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APPLICATION OF GREY-MARKOV MODEL FOR FORECASTING NIGERIA ANNUAL RICE PRODUCTION

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ABSTRACT

In this paper, Grey system model (GM(1,1)) and Grey-Markov model that forecast Nigeria annual Rice production have been presented. The data used in the research were collected from the archive of Central Bank of Nigeria for a period of Six years (2010-2015). The fitted models showed high level of accuracy. Hence, the models can be used for food security plans of the nation.

Keywords: Rice, Production, Grey-System Model, Grey-Markov Model, Forecasting, Nigeria

INTRODUCTION

Accurate and reliable seasonal forecasts of crop yields are among the most valuable pieces of information that stakeholders such as farmers, commodity traders, and government officials can have at their disposal to make strategic decisions in their respective roles (Bruno and Lin, 2018) The prediction of annual crop production is one of the most desirable yet challenging tasks for every nation. Nowadays, due to the unpredictable climatic changes, farmers are struggling to obtain a good amount of yield from the crops (Khosla *et al.,* 2020). Reliable and dependable estimation of crop yields before growing season begins is important information for the government in decision making and policies formulation. Reliable information about crop estimated has been presented in the following researches (Burgueno *et al.,* 2011; Russello, 2018; Saeed and Lizhi, 2019; Syngenta, 2018; You *et al.,* 2017; Zhang *et al.,* 2017). At present time, one of the most important sectors of Nigerian economy is agriculture. It is also the major means of livelihood of many homes in the country. With population of about Two hundred million people, more than 70% of the population is engaged in agricultural activities. To feed the increasing population of Nigeria, there is a need to incorporate the latest technology and tools in the agricultural sector. It is on this note, that this study has adopted GM(1,1) and Grey-Markov model. The GM(1,1) and Grey-Markov models have proven track record of high level of accuracy in forecasting (Li Q *et al.*, 2007;Mao and Sun, 2011; Yong *et al.*, 2016; Xin *et al.*,2018). GM(1,1) forecasting model is a viable and powerful mathematical tool because of its ability to use small sample size and make long time forecasting with minimal error(Jian-Yi and Ying, 2014; Wei and Jian-Min, 2013;Yong and Yang, 2016).

MATERIALS AND METHODS

The Grey-System Model GM(1,1)

The grey GM(1,1) model can make use of the discrete data series to establish an equation of grey continuous differential equation by adding these data from the first in Accumulating Generation Operator (AGO), and the equation can then be solved to perform forecast (Li Q *et al.,* 2007).

Let the raw data series be represented by $x^{(0)}(k), k=1,2,3,...,n, x^{(0)}(k) \geq 0$ which can also be represented as:

$$X^{(0)}(k)=\left(x^{(0)}(1), x^{(0)}(2), x^{(0)}(3),..., x^{(0)}(n)\right) \tag{1}$$

Let the accumulated generating sequence be represented as:

$$X^{(1)}(k)=\left(x^{(1)}(1), x^{(1)}(2), x^{(1)}(3),..., x^{(1)}(n)\right) \tag{2}$$

Where $X^{(1)}(k)=\sum_{i=1}^{k} x^{(0)}(i), k=1,2,...,n$ (3)

$X^{(1)}(k)$ is called accumulated generating operation of $X^{(0)}(k)$ denoted as 1-AGO
By differentiating $X^{(1)}$, a whitened differential equation is obtained

$$\frac{dx^{(1)}}{dt}+ax^{(1)}=b \tag{4}$$

Where a and b are parameters to be identified. a is called developing coefficient and b is grey input.

The difference form is given as:

$$x^{(0)}(k)+ax^{(1)}(k)=b \tag{5}$$

Equation (5) represents the original form of the GM(1,1) model. The symbol GM(1,1) stands for first order Grey Model in one variable.

Equation (6) is the solution of equation (4)

$$\hat{x}^{(1)}(k+1)=\left(x^{(1)}(0)-\frac{b}{a}\right)e^{-ak}+\frac{b}{a} \tag{6}$$

Equation (6) is the time response function while parameters a and b are estimated using Least Square Method as follows:

$$\begin{bmatrix} a \\ b \end{bmatrix}=\left[B^T B\right]^{-1} B^T Y \tag{7}$$

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$$\text{Where, } B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ -z^{(1)}(4) & 1 \\ . & . \\ . & . \\ . & . \\ -z^{(1)}(n) & 1 \end{bmatrix} \tag{8}$$

$$z^{(1)}(k) = \frac{x^{(1)}(k) + x^{(1)}(k-1)}{2}, (k = 2,3,4....,n) \tag{9}$$

$$Y = \left[x^{(0)}(2), x^{(0)}(3), x^{(0)}(4), ..., x^{(0)}(n)\right]^T \tag{10}$$

The reduction value of equation (6) is given in equation (11) below:

$$\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k) = \left(1 - e^{a}\right)\left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} \tag{11}$$

Prediction Accuracy Test

To determine the accuracy of our forecast, we shall adopt mean absolute percentage error (MAPE). This tool is often used for determining prediction accuracy showing the same characteristics i.e the smaller the value, the higher the prediction accuracy.

MAPE

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \times 100\% \tag{12}$$

Where;

$\hat{y}_i$ is the Grey Model predicted value.

y_i is the Grey-Model actual value.

n is the number of prediction samples (Xin Z *et al.*, 2018)

Lewis (1982) divided the prediction accuracy of models into four grades and the division of prediction accuracy grades is shown in the table below:

Table 1

MAPE	Prediction Accuracy
$< 10\%$	High
$10\% - 20\%$	Good
$20\% - 50\%$	Feasible
$> 50\%$	Low

Grey- Markov Model for forecasting Nigeria Annual Rice Production

The Grey-Markov model (GMM) is an extension of Grey Model (GM) to further reduce prediction errors. In Grey model, the problems of poor fitting degree and low prediction accuracy may emerge when the change range of original data is too large. However, these problems can be well resolved by adopting Markov chain which can narrow down the prediction interval and improve the prediction accuracy. Markov stochastic process improves these limitations of Grey model because it reflects the stochastic volatility impact on elements by determining the transfer law of states (Ducan *et.al*, 1998).

Building the Grey-Markov Model

First step in building the GMM is to divide the residual errors into q states where each state satisfies the equi-probability principle and is defined as$R_1, R_2, \dots, R_q$. Next, the construction of the transition matrix is done by determining the probability from state R_i to state R_j which results in the transition matrix$P$.

$$P^{(1)} = \begin{bmatrix} p^{(1)}_{(11)} & p^{(1)}_{(12)} & \cdots & p^{(1)}_{(1q)} \\ p^{(1)}_{(21)} & p^{(1)}_{(22)} & \cdots & p^{(1)}_{(2q)} \\ \vdots & \vdots & \cdots & \vdots \\ p^{(1)}_{(q1)} & p^{(1)}_{(q2)} & \cdots & p^{(1)}_{(qq)} \end{bmatrix} \tag{13}$$

$$P^{(m)} = \begin{bmatrix} p^{(m)}_{(11)} & p^{(m)}_{(12)} & \cdots & p^{(m)}_{(1q)} \\ p^{(m)}_{(21)} & p^{(m)}_{(22)} & \cdots & p^{(m)}_{(2q)} \\ \vdots & \vdots & \cdots & \vdots \\ p^{(m)}_{(q1)} & p^{(m)}_{(q2)} & \cdots & p^{(m)}_{(qq)} \end{bmatrix} \tag{14}$$

Where $P_{ij}^{(m)} = \dfrac{M_{ij}^{(m)}}{m_i}, (i, j = 1,2,3,\dots,L)$, $M_{ij}^{(m)}$ stands for the transition from R_i to R_j in m steps and m_i is the number of state R_i.

Next, the residual error must be confirmed.

Let the interval median in $[R_{i-}, R_{i+}]$ be residual error forecasting value as follows:

$$\hat{e} = \frac{1}{2}[R_{i-} + R_{i+}] \tag{15}$$

Hence, the Grey-Markov model is obtained as:

$$\hat{Y}(k+1) = [1 + \hat{e}]\hat{x}^{(0)}(k+1) \tag{15}$$

Where, $\hat{x}^{(0)}(k+1) = \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k)$

Hence:

$$\hat{Y}(k+1) = \left[1 + \frac{1}{2}(R_{i-} + R_{i+})\right]\hat{x}^{(0)}(k+1) \tag{17}$$

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RESULTS AND DISCUSSION

Application of Grey System Model for Forecasting Nigeria Annual Yam Production

The data used in this research were collected from the archive of Central Bank of Nigeria for the period of Six years (2010-2015). It is the original data sequence of annual rice production in Nigeria. It is presented in table 2 below.

Table 2: Nigeria Annual Rice Production from 2010 to 2015

S/N	Year of Production	Rice Production('000 Tonnes)
1	2010	5420.19
2	2011	5690.19
3	2012	5971.90
4	2013	6209.90
5	2014	6464.73
6	2015	6725.6

Source: Central Bank of Nigeria Annual Report 2015

We begin the application by substituting the raw data in table 2 into equation (1), to obtain equation 18 below

$$X^{(0)} = (5420.19,\ 5690.19,\ 5971.90,\ 6209.9,\ 6464.73,\ 6725.6) \tag{18}$$

using equation (2) we obtain the accumulated generating sequence from equation (18) as given below:

$$X^{(1)} = (5420.19, 11110.38,\ 17682.28,\ 23292.18,\ 29756.91,\ 36482,51) \tag{19}$$

Using equation(9), we obtain equation 20 below

$$Z^{(1)} = (8265.285,\ \ 14096.33,\ \ 20187.23,\ \ 26524.45,\ \ 33119.71) \tag{20}$$

using equation (10) , we obtained equation (21)

$$Y = \begin{bmatrix} 5690.19 \\ 5971.90 \\ 6209.90 \\ 6464.73 \\ 6725.60 \end{bmatrix} \tag{22}$$

And using equation(8), we obtained equation (22)

$$B = \begin{bmatrix} -8265.29 & 1 \\ -14096 & 1 \\ -20187.23 & 1 \\ -2654.45 & 1 \\ -33119.71 & 1 \end{bmatrix} \tag{22}$$

equation (23) is obtained using equation (7) by the help of Maple 17 software

$$\hat{a} = \begin{bmatrix} -0.0412 \\ 5370 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \tag{23}$$

Where $a = -0.0412$, $b = 5370$

Substituting for a and b in equation (6), we obtained equation (24) below:

$$\hat{x}(k+1) = 135759.999582e^{0.0412k} - 130339.8058 \tag{24}$$

Evaluating equation (24) for $k = 0,1,...,5$ we obtained equation (25) below:

$$\hat{X}^{(1)} = (5420.19, 11130.33, 17080.63, 23281.21, 29742.58, 36475.73) \tag{25}$$

We compute the simulated value for our model using equation (26) below:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1) \tag{26}$$

The simulated values are presented in equation(27) below

$$\hat{X}^{(0)} = (5420.19, 5710.14, 5950.30, 6200.58, 6461.38, 6733.14) \tag{27}$$

Equation (27) is the simulated values from 2010-2015 and is presented in Table 3 below.

Table 3: Comparison of Actual and Grey simulated value for Nigeria Rice Production from year 2010-2015.

S/N	Year Of Production	Actual Rice Production ('000 Tonnes)'	Grey Simulated Rice Production('000 Tonnes)	Residual Error	Relative Error(%)
1	2010	5420.19	5420.19	0	0
2	2011	5690.19	5710.14	-19.95	-.035
3	2012	5971.90	5950.30	21.6	0.36
4	2013	6209.90	6200.58	9.32	0.15
5	2014	6464.73	6461.38	3.35	0.05
6	2015	6725.60	6733.14	-7.54	-0.11

Using equation (12), we observed from Table 3 that:

$MAPE = 1.02\%$ which is the error of the forecast and it is described as high accuracy (Lewis, 1982) . Hence the accuracy is calculated as:

$ACCURACY = 100\% - 1.02\% = 98.98\%$ this shows that the simulated accuracy is high.

Application of Grey-Markov Model for Prediction Nigeria Annual Rice Production

State Partition: due to the small sample size in this study, the error can be divided into three states respectively, using E_1, E_2, E_3 as shown in the table below:

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Table 4: State division for the error states

STATE	$E_1(\%)$	$E_2(\%)$	$E_3(\%)$
ERROR RANGE	$-0{,}35 \sim -0.113$	$-0.113 \sim 0.124$	$0.124 \sim 0.360$

From Table (4), we obtained the error states for Table (5)

Table 5: Error states for the simulated values Nigeria Annual Rice Production

S/N	Year Of Production	Actual Rice Production ('000 Tonnes)'	Grey Simulated Rice Production('000 Tonnes)	Relative Error (%)	Error State
1	2010	5420.19	5420.19	0	E_2
2	2011	5690.19	5710.14	-.035	E_1
3	2012	5971.90	5950.30	0.36	E_3
4	2013	6209.90	6200.58	0.15	E_3
5	2014	6464.73	6461.38	0.05	E_2
6	2015	6725.60	6733.14	-0.11	E_2

Equation (17) which is Grey-Markov model will be used to simulate Nigeria Annual Rice Production from 2010-2015.

Therefore,

$$\hat{Y}_1(k+1) = \left[1 + \tfrac{1}{2}(R_{i-} + R_{i+})\right]\hat{x}^{(0)}(k+1) \tag{28}$$

$k = 0,1,2,3,4,5$

For $k = 0$

$$\hat{Y}_1 = \left[1 + \tfrac{1}{2}(-0.113\% + 0.124\%)\right] * 5420.19 = 5420.19 \tag{29}$$

for $k = 1$

$$\hat{Y}_2 = \left[1 + \tfrac{1}{2}(-0.35\% - 113\%)\right] * 5710.14 = 5697.00 \tag{30}$$

$k = 2$

$$\hat{Y}_3 = \left[1 + \tfrac{1}{2}(0.124\% + 0.36\%)\right] * 5950.3 = 5962.20 \tag{31}$$

$k = 3$

$$\hat{Y}_4 = \left[1 + \tfrac{1}{2}(0.124\% + 0.36\%)\right] * 6200.58 = 6212.98 \tag{32}$$

$k = 4$

$$\hat{Y}_5 = \left[1 + \tfrac{1}{2}(-0.113\% + 0.124\%)\right] * 6461.38 = 6461.38 \tag{33}$$

$k = 5$

$$\hat{Y}_6 = \left[1 + \tfrac{1}{2}(-0.113\% + 0.124\%)\right] * 6733.14 = 6733.14 \tag{34}$$

We obtained the Grey-Markov Simulated Values presented in Table (6)

Table 6: Comparison of Actual and Grey-Markov Simulated value for Nigeria annual Rice Production for the period 2010-2015

S/N	Year of Production	Actual Rice Production ('000 Tonnes)'	Grey-Markov Simulated Rice Production('000 Tonnes)	Residual Error	Relative Error (%)
1	2010	5420.19	5420.19	0	0
2	2011	5690.19	5697.00	-6.81	-0.12
3	2012	5971.90	5962.20	9.7	0.16
4	2013	6209.90	6212.98	-3.08	-0.05
5	2014	6464.73	6461.38	3.35	0.05
6	2015	6725.60	6733.14	-7.54	-0.11

Using equation (12), we observed from Table (6) that,

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{y_i - \hat{y}_i}{y_i}\right| \times 100\% = 0.49\%$$

$$ACCURACY = 100\% - 0.49\% = 99.5\%$$

The above figure also indicates high accuracy level.

Table 7: Comparison of actual value, Grey model and Grey-Markov model Simulated Values for the annual Production

S/N	Year Of Production	Actual Rice Production ('000 Tonnes)'	Grey Simulated Rice Production('000 Tonnes)	Grey-Markov Simulated
1	2010	5420.19	5420.19	5420.19
2	2011	5690.19	5710.14	5697.00
3	2012	5971.90	5950.30	5962.20
4	2013	6209.90	6200.58	6212.98
5	2014	6464.73	6461.38	6461.38
6	2015	6725.60	6733.14	6733.14

Grey Forecasting for Nigeria Annual Rice Production from 2016 -2023

To make forecast from 2016-2022, we evaluate equation (18) for $k = 6,7,8,9,10,11,12,13$

Thus, we have

$$\hat{X}^{(1)} = (43492.07, 50803.53, 58422.51, 66361.94, 74635.32, 83256.67, 92240.65, 101602.49) \quad (35)$$

We then compute the forecast values using equation (26) that is:

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1)$$

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$$\hat{X}^{(0)} = (7016.34, 7311.46, 7618.98, 7939.44, 8273.37, 8621.51, 8983.97, 9361.84) \quad (36)$$

The forecasted values of equation (36) is presented in table 8 below

Table 8: Grey forecasted values for Nigeria Annual Rice production from year 2020-2023

Year of Production	Grey forecast Rice Production ('000 Tonnes)
2016	7016.34
2017	7311.46
2018	7618.98
2019	7939.44
2020	8273.37
2021	8621.51
2022	8983.97
2023	9361.84

Grey-Markov Model Prediction from 2016-2023

To achieve this, we obtained the error state for year 2016 to 2023 through the use of equation (37) to (44) and the information in table (5) after which we use equation (16) and the error states to the prediction from 2016 to 2023 as presented in table (9)

From Table 5, we construct transition probability matrix using equation (13)

$$P^{(1)} = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \quad (38)$$

The two steps, three steps, four steps up to the nine steps the transition probability matrix is calculated using equation (14), hence we have them respectively:

$$P^{(2)} = \begin{bmatrix} 0 & 0.50 & 0.50 \\ 0.25 & 0.25 & 0.5 \\ 0.25 & 0.50 & 0.25 \end{bmatrix} \quad (38)$$

$$P^{(3)} = \begin{bmatrix} 0.25 & 0.50 & 0.25 \\ 0.125 & 0.375 & 0.50 \\ 0.25 & 0.375 & 0.375 \end{bmatrix} \quad (39)$$

$$P^{(4)} = \begin{bmatrix} 0.25 & 0.375 & 0.375 \\ 0.188 & 0.438 & 0.375 \\ 0.188 & 0.375 & 0.438 \end{bmatrix} \quad (40)$$

$$P^{(5)} = \begin{bmatrix} 0.188 & 0.375 & 0.438 \\ 0.219 & 0.406 & 0.375 \\ 0.188 & 0.406 & 0.406 \end{bmatrix} \quad (41)$$

$$P^{(6)} = \begin{bmatrix} 0.188 & 0.406 & 0.406 \\ 0.203 & 0.391 & 0.406 \\ 0.203 & 0.406 & 0.391 \end{bmatrix} \quad (42)$$

$$P^{(7)} = \begin{bmatrix} 0.203 & 0.406 & 0.391 \\ 0.195 & 0.398 & 0.406 \\ 0.203 & 0.398 & 0.398 \end{bmatrix} \tag{43}$$

$$P^{(8)} = \begin{bmatrix} 0.203 & 0.398 & 0.398 \\ 0.199 & 0.402 & 0.398 \\ 0.199 & 0.398 & 0.402 \end{bmatrix} \tag{44}$$

To achieve this, we need to first find the error states of each of the year that is, from 2016 to 2023.

From table 5, we observed that 2015 is in error state E_2, this implies that, the initial state vector for the Grey-Markov prediction is

$$V_0 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \tag{45}$$

To obtain the error state for year 2016, we multiply equation (45) by equation (37).

$$V_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 \end{bmatrix} \tag{46}$$

It is observed from equation (46) that state 1 and state 2 have equal probability of error state, however, we choose error state1(E_1) for the 2016

To obtain the error state for year 2017, we multiply equation (46) by equation (38).

$$V_2 = \begin{bmatrix} 0.5 & 0.5 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0.50 & 0.50 \\ 0.25 & 0.25 & 0.5 \\ 0.25 & 0.50 & 0.25 \end{bmatrix} = \begin{bmatrix} 0.125 & 0.375 & 0.5 \end{bmatrix} \tag{47}$$

As it can be observed, error state 3 has the highest probability, therefore 2017 has error state3 (E_3)

To obtain the error state for year 2018, we multiply equation (47) by equation (39).

$$V_3 = \begin{bmatrix} 0.125 & 0.375 & 0.5 \end{bmatrix} \begin{bmatrix} 0.25 & 0.50 & 0.25 \\ 0.125 & 0.375 & 0.50 \\ 0.25 & 0.375 & 0.375 \end{bmatrix} = \begin{bmatrix} 0.203 & 0.391 & 0.406 \end{bmatrix} \tag{48}$$

As it can be observed, error state 3 has the highest probability; therefore 2018 has error state 3

Following similar process we obtained error states for 2019, 2020, 2021, 2022 and 2023 as E_3,E_3,E_2, E_2 and E_2 respectively

Using equation 17 and error states obtained for the respective years, we have the predictions from 2016 to 2023 respectively

$$\hat{Y}_7(2016) = \left[1 + \tfrac{1}{2}(-0.35\% - 0.113\%)\right] * 7016.34 = 7000.20$$

$$\hat{Y}_8(2017) = \left[1 + \tfrac{1}{2}(0.124\% + 0.36\%)\right] * 7311.46 = 7326.08$$

$$\hat{Y}_9(2018) = \left[1 + \tfrac{1}{2}(0.124\% + 0.36\%)\right] * 7618.98 = 7632.23$$

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$$\hat{Y}_{10}(2019) = \left[1 + \tfrac{1}{2}(0.124\% + 0.36\%)\right] * 7939.44 = 7955.32$$

$$\hat{Y}_{11}(2020) = \left[1 + \tfrac{1}{2}(0.124\% + 0.36\%)\right] * 8273.37 = 8289.92$$

$$\hat{Y}_{12}(2021) = \left[1 + \tfrac{1}{2}(-0.113\% + 0.124\%)\right] * 8621.51 = 8621.5$$

$$\hat{Y}_{13}(2022) = \left[1 + \tfrac{1}{2}(-0.113\% + 0.124\%)\right] * 8983.97 = 8983.97$$

$$\hat{Y}_{14}(2023) = \left[1 + \tfrac{1}{2}(-0.113\% + 0.124\%)\right] * 9361.84 = 9361.84$$

These values are presented in Table 9, below.

Table 9: Grey-Markov prediction value from 2016-2023.

Year of Production	Grey-Markov Rice Production ('000 Tonnes)
2016	7000.20
2017	7326.08
2018	7632.23
2019	7955.32
2020	8289.92
2021	8621.50
2022	8983.97
2023	9361.84

CONCLUSION

The research has applied Grey system theory to forecast Nigeria annual Rice Production with level of accuracy using empirical data obtained from the archive of Central Bank of Nigeria. This was possible due to the mathematical prowess of Grey system model that not only requires minimal data but also has the capability of long-time forecasting with minimal error. The results showed that the model is reliable and dependable. Thus, information from this research could serve as a guide to the government and the farmers for better planning and sustainable Rice production in the country.

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