International Journal of Mathematical Analysis and Modelling

(Formerly Journal of the Nigerian Society for Mathematical Biology)

Volume 5, Issue 1 (May), 2022

ISSN (Print): 2682 - 5694
ISSN (Online): 2682 - 5708

International Journal of Mathematical Analysis and Modelling
Volume 5, Issue 1, May 2022, Pages 121 – 127

Analysis of drought and flood occurrence using markov chain

O. K. Musa*, L. Adamu*†, E. N. Didigwu‡, A. Abdullahi§, and S. D. Yakubu**

Abstract

Flood and drought are among the most common natural disasters affecting the world. In this paper, Markov model has been used to analyse and predict flood and drought occurrences in Birnin Kebbi, Nigeria. The Standardized Precipitation index(SPI) was used to classify the annual rainfall of Birnin Kebbi into three states (flood, normal and drought). After some successful iterations of the model, the model stabilized to equilibrium probabilities, revealing that in the long-run 20% of the years in Birnin Kebbi will experience flood, 60% will experience normal rainfall and 20% will experience drought. It was also observed that, a drought year cannot be followed by a flood year and the probability of a drought year to be followed by a normal year is high while the probability of a normal year to be followed by a drought year and a drought year to be followed by another drought year is extremely small. Results from this research is an important information to the government and people of Kebbi state for better understanding of rainfall dynamics in their locality.

Keywords: markov model; annual rainfall, standardized precipitation index, transition probability, equilibrium probabilities.

1. Introduction

Floods and droughts are among the most disastrous natural hazards in the world ([1] and [4]). Drought occurs when there is significant rainfall deficit that causes hydrological imbalances and affects the land productive systems. Drought practically occurs in all climatic regions with both high and low mean rainfalls [11]. It can result in damaging agricultural production and the natural environment [2].

* Department of Mathematics, Federal University of Technology, Minna, Nigeria
† Corresponding Author
‡ Department of Mathematics, Enugu State University of Science and Technology, Enugu, Nigeria
§ Department of Mathematics, Faculty of Science, Air Force Institute of Technology, Kaduna, Nigeria
** Department of Mathematical Science, Ibrahim Badamasi Babagida University, Lapai, Nigeria

Drought is often seen as a "creeping" phenomenon with slow onset and cessation. As a result, an effective drought monitoring system is the most important tool for developing and implementing efficient mitigation strategies. However, not only can the onset of drought conditions be rapid, an indication of how long drought conditions may continue will enable improved planning and resource allocation. For this reason, a capability to accurately forecast the onset, persistence and cessation of drought conditions will enable more effective drought mitigation strategies to be developed [10]. [12] defines flood as significant rise of water level in a stream, lake, reservoir or coastal region. Also, [8] defined flood as excessive water run-off or the rise in water level in a particular area which is more than what the particular environment can absorb. Flood and drought are one of the most devastating natural disasters in the world. In Nigeria, the story is not different as these has caused serious havoc to our environment and the economy. However, little or nothing can be done to avoid its occurrence but prior information such as prediction of flood and drought can assist the farmers, stakeholders and the general public to take adequate proactive measure in order to mitigate their effects. Hence, the result from this model will provide adequate information about flood and drought in some upcoming years for the studied areas. According to NEMA, in 2012 alone, floods caused more than N2.6 trillion in economic damage, much of which could be attributed to large-scale transboundary floods (from rivers Niger and Benue). In 2019, about 126 deaths were recorded, over 48,000 people were displaced and property worth millions of Naira destroyed across the country (Nigeria Hydrological Services Agency, 2020).The Standard Precipitation Index (SPI) was introduced by [7] as measure of the precipitation deficit that is uniquely related to probability. It can be calculated for any accumulation timescale, usually from monthly precipitation observations, and is typically expressed as SPI-n, where n is the number of months of accumulation. The time series is analogous to a moving average in the sense that a new value is calculated each month and is auto-correlated to previous months depending on the accumulation timescale [10].

2. Materials and method

Birnin Kebbi is the capital city of Kebbi state, is located in north-western Nigeria on (12° 27′ N and 4° 11′ E).The data used for this research work were obtained from the National Oceanic and Atmospheric Administration (NOAA) for the period of thirty years (1991 to 2020). After which the SPI was used to classify the rainfall into states

2.1. Model formulation

Suppose that the SPI rainfall classification for a year is considered as a random variable x, the collection of these random variables over the years constitute a stochastic process X_n, n=0,1,2,3,…, We assume that this stochastic process satisfies Markov property. Let the annual rainfall be modelled by a three-state Markov model based on the SPI classification

State 1: Flood (SPI Value ≥1)
State 2: Normal (-0.99≤SPI Value ≤0.99)

State 3: Drought (SPI Value ≤ -1)
The transition probability matrix is represented by

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ p_{31} & P_{32} & P_{33} \end{bmatrix}. \tag{1}$$

Following [5], let $P^{(n)}$ be the probability state vectors of the Markov chain, where n=0,1,2,3,… and let $P_i^{(n)}$ be the probability that the annual rainfall is in the ithstate at the nth year. $P^{(0)}$ is the initial state vector of the Markov chain and $p^{(n)}$ is the state vector at the nth year. Then, by induction we have that

$$P^{(n)} = P^{(n-1+1)} = P^{(n-1)} \cdot P = P^n \tag{2}$$

On iteration we have,

$$P^{(n)} = P^{(0)} P^n . \tag{3}$$

That is, the initial state vector $P^{(0)}$ and the transition matrix P determine the state vector $P^{(n)}$ at the nth year. If we now let

$$P^{(n)} = \begin{bmatrix} P_1^n & P_2^n & P_3^n \end{bmatrix}. \tag{4}$$

Denote the probabilities of finding the annual rainfall in any of the three states at the nth year and also let

$$P^{(0)} = \begin{bmatrix} P_1^0 & P_2^0 & P_3^0 \end{bmatrix}. \tag{5}$$

Denote the initial state vector and its elements, the Markov chain model for annual rainfall based on SPI classification in the study area can be represented by

$$\begin{bmatrix} P_1^n & P_2^n & P_3^n \end{bmatrix} = \begin{bmatrix} P_1^0 & P_2^0 & P_3^0 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}^n \tag{6}$$

Limiting State Probability

The limiting state probability is represented by equation (7) below and is obtained when n → ∞ in equation (3).

$$\pi = \pi P, \tag{7}$$

where π = $\begin{bmatrix} \pi_1 & \pi_2 & \pi_3 \end{bmatrix}$

$$\pi = \sum_{i=1}^{3} \pi_i = 1. \tag{8}$$

These equations will be use to find the limiting state probabilities for our model.

3. Results and discussion

SPI classification	frequency	State
SPI value ≥ 1	6	Flood
-0.99≤ SPI value≤ 0.99	18	Near normal
SPI value ≤ -1	6	Drought

Table 1: A summary of SPI annual rainfall classification of BerninKebbi.

From Table 1, we obtained the transition count presented in equation (9)

$$C = \begin{bmatrix} 2 & 2 & 2 \\ 4 & 10 & 3 \\ 0 & 5 & 1 \end{bmatrix}. \tag{9}$$

The probability transition matrix obtained from equation (9) is presented below

$$P = \begin{bmatrix} 0.333 & 0.333 & 0.333 \\ 0.235 & 0.588 & 0.176 \\ 0 & 0.833 & 0.167 \end{bmatrix}. \tag{10}$$

Calculating P^n, on iteration we have

$$P^2 = \begin{bmatrix} 0.189 & 0.584 & 0.225 \\ 0.216 & 0.571 & 0.211 \\ 0.196 & 0.629 & 0.174 \end{bmatrix} \quad (11)$$

$$P^4 = \begin{bmatrix} 0.206 & 0.585 & 0.205 \\ 0.205 & 0.584 & 0.206 \\ 0.207 & 0.582 & 0.207 \end{bmatrix} \quad (12)$$

$$P^8 = \begin{bmatrix} 0.206 & 0.583 & 0.205 \\ 0.205 & 0.583 & 0.205 \\ 0.206 & 0.583 & 0.206 \end{bmatrix} \quad (13)$$

$$P^{20} = \begin{bmatrix} 0.204 & 0.577 & 0.204 \\ 0.204 & 0.577 & 0.203 \\ 0.203 & 0.578 & 0.203 \end{bmatrix} \quad (14)$$

$$P^{30} = \begin{bmatrix} 0.202 & 0.573 & 0.202 \\ 0.202 & 0.572 & 0.202 \\ 0.202 & 0.573 & 0.202 \end{bmatrix} \quad (15)$$

$$P^{50} = \begin{bmatrix} 0.199 & 0.563 & 0.199 \\ 0.199 & 0.563 & 0.199 \\ 0.199 & 0.563 & 0.199 \end{bmatrix} \quad (16)$$

Limiting State Probabilities

As n increases, p^n gets closer to equation (16) that is, n≥50 the transition probabilities stabilise to equation (16), and from equation (3) with the initial state probability vector

$$P^0 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$P^{(n)} = P^0 P^n = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.199 & 0.563 & 0.199 \\ 0.199 & 0.563 & 0.199 \\ 0.199 & 0.563 & 0.199 \end{bmatrix}$$

$$P^{(n)} = [0.199 \quad 0.563 \quad 0.199]. \quad (16)$$

Correcting to one decimal place, we have:

$$P^{(n)} = [0.2 \quad 0.6 \quad 0.2].$$

This is the probability of finding the annual rainfall based on SPI classification fall in any of three states for large n (1.e n ≥ 50).

From equation (3), we obtained the limiting state probability vector that is equation (7).
Thus: $\pi = \pi P\ (0.2\ \ 0.6\ \ 0.2)$.

The interpretation of this result is that, in the long-run 20% of the years during rainy season in BirninKebbi will experience flood, 60% will experience normal rainfall while 20% will experience drought.As it can be seen from equation (10) (transition probability matrix) a drought year cannot be followed by a flood yearand the probability of a drought to be followed by a normal year is high while the probability of a normal year to be followed by a drought and a drought year to be followed by another drought year is extremely small.

4. Conclusion

The rainfall pattern of Birnin Kebbi, Kebbi state has been analysed using Markov Chain with the help of Standardized Precipitation Index (SPI). Results from the research is an important information to the government and people of Kebbi state for better understanding of rainfall dynamics and viable agricultural production.

References

[1] Bates D., Maechler M., Bolker B., and Walker S. (2014). Linear mixed-effects models using Eigen and S4. R Package Version 1:1–23.

[2] Gidey E., Dikinya O., Sebego R., Segosebe E., and Zenebe A. (2018). Analysis of the long-term agricultural drought onset, cessation, duration, frequency, severity and spatial extent using vegetation health index (VHI) in Raya and its environs, northern Ethiopia. Environmental Research System 7 (13). https://doi.org/10.1186/s40068-018-0115-z.

[3] Guha-Sapir D., Vos F., Below R., and Ponserre S. (2012). Annual disaster statistical review 2011: The numbers and trends. Brussels: Centre for Research on the Epidemiology of Disasters, Institute of Health and Society (IRSS), and Universitécatholique de Louvain.

[4] IPCC. (2007). Summary for Policymakers. In: Climate Change 2007: Impacts, Adaptation and Vulnerability. Contribution of Working Group II to the Fourth Assessment Report of the Inter-governmental Panel on Climate Change, M.L. Parry, O.F. Canziani, J.P. Palutikof, P.J. van der Linden and C.E. Hanson, Eds., Cambridge University Press, Cambridge, UK, 7-22.

[5] Lawal A. (2017). Stochastic model of rainfall precipitations for crop production in some selected states of north central Nigeria. Unpublised Phd thesis, Department of Mathematics, Federal University Technology, Minna, Nigeria.

[6] Lawal A., Abubakar U.Y., Danladi H., and Andrew S. G. (2016). Markovian approach for the analysis and prediction of weekly rainfall pattern in Makurdi, Benue State, Nigeria. J. Appl. Sci. Environ. Manage Vol. 20 (4) 965-971.

[7] McKee T. B., Doesken N. J., and Kliest J. (1993). The relationship of drought frequency and duration to time scales. In: Proceedings of the 8th Conference on Applied Climatology, Anaheim, CA, America. Meteorol. Soc. Boston, pp. 179–184.

[8] Nelson D. R., Adger W. N., and Brown K. (2007). Adaptation to environmental change: contributions of a resilience framework. Annual Review of Environment and Resources, 32(1): 395-419.

[9] Nigeria Hydrological Services Agency, (2020). Annual Flood Outlook. https://www.google.com/url?sa=t&source=web&rct=j&url=https://nihsa.gov.ng/wp-content/uploads/2020/06/2020-NIHSA-Annual-Flood-Outlook-AFO-5-2.pdf&ved=2ahUKEwiS7rSEx6LzAhWsDmMBHV82DVQQFnoECBUQAQ&usg=AOvVaw1JaPlygzezDG70hkgW0tXJ.

[10] Singleton A. (2012). Forecasting drought in europe with the standardized precipitation index: an assessment of the performance of the european centre for medium range weather forecasts variable resolution ensemble prediction system. Journal of the International Society for the Prevention and Mitigation of Natural Hazards, DOI 10.1007/s11069-019-03665-6.

[11] Um M., Kim Y., Park D., and Kim J. (2017). Effects of different reference periods on drought index estimations from 1901 to 2014. Hydrology and Earth System Sciences, 21: 4989–5007.