac{50}{10} = 5$$

Example 2) Suppose an effort of 4 N is used to raise a load of 40 N, what is it M.A?

$$\text{M.A} = \frac{load}{effort} = \frac{l}{e} = \frac{40}{4} = 10$$

8.2.3 Velocity Ratio (V.R)

The velocity ratio of a machine is the ratio of the distance moved by the effort to the distance moved by the load.

$$VR = \frac{distance\ moved\ by\ effort}{distance\ moved\ by\ load}$$

Here again the same units appear on the top and bottom of the equation, so VR is a number without unit.

8.2.4 Efficiency of a Machine (e)

Due to outside forces, such as friction, energy can be lost and output can be lessened.

$$\%Efficiency = \frac{work\ output}{work\ input} \times 100\%$$

-The more output you are able to achieve, the more the efficiency the machine.

-The more the friction acting on the machine, the less the efficiency of the machine will be.

8.2.5 Relationship Between M.A, V.R. and e of a Machine

Machine is a device which multiplies the force or multiplies the speed or it changes the direction of effort. There are some terms related to machines like load, effort, velocity ratio, mechanical advantage and efficiency. Load is the output which is overcome by machine whereas the effort is the input to the machine.

Let a machine overcome a load ‘L’ by the application of an effort E. In time ‘t’. Let the displacement of effort of dE and displacement of load be d1

Work input = Effort ×displacement of effort = E × dE

Work output = Load × displacement of load = L × dL

$$Efficiency(\eta) = \frac{work\ output}{work\ input}$$

$$= \frac{\mathrm{L} \times \mathrm{dL}}{\mathrm{E} \times \mathrm{dE}} \qquad (1)$$

Also, mechanical advantage (M.A)

$$= \frac{\mathrm{Load(L)}}{\mathrm{Effort(E)}} \qquad (2)$$

And Velocity Ratio (V.R)

$$= \frac{\mathrm{dE}}{-\mathrm{dL}} \qquad (3)$$

By equation 1, 2 and 3

$$Efficiency(\eta) = \frac{\mathrm{M.A}}{\mathrm{V.R}}$$

$$\mathrm{M.A} = \mathrm{V.R} \times Efficiency(\eta)$$

Thus, the mechanical advantage of the machine is equal to the product of its efficiency and Velocity ratio.

Note:
Mechanical advantage (M.A.), efficiency (η) and velocity ratio are interconnected with each other. Basically, mechanical advantage of machine is equal to the product of its efficiency and velocity Ratio. Therefore,

$$\text{M.A} = \text{V.R} \times \eta$$

Example 3) In a machine, a load of 400 N is moved by 2 m by an effort of 60 N moving through 20 m. Cal. (i) the V.R. (ii) M.A. (iii) Efficiency (iv) Wasted energy

Solution

i) $\text{V.R.} = \dfrac{\text{distance moved by effort}}{\text{distance moved by load}} = \dfrac{20}{2} = \mathbf{10}$

ii) $\text{M.A.} = \dfrac{\text{load}}{\text{effort}} = \dfrac{400}{60} = \mathbf{6.7}$

iii) $\text{Efficiency} = \dfrac{\text{M.A} \times 100\%}{\text{V.R}} = \dfrac{6.7 \times 100}{10} = \mathbf{67\%}$

iv) Wasted energy = 1200 – 800 = **400 J**

Example 4) A machine whose efficiency is 75% is used to lift a load of 100 N. Cal. the effort put into the machine if it has a V.R. of 4.

Solution

$$e = \frac{\text{M.A}}{\text{V.R}} \times 100\%$$

Make M.A. subject formula

$$e \times V.R = M.A. \times 100$$

$$M.A. = \frac{e \times V.R}{100\%}$$

$$\text{But } M.A. = \frac{l}{e}$$

Substitute and make E subject formula

$$E = \frac{L \times 100}{e \times V.R} = \frac{100 \times 100}{75 \times 4} = \mathbf{33.33\ N}$$

Example 5) A machine with a velocity ratio 30 moves a load of 3000 N when an effort of 200 N is applied. What is the efficiency of the machine?

Solution:

V.R. = 30, Load = 3000 N, effort = 200 N, e = ?

$$e = \frac{M.A}{V.R} \times 100\%$$

$$\text{But } M.A. = \frac{l}{e} = \frac{3000}{200} = \mathbf{15}$$

$$\therefore e = \frac{15}{30} \times 100\% = 0.5 \times 100\% = \mathbf{50\%}$$

8.3 The Lever System

A lever is a straight bar or other rigid structure supported at a fulcrum in such a way that a small force (or effort) can balance or move a much larger load.

8.3.1 Forces at Fulcrum (Reaction)

In a lever system, there must be a force or reaction at the fulcrum to balance the net (upward and downward) forces on the system.

8.3.2 Types of Levers

First Class lever System

The fulcrum is between the effort and the load. Examples, scissors, pliers, seesaw, your neck, a jack, etc.

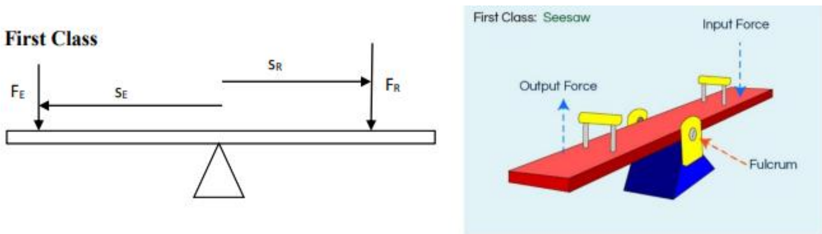


Fig 8.2

Second Class lever system

The load is between the effort and the fulcrum. Examples, wheel barrow, nutcrackers, the ball of your foot, etc.

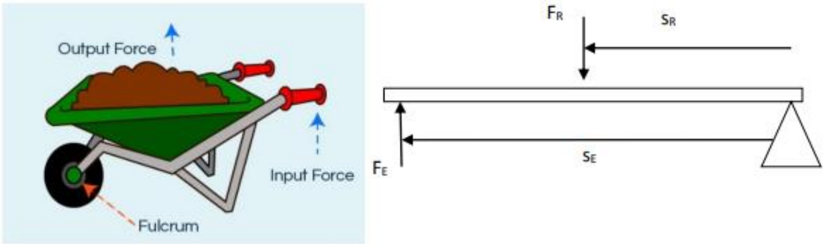


Fig. 8.3

Third class lever system

The effort is between the load and the fulcrum e.g. forearm, a hockey stick, fishing pole, your forearm, etc.

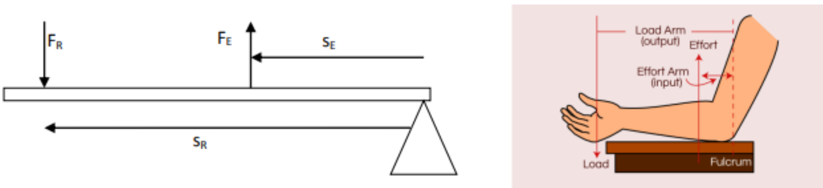


Fig. 8.4

8.4 Exercise

1) State the law of conservation of energy

2) Define machine and list four types of machine

3) Differentiate between mechanical advantage and velocity ratio

4) Express the relationship between mechanical advantage, velocity ratio and efficiency of a machine
5) Discuss three classes of lever system with relevant examples.

6) If a force of 20 N level up a load of 100 N, what is the mechanical advantage?

7) Suppose an effort of 6 N is used to raise a load of 50 N, what is it M.A?

8) A block and tackle consisting of 5 pulleys is used to raise a load of 400 N through a height of 10 m. If the work done against friction is 1000 J. Calculate (a) the work done by the effort (b) the efficiency of the system (c) the effort applied

9) A machine of V.R 5 is 80% efficiency. What effort would be needed in order to lift a load of 2000 N with the aid of this machine?

CHAPTER NINE: PROJECTILE

BY

DR. M. I. KIMPA

9.1 Concept of Projectile

Projectile motion is a special case of two-dimensional motion. A particle moving in a vertical plane with an initial velocity and experiencing a free-fall (downward) acceleration, displays projectile motion. Some examples of projectile motion are the motion of a ball after being hit/thrown, the motion of a bullet after being fired and the motion of a person jumping off a diving board. For now, we will assume that the air, or any other fluid through which the object is moving, does not have any effect on the motion. In reality, depending on the object, air can play a very significant role. For example, by taking advantage of air resistance, a parachute can allow a person to land safely after jumping off an airplane.

9.2 Motion of a Projectile

Projectiles are objects that are given an initial velocity and subsequently travel along their trajectory (flight path) due to their own inertia.

Vectors describe the velocity, acceleration, and forces that act upon a projectile in terms of direction and magnitude. The principles of vector addition are used to understand and predict the trajectory of projectiles as well as other applications of two-dimensional motion, such as circular motion or the elliptical orbits of planets and comets. Therefore, vector addition is an important subject in the field of mechanics—a branch of physics that studies how physical bodies behave when subjected to forces or displacements.

Projectiles tend to follow a parabolic trajectory. If you draw a line that follows the movement of a ball after you throw it, you would see the shape of a parabola. The shape of the parabola depends on the initial speed and the release angle, but all projectiles launched at an angle follow this parabolic curve.

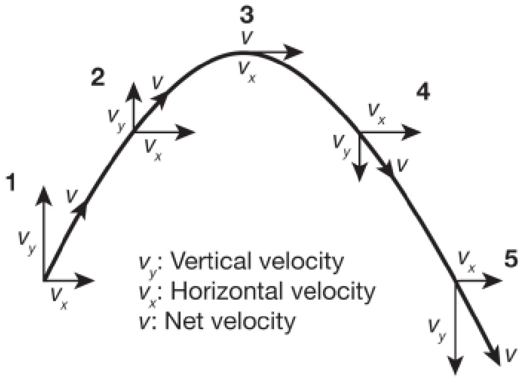


Fig. 9.1

To understand the motion of a projectile, it helps to consider the object as moving in two dimensions: the vertical (y) direction and the horizontal (x) direction. The velocity of the projectile at any given time can be broken down or resolved into a vector in the x direction and a vector in the y direction. The magnitudes of these vectors are independent of one another. Gravity only affects the vertical component of the velocity, not the horizontal component.

Consider Figure 1. When the projectile is launched, the velocity, v, consists of two independent, perpendicular components: vx and vy. If air resistance is negligible, the horizontal component of the velocity (vx) remains constant, whereas the vertical component of the velocity (vy) changes due to gravitational acceleration. The initial value for vy decreases as the projectile travels to the highest point in the parabolic arc and then increases in the opposite direction as the projectile descends. If air resistance is negligible, the vertical velocity of the projectile when it returns to the elevation from which it was launched will have the same magnitude as when the projectile was launched, but the direction will have turned 180°.

A two-dimensional curved motion, with constant acceleration of a particle thrown obliquely into air, is known as a projectile motion.

9.3 Time of Flight

Time of flight (T) is the time required for a projectile to return to the same level from which it was projected.

$$T = \frac{2u\sin\Theta}{g}$$

9.4 Maximum Height

The Maximum height (H) is the highest vertical distance attained as measured from the horizontal projection plane.

$$H = \frac{u^2\sin^2\Theta}{2g}$$

9.5 Range

Range (R) is the horizontal distance from point of projection to the point where the projectile hits the projection plane again

$$R = \frac{u^2 \sin 2\Theta}{g}$$

Example 1) A particle is projected at an angle of 60^0 to the horizontal with a speed of 20 m/s. Calculate the (a) total time of flight of the particle (b) speed of te particle at its maximum height ($g = 10\ m/s^2$).

Solution

$g = 10\ m/s^2$, $u = 20\ m/s$, $\Theta = 60^0$ $T = ?$

a) Time of flight $= \frac{2u\sin\Theta}{g} = \frac{2 \times 20 \times 0.8660}{10} =$ **3.46 sec.**

b) Max Height = At max height, the speed of the particle is 0.

Example 2) A lawn tennis ball is projected with a speed of 15 m/s. If the ball attains a range of 19.5 m, calculate the angle of projection.

Solution

$u = 15\ m/s$, $R = 19.5\ m$ $g = 10\ m/s2$

$$\text{Range} = \frac{u^2 \sin 2\Theta}{g}$$

$$19.5 = \frac{15^2 \sin 2\Theta}{10} = 195 = 225 \sin 2\Theta$$

$\sin 2\Theta = 0.8666$

$\Theta = 30^0$

Example 3) A projectile is fired with initial vel. of 100 m/s at an angle of 30^0 with horizontal. Cal (i) time of flight (ii) max. height (iii) range

i) Time of flight $= \frac{2u\sin\Theta}{g} = \frac{2 \times 100 \text{ x } 0.5}{10} = \mathbf{10\ sec.}$

ii) Max. height $= \frac{u^2\sin^2\Theta}{2g} = \frac{100^2 \times \sin^2 0.5}{20} = \mathbf{125\ m}$

iii) Range $= \frac{u^2\sin 2\Theta}{g} = \frac{100^2 \times \sin 60}{10} = \mathbf{866\ m}$

9.6 Exercise

1) Define projectile

2) Define the following terms:
a) Time of flight
b) Maximum height
c) Range

3) An object is projected with a velocity of 100 m/s from the ground level at an angle Θ to the vertical. If the total time of flight of the projectile is 10 s, cal. Θ ($g = 10\ m/s^2$)

4) A particle is projected at an angle of 600 to the horizontal with a speed of 20 m/s. calculate (a) total time of flight of the particle (b) speed of the particle at its maximum height ($g = 10\ m/s^2$).

5) A ball is thrown with speed of 100 m/s attains a height of 150 m, Calculate the time of flight ($g = 10\ m/s^2$).

CHAPTER TEN: FRICTIONAL FORCE

BY

DR. M. I. KIMPA

10.1 Concept of Friction

Friction is a force that resists motion of sliding or rolling of one object moving relative to another. It is a result of the electromagnetic attraction between the charged particles of two touching surfaces. We find and use it everywhere and every day whenever objects come into contact with each other. Although it always acts in the always acts in the direction opposite to the way an object wants to slide.

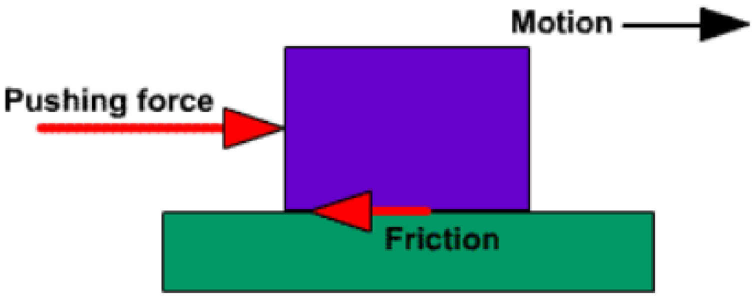


Fig. 10.1

For example, we use car brakes if we want to stop or slow down because of the friction created between the brakes and wheels that slow/stop the car down.

10.2 Types of Friction

10.2.1 Rolling Friction

Rolling friction is the force that resists motion when an object rolls on a surface. Technically it's not friction; its **'rolling resistance'** since when a body rolls perfectly upon a surface, on paper, there is no sliding friction between that object and surface. But due to elastic properties in real life, both the bodies and the surface experience deformations due to contact between the bodies. Since the surface of contact is very small in real life hence the net normal force is also small and it is not enough like the static friction to prevent a body from sliding and keep it stationary and static friction increases with the increase in the external force; therefore, rolling friction is usually less than the static.

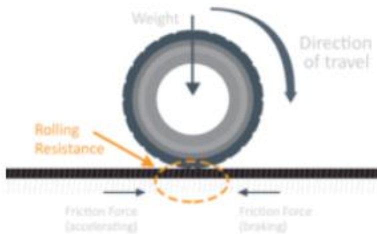


Fig. 10.2

10.2.2 Sliding friction

Sliding friction is the frictional force between two surfaces that are rubbing against each other. It's a very easy and common concept. It's hard to find a perfectly smooth surface in the real life, therefore when an object slides on any surface, it undergoes a backward force because of the relative motion between the two adjacent surfaces. It always acts against the motion. For a static situation, the applied force that tries to slide the object is always equal to the force of friction acting on the object. There comes a certain moment that the object starts moving in the direction of the external force. This happens when we gradually increase the applied force. The force of friction that acts against the motion remains constant.

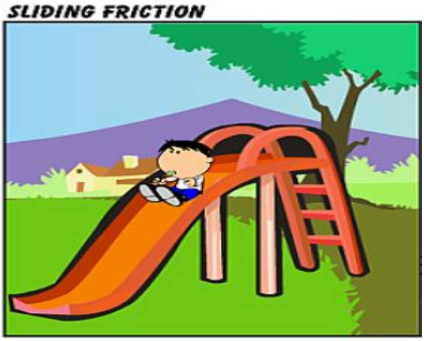


Fig. 10.3

10.2.3 Fluid Friction

When fluid layers are moving relative to each other, a type of friction occurs which is known as 'fluid friction'. The internal resistance to the flow of fluids is termed 'viscosity'; in simple terms, the viscosity is nothing but 'thickness' of a fluid.

Fig. 10.4

10.3 Advantages and Disadvantages of Friction

10.3.1 Advantages of Friction

1. Friction helps us to walk on the ground
2. We can write on a paper or on a board due to friction
3. Friction helps in stopping a vehicle on applying the brakes.
4. It helps in generating heat when we rub our hands together
5. Asteroids burn in the atmosphere before reaching the earth due to friction and saves lives on earth.

10.3.2 Disadvantages of friction

1. Friction produces a lot of heat in various parts of the machinery and this leads to wastage of energy as heat
2. Opposes motion, hence more energy is needed to overcome friction
3. Noise production in machines is irritating as well as leads to energy loss
4. Forest fires are caused due to friction between branches of trees
5. Lot of effort and money goes in using techniques like greasing and oiling to overcome friction and usual wear and tear caused by it

10.4 Laws of Friction

- Friction is proportional to the normal force exerted between the surfaces.
- Friction does not depend on the area of contact.
- Friction force depends on the type of surfaces in contact.
- The coefficient of static friction is greater than the coefficient of kinetic friction.

10.5 Method of Measuring Coefficient of Static Friction

An easy way to measure the coefficient of static friction is to place two objects together and then tilt them until the top one slide. The angle at which one object starts to slip on the other is directly related to the coefficient.

When the two objects are horizontal there is no frictional force. As the objects are slowly tilted, the force of static friction must increase from zero to counteract the component of the force of gravity that acts along the interface.

Eventually, as the angle increases, that component of the force of gravity exceeds the maximum value of the force of static friction, and the top object slides off.

$$F_s = f_{s\,max} = \mu_s N$$

Because this is the angle at which the force of static friction equals its maximum value.

Use a coordinate system with +x down the slope and +y perpendicular to the slope.

Split the force of gravity into x and y components.

Apply Newton's second law twice.

$$\sum F_x = ma_x = o \quad , \sum F_y = ma_y = o$$

$$mgsin(\theta) - F_s = 0 \quad , \quad N - mgcos(\theta) = 0$$

$$mgsin(\theta) = \mu_s N \quad , \quad N = mgcos(\theta)$$

Substitute the second expression into the first:

$$mgsin(\theta) = \mu_s mgcos(\theta)$$

The factors of mg cancel. Re-arranging gives

$$\frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta) = \mu_s$$

So, the coefficient of static friction is equal to the tangent of the angle at which the objects slide.

10.6 Method of Measuring Coefficient of Dynamic Friction

Friction is a resistive force that prevents two objects from sliding freely against each other. The coefficient of friction (**fr**) is a number that is the ratio of the resistive force of friction (**Fr**) divided by the normal or perpendicular force (**N**) pushing the objects together. It is represented by the equation:

$$f_r = \frac{F_r}{N}$$

There are different types and values for the coefficient of friction, depending on the type of resistive force. You can determine the coefficient of friction through experiments, such as measuring the force required to overcome friction or measuring the angle at which an object will start to slide off an incline.

10.7 Reduction of Friction

The following are the different methods that are used for reducing friction

i. For objects that move in fluids such as boats planes, cars, etc, the shape of their body is streamlines in order to reduce the friction between the body of the objects as the fluid

ii. By polishing the surface, as polishing makes the surface smooth and friction can be reduced.

iii. Using lubricant such as oil or grease can reduce the friction between the surfaces.

iv. When objects are rolled over the surface, the friction between the rolled object and the surface can be reduced by using ball bearing.

10.8 Exercise

1) Define friction and explain three types of friction
2) Enumerate four advantages and four disadvantages of frictional force
3) What are the laws of friction
4) State three methods of reducing friction.

CHAPTER ELEVEN: ELASTICITY

BY

DR. S. TAUFIQ

11.1 Concept of Elasticity

Elasticity is the ability of a body to resist a distorting influence or deforming force and to return to its original size and shape when that influence or force is removed. Solid objects will deform when adequate forces are applied on them. If the material is elastic, the object will return to its initial shape and size when these forces are removed.

The physical reasons for elastic behaviour can be quite different for different materials. In metals, the atomic lattice changes size and shape when forces are applied (energy is added to the system). When forces are removed, the lattice goes back to the original lower energy state. For rubbers and other polymers. Elasticity is caused by the stretching of polymer chains when forces are applied.

11.2 Stress

The internal resistive force per unit area of the body is called stress.

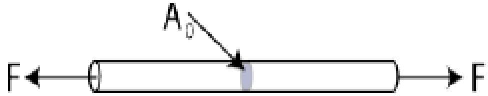


Fig. 11.1

$$Stress(\sigma) = \frac{Force}{Cross\ sectonal\ area} = \frac{F}{A_o}$$

11.3 Strain

If a wire of length L increases in length by 'l' then the ratio (l/L) is called **strain.**

Fig. 11.2

$$Strain = \frac{Elongation}{Original\ length} = \frac{\Delta L}{L}$$

11.4 Young Modulus

For materials whose length is much greater than the width or thickness, we are concerned with the longitudinal modulus of elasticity, or Young Modulus (Y).

$$\text{Young Modulus (Y)} = \frac{\text{normal stress}}{\text{longitudinal strain}}$$

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

11.5 Hooke's Law

It states that with in elastic limits, stress is proportional to strain. Within elastic limits, tension is proportional to extension. So, Stress ∝ Strain

$$A \propto {}^{I}/_{L}$$

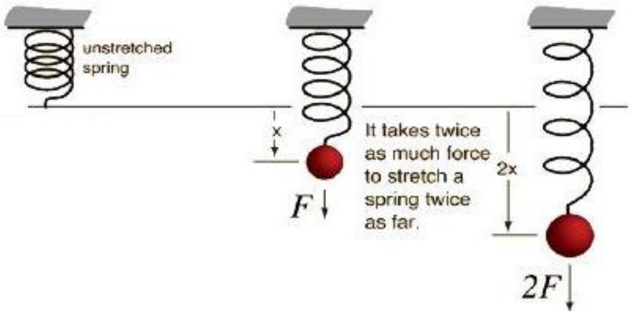


Fig. 11.3

$$F_{sprngs} = -kx$$

Spring constant k

$$k = \frac{\Delta F}{\Delta x}$$

11.6 Energy Stored in an Elastic Material

Elastic potential energy is the potential energy stored by stretching or compressing an elastic object by an external force such as the stretching of a spring. It is equal to the work done to stretch the spring which depends on the spring constant k and the distance stretched.

According to Hooke's law, the force applied to stretch the spring is directly proportional to the amount of stretch.

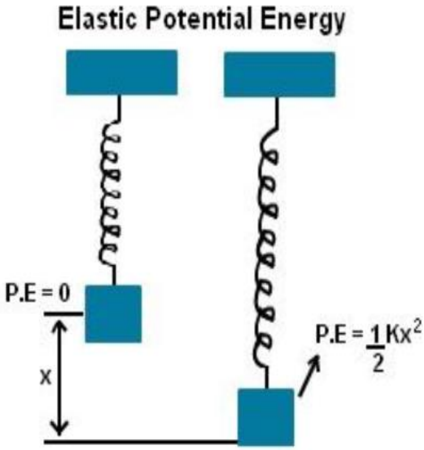


Fig. 11.4

In other words; force required to stretch the spring is directly proportional to the displacement. It is given as:

$$F = kx$$

Wherein,

k = spring constant
x = displacement

The elastic potential energy of the stretch spring is given as

$$P.E = \frac{1}{2}kx^2$$

Where,

P.E = elastic potential energy and its expressed in joules

Example 1) A force of 0.8 N stretches an elastic spring by 2 cm. Find the elastic constant of the spring.

Solution

f = 0.8 N,   e = 2 cm = 0.02 m,   k = ?

From Hooke's law:

$$F = ke$$

$$0.8 = k \times 0.02$$

$$k = \frac{0.8}{0.02} = \mathbf{40\ Nm^{-1}}$$

Example 2) A spring is stretch 4 cm by a force of 15 N. What is the work done by the force?

Solution

e = 4 cm = 0/04 m,   f = 15 N, Work done (W) = ?

$$W = \tfrac{1}{2} Fe = \tfrac{1}{2} \times 15 \times 0.04 = \mathbf{0.3J}$$

Example 3) A spiral spring is compressed by 0.02 m. Calculate the energy stored in the spring if the force constant is 400 N/m

Solution

e = 0.02 m,   k = 400 N/m,   Energy stored (W) = ?

$$W = \tfrac{1}{2} fe = \tfrac{1}{2} Ke$$

$$= \tfrac{1}{2} \times 400 \times 0.02 = \mathbf{0.08\ J}$$

Example 4) Suppose a 2 kg mass is attached to the end of a vertical wire of length 2 m and area 3.2 x $10^{-7}m^2$, and the extension is 0.60 mm. find (i) stress (ii) strain (iii) young modulus.

Solution

m = 2 kg, l = 2 m, A = 3.2×10^{-7} m2, e = 0.6 mm = 0.0006 m

stress = ? strain = ? young modulus = ?

i) Stress $= \frac{F}{A} = \frac{m \times g}{A} = \frac{2 \times 10}{3.2 \times 10^{-7}} = \mathbf{6.25 \times 10^{7}\ N/m^{2}}$

ii) Strain $= \frac{e}{l} = \frac{0.0006}{2} = \mathbf{3 \times 10^{-4}}$

iii) Young modulus $= \frac{\text{stress}}{\text{Strain}} = \frac{6.25 \times 10^{7}}{3 \times 10^{-4}} = \mathbf{2.08 \times 10^{11}\ N/m^{2}}$

11.7 Exercise

1) Define elasticity

2) differentiate between stress, strain and young modulus

3) State Hooke's law and mention the condition under which it is obeyed

4) A force of 2 N stretches an elastic material by 30 mm. What additional force will stretch the material 35 mm? Assume that the elastic limit is not exceeded.

5) If a force of 100 N stretches a spring by 0.1 cm, what is its elastic constant?

6) A spring is stretch 6.2 cm by a force of 20 N. What is the work done by the force?

7) A force of 10 N stretches an elastic spring by 0.04 m. Find the elastic constant of the spring.

8) A spring of natural length 1.5 m is extended 0.005 m by a force of 0.8 N, what will its length be when the applied force is 3,2 N?

CHAPTER TWELVE: PARTICLE NATURE OF MATTER

BY

DR. M. I. KIMPA

12.1 Structures of Matter

Solids, liquids and gases are made of atoms; the diameter of whose is about 10^{-10} m. In recent year's direct visual evidence of atoms have been obtained by scientists using powerful microscopes (e.g. atomic force microscope). For details of the internal structure of atoms these microscopes are not yet sufficiently powerful. In this context, **an atom is the smallest indivisible component of a chemical element**. It is important to note that an atom has its constituents, the electrons, protons, and neutrons; hence an atom is not the fundamental component of matter.

Atoms are made up of positive and negative electric charges, and the attraction and repulsion of these charges are the basis of all the chemical and physical phenomena observed in solids, liquids, and gases. The arrangement of electric charges in the atoms remained a mystery until the discovery of the nucleus by Rutherford, while Niels Bohr explained the factors determining the characteristic colours of light emitted by atoms.

12.1.1 J.J. Thomson's Model (Plum-pudding Model)

The features of this model are:

1. All the atoms contain electrons, which are negatively charged.

2. Atoms are electrically neutral entities; every atom thus contains enough positively charged matter to balance the negative charges of the electrons.
3. Electrons are several times much lighter than the whole atom, implying that the positively charged constituent provides nearly all the entire mass of the atoms.
4. Thomson proposed that atoms are uniform spheres of positively charged matter in which electrons are embedded.

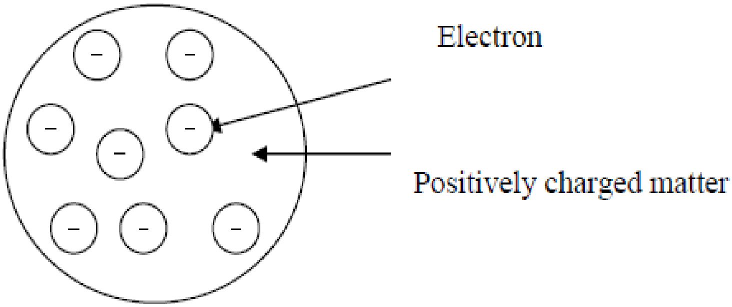


Fig. 12.1: The Thomson's model of an atom

12.1.2 Rutherford's Model

The model explains the experimental results of Geiger and Marsden (1913) wherein some of the alpha (α) particles directed at the atoms (tiny metallic (gold) foil) were scattered through large angles. The Rutherford's scattering experiment shown in Fig. 2, as conducted by Geiger and Marsden, disapproved the Thomson's model of an atom.

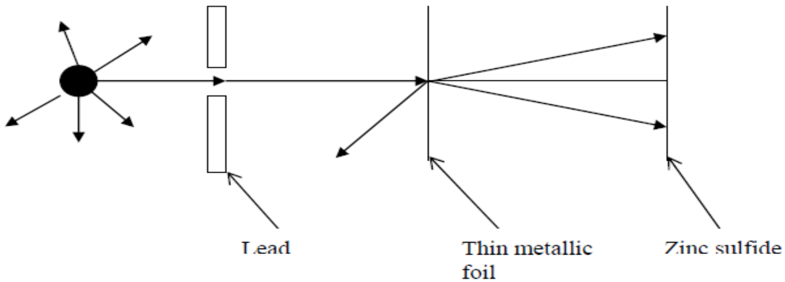


Fig 12.2: The Rutherford's scattering experiment as conducted by Geiger and Marsden.

The key features of the Rutherford's model are:

1. In this model an atom comprises a tiny nucleus, in which its positive charge and nearly all of its mass are concentrated, with the electrons some distance away, as shown in Fig. 3.

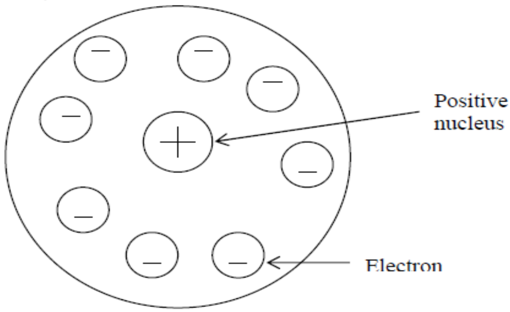


Fig. 12.3: The Rutherford's model of an atom.

2. The large scattering of the alpha (α) particle (~7000 times more massive than an electron) is due to intense electric field (and hence force) it encounters at the nucleus.
3. The atomic electrons, because of their very small mass do not affect the motion of the incident alpha (α) particles.

4. The deflection an alpha (α) particle experiences when it nears the nucleus depends on the magnitude of the nuclear charge. From this fact, the nuclear charges of various atoms can be estimated by comparing the relative scattering angles, showing that:
   i. All the atoms of a given element have the same nuclear charge.
   ii. The nuclear charge increased regularly from element to element in the Periodic Table.
   iii. The nuclear charge = $+eZ$. (e is the electronic charge and has a magnitude of 1.6x10-19 Coulombs, Z is the atomic number of the element and is equivalent to the number of the protons in the nuclei of the atoms).
5. From the Rutherford's nuclear model, the scattering (deflection) angle (θ) of the alpha (α) particles can be calculated. Shown in Fig. 4 is the trajectory of the scattered alpha particles. The trajectories are hyperbolas.

The perpendicular distance between the nucleus and the original (undeflected) line of motion is called the **impact parameter** (b).

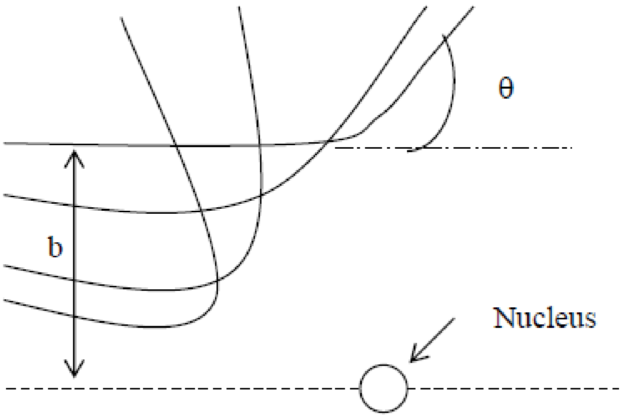


Fig. 12.4: The trajectories of scattered alpha particle

According to classical mechanics, the angle of deflection (α) can be expressed in terms of the impact parameter (*b*), the energy (*E*) of the α-particle, and the charge of the nucleus (*Ze*) as

$$\theta = 2cot^{-1}\left(\frac{2\pi\varepsilon_0 Eb}{Ze^2}\right)$$

ε_0 is the dielectric constant of free space and has a value of $8.85 \mathrm{x} 10^{-12}$ $C2m^{-2}N^{-1}$

In order to undergo sufficiently large deflection, the α-particle must strike an atom within the impact parameter, 10^{-13}m or less.

Limitations of Rutherford's Model:
The two limitations of the model are:

1. It could not account for the stability of the atom.

2. It predicts that electrons (due to ever changing radii of circular orbits) will radiate electromagnetic (em) waves of all frequencies, that is, a continuous spectrum of these waves will result. Experimental evidence, on the contrary show that atomic spectra are not continuous but discrete, i.e., are single spectral lines corresponding to specific frequencies

12.1.3 Bohr's Model

We have noted that Rutherford proposed an atom comprising a central positively charged nucleus surrounded by negative charges, identified as electrons. The nucleus must exert an attraction on the electrons, so it is necessary to postulate that they would circulate in orbits in a way that the centrifugal force balances the electrostatic attraction. From the classical theory of electromagnetism, when an electron (or a charge) moves in an orbit, it emits electromagnetic (em) waves. Thus from classical theory we expect electrons circulating around the nucleus to emit em radiations of the same frequency as the orbital frequency. **This would involve loss of energy and an approach of the electron nearer the nucleus. A consequence is a continual change of frequency and also a final collapse of the electron into the nucleus-neither of which is observed to happen.** Atoms, when they do, emit radiations of fixed frequency. This prediction of Rutherford's model is not in accord with experiment.

Bohr (1915) proposed a model of the atoms as follows:

i. The electrons exist only in stable circular orbits of fixed energy, the angular momentum of an electron in an orbit being an integral multiple of $h/_{2\pi}$ ☐ , where h is Planck's constant.

ii. An electron will emit or absorb energy only when making a transition (change) from one orbit to another possible orbit.

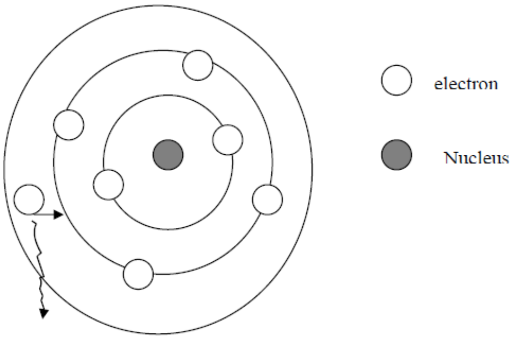


Fig. 12.5: Bohr's model of an atom showing electrons in stable orbits and an electron making a transition from one orbit to the other resulting in radiation of energy

Bohr's hypothesis was an attempt to explain the possible electron orbits, the energy differences between which would account for the observed spectral lines. Let us now consider a single-electron atom. If m is the mass of the electron moving with a velocity v in the circular orbit or radius r around a fixed nucleus of charge $+Ze$, then from Coulomb's law we obtain the attractive force (F_e) between the electron and the nucleus as:

$$F_c = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

Where ε_0 is the permittivity of free space.

(Recall Coulomb's law, as you learnt in electrostatics, is the law that gives the force between charged particles).

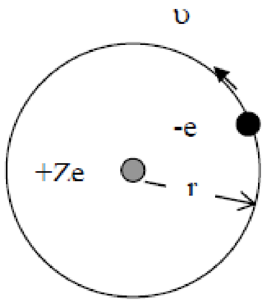


Fig 12.6: A single-electron orbiting the nucleus with a velocity v.

Assuming Newton's law of motion (i.e., force is the product of mass and acceleration) applies then the centrifugal force F_{cf} on the electron is $mv^2/_r$. If the electron remains in its orbit, these forces (electrostatic and centrifugal) must be then equal or balance out, i.e.,

$$mv^2/_r = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$$

So that,

$$v^2 = \frac{Ze^2}{4\pi\varepsilon_0 mr}$$

Equation (1.4) shows that for any particular value of *r* there is a particular value of v. The energy of the electron consists of the potential energy (P.E) and kinetic energy (K.E) components.

$$K.E = \frac{1}{2}mv^2 \; and \; P.E = -\frac{Ze^2}{4\pi\varepsilon_0 r} = -mv^2$$

(Note: Reference for P.E. point is at taken as the zero position). The total energy (T) is the sum of P.E and K.E, thus

$$T = K.E + P.E = \frac{1}{2}mv^2 - mv^2 = -\frac{1}{2}mv^2$$

If we now introduce Bohr's condition that the angular momentum (mvr) of an electron can have only certain value which is an integral multiple of $h/_{2\pi}$ then

$$(mvr) = \frac{nh}{2\pi}$$

Where n=1,2,3,4,..... is an integer. Combining equations, we obtain

$$v = \frac{Ze^2}{2nh\varepsilon_0}$$

And the total energy is

$$T = -\frac{1}{2}mv^2 = \frac{Z^2me^4}{8h^2{\varepsilon_0}^2}\left(\frac{1}{n^2}\right)$$

Inserting the appropriate numerical values of the constants, the total energy becomes

$$T = -2.18\frac{Z^2}{n^2} \times 10^{-18} J$$

n is called the principal quantum number.

As an electron jumps from an outer orbit (say *n2*) to an inner orbit (say, *n1*) nearer the nucleus, there will be a decrease in its total energy equal to the difference in its energiesin the two orbits. This change in energy ($\square\Delta E = E2 - E1$) is expressed as

$$(\square\Delta E - 2.18 \times 10^{-18} Z^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) J$$

This energy is released as a quantum of e.m. radiation (photon) of frequency $v = \frac{\Delta E}{h}$ or

Wavelength $\lambda = \frac{c}{v} = \frac{ch}{\Delta E}$

When we combine the equations, we can obtain the expression for *r*, i.e.,

$$r = n^2 \frac{h^2 \varepsilon_0}{\pi Z m e^2}$$

Equation (1.9) shows that $r\alpha n^2$ for hydrogen atom (*Z=1*), the radius of the first orbit
(*n=1*) we get

$$r = \frac{h^2 \varepsilon_0}{\pi m e^2}$$

This is called Bohr's radius and its numerical value is $0.53\text{x}10^{-10}$ (or 0.53Å).

12.2 Molecules

A molecule is in general a group of two or more atoms that are chemically bonded together that is, tightly held together by attractive forces. A molecule can be defined as the smallest particle of an element or a compound that is capable of an independent existence and shows all the properties of that substance. Atoms of the same element or of different elements can join together to form molecules.

12.2.1 Molecules of Elements

The molecules of an element are constituted by the same type of atoms. Molecules of many elements, such as argon (Ar), helium (He) etc. are made up of only one atom of that element. But this is not the case with most of the nonmetals. For example, a molecule of oxygen consists of two atoms of oxygen and hence it is known as a diatomic molecule, O_2. If 3 atoms of oxygen unite into a molecule, instead of the usual 2, we get ozone. The number of atoms constituting a molecule is known as its atomicity. Metals and some other elements, such as carbon, do not have a simple structure but consist of a very large and indefinite number of atoms bonded together. Let us look at the atomicity of some non-metals.

Table 12.1: Atomicity of Element

Types of Element	Name	Atomicity
Non metals	Argon Helium Oxygen Hydrogen Nitrogen Chlorine Phosphorus Sulphur	Monoatomic Monoatomic Diatomic Diatomic Diatomic Diatomic Tetra-atomic Poly-atomic

12.2.2 Molecules of Compounds

Atoms of different elements join together in definite proportions to form molecules of compounds. Few examples are given in Table 3.4.

Table 12.2: Molecules of some Compound

Compound	Combining Element	Ratio by Mass
Water	Hydrogen, oxygen	1:8
Ammonia	Nitrogen, Oxygen	14:3
Carbondioxide	Carbon, Oxygen	3:8

12.3 States of Matter

The ice, soft drink, and bubbles are examples of the three familiar states of matter: solids, liquids, and gases. The ice is a solid, the soft drink is a liquid, and the bubbles are filled with a gas. These are the three states of matter that usually occur on earth.

Stars, such as the sun, are made of another state of matter called plasma. The plasma state is similar to the gas state and usually occurs at high temperatures. A plasma is made atoms that have been broken apart and contains electrically charged particles. Although plasma can be found in lightening and in fluorescent lights, the plasma state is not common on earth.

12.3.1 Particles in Matter

Matter is made of very small particles called atoms. Atoms can combine to form molecules, which are also very small particles of matter. All objects, such as flower, are made of these particles.

In all objects and materials, these tiny particles of matter are always in motion. Even though the flower is not moving, the atoms and molecules in the flower are always moving. Some particles move to the left or the right, some move up and down, and some move in other directions.

Particles in matter move in a type of motion called random motion. In random motion, particle can move in any direction and can have different speeds. In any object, the number of particles moving in one direction is always equal to the number of particles moving in the opposite direction. As particles move, they also collide with other particles. These collisions can change a particle's direction of motion and its speed.

Particle Attract

As they are moving, atoms and molecules usually exert a pull, or an attractive force, on each other. These forces tend to pull particles closer together. Atoms contain positively charged protons and negatively charged electrons. These electric charges can cause attraction between the atoms and molecules in matter. If particles move closer together, the attractive forces between them become stronger. As they move farther apart, the attraction between them becomes weaker.

12.3.2 Solids

A solid is matter with a fixed shape and a fixed volume.

The Forces between Particles in a Solid

The motion of the particles and the strength of the attractive forces between them determine whether a substance is a solid, a liquid, or a gas. As in all matter, the particles in a solid always are in motion. However, the particles in a solid are so close together that the attractive forces between them are strong.

The Motion of Particle in a Solid

The particle in a solid are attracted to each other by strong forces that keep the particle close together. Because the forces between particles are strong, particles in a solid cannot moe very far from each other. Each particle moves only a short distance back and forth between neighbouring particles. As a result, the particles in a solid stay in nearly the same position, vibrating back and forth in all directions. Because the particles in a solid don't move from one place to another, the shape and volume of the solid remain fixed.

12.3.4 Liquid

Unlike a solid, a liquid can flow and does not have a fixed shape. A liquid has the shape of the container in which is placed. A liquid is matter with a fixed volume not a fixed shape.

The Forces between Particles in a Liquid

The attractive forces between particles in a liquid are weaker than they are in a solid. These forces are not strong enough to keep the particles in fixed positions. As a result, the particles in a liquid move more freely than they do in a solid.

Motion of particles in a Liquid

In a solid, a particle stays in one place and moves a short distance back and forth. In a liquid, particles can move past neighbouring particles. Because the particles in the liquid can move from one place to another, a liquid cand flow and change shape. However, the forces between particles in a liquid are strong enough to keep the particles close to each other. This causes the volume of the liquid to remain fixed.

12.3.4 Gas

Every second you are surrounded by a gas- The air around you. When you breathe, you force this gas to flow into and out of your lungs. Even though a gas can flow, it is different from a liquid. A gas is matter that has no fixed volume and no fixed shape.

Changes in Shape and Volume

Gases and liquid do not have fixed shape. Unlike solids or liquid, a gas also does not have a fixed volume. If any amount of gas is put in a container, the gas expands until it fills the container. This means that the shape and volume of the gas depends on the shape and volume of the container the gas is in.

Forces between particles in a Gas

The particles in a gas are much farther apart than the particles in a solid or a liquid. Because the particles in a gas are so far apart, the forces between these particles are weak. As a result, the particles in a gas are not held together and move freely past each other.

Motion of Particles in Gas

Inside the container, gas particle moves in random motion from place to place. As gas particle move, they collide with each other and with the sides of the container. Between collisions, a gas particle moves in a straight line. However, a collision can make the particle move in different direction and can also change its speed. Because the particles in a gas can move freely from place to place, a gas does not have a fixed shape.

Unlike particles in solids and liquids, gas particles are not held together by attractive forces. As a result, gas particle spread out until they are evenly distributed throughout a container. No matter how large the container, the volume of a gas is always the same as the volume of the container the gas is in.

12.4 Changes in States of Matter

Changes in energy can cause matter to change from one state to another. Changes in the weather can cause changes in states of matter. In early spring, snow melts into liquid water. A summer rain leaves puddles of water on the sidewalk, but the warm Sun makes the puddles evaporate into a gas.

12.4.1 Temperature, Thermal Energy and Heat

Ice and liquid water are different states of the same substance. They both are made from particles that are water molecules. But how can an ice cube change into liquid water? A change from one state of matter to another is a result of two things. One is changes in the motion of the particles. The other is the strength of the forces among particles.

Moving Particles and Kinetic Energy

Recall that moving objects, such as a car or a train, have kinetic energy. The kinetic energy of an object increases as its speed increases. Even when an object is not moving, the particles in the object are in random motion. As a result, these particles also have kinetic energy. For example, the particles in the balloon

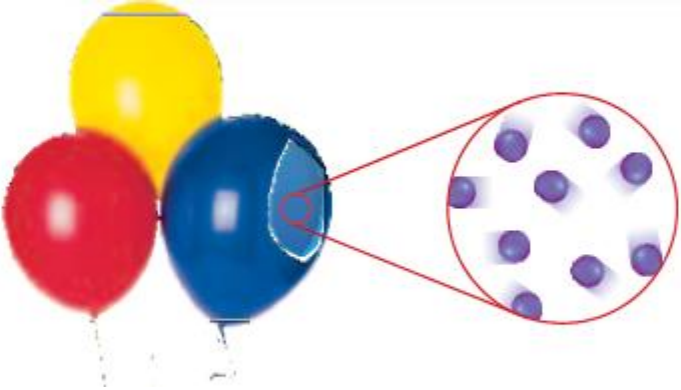


Fig. 12.7

The gas particles inside the balloon have energy because they are moving.

Temperature and Average Kinetic Energy

Temperature is a measure of the average kinetic energy of the particles in a material. This means that the average kinetic energy of particles inside the balloon is greater than the average kinetic energy of particles outside. As a result, particles inside the balloon are moving faster on average than particles outside the balloon. Particles in matter move faster as the temperature increases.

Measuring Temperature

One way to measure temperature is to use a liquid thermometer. Some thermometers have a red liquid inside a glass tube. When the liquid gets warmer, the particles in the liquid begin to move faster. The particles then get farther apart and take up more space. This causes the liquid to expand and the liquid rises in the tube.

The marks on a thermometer tell you the temperature in degrees. The range between the temperatures at which water freezes and boils on the different scales is shown. This range is divided into 180 degrees on the Fahrenheit scale. It is divided into 100 degrees on the Celsius and Kelvin scales. The Fahrenheit scale is widely used in the United States, but the Celsius scale is usu- ally used in other countries. The Celsius and Kelvin scales are used in science.

Particles of Matter and Potential Energy

In addition to having kinetic energy, the particles in a substance have potential energy as a result of the forces that they exert on each other. Potential energy decreases as particles get closer together and increases as particles get farther apart. A ball held above the ground has potential energy**.** The amount of potential energy depends on the distance between the ball and Earth. If you let the ball go, its potential energy decreases as it gets closer to the ground. In the same way, the potential energy of particles in matter decreases when the particles are closer together.

Thermal Energy

A substance also has thermal energy. **Thermal energy** includes both the kinetic energy and potential energy of the particles. Different states of matter have different amounts of thermal energy. Compared to the solid state, the particles of a substance in the gas state move faster and are farther apart. These particles have more kinetic and potential energy than the particles in the solid state. This means that the thermal energy of the substance in the gas state is greater than the thermal energy in the solid state. For any given substance, the particles have the most thermal energy in the gas state and the least thermal energy in the solid state.

Adding and Removing Thermal Energy

Thermal energy can be added to a material or **removed** from a material. When you heat a pot of water on a stove, thermal energy is added to the water. Thermal energy flows into a material when it is heated. When a warm bottle of water cools in a refrigerator, thermal energy is removed from the water. Thermal energy flows out of a material when it is cooled.

Thermal Energy and Changes in State

When thermal energy is added to a material, the thermal energy of the material increases, adding thermal energy can cause the potential energy and the kinetic energy of the particles in a material to increase. If the kinetic energy increases, then the temperature of the material increases. However, when only the potential energy increases, the temperature of the mate- rial doesn't change. Instead, the material changes from one state of matter to another. To change a material from one state of matter to another, thermal energy must flow into or out of the material.

Changes between the Solid and Liquid States

The particles that make up the liquid steel and the solid steel. The difference between the liquid and the solid depends on the movement of the particles and the thermal energy they contain. Particles in the liquid steel move faster and have more thermal energy. Particles in the solid steel move more slowly and have less thermal energy. Thermal energy must be added to a material or taken away to change it from one state of matter to another.

Melting

Melting occurs when a solid changes into a liquid. When you heat a solid, thermal energy flows into the solid. Then the temperature of the solid increases until the temperature reaches the melting point. The **melting point** of a material is the temperature at which the material changes from a solid to a liquid.

The temperature of a solid material changes as it is heated and thermal energy is added. At first, the temperature of the solid increases. But when the temperature reaches the melting point, the temperature of the material stops increasing. As the material changes from a solid to a liquid, the temperature stays constant at the melting point.

Energy Changes during Melting

Thermal energy still is being added to the material as it melts. Because the temperature is not changing, the average kinetic energy of the particles doesn’t change. Instead, the added thermal energy causes only the potential energy of the particles to increase.

When the potential energy of the particles increases, the arrangement of the particles in the material changes. In most materials, particles move farther apart. The new arrangement causes the attractive forces between particles to become weaker. When melting occurs, these forces have become weak enough that the particles can move past each other. After the solid has changed completely into a liquid, adding thermal energy causes the temper- ature of the liquid to increase.

Freezing

Freezing occurs when a liquid change into a solid. When a material cools, thermal energy flows out of the material. The temperature of the material decreases until the freezing point is reached. The **freezing point** is the temperature at which the liquid changes to a solid. As thermal energy continues to flow out of the material, the temperature remains constant at the freezing point. After all the liquid has changed to a solid, the temperature decreases once again as thermal energy is removed.

Freezing: The Opposite of Melting

Freezing is the opposite of melting. For any material, the freezing point is the same as the melting point. While freezing is occurring, thermal energy is being removed from the material. The temperature remains constant, so the average kinetic energy of the particles doesn’t change. Instead, the potential energy of the particles decreases. In most materials this means that the particles move closer together. Then the forces between the particles become strong enough for the particles to be held in fixed positions. The liquid becomes a solid.

Changes between Liquids and Gases

If you heat a pot of water on the stove, you will notice bubbles forming in the water. Tiny water droplets in the form of steam rise into the air. Water in its invisible gas form, called water vapor, also rises from the pot. The liquid is changing to a gas.

Vapourization and Boiling

When liquid water is heated, its temperature rises until it reaches 100°C. At this temperature, liquid water changes into water vapor. The change from a liquid to a gas is called **vaporization.** When vaporization occurs, the attractive forces between particles are too weak to keep particles close to each other. Particles spread out and move independently. Vaporization can occur within a liquid and at the surface of a liquid. Vaporization that occurs within a liquid is called **boiling.** When a liquid boil, bubbles form within the liquid. These bubbles contain particles of the material in the gas state.

The **boiling point** is the temperature at which boiling occurs in a liquid, the temperature doesn't change while a liquid is boiling. Boiling ends after the liquid has changed to a gas. If thermal energy continues to be added, then the temperature of the gas will continue to rise.

Evaporation

Vaporization that occurs at the surface of a liquid is called **evaporation.** Evaporation occurs during boiling and at tempera- tures below the boiling point. Recall that particles in a material move at different speeds. Some particles at the liquid's surface are moving much faster than other particles. Some of these particles are moving so fast that the attractive forces aren't strong enough to keep them at the surface of the liquid. These fast-moving particles escape into the space above the liquid. Above the liquid, the particles are far apart and the attractive forces between them are weak. These particles move independently and are in the gas state.

During evaporation, the fastest particles leave the surface of the liquid. The particles that remain have less kinetic energy. This means that the average kinetic energy of the liquid decreases. As a result, the liquid cools as evaporation occurs. You experience this cooling effect when perspiration evaporates from your skin.

Pressure and the Boiling Point

The boiling point of a liquid depends on the types of atoms and molecules that make up the liquid. The boiling point also depends on the pressure exerted on the liquid. Recall the air around you exerts pressure. This pressure is exerted on a pot of water heating on a stove. For the water to boil, bubbles containing water vapor must form in water. The pressure exerted on the water by the air makes it harder for these bubbles to form. As air pressure increases, the water must be heated to a higher temperature before bubbles of water vapor form. This means that the boiling point of a liquid increases as the pressure on the liquid increases. As the pressure on the liquid decreases, the boiling point decreases.

Condensation

On a hot day, you might see drops of water on the outside of a glass of ice-cold water. These drops of water come from the air surrounding the glass. The air contains water vapor—a gas. The cold glass cools the air next to it. When the water vapor in the air next to the glass becomes cool enough, it changes from a gas to a liquid. The change from a gas to a liquid is called **condensation.** Early in the morning, you might have noticed dew on the grass. During the night, blades of grass cool more quickly than the air. When their temperature becomes low enough, condensation occurs and water droplets form.

Condensation— The Reverse of Vapourization

For condensation to occur, thermal energy must be removed from a gas. This causes the gas particles to move more slowly and the temperature of the gas to decrease. The gas continues to cool as thermal energy continues to be removed. Finally its temperature becomes low enough for condensation to occur. Then particles move slowly enough so that the attractive forces are able to keep the particles close together. As a result, a liquid form.

Adding Thermal Energy

As the container is heated, the temperature of the ice increases. The temperature of the ice continues to rise until the melting point of ice is reached. The temperature stays constant as the ice begins to melt and change from a solid to a liquid. Even though the temperature isn't changing, thermal energy must be added to the ice to change all the solid ice to liquid water.

After all the ice has melted, the temperature of the water begins to increase as the container is heated. When the water temperature reaches the boiling point of water, the temperature stops increasing. As the container continues to be heated, liquid water changes to water vapor. Finally, all the liquid water changes to water vapor. Adding more thermal energy then causes the temperature of the water vapor to increase.

Removing Thermal Energy

Ice can be melted to form water by heating the ice. The water that is formed can be changed back into ice by removing thermal energy and cooling the water. This means that the changes between states of matter are reversible.

Changes between Solids and Gases

Dry ice is solid carbon dioxide. At room temperature, dry ice absorbs thermal energy and changes directly into a colourless gas. **Sublimation** is the change of a solid to a gas without going through the liquid state. The thick fog around the dry ice is caused by the cold carbon dioxide gas that causes water vapor in the air to condense into small droplets. For sublimation to occur, thermal energy must be added to a solid.

The opposite of sublimation is **deposition,** the change of a gas to a solid without going through the liquid state. For deposition to occur, thermal energy must be removed from a gas. When the leaf becomes cold enough, water vapor in the air surrounding the leaf loses enough thermal energy to change into a solid.

Changes in Energy among States of Matter

The state of matter of a substance depends on the amount of thermal energy a substance contains. For a material to change from one state of matter to another, thermal energy must be added to the material or removed.

12.5 Exercise

1) Define matter
2) Outline the major features of J.J. Thomson's model of atom
3) Discuss two limitations of Rutherford's model of an atom
4) What are the contributions of Bohr in the structure of an atom?

CHAPTER THIRTEEN: FLUID MOTION
BY
DR. M. I. KIMPA

13.1 Fluid Motion

13.1.1 General Characteristics of Fluid Flow

Steady Flow

A steady flow is the one in which the quantity of liquid flowing per second through any section, is constant. This is the definition for the ideal case. True steady flow is present only in Laminar flow. In turbulent flow, there are continual fluctuations in velocity. Pressure also fluctuates at every point. But if this rate of change of pressure and velocity are equal on both sides of a constant average value, the flow is steady flow. The exact term use for this is mean steady flow. Steady flow may be uniform or non-uniform.

Uniform flow

A truly uniform flow is one in which the velocity is same at a given instant at every point in the fluid. This definition holds for the ideal case. Whereas, in real fluids velocity varies across the section. But when the size and shape of cross section are constant along the length of channels under consideration, the flow is said to be uniform.

Non-uniform flow

A non-uniform flow is one in which velocity is not constant at a given instant.

Unsteady Flow

A flow, in which quantity of liquid flowing per second is not constant, is called unsteady flow. Unsteady flow is a transient phenomenon. It may be in time become steady or zero flow. For example when a valve is closed at the discharge end of the pipeline. Thus, causing the velocity in the pipeline to decrease to zero. In the meantime, there will be fluctuations in both velocity and pressure within the pipe. Unsteady flow may also include periodic motion such as that of waves of beaches. The difference between these cases and mean steady flow is that there is so much deviation from the mean. And the time scale is also much longer.

Laminar and Turbulent flow

Laminar flow: A laminar flow is one in which fluid flow is in the form of layers and there is no intermixing of fluid particles or molecular momentum transfer. In laminar flow the motion of the particles of fluid is very orderly with all particles moving in straight lines parallel to the pipe walls.

Turbulent flow: A turbulent flow is one in which there is high order of intermixing of fluid particles.

Rotational and irrotational flow

Rotational flow: If the fluid particles rotate about their axis or centre of mass.

13.2 Bernoulli's Theorem and Application

In the 18th century, the Swiss physicists Daniel Bernoulli derived a relationship between the velocity of a fluid and the pressure it exerts. Qualitatively, Bernoulli's principle states that swiftly moving fluids exert less pressure than slowly moving fluids.

Bernoulli's principle is extremely important in our everyday life. It is the primary principle which leads to lift on an airplane wing and allows the plane to fly. It is the primary reason a sailboat can sail into the wind. It is the primary reason a baseball can curve. It is an important reason that smoke is drawn up a chimney.

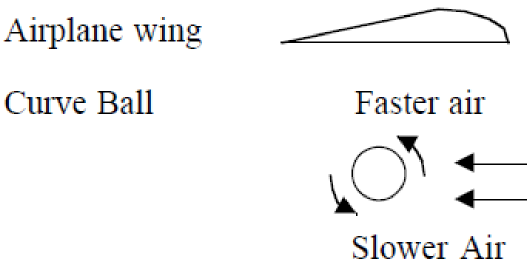


Fig. 13.1

Bernoulli's equation is really a consequence of a fundamental principle of physics: the conservation of energy. It can be derived using energy principles.

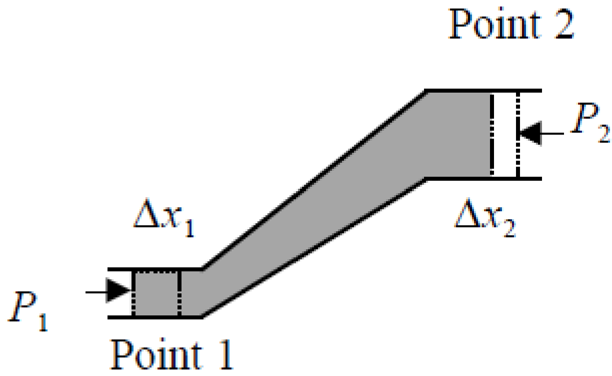


Fig. 13.2

Consider a fluid moving through a pipe. The pipe's cross sectional area changes, and the pipe changes elevation. At one point the pipe has a cross sectional area of A_1, a height of y_1, a pressure of P_1, a velocity of v_1 and moves a distance of Δx_1 in a time of Δt. At another point P_1 along the pipe these quantities are given by A_2, y_2, P_2, v_2, and Δx_2.

We are going to push a certain amount of fluid up the pipe from point 1 to point 2. P_1 is opposite in direction from P_2 because the rest of the fluid pushes to the left of fluid at point 2 and to the right of the fluid at point 1.

Recall, the equation that fundamental

$$W_{Nc} = \Delta K + \Delta U$$

We want to look at each of these terms individually to derive Bernoulli's equation.

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}\rho V_2 v_2^2 - \frac{1}{2}\rho V_1 v_1^2$$

$$\Delta U = mgy_2 - mgy_1 = \rho V_2 mgy_2 - \rho V_1 mgy_1$$

$$W_1 = F\Delta x_1 = P_1 A_1 \Delta x_1$$

$$W_2 = F\Delta x_2 = P_2 A_2 \Delta x_2$$

$$W_{Nc} = W_1 + W_2 = P_1 A_1 \Delta x_1 - P_2 A_2 \Delta x_2$$

$$W_{Nc} = \Delta K + \Delta U$$

$$P_1 V_1 - P_2 V_2 = \frac{1}{2}\rho V_2 v_2^2 - \frac{1}{2}\rho V_1 v_1^2 + \rho V_2 g y_2 - \rho V_1 g y_1$$

We know that since this this is an incompressible fluid, $V_1 = V_2$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g y_2 - \rho g y_1$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

This is Bernoulli's Equation.

Remember, that Bernoulli's principle applies when a fluid is moving. The pressure is not the same if the velocities are different. Pascal's principle applies when the fluid is stationary. A pressure applied at one point is transferred to every point of the fluid. Pascal's principle can be used for something moving slowly or something that moves a little then stops like a hydraulic jack.

As shown, for a large storage tank of height *h* which has a small pipe open at the bottom of it, the pressure at the top and at the small opening is atmospheric pressure. This is actually an important point for solving problems.

The pressure of any part of a container open to the atmosphere is the same pressure as the atmosphere for the entire section of container that is at the same height as the opening and has the same cross sectional area as the opening. Also, in this case, the velocity at the top of the storage tank is basically zero because it has such a huge cross sectional area, so Bernoulli's equation gives the velocity at the bottom of the tank to be:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

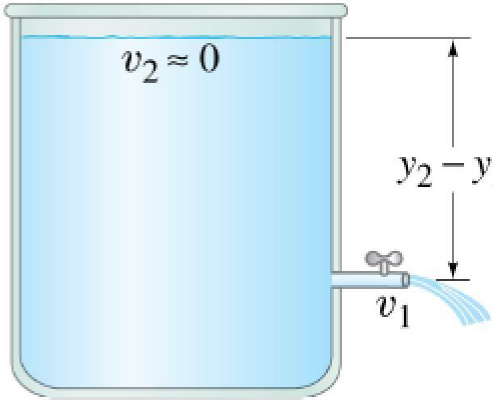
$$P_1 = P_2 = 1atm, v = 0, y_1, -y_2 = h$$

So,

$$v_2 = \sqrt{\{2gh\}}$$

Applications of Bernoulli's Principle: Torricelli, Airplanes, Baseballs, Blood Flow

Using Bernoulli's principle, we find that the speed of fluid coming from a spigot on an open tank is:



$$\frac{1}{2}\rho v_1^2 + \rho g y_1 = \rho g y_2$$

Or,

$$v_1 = \sqrt{2g(y_2 - y_1)}$$

This is called Torricelli's theorem

Airplanes

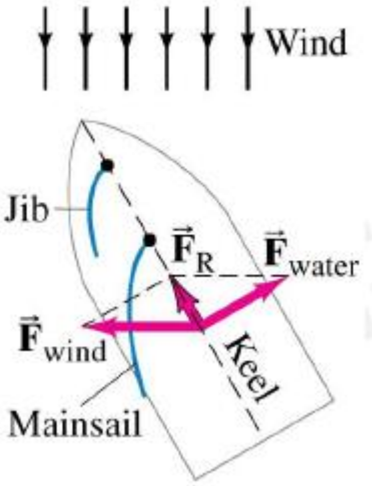


Fig. 13.3

A sailboat can move against the wind, using the pressure differences on each side of the sail, and using the keel to keep from going sideways.

Baseballs

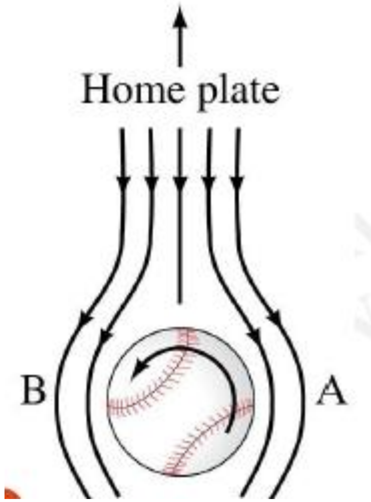


Fig. 13.4

A ball's path will curve due to its spin, which results in the air speeds on the two sides of the ball not being equal.

Blood Flow

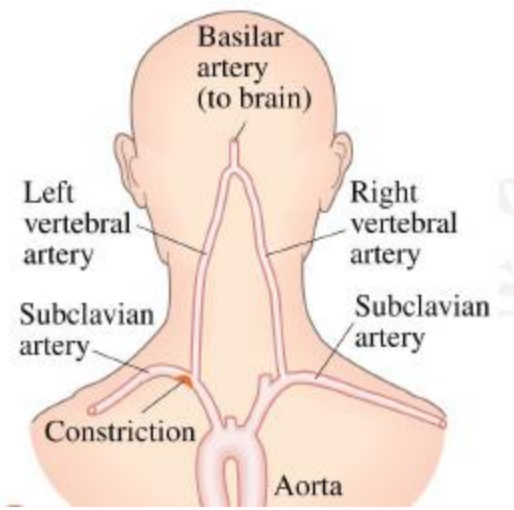


Fig. 13.5

A person with constricted arteries will find that they may experience a temporary lack of blood to the brain as blood speeds up to get past the constriction, thereby reducing the pressure.

13.3 Poiseuille's Equation

Consider a solid cylinder of fluid, of radius r inside a hollow cylindrical pipe of radius R.

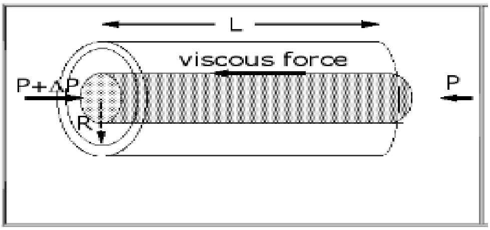


Fig. 13.6

The driving force on the cylinder due to the pressure difference is:

$$F_{presure} = \Delta P(\pi r^2)$$

The viscous drag force opposing motion depends on the surface area of the cylinder (length L and radius r)

$$F_{viscosity} = -\eta(2\pi L)\frac{dv}{dr}$$

In an equilibrium condition of constant speed, where the net force goes to zero.

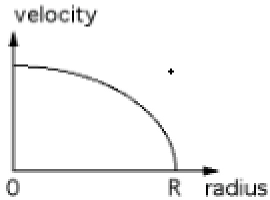
$$F_{presure} + F_{viscosity} = o$$

$$\Delta P(\pi r^2) = \eta(2\pi r L)\frac{dv}{dr}$$

So,

$$\frac{dv}{dr} = \frac{\Delta P(\pi r^2)}{\eta(2\pi r L)} = \left(\frac{\Delta P}{2\eta L}\right).r$$

We know that emperically that the velocity gradient should look like this:



At the centre

$$.r = o$$

$$.\frac{dv}{dr} = o$$

$$.v \text{ is at maximum}$$

At the edge

$$.r = R$$

$$.v = o$$

From the velocity gradient equation above, and using the empirical velocity gradient limits, integration can be made to get an expression for the velocity.

$$\frac{dv}{dr} = \left(\frac{\Delta P}{2\eta L}\right).r$$

Rewriting

$$\int_v^0 dv = \left(\frac{\Delta P}{2\eta L}\right).\int_r^R rdr$$

$$V(r) = \left(\frac{\Delta P}{4\eta L}\right)[R^2 - r^2]$$

This has a parabolic form as expected. Now the equation of continuity giving the volume flux for a variable speed is:

$$\frac{dv}{dt} = \int V.dA$$

Substituting the velocity profile equation and the surface area of the moving cylinder:

$$\frac{dv}{dt} = \int V.dA = \int_r^R \left(\frac{\Delta P}{4\eta L}\right)[R^2 - r^2].(2\pi r dr)$$

$$= \left(\frac{\pi.\Delta P}{2\eta L}\right)\int_r^R [R^2 r - r^3]\,dr$$

$$= \left(\frac{\pi.\Delta P}{2\eta L}\right)\left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

Poiseuille's equation

$$= \frac{\pi.\Delta P.R^4}{8\eta L}$$

13.4 Exercise

1) State six characteristics of fluid flow

2) Outline three applications of Bernoulli's principle

CHAPTER FOURTEEN: KINETIC THEORY OF GAS

BY

DR. S. TAUFIQ

14.1 Gas Law

Based on the kinetic theory of gases, scientists were able to describe how gases behave and change using mathematical equations. There are 4 variables that work together to determine the behaviour of gases – temperature, pressure, volume, and the number of particles.

14.2 Boyle's Law

Relationship between the volume of a gas and its pressure

$$p \times V = k$$

Where **k** is a constant at a specific temperature for a given amount of gas

Note: For Boyle's law to hold, the amount of gas (moles) must not be changed

•Temperature must also be constant

Boyle's law means if we know the volume of gas at a given pressure, we can predict the new volume if the pressure is changed provide that neither temperature nor amount of gas is changed

If we represent initial pressure and volume as

$$P_1 and\ V_1$$

And final pressure and volume as

$$P_2 and\ V_2$$

Using Boyle's Law

$$P_1V_1 = k$$

$$P_2V_2 = k$$

$$P_1V_1 = k = P_2V_2$$

$$P_1V_1 = P_2V_2$$

Application of Boyle's Law

i) It is used in finding the pressure of a given mass of gas at constant temperature when the volume is known

ii) It is also used in determining the volume of a given mass of gas at constant temperature when the pressure is known.

14.3 Charles's Law

Relationship between the volume of a gas and temperature

$$V = bT$$

Where **b** is constant at constant pressure and number of moles

$$V = bT$$

Or

$$\frac{V}{T} = b$$

$$\frac{V_1}{T_1} = b$$

$$\frac{V_2}{T_2} = b$$

$$\frac{V_1}{T_1} = b = \frac{V_2}{T_2}$$

So,

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

14.4 Pressure Law

If **volume** doesn't change, then as temperature increases, pressure also increases. They are **directly related**. If a gas is in a fixed container, as the **temperature increases** the molecules collide more frequently with the walls of the container causing **increased pressure**.

$$\frac{P}{T} = k$$

$$\frac{P_1}{T_1} = k$$

$$\frac{P_2}{T_2} = k$$

$$\frac{P_1}{T_1} = k = \frac{P_2}{T_2}$$

So,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

14.5 Ideal Gas Equation

In reality, an ideal gas **does not exist**. In this unit however, we are going to assume that gases behave ideally. This will make our math easier & is a close approximation. Real gases behave like an ideal gas at **high temperature** & at **low pressure.**

Pressure (P) time's volume (V) equals the number of moles (n) times the **ideal gas constant (R)** times the temperature in Kelvin (T)

According to
Boyle's Law PV = k or V= k/P (at constant T and n)

Charles's Law V = bT (at constant P and n)

Avogadro's Law V = an (at constant T and P)

$$V = R\left(\frac{T_n}{P}\right)V$$

$$PV = nRT$$

14.6 Dalton's Law of Partial Pressures

Dalton's Law of Partial Pressures state that Equal amounts of gas at the same temperature and volume have equal pressure. The **total pressure** inside a container is equal to the **partial pressure** due to each gas.

$$P_{total} = P_1 + P_2 + P_3 \ldots ..$$

$$P_{total} = P_1 + P_2 + P_3 = n_1\left(\frac{RT}{V}\right) + n_2\left(\frac{RT}{V}\right) + n_3\left(\frac{RT}{V}\right)$$

$$= (n_1 + n_2 + n_3)\left(\frac{RT}{V}\right)$$

$$= n_{total}\left(\frac{RT}{V}\right)$$

It is the total number of moles of gase in the mixture that is important not the identity of the molecule.

For instance, we can find the pressure in the fourth container by **adding** up the pressure in the **first three** containers.

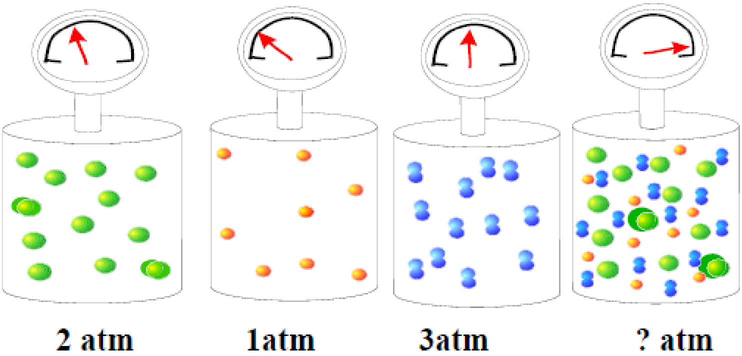


Fig. 14.1

14.7 Avogadro's Law

Relates the volume and the number of moles of a sample gas at constant temperature and pressure

$$V = na$$

$$\frac{V}{n} = a$$

$$\frac{V_1}{n_1} = a$$

$$\frac{V_2}{n_2} = a$$

$$\frac{V_1}{n_1} = a = \frac{V_2}{n_2}$$

So,

$$\frac{V_1}{n_1} = \frac{V_2}{n_2}$$

14.8 Graham's Law of Diffusion

According to the Graham's law, At constant temperature and pressure, the rates of diffusion or effusion of different gases are inversely proportional to the square root of their density.

Let at constant temperature and pressure, the rate of diffusion or effusion of gas molecules equal r and density equal d. according to Graham's law,

$$r = {}^{k}/_{\sqrt{d}}$$

Where k is constant

$$r_1 = {}^{1}/_{\sqrt{d_1}}$$

$$r_2 = {}^{1}\!\Big/\!\sqrt{d_2}$$

So,

$$\frac{r_1}{r_2} = \frac{\sqrt{d_1}}{\sqrt{d_2}}$$

14.9 General Gas Equation

This equation holds when the amount of gas (moles) is held constant.

$$\frac{PV}{T} = k$$

$$\frac{P_1V_1}{T_1} = k$$

$$\frac{P_2V_2}{T_2} = k$$

$$\frac{P_1V_1}{T_1} = k = \frac{P_2V_2}{T_2}$$

So,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Example 1) 150 cm^3 of air at atmospheric pressure, that is 760 mm of mercury, is put under a pressure of 2.0 m of mercury. What is the volume of the air at this pressure?

Solution

$V_1 = 150\ cm^3$, $P_1 = 760$ mm, $P_2 = 2.0$ m, $V_2 = ?$

$$V_2 = \frac{P_1V_1}{P_2} = \frac{760 \times 150}{2000} = \mathbf{57\ cm^3}$$

Example 2) 0.6 m^3 of air is under a pressure of 250 KN/m^2. What pressure will be needed to reduce the volume to 0.2 m^3?

Solution

$P_1 = 250\ KN/m^2$, $V_1 = 0.6\ m^3$, $V_2 = 0.2\ m^3$

:. $$P_2 = \frac{P_1V_1}{V_2} = \frac{250 \times 0.6}{0.2} = \mathbf{750\ KN/m^2}$$

Example 3) A fixed mass of gas of volume 546 cm^3 at 0^0C is heated at constant pressure. What is the volume of this gas at 100^0C?

Solution

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$0^0C = 273K$

$100^0C = 373K$

$$V_1T_2 = V_2T_1$$

$$V_2 = \frac{V_1T_2}{T_1} = \frac{546 \times 373}{273} = \mathbf{746\ cm^3}$$

Example 4) If the volume of a given mass of gas at 0^0C is 36 cm^3, determine the temperature at which its volume becomes 48 cm^3, assuming that the pressure remains unchanged.

Solution

$V_1 = 36\ cm^3$, $V_2 = 48\ cm^3$, $T_1 = 273K$, $T_2 = ?$

$$T_2 = \frac{V_2T_1}{V_1} = \frac{48 \times 273}{36} = 364K = \mathbf{91^0C}$$

14.10 Exercise

1) State Boyle's law and Charles law
2) Express the equation for Ideal gas equation
3) State Avogadro's law
4) A fixed mass of gas with volume 600 cm^3 at 0^0K is heated at constant pressure. Calculate the volume of the gas at 130^0C
5) The pressure of a gas at a constant volume is 100 cmHg at 27^0C. Calculate the pressure at 87^0C
6) The temperature of a gas of volume 50 cm^3 is 32^0C at pressure of 300 mmHg. Calculate the temperature when the volume is 80 cm^3 at pressure 360 mmHg

ANSWERS TO QUESTIONS

CHAPTER ONE

Q4)	C				
Q5)	B				
Q6)	B				
Q7)	C				
Q8(a)	-0.1 m	(b)	0.025	(c)	3000 m
Q9(a)	5,100 secs	(b)	10,800 secs	(c)	72,000 secs

CHAPTER TWO

Q6)(a)	3×10^{-10} s	(b)	1.26×10^{4} m		
Q7(a)	1.92 km	(b)	4.57 m/s		
Q8(a)	-8 m/s^2	(b)	-9 m/s	(c)	7 m/s
Q9(a)	-12.7 m/s^2	(b)	-2.3 m/s^2		

CHAPTER THREE

Q6(a)	16 N	(b)	4 N

CHAPTER FOUR

Q3)	1.5 sex
Q4)	5400 m/s

CHAPTER FIVE

Q4) 1.96 x 10^5 dyne or 1.96 N

Q5(a) 534 N (b) 54.4 kg

Q6(a) 12 N (b) 3 m/s^2

Q7(a) 2.5 N (b) 5 N

CHAPTER SIX

Q6) a

Q8(a) 118 N (b) 32^0

CHAPTER SEVEN

Q4(a) 317 J (b) 176 J (c) 0 (d) 0 (e) 141 J

Q5(a) 51 J (b) 69 J

Q6(a) 9000 J (b) 300 N

Q7(a) 75000 J (b) 25000 W (33.5 hp) (c) 33300 W (44.7 hp)

Q8) 1.8 KW and 2.4 hp

Q9) 1.72 kJ

Q10) 7.5 m/s

CHAPTER EIGHT

Q6) 5

Q7) 83

Q8(a) 500 J (b) 80% (c) 100 N

Q9) 500 N

CHAPTER NINE

Q4(a) 3.5 secs (b) 0

Q5) 11.07 secs

CHAPTER ELEVEN

Q4) 2.33 N

Q5) 100,000 N/m

Q6) 0.038 J

Q7) 250 N/m

Q8) 0.0152

CHAPTER FOURTEEN

Q4) 885.71 cm^3

Q5) 120 cm Hg

Q6) 3.12.6^{0}C

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