**STUDY OF THE BEHAVIOUR OF FLUID PARTICLES IN SPHERICAL REGION USING DIFFUSION MAGNETIC RESONANCE IMAGING EQUATION**

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**ABSTRACT**

*In the petroleum industry, Nuclear Magnetic Resonance (NMR) technology has been applied in several ways to provide vital information about petro-physical properties of reservoirs. However, there is need to study the molecular behaviours of particles that form the fluids in a reservoir. In this research work, diffusion magnetic resonance equation has been applied* *in spherical coordinates and solved analytically using the method of separation of variables and solution of Legendre equation by Frobenius method. The fluids considered in this research work are glycerine, kerosene and water. From the results obtained, it can be deduced that Magnetization of each fluid decreases as radial adjustment decreases.* *Also, the decrease in radial adjustment enhanced the vibration of each fluid but later declined at some points. The relaxation points of kerosene and water are at the same radial adjustment with the Magnetization values (1.33  and 2.3) but that of glycerine differs with the Magnetization values of (2.9), On the whole, glycerine relaxed faster than kerosene and water.*

**Keywords:** Spherical Coordinate, Relaxation Times, Diffusion Coefficient

**1.0 INTRODUCTION**

The application of nuclear magnetic resonance measurement in petroleum industry gives better understanding of the interaction between fluids in the reservoirs and rock properties and one of the best tools for quantifying fluid properties, reservoir properties as well as determining reservoir productivity (Olaide *et al.,* 2020). Due to its non-invasiveness, nuclear magnetic resonance techniques have proved to be a powerful and reliable tool in studying flow in restricted geometries. It is particularly useful for studying diffusion because it can provide self-diffusion coefficient accurately for the individual components or multi-components systems in a few minutes of time, where traditional radioactive tracer techniques can take a week for each component and require isotropic substitution (Awojoyogbe *et al.,* 2011). Dada *et al.* (2010) applied analytical technique in form of a plane wave to transform the time dependent Bloch NMR flow equation to diffusion advection equation for the qualitative analysis of nuclear magnetization. They consider the perfusion of any specific blood component as one dimensional since blood flow within the vessels is directional. Yusuf *et al.* (2010) used analytical solution of magnetic resonance imaging (MRI) equation to study the general behaviour of fluid flow in human living tissues. Yusuf *et al.* (2019) used magnetic resonance imaging to image the materials that are causing obstructions of fluid in a cylindrical pipe. Mallin *et al.* (2011) used NMR pulse to determine the properties of glycerine and mineral oil. The properties in this experiment are spin lattice relaxation time () and spin-spin relaxation time (). Therefore, this research work is aimed at studying the behaviour of fluid particles of glycerine, kerosene and water in spherical region using diffusion magnetic resonance equation.

**2.0 Mathematical Formulation**

The dynamics of the magnetization of fluid flow using relaxation rate is described by the Bloch equations given as:

 (2.1)

 (2.2)

 (2.3)

Thus, to study the diffusion process of magnetization in a fluid flow at a uniform velocity v, which is constant in time, we need to consider the advection equation that describes such process which is given as:

 (2.4)

 (2.5)

Now, assuming, then this implies. Then equation (2.1), (2.2) and (2.3) becomes:

 (2.6)

 (2.7)

 (2.8)

where Component of transverse magnetization along -axis

Component of transverse magnetization along y-axis

Component of magnetization along the z-axis

Equilibrium magnetization

Gyro-magnetic ratio of fluid spins

= Radio-frequency (RF) magnetic field

Longitudinal or spin lattice relaxation time

Transverse or spin-spin relaxation time

The fluid velocity

t = time

Now, making the subject in equation (2.7) and substituting into equation (2.8) we have:

 (2.9)

where , and 

Thus, equation (2.9) is the general equation for fluid flow in magnetic resonance imaging (Awojoyogbe, 2004).

Then, from equation (2.9), if provided that:

 (2.10)

Hence, equation (2.9) is reduced to:

 (2.11)

Hence, the parameter D is called diffusion coefficient that is accurately defined in terms of MRI fluid flow which is an intrinsic part of the Bloch nuclear magnetic resonance equation and the function is called forcing function. If the forcing function is non-zero, then we have forced vibrating system otherwise the system is undergoing freely vibration (Awojoyogbe *et al.*2011).

Thus, equation (2.11) can be expressed in three-dimensions as:

 (2.12)

Now, transforming equation (2.12) to the spherical coordinates system defined as we have:

 (2.13)

Now, assuming that, this implies, so, equation (2.13) is reduced to:

 (2.14)

Let the general solution of equation (2.14) be of the form:

 (2.15)

Then, from equation (2.14) we first consider the following:

 (2.16)

Now, applying the method of separation of variable to equation (2.16), we have the following:

 (2.17)

 (2.18)

Then, solving equation (2.17) and (2.18) and substituting the results into equation (2.15) we have

 (2.19)

where the functions  and  are the Legendre functions of the first and second kinds and the function is the radio-frequency field applied to perturb the molecules of the fluid. Now, since must be bounded at  and  either we must choose  in equation (2.19), then we obtained the bounded solution as:

 (2.20)

Then, the boundary conditions to be imposed are:

 (2.21)

Now, simplifying equation (2.20) further and applying the boundary condition we have the final solution of the magnetization  as:

 (2.22)

**3.0 Results and Discussion**

From the solution obtained, the graphs of the fluids under consideration namely glycerine, kerosene and water were plotted using Maple 17 software. The plotting was done in 3-dimensions with the Magnetization (), plotted against angle of inclination (θ) and radial adjustment (r). Other peculiar quantities like the relaxation times and coefficient of diffusion of each of the fluid considered were used for the plotting.

**3.1 Graphs of Glycerine with Radial Adjustments**

The graphical illustration presented in figure (3.1 to 3.6) show the effects of radial adjustments on the magnetization and glycerine.

|  |  |
| --- | --- |
| Figure 3.1 plot of  against (θ = 0..2π) and  (r = 0..35) | Figure 3.2 plot of  against (θ = 0..2π) and  Radial adjustment (r = 0..30) |
| Figure 3.3 plot of  against (θ = 0..2π) and  Radial adjustment (r = 0..25) | Figure 3.4 plot of  against (θ = 0..2π) and  Radial adjustment (r = 0..20) |
| Figure 3.5 plot of  against (θ = 0..2π) and  Radial adjustment (r = 0..15) | Figure 3.6 plot of  against (θ = 0..2π) and  Radial adjustment (r = 0..10) |

In figure 3.1 to 3.6, it was observed that the decrease in radial adjustment results to decrease in magnetization and vibration increases and later declined at figure 3.5 and Figure 3.6 shows that the fluid (glycerine) is completely at rest, which is the relaxation point because at that figure the surface of graph are completely flat at all angles and vibration vanishes.

**3.2 Graphs of Kerosene with Radial Adjustments**

Figure 3.7 to 3.12 shows the effect of radial adjustments on magnetization and kerosene.

|  |  |
| --- | --- |
| Figure 3.7 plot of  against (θ = 0..2π) and  (r = 0..15) | Figure 3.8 plot of  against (θ = 0..2π) and  Radial adjustment (r = 0..10) |
| Figure 3.9 plot of  against (θ = 0..2π) and  Radial adjustment (r = 0..5) | Figure 3.10 plot of  against (θ = 0..2π) and Radial adjustment (r = 0..4) |
| Figure 3.11 plot of  against (θ = 0..2π) and Radial adjustment (r = 0..3) | Figure 3.12 plot of  against (θ = 0..2π)  and Radial adjustment (r = 0..2) |

It was observed from figure 3.7 to 3.12 that the decrease in radial adjustment results to the decrease in magnetization and increases vibration and later declined at figure 3.11 which leads to the relaxation point of kerosene at figure 3.12. It is also noted that the pattern of radial adjustment from figure 3.7 to 3.9 is changed at figure 3.10 to 3.12 because the fluid under consideration (kerosene) changes his behaviours immediately after figure 3.9.

**3.2 Graphs of Water with Radial Adjustments**

Figure 3.13 to 3.18 shows the effect of radial adjustments on magnetization and water.

|  |  |
| --- | --- |
| Figure 3.13 plot of  against (θ = 0..2π)  and (r = 0..15) | Figure 3.14 plot of  against (θ = 0..2π) and Radial adjustment (r = 0..10) |
| Figure 3.15 plot of  against (θ = 0..2π) and Radial adjustment (r = 0..5) | Figure 3.16 plot of  against (θ = 0..2π) and Radial adjustment (r = 0..4) |
| Figure 3.17 plot of  against (θ = 0..2π) and Radial adjustment (r = 0..3) | Figure 3.18 plot of  against (θ = 0..2π)  and Radial adjustment (r = 0..2) |

Now, figure 3.13 to 3.18 shows that the decrease in magnetization is as results of decreasing in radial adjustment and vibration increases and later decline at figure 3.17 which lead to the relaxation point of water at figure 3.18 when radial adjustment continue reducing. It is also observed that the arrangement of radial adjustment of water follows the same pattern as that of kerosene.

**4.0 Conclusion**

The fluids considered in this research work are glycerine, kerosene and water. The effects of radial adjustments for each of the fluids considered are shown on the graphs analyzed. From the results obtained, it can be concluded that Magnetization of each fluid decreases as radial adjustment decreases. Also, the decrease in radial adjustment enhanced the vibration of each fluid but later declined at some points. The relaxation points of kerosene and water are at the same radial adjustment but that of glycerine differs. On the whole, glycerine relaxed faster than kerosene and water.

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