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Effects of Relaxation Times from the Bloch Equations on Age Related Changes in White and Grey Matter

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Abstract

This research work presented the analytical method of using T_1 and T_2 relaxation rates of white matter and grey matter to distinguish the passage of time on human organs. A time dependent model equation evolved from the Bloch Nuclear Magnetic Resonance equation was solved under the influence of the radio frequency magnetic field $[rfB_1(x,t) \neq 0]$ and in the absence of radio frequency magnetic field $[rfB_1(x,t) = 0]$. The general solution was considered in three cases. Analysis of the solutions obtained revealed that the rate of decrease of the white matter was faster than that of the grey matter. Between 100 and 400 seconds the difference is more noticeable.

Keywords: Forcing function, Grey Matter, White matter, Spin lattice and Spin-spin relaxation times.
MSC2010: 65L05, 65L06, 65L10.

1.0 Introduction

This study used the Bloch Nuclear Magnetic Resonance (NMR) equations to show the inter relationship between white and grey matter of the brain in determining the length of time a human organ had existed. This is an analytical approach which is an alternative method to the statistical methods many researchers in this field of biomedicine had applied. The mathematical processes involved in using relaxation rates as powerful tools to show the inter relationship between white and grey matter (in the brain) was examined. The need to study aging organs is very important because the study of aging is more than grey hair and wrinkles of the human skin. The study of aging could help humans understand aging and age-related diseases in order to identify new target

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for drugs which might likely help humans to live better.

Biomedicine has shown from discoveries of the twentieth century that hydrogen atoms in water molecules can produce nuclear magnetic resonance phenomena. The discovery further illustrated that nuclear magnetic resonance is required to get the information from the distribution of water molecules in the human body, through which the internal anatomy of the human body could be mapped accurately, Jimmy *et al.* [1]. Nuclear Magnetic Resonance (NMR) imaging also known as Magnetic resonance imaging (MRI) is a noninvasive medical imaging technique used by doctors to form the pictures of the anatomy and the physiological processes of the body. MRI is used extensively in modern medical diagnosis analysis and scientific research because of its excellent soft tissue unrivaled imaging quality and non-radiating capabilities. With MRI, the macro-structural aging changes in human organs, tissues, and cells are visibly determined, Ponti *et al.* [2].

Fuzzy S-sets can also be helpful in addressing the uncertainty in the concepts of aging organs through integrating diverse information from various imaging modalities and biomarkers, providing a more comprehensive picture of organ health and aging. Standardizing Fuzzy S-set-based methods could improve consistency in interpreting medical images and support clinicians in making informed decisions about patient care, Elijah & Muhammad [3].

Research has shown that at the microscopic level, the concept of aging organs is not still clear as the individual cells that make up the human organs wear out and require replacement on a regular basis, that is, many tissues regenerate over time but the rate of the regeneration varies. Now taking the brain as a case study in this work, most cells in the brain or neurons are not replaced as human increase in age but they are in general as old as the human body. These long-lived brain cells are more vulnerable to age-related wear and tear than those that only last a few days like those cells in the human intestine that gets replaced after just a few days, Jo *et al.* [4].

According to Medlineplus [5], aging changes happen throughout the human body cells, tissues and organs and these changes affect the functioning of all the body systems. In the human body cells, there were reports that, with age, the cells become larger and are less able to divide and multiply. It was also visibly shown that many cells lose their ability to function or they begin to malfunction as an indication to aging changes.

Waste product was said to build up in the tissue as aging continues which has caused tissue changes such as connective tissue changes. This change makes the tissue stiffer causing the organs, blood vessels and airways more rigid. As a result of aging changes many tissues are said to lose mass while some tissue become rigid. Also, because of the aging changes in human body cells and tissues, human organs also change with increase in age. These aging changes in organs appear slowly and over a long period, Kirsty *et al.* [6].

Physiologists have discovered that the performance of many organs such as the heart, kidneys, brain or lungs shows a systematic decline throughout the aging process over the life span. The systematic decline is due to loss of cells from these human organs. The changes that appear in the individual cells and the whole organs make the body to change with aging, Ann [7].

Some groups of researchers have clearly shown that the way human organs and tissues age could give us more detailed information on the changes in our body. It was also claimed that many researchers tend to be more interested in the different changes between human chronological age and human biological age as new research might be able to predict which individual human organs would be the first to die off. It was also published in Nature Medicine at the Stanford University School of Medicine that just as people have an individual genotype, so do they have ageotype and this gives a new framework to think about aging and could possibly help physicians to identify the most important thing to target to extend healthy life. It was explained that the hope is that once humans could identify their ageotype, that is, main cause of individual organs aging, it gives room for precautionary interventions - exercise or diet or medications, Sharon [8].

Medlineplus [5] claimed that some theories on aging have suggested that aging is caused by injuries from wear and tear on the body or byproducts of metabolism. Also, in another theory aging was viewed as a predetermined process controlled by genes. Aging is said to be a complex process that varies as to how it affects different organs of the human body. Most people who study aging suggested that aging depends on the interaction of many lifelong influences such as influences due

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to heredity, environment, culture, diet, leisure, exercise and past illnesses. Some aging processes are common at later stages in life. As humans grow older, the human body experiences notable changes.

The applicability and usefulness of Lomax-Weibull distribution was illustrated using two lifetime data sets obtained from literature. The Lomax-Weibull distribution can be used to also model the lifespans of specific organs like hearts, kidneys and livers considering factors like individual health, medical conditions and genetic predispositions. The model can be utilize to stimulate potential organ failure risks under various environmental changes informing preventive strategies, Osagie & Osemwenkhae [9].

In a research study carried out by Maitre *et al.* [10], it was proposed that Magnetic Resonance Imaging (MRI) is a useful tool to study brain growth, but its realism can be limited by time and medical constraints. They initiated a study to find out if MRI relaxometry of the deep nuclei would reflect the influence of gestational age at birth on structures essential to monitor development, regardless of postnatal age at the time of imaging.

Neuroimaging is an attractive tool for assessing environmental influences on neonatal brain growth and organization as it is non-invasive and has direct anatomical specificity. While ultrasound is routinely used for diagnostic and counseling purposes, a magnetic resonance imaging (MRI) using conventional technique offers a T_1 and T_2 quantitative advantage. Magnetic resonance (MR) relaxometry, which measures the time it takes for nuclear spins to return to thermal equilibrium after being disturbed, conveys information about the molecular environment of water in tissues. Hence, MR Relaxometry provides a quantitative measure of central nervous system organization that can be incorporated into predictive models of brain growth, Siemonsen *et al.* [11]

Many groups have investigated hydrogen proton T_1 and T_2 quantitative MRI (Q-MRI) relaxometry as tools for measuring age changes caused by diseases affecting the brain, including multiple sclerosis, cerebral neoplasia, epilepsy, stroke, dementia, schizophrenia, depression, human immunodeficiency virus infection, cerebral ischemia and other conditions. Age-related changes of the mean T_1 or mean T_2 of selected brain regions have also been investigated to be a function of ages of human organs, Suzuki *et al.* [12]

MRI test has been a great tool for getting the images of deeper tissues of the brain such as the white matter (WM). The MRI test would easily take the pictures of the inside of the human brain and can show any damage to the organs and tissues in the brain. Medical imaging diagnoses have been greatly advanced to easily detect white matter diseases. WM is a deep tissue of the brain that helps human think fast, walk straight and prevents human from falling. This tissue contains millions of nerve fibers that connect other parts of the brain and spinal cord and signals the nerves in the body to talk to one another. When the WM tissue becomes diseased, those signals that help the human body to think fast, walk straight without falling would never get through to the nerves in the body, then the human body stops working as it should. This leads to a disease the medical professionals called white matter disease; white matter disease is the wearing out of tissue in the deepest region of the brain as a result of some physiological changes to the body that include aging of the human organs. White matter diseases happen in older or elderly people, Medlineplus [5].

Gwenaëlle *et al.* [13] used MRI scans to look at changes in the brain structure of 484 healthy people, ranging in age from 8 to 85 years. The researchers compare the patterns of grey matter damage observed in the brain scans of people with Alzheimer and people with schizophrenia with a specific network in the grey matter found to be vulnerable to aging as they identified from the MRI data of the healthy human brains used in their researches. They suggested that this region of the brain may play a role in the development of the Alzheimer's brain disorder because there were similarities from their comparison. The symptoms of Alzheimer's diseases first appear after age 60.

Cho *et al.* [14] understudied the ten structures of the white matter and grey matter of the brain with a goal to determine the expected normal range of variation in spin-lattice relaxation time (T_1) of brain tissue in vivo, as a function of age. An inversion recovery method was used to map T_1 transversely, at the level of the basal ganglia, in a study population of 115 healthy subjects (ages 4 to 72; 57 male and 58 female). They conducted a least-squares regression analysis which shows that T_1 varied as a function of age in pulvinar nucleus ($R^2 = 56\%$), anterior thalamus ($R^2 = 51\%$),

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caudate ($R^2 = 50\%$), frontal white matter ($R^2 = 47\%$), optic radiation ($R^2 = 39\%$), putamen ($R^2 = 36\%$), genu ($R^2 = 22\%$), occipital white matter ($R^2 = 20\%$) (all $p < 0.0001$), and cortical gray matter ($R^2 = 53\%$) ($p < 0.001$). They postulated that there were no significant differences in T_1 between men and women.

Another study stated that the tissue can be characterized by two different relaxation times – T_1 and T_2 . T_1(Longitudinal relaxation time) is the time constant which determines the rate at which excited protons return to equilibrium. It is a measure of the time taken for spinning protons to realign with the external magnetic field. The T_1 relaxation time, also known as the spin-lattice relaxation time, is a measure of how quickly the net magnetization vector (NMV) recovers to its ground state. The return of excited nuclei from the high energy state to the low energy or ground state is associated with loss of energy to the surrounding nuclei. Nuclear magnetic resonance (NMR) was originally used to examine solids in the form of lattices, hence the name "spin-lattice" relaxation, Rock [15].

In a study carried out by Yusuf [16], the two different relaxation times T_1 and T_2 were used to study the general analysis of physiological flow in human body. The results show that the two relaxation parameters are very vital in describing and analysing the flow of fluids in human body and the surrounding tissues.

Badve *et al.* [17] described Magnetic Resonance Fingerprinting (MRF) as a method which simultaneously and rapidly measures multiple tissue properties, with initial application in measuring T_1 and T_2. This technique is based on the premise that acquisition parameters can be varied in a pseudo-random manner such that each combination of tissue properties will have a unique signal evolution. Using the Bloch equations, a dictionary of all possible signal evolutions can be created that includes all known acquisition parameters and all possible range of values and combination of the properties of interest.

Anna *et al.* [18] proposed in their research work on human brains, that the relaxation rate measurements were present for both grey and white matter in normal human brain. These measurements were made using a 3.0 T magnetic resonance imaging (MRI) scanner in normal adults with no clinical evidence of neurological disease. Nineteen human beings, 8 females and 11 males, were studied for $T_(1)$ and T_2 measurements. Measurements were made using a saturation recovery method forT_1, a multiple spin-echo experiment forT_2, and a fast low-angle shot (FLASH) sequence with 14 different echo times for T_2 Results of the measurements were summarized as follows: Average T_1values measured for grey matter and white matter were 1331 msec and 832 msec, respectively. Average T_2values measured for grey matter and white matter were 80msec and 110 msec, respectively. The average T_2 values for occipital and frontal grey matter were 41.6msec and 51.8 msec, respectively. Average T_2values for occipital and frontal white matter were 48.4msec and 44.7 msec, respectively. It was revealed from ANOVA test that for both grey and white matter there were no significant differences in T_1 from one location in the brain to another. T_2 in occipital grey matter was significantly higher ($0.0001 < P < 0.0375$) than the rest of the grey matter, while $T_(2)$ in frontal white matter was much lower ($P < 0.0001$), Janaka *et al.* [19].

In another research the spin-lattice relaxation time (T_1) of human brain tissue has been used as an indicator of brain development or of brain maturation. In general, T_1 decreases with age until a critical age is reached, at which point T_1 begins to increase. The critical age ranges from 35.9 years in the occipital white matter to 60.4 years in the cortical grey matter; both are neuron cells of the deeper tissues found in the human brain, Yulin *et al.* [20].

Josef *et al.* [21] researched that the T_1 and T_2 relaxometry on patients with Parkinson disease, patients with multiple system atrophy and age-matched healthy control patients. The values from healthy humans were compared with total iron concentration as estimated from the histochemical studies. They further observed the T_1 and T_2 changes found in patients with Parkinson disease and patients with multiple system atrophy were then interpreted in terms of possible or known pathologic changes in ferritin and other forms of iron.

Antonio *et al.* [22] proposed that ageing is a stochastic process which combines predictable and random effects that eventually lead to the accumulation of unrepaired cellular damage, weakening cellular repair and compensatory mechanisms. Much of the individual variation in ageing is due to

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the lifestyle and the effects of the environment, with genes responsible for only 25% of variability. They postulated that the brain cells are particularly susceptible to the accumulated effects of ageing.

Another group also researched the living human brain and stated their findings keenly and aptly that at low nuclear MR frequencies, age-related factors constitute an important influence on the T_1 relaxation time. They further demonstrated the importance of understanding the variation of T_1 and T_2 in relation to age with the localization in the brain. They explained that for instance, in the investigation of diseases, the contrast resolution between the tissues changed by disease and the normal tissue might be different, depending on the normal tissue variation in relaxation times, Agartz *et al.* [23].

Considering the weight of the human brain under investigation, the protein content of human white matter decreases continuously with age over the period from 33 to 92 years of age. In cortical grey and white matter regions, cerebral blood flow, cerebral blood volume and oxygen consumption decrease with age at a rate of about 0.5% per year. It was declared that these various results suggest that healthy aging persons can show subtle brain changes that do not manifest clinical symptoms but which may be associated with occult pathologic processes Grant *et al.* [24].

It was deduced that newer techniques of quantitative MR imaging are more objective and potentially more sensitive to subtle brain changes than are conventional MR imaging techniques. Quantitative MR imaging techniques have led to remarkable success in identifying brain abnormalities associated with fragile-X syndrome, schizophrenia, multiple sclerosis, phenylketonuria, epilepsy, and human immunodeficiency virus (HIV) associated dementia. Their findings suggest that quantitative MR imaging may give the clinician sufficient sensitivity and accuracy to characterize the processes of brain maturation in healthy children more objectively or perhaps to identify with greater surety those children who are developmentally delayed. They proposed that the reduction in brain T_1 with increasing age is related to a reduction in brain tissue water content. Brain water content decreases from 88% at birth to 82% at 6 months of age, as a result of a nearly 50% increase in dry weight of the brain. Brain water content continues to decrease, though at a slower rate, for several more years of human life, Grant *et al.* [25].

Awojoyogbe [26], in his work on analytical solution of the time – dependent Bloch NMR flow equations: A Translational Mechanical Analysis, explained that solutions to the Bloch NMR equations always exist when appropriate mathematical techniques are used. This reduces the difficulty encountered when revealing the hidden applications of the blood flow parameters.

From the various literature reviews undertaken so far, it is obvious that most of the researches were undertaken through clinical data collated over time and statistical methods and approaches were adopted. Therefore, this research work is undertaken to investigate, analytically as obtained from the Bloch NMR equations, the effects of T_1 and T_2 relaxation rates of white and grey matter on age-related changes or the length of time a human organ (brain) had existed.

2.0 Mathematical Formulation

By applying the kinetic theory of moving fluids on the Bloch Magnetic Resonance Equations as evolved by Awojoyogbe [26], the general flow equation is

$$v^2\frac{\partial^2 M_y}{\partial x^2}+2v\frac{\partial^2 M_y}{\partial x\partial t}+v\left[\frac{1}{T_1}+\frac{1}{T_2}\right]\frac{\partial M_y}{\partial x}+\left[\frac{1}{T_1}+\frac{1}{T_2}\right]\frac{\partial M_y}{\partial t}+\frac{\partial^2 M_y}{\partial t^2}+\left[\frac{1}{T_1T_2}+\gamma^2\beta_1^2\right]M_y=\frac{M_0\gamma\beta_1}{T_1} \tag{1}$$

where M_0 = equilibrium magnetization;
M_x = component of transverse magnetization along the x-axis;
M_y = component of transverse magnetization along y-axis;
M_z = component of magnetization along the field (z-axis);
γ = gyro-magnetic ratio of fluid spins;
$\beta_1(t)$ = radio-frequency (RF) magnetic field;
T_1 = Longitudinal or spin lattice relaxation time;

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T_2 = Transverse or spin-spin relaxation time;
Time-dependent Bloch-flow equation is evolved from equation (1) as follows:

$$\frac{\partial^2 M_y}{\partial t^2} + \left[\frac{1}{T_1} + \frac{1}{T_2}\right]\frac{\partial M_y}{\partial t} + \left[\frac{1}{T_1 T_2} + \gamma^2\beta_1^2\right] M_y = \frac{\gamma\beta_1 M_0}{T_1} \tag{2}$$

provided;

$$v^2\frac{\partial^2 M_y}{\partial x^2} + 2v\frac{\partial^2 M_y}{\partial x \partial t} + v\left[\frac{1}{T_1} + \frac{1}{T_2}\right]\frac{\partial M_y}{\partial x} = 0 \tag{3}$$

Equation (2) is a time dependent equation.
Now for convenience the non-homogeneous equation (2) can be written as

$$\frac{d^2 M_y}{dt^2} + m\frac{dM_y}{dt} + fM_y = B_0 \sin\omega t \tag{4}$$

where $m = \frac{1}{T_1} + \frac{1}{T_2}$;
$f = \frac{1}{T_1 T_2} + \gamma^2\beta_1^2$;
$B_0 = \frac{M_0}{T_1}$; and
$\sin\omega t = \gamma\beta_1(t)$
Since the equation is a differential equation of order two, its general solution can be represented as the sum
$M_y = M_{y_c} + M_{y_p}$
where $M_{y_c}(t)$; is the complementary solution of the homogeneous form of equation (4) given as:

$$\frac{d^2 M_{y_c}}{dt^2} + m\frac{dM_{y_c}}{dt} + fM_{y_c} = 0 \tag{5}$$

$M_{y_p}(t)$; is the particular solution of equation (4) given as:

$$\frac{d^2 M_{y_p}}{dt^2} + m\frac{dM_{y_p}}{dt} + fM_{y_p} = B_0 \sin\omega t \tag{6}$$

Note that the equation (4), is a non-homogeneous constant second order equation in which t can be regarded as time and $B_0 \sin\omega t$ as an external force to the system which describes the motion of the particles in the matter being considered. If the external force is zero ($B_0 \sin\omega t = 0$) then it implies that the particles in the system vibrate freely.
Now applying the method of solving oscillatory model equations by Alan [27], then it follows that the model differential equation can be written as

$$\frac{d^2[M_{y_c} + M_{y_p}]}{dt^2} + m\frac{d[M_{y_c} + M_{y_p}]}{dt} + f[M_{y_c} + M_{y_p}] = B_0 \sin\omega t \tag{7}$$

which is equivalent to:

$$\frac{d^2 M_{y_c}}{dt^2} + m\frac{dM_{y_c}}{dt} + fM_{y_c} + \frac{d^2 M_{y_p}}{dt^2} + m\frac{dM_{y_p}}{dt} + fM_{y_p} = B_0 \sin\omega t \tag{8}$$

It was observed that the first three terms of the equation (8) vanish, the solution $M_{y_c}(t)$ is called the complementary solution while the solution $M_{y_p}(t)$ is the particular integral.
Now, introducing the notation $m = 2\Psi$ and notation $f = \Omega^2$, then the characteristic equation to the homogeneous part of the model equation becomes

$$\lambda^2 + 2\Psi\lambda + \Omega^2 = 0 \tag{9}$$

with the roots obtained as equations (10) and (11)

$$\lambda_1 = -\Psi + (\Psi^2 - \Omega^2)^{\frac{1}{2}} \tag{10}$$

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$$\lambda_2 = -\Psi - (\Psi^2 - \Omega^2)^{\frac{1}{2}} \tag{11}$$

To further simplify the notation, we set $r^2 = \Psi^2 - \Omega^2$, so that

$$\lambda_1 = -\Psi + r \text{ and } \lambda_2 = -\Psi - r \tag{12}$$

The complementary solution then will correspond to the following three cases D being the discriminant.
Case I: $D > 0$, $\left(\Psi^2 \gg \Omega^2\right) \implies \Omega^2$ is negligible.
The complementary function $M_{y_c}(t)$ is non oscillatory and it obtained as

$$M_{y_c}(t) = e^{-\Psi t}\left(c_1 e^{rt} + c_2 e^{-rt}\right) \tag{13}$$

Case II: $D < 0$, $\left(\Psi^2 \ll \Omega^2\right) \implies \Psi^2$ is negligible.
Now setting $r^2 = -\tau_0^2$ the complementary function was observed to be oscillatory and was obtained as

$$M_{y_c}(t) = e^{-\Psi t}\left(c_1 \cos \tau_0 t + c_2 \sin \tau_0 t\right) \tag{14}$$

Case III: $D = 0$, $\left(\Psi^2 = \Omega^2\right)$
Here the complementary function is non oscillatory and was obtained as

$$M_{y_c}(t) = e^{-\Psi t}\left(c_1 + c_2 t\right) \tag{15}$$

Considering the last three terms of the equation (10), the form of the particular integral $M_{y_p}(t)$ can be determined since it has constant coefficients and the non-homogeneous term $B_0 \sin \omega t$. The non-homogeneous term can be obtained by differentiating the particular integral $M_{y_p}(t)$ of the form

$$M_{y_p}(t) = A \sin \omega t + B \cos \omega t \tag{16}$$

Now by substituting the above equation (16) into the non-homogeneous part of equation (**??**)

$$\implies \frac{d^2(A \sin \omega t + B \cos \omega t)}{dt^2} + 2\Psi \frac{d(A \sin \omega t + B \cos \omega t)}{dt} + \Omega^2(A \sin \omega t + B \cos \omega t) = B_0 \sin \omega t \tag{17}$$

$$-A\omega^2 \sin \omega t - B\omega^2 \cos \omega t + 2\Psi(A\omega \cos \omega t - B\omega \sin \omega t) + \Omega^2(A \sin \omega t + B \cos \omega t) = B_0 \sin \omega t \tag{18}$$

Collecting like terms;

$$[(\Omega^2 - \omega^2)A - 2\Psi\omega B] \sin \omega t + [(\Omega^2 - \omega^2)B + 2\Psi\omega A] \cos \omega t = B_0 \sin \omega t \tag{19}$$

Comparing the coefficients of $\sin \omega t$ and $\cos \omega t$

$$(\Omega^2 - \omega^2)A - 2\Psi\omega B = B_0 \tag{20}$$

$$(\Omega^2 - \omega^2)B + 2\Psi\omega A = 0 \tag{21}$$

Now solving for A and B simultaneously from the equation (20) and equation (21) gives

$$A = \frac{B_0(\Omega^2 - \omega^2)}{(\Omega^2 - \omega^2)^2 + 4\Psi^2\omega^2} \tag{22}$$

$$B = -\frac{2B_0\omega\Psi}{(\Omega^2 - \omega^2)^2 + 4\Psi^2\omega^2} \tag{23}$$

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The required particular integral is obtained through substituting for A and B in equation (16)

$$M_{y_p}(t) = \frac{B_0(\Omega^2 - \omega^2)}{(\Omega^2 - \omega^2)^2 + 4\Psi^2\omega^2} \sin\omega t - \frac{2\Psi\omega B_0}{(\Omega^2 - \omega^2)^2 + 4\Psi^2\omega^2} \cos\omega t \tag{24}$$

Equation (16) can be expressed as

$$M_{y_p}(t) = (A^2 + B^2)^{\frac{1}{2}} \left[\frac{A}{(A^2 + B^2)^{\frac{1}{2}}} \sin\omega t + \frac{B}{(A^2 + B^2)^{\frac{1}{2}}} \cos\omega t \right] \tag{25}$$

Also, defining angle ϕ such that

$$\sin\phi = \frac{A}{(A^2 + B^2)^{\frac{1}{2}}}, \text{ and } \cos\phi = \frac{B}{(A^2 + B^2)^{\frac{1}{2}}} \tag{26}$$

From the trigonometric identity, $\cos(\omega t - \phi) = \cos\omega t \cos\phi + \sin\omega t \sin\phi$, the particular integral $M_{y_p}(t)$ can be expressed in the form

$$M_{y_p}(t) = \frac{B_0}{\left[(\Omega^2 - \omega^2)^2 + 4\Psi^2\omega^2\right]^{\frac{1}{2}}} \cos(\omega t \ - \phi) \tag{27}$$

This gives the general solution to equation(3) by substituting for $M_{y_c}(t)$ and $M_{y_p}(t)$ as

$$M_y(t) = M_{y_c}(t) + \frac{B_0}{\left[(\Omega^2 - \omega^2)^2 + 4\Psi^2\omega^2\right]^{\frac{1}{2}}} \cos(\omega t \ - \phi) \tag{28}$$

The complementary solution $M_{y_c}(t)$ is one of the Cases I – III as obtained from equations (13) to (15).
Therefore,
For case I, the general solution is given as

$$M_y(t) = e^{-\Psi t}\left(c_1 e^{rt} + c_2 e^{-rt}\right) + \frac{B_0}{\left[(\Omega^2 - \omega^2)^2 + 4\Psi^2\omega^2\right]^{\frac{1}{2}}} \cos(\omega t \ - \phi) \tag{29}$$

For case II, the general solution is given as

$$M_y(t) = e^{-\Psi t}(c_1 \cos\tau_0 t + c_2 \sin\tau_0 t) + \frac{B_0}{\left[(\Omega^2 - \omega^2)^2 + 4\Psi^2\omega^2\right]^{\frac{1}{2}}} \cos(\omega t \ - \phi) \tag{30}$$

For case III, the general solution is given as

$$M_y(t) = e^{-\Psi t}(c_1 + c_2 t) + \frac{B_0}{\left[(\Omega^2 - \omega^2)^2 + 4\Psi^2\omega^2\right]^{\frac{1}{2}}} \cos(\omega t \ - \phi) \tag{31}$$

Substituting back $m = 2\Psi$ and notation $f = \Omega^2$
For Case I, the general solution is given as

$$M_y(t) = e^{-\Psi t}\left(c_1 e^{rt} + c_2 e^{-rt}\right) + \frac{B_0}{\left[(f^2 - \omega^2)^2 + m^2\omega^2\right]^{\frac{1}{2}}} \cos(\omega t \ - \phi) \tag{32}$$

For Case II, the general solution is given as

$$M_y(t) = e^{-\Psi t}(c_1 \cos\tau_0 t + c_2 \sin\tau_0 t) + \frac{B_0}{\left[(f^2 - \omega^2)^2 + m^2\omega^2\right]^{\frac{1}{2}}} \cos(\omega t \ - \phi) \tag{33}$$

For Case III, the general solution is given as

$$M_y(t) = e^{-\Psi t}(c_1 + c_2 t) + \frac{B_0}{\left[(f^2 - \omega^2)^2 + m^2\omega^2\right]^{\frac{1}{2}}} \cos(\omega t \ - \phi) \tag{34}$$

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3.0 Results and Discussion

3.1 Presentation of results for case I

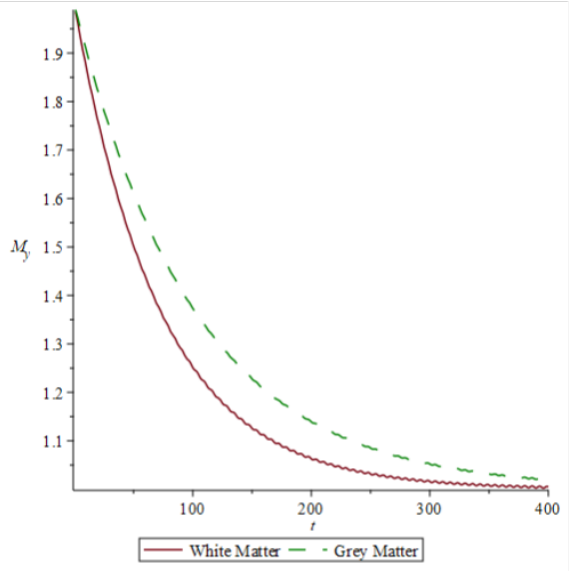

Figure 1: **Comparing White Matter and Grey Matter for case I**

For this case, the general solution was not oscillatory. The mathematical analysis of this case showed that it was overdamped since the roots of the characteristics equation were real and distinct. It was observed from the diagram that the difference between the grey matter and white matter was more pronounced. The slope of the white matter was steeper than that of the grey matter with time. The grey matter decreased but at a slower rate to the white matter.

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3.2 Presentation of results for case II

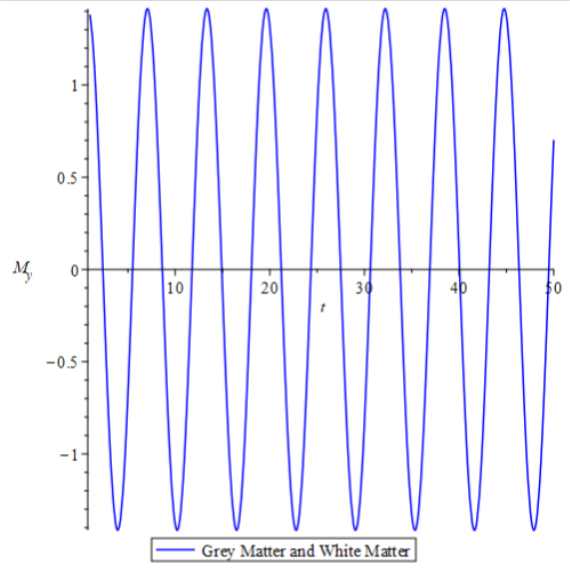

Figure 2: **Comparing White Matter and Grey Matter for case II**

The second case solution was oscillatory and it was under damped since the roots of the characteristic equation were complex. The grey matter and white matter exhibited to and fro movements. Hence, there was no clear difference between the grey matter and white matter for as they both overlap.

3.3 Presentation of results for case III

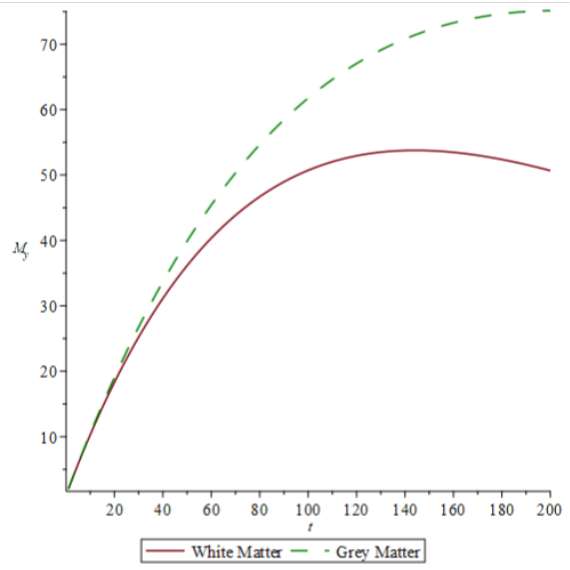

Figure 3: **Comparing White Matter and Grey Matter for case II**

This case was critically damped since the mathematical analysis showed that the characteristics roots were real and equal. The solution here was also like the Case I, which was not oscillatory. This Case represented the boundary between the over damped behaviour of Case I and the oscillatory behaviour of case II. In this case, it was observed that the graphs of the grey matter and white matter differ, the grey matter increased faster than the white matter.

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4.0 Discussion of Results

In Figure 1, T_1 and T_2 values of both the white matter and the grey matter of the brain were applied to plot the graph of magnetization against time. With this, the radio frequency force field is more than the required amount to restrict the motion of the excited protons and it explains the behaviour of the white matter and grey matter. The pattern of change is such that the rate of decrease of the white matter is faster than that of the grey matter and the implication is that as human being grows older the white and grey matter decline. As a result of this shortage, human being experiences difficulty in problem solving, planning and memory loss. This also means that older tissues have longer relaxation times and are more overdamped which explains why MRI signals from older tissues will decay more quickly and have no oscillations.

From the Figure 2, there is no noticeable difference between the white matter and grey matter as they both exhibit an oscillatory motion with the same amplitude. In this case, the radio frequency field is less than the required amount to restrict the movement of the excited protons. The white matter and grey matter behaviour explain that there are times when the magnetization (M_y) equals zero, which can be as a result of any of the following reasons: I. The patient not properly aligned with the magnetic field. This can cause the magnetization to be cancelled out. II. The patient has moved during the scan. This can also cause the transverse magnetization to be lost. III. There is a technical problem with the MRI scanner. This could be a problem with the RF pulse, the receiver or the data acquisition system.

In this situation, the transverse magnetization of zeros for both white matter and grey matter can be used to provide more hidden information about T_1 and T_2 parameters of the white matter and grey matter.

Lastly in Figure 3, it was observed that the rate of decrease of the white matter is faster than that of the grey matter. This implies that at early childhood, there is a rapid increase in grey matter volume and slow increase in white matter volume. This rapid increase in the grey matter can lead to improved motor skills such as walking, running and playing sports. The slow increase in the white matter can lead to improved skills such as problem-solving, reasoning, and learning. Here the radio frequency field is exactly the required amount to bring the excited protons back to equilibrium.

5.0 Conclusion

In this paper, it has been shown through analytical method that T_1 and T_2 relaxation rates can be used to distinguish the passage of time (aging) on human brain by interpreting the behaviours of white and grey matter. The transverse magnetization of white and grey matter was used to explain the difference in the way the water molecules in these two types of brain tissue behave when they are exposed to a magnetic field. NMR Bloch equations which were used revealed results that were in agreement with those obtained by other scientists who have researched on the pattern of changes of the white matter and grey matter in the human brain by collating statistical data.

Case I and Case III agreed with previous studies that white matter and grey matter develop throughout childhood and adolescence then reach peak values at early adulthood before they begin to decline at different rates. The pattern of change was such that the rate of decrease of the white matter was faster than that of the grey matter and was more noticeable.

Also, case II showed that the white matter and grey matter volume coincided at certain periods during brain development. At this, both white matter and grey matter were oscillatory.

On the whole, MRI has proven to be a very useful tool for studying developments in the human brain.

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References

[1] D. B. Jimmy and H. G. Parkes. 14 - nmr studies of body fluids and tissue extracts. NMR in Physiology and Biomedicine, 1994. doi: https://doi.org/10.1016/B978-0-12-283980-1.50019-3. URL https://www.sciencedirect.com/science/article/pii/B9780122839801500193.

[2] F. Ponti, A. Santoro, D. Mercatelli, C. Gasperini, M. Conte, M. Martucci, L. Sangiorgi, C. Franceschi, and A. Bazzocchi. Aging and imaging assessment of body composition: From fat to facts. Frontiers in Endocrinology, 10(861), 2020. doi: https://doi.org/10.3389/fendo.2019.00861.

[3] E. E. Elijah and U. F. Muhammad. A novel approach for normal parameter reduction algorithm of soft set using unit similarity matrix. International Journal of Mathematical Sciences and Op timization: Theory and Applications, 9(1):43 – 58, Jul. 2023. doi: 10.6084/zenodo.8218007. URL http://ijmso.unilag.edu.ng/article/view/1809.

[4] W. Jo, W. Phillip, H. H. Ian, N. Dinuli, W. Zimu, L. W. Robyn, and R. Joanne. Fac tors associated with brain ageing – a systematic review. BMC Neurology, 21(1), 2021. doi: https://doi.org/10.1186/s12883-021-02331-4.

[5] Medlineplus. Aging changes in organs, tissue and cells. 2022. URL https://medlineplus.gov/ency/article/004012.htm.

[6] L. S. Kirsty, D. B. Ratan, A. B. Bruce, D. Henrik, and F. Jonas. Retrospective birth dating of cells in humans. Cell, 122(Issue 1), 2005. ISSN 0092-8674. doi: https://doi.org/10.1016/j.cell.2005.04.028.

[7] P. Ann. The organs in our body may age differently. 2015. URL Retrieved from https://www.healthline.coom/health-news/the-organs-in-0ur=body-may-age-differently 091715Why-We-Need-to-Study-Aging.

[8] B. Sharon. People body systems age at different rates. 2020. URL Retrieved from Scientific American: https://www.scientificamerican.com/article/peoples-body-systems-age-at different-rates/.

[9] Sunday A. Osagie and Joseph E. Osemwenkhae. Lomax-weibull distribution with prop erties and applications in lifetime analysis. International Journal of Mathematical Sci ences and Optimization: Theory and Applications, 2020(1):718 – 732, Jul. 2020. URL http://ijmso.unilag.edu.ng/article/view/969.

[10] N. L. Maitre, J. C. Slaughter, A. R. Stark, J. L. Aschner, and A. W. Anderson. Validation of a brain mri relaxometry protocol to measure effects of preterm birth at a flexible postnatal age. BMC Pediatr, 14(84), 2014. doi: https://doi.org/10.1186/1471-2431-14-84.

[11] S. Siemonsen, J. Finsterbusch, J. Matschke, A. Lorenzen, X. Q. Ding, and J. Fiehler. Age-dependent normal values of t2* and t2' in brain parenchyma. AJNR. American Journal of Neuroradiology, 29 (5), 2008. doi: https://doi.org/10.3174/ajnr.A0951.

[12] S. Suzuki, O. Sakai, and H. Jara. Combined volumetric t1, t2 and secular-t2 quantitative mri of the brain: age-related global changes (preliminary results). Magnetic Resonanc Imaging, 24(7), 2006. doi: https://doi.org/10.1016/j.mri.2006.04.011.

[13] D. Gwena¨elle, R. G. Adrian, K. T. Christian, T. W. Lars, P. Eugene, E. Andreas, B. W. Kris tine, J. Anthony, G. Achim, U. M. Andreas, M. M. Paul, M. F. Anders, M. S. Stephen, and J. Heidi. A common brain network links development, aging, and vulnerability to disease. Proceedings of the National Academy of Sciences of the United States of America, 111(49), 2014. doi: https://doi.org/10.1073/pnas.1410378111.

[14] S. Cho, D. Jones, W. E. Reddick, R. J. Ogg, and R. G. Steen. Establishing norms for age-related changes in proton t1 of human brain tissue in vivo. Magn Reson Imaging, 15(10):1133–43, 1997. doi: 10.1016/s0730-725x(97)00202-6. PMID: 9408134.

[15] P. Rock, U. Bashir, and A. Murphy. T2 relaxation times. reference article, radiopaedia.org (ac cessed on 08 jul 2023). 2021. doi: https://doi.org/10.53347/rID-16494. URL Retrieved from https://radiopaedia.org/articles/16494.

[16] S.I. Yusuf, Y.M. Aiyesimi, and O.B. Awojoyogbe. An analytical investigation of bloch nuclear magnetic resonance flow equation for the analysis of general fluid flows. Nigeria Journal of Mathematics and Applications, 20, 2010.

[17] C. Badve, A. Yu, M. Rogers, D. Ma, Y. Liu, M. Schluchter, J. Sunshine, M. Griswold, and V. Gulani. Simultaneous t1 and t2 brain relaxometry in asymptomatic volunteers using magnetic resonance fingerprinting. Tomography (Ann Arbor, Mich.), 1(1), 2015. doi: https://doi.org/10.18383/j.tom.2015.00166.

[18] H. M. Anna, N. E. van Haren, H. G. Schnack, R. S. Kahn, and H. E. Hulshoff Pol. Human brain changes across the life span: A review of 56 longitudinal magnetic resonance imaging studies. Human Brain Mapping, 33(8), 2012. doi: https://doi.org/10.1002/hbm.21334.

[19] P. W. Janaka, K. H. Scott, R. Scott, and S. B. William. Nmr relaxation times in the human brain at 3.0 tesla. J. Magn. Reson. Imaging, 9(4), 1999. doi: https://doi.org/10.1002/(SICI)1522 2586(199904)9:4¡531::AID JMRI4¿3.0.CO;2-L.

[20] G. Yulin, R. I. Grossman, J. S. Babb, M. L. Rabin, L. J. Mannon, and D. L. Kolson. Age-related total gray matter and white matter changes in normal adult brain. part ii: quantitative magnetization transfer ratio histogram analysis. AJNR. American Journal of Neuroradiology, 23(8), 2002.

[21] V. Josef, R. Andrea, A. B. Rodney, C. Margherita, M. Claudio, L. Marco, and P. Gi anni. T1 and t2 in the brain of healthy subjects, patients with parkinson disease, and pa tients with multiple system atrophy: relation to iron content. Radiology, 211(2), 1999. doi: https://doi.org/10.1148/radiology.211.2.r99ma53489.

[22] G. Antonio, L. Santelli, V. Tomassini, R. Bosnell, S. Smith, N. De Stefano, and H. Johansen-Berg. Age-related changes in grey and white matter structure throughout adulthood. NeuroImage, 51(3), 2010. doi: https://doi.org/10.1016/j.neuroimage.2010.03.004.

[23] I. Agartz, J. S¨a¨af, L. O. Wahlund, and L. Wetterberg. T1 and t2 relaxation time estimates in the normal human brain. Radiology, 181(2), 1991. doi: https://doi.org/10.1148/radiology.181.2.1924801.

[24] S. R. Grant, S. A. Gronemeyer, and J. S. Taylor. Age-related changes in proton t1 values of normal human brain. Journal of Magnetic Resonance Imaging: JMRI, 5(1), 1995. doi: https://doi.org/10.1002/jmri.1880050111.

[25] S. R. Grant, R. J. Ogg, W. E. Reddick, and P. B. Kingsley. Age-related changes in the pediatric brain: quantitative mr evidence of maturational changes during adolescence. AJNR. American Journal of Neuroradiology, 18(5), 1997.

[26] O.B. Awojoyogbe. Analytical solution of the time dependent bloch nmr flow equations. a translational mechanical analysis. Physica, A(339), 2004.

[27] J. Alan. Advanced engineering mathematics. section 6.2: Oscillatory solutions harcourt/ academic press, a harcourt science and technology company; 7th edition. pages 285-287. 2002.