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Mathematical Analysis of Draw-Down Level and Its Effect On Crop Yield

[1]M. D. Shehu, [2] O. P. Evans, [3]H. O. Ahmed and [4]I. A. Adeniyi

Abstract

In this paper, we formulated mathematical equations for groundwater flow considering the Draw-down levels in an unconfined Aquifer system, using the parameters; well pumping rate, hydraulic conductivity, transitivity, radius of influence. Different draw-down levels for different wells were determined in an unconfined aquifer system. Relating the draw-down level to crop production (Maize yield) shows that the draw-down has a polynomial relationship with maize yield which can be used to predict optimum maize yield within an unconfined aquifer system.

1. Introduction

Groundwater is a major source of water domestic needs and irrigation purposes. Ground water is the cheapest and simplest source of water supply because it requires little or no treatment (Thangarajan, 2007) the high demand for water especially for irrigation can lead to over exploitation, the continuous decline in the water level is as a result of the increased groundwater discharge which has led to the deterioration of water quality and widespread of dried wells (Egharevba, 2015), consider groundwater as one of the major water sources needed to supply moisture for crop growth, that is, Groundwater can be used for irrigation and

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[1,2,3]Department of Mathematics and Statistics, Federal University of Technology, Niger State, Nigeria
[4]Department of Statistics and Mathematical Sciences, Kwara State University Molete, Nigeria

can be a natural source of water in arid region. He further stated that, if it is not in the required proportion it results to faulty design of irrigation. To effectively manage groundwater we must have a good knowledge of the aquifer system, identify the practical ways to control the water level, if not well managed it affects the agricultural activities of an area (Adeboye and Shehu, 2015)

Adeboye and Shehu, 2015, formulated a groundwater flow equation using Darcys law, and stressed that adequate attention must be given to the draw-dawn level as it has significant effect on the Agricultural productivity within an aquifer system.

In this work we extended the work of Adeboye and Shehu, 2015 by formulating a mathematical model to access the effect of draw down level on crop yield within an aquifer region.

2. Model Formulation

We define a total draw down, D_w, of each well within an aquifer system as:

$$D = D_{aquifer} + D_{well\ loss} \tag{1}$$

$$D = \alpha Q + \beta Q^2 \tag{2}$$

where,
D is total draw down.
Q is well pumping rate.
αQ is formation.
βQ^2 is the well loss.
β is constant based on the well condition according to Asawa, (2008).

$$\alpha = \frac{In(\frac{R}{r})}{2\pi T}$$

where R is radius of influence and r is radius of well.
The total drawdown is given as:

$$D = \frac{Q_1}{2\pi T} In(\frac{R}{r}) + \beta Q_i^2$$

where,
r is radius of well and β is constant based on the well condition.
To obtain the theoretical drawdown of wells, the pumping rates for unconfined aquifer was calculated as follows:
The pumping rate of an unconfined aquifer is given as

$$Q = \frac{\pi K(2H - D_u)D_u}{In(\frac{R}{r})}$$

To attain maximum yield, the drawdown, Du, will be equal to the total thickness, H, of the unconfined aquifer.
That is, $D_u = H$

$$\implies Q = \frac{\pi K(2D_u - D_u)D_u}{In(\frac{R}{r})}$$

$$Q = \frac{\pi K(D_u)D_u}{In(\frac{R}{r})}$$

$$Q = \frac{\pi K D_u^2}{In(\frac{R}{r})} \tag{3}$$

The theoretical drawdown of an unconfined aquifer can be obtained from (8)

$$D_u = \left(\frac{QIn(\frac{R}{r})}{\pi K}\right)^{0.5}$$

where, D_u is theoretical drawdown for a unconfined aquifer.
Q is pumping rate.
K is hydraulic conductivity of the aquifer medium.
The draw-down model describe above was used to obtain twenty draw-down levels with corresponding crop yields and water-use efficiency. To investigate the effect of draw-down levels and water-use efficiency on crop yield, we shall use polynomial regression models of order 2. The general form of the saturated regression model is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_2 + \beta_4 x_2^2 + \epsilon \tag{4}$$

Where y is the crop yield, x_1 and x_2 are the draw-down level and water-use efficiency respectively, $\beta_0, \beta_1, \beta_2, \beta_3$ and β_4 are regression coefficients; and ? is the error term.
We use the method of least square and ridge regression to estimate the model coefficients. The model was assessed using various tools such as coefficient of determination, t test and F tests. The t statistic tests whether or not the estimated coefficient is significantly different from 0 while the F-test is used to test how suitable is the model obtained. P values were used to make determine significance or non-significance at level of significance of 0.05.

3. Results and Discussions

Table 1: Draw-down level, crop yield and water-use efficiency data

Draw-down level	Yeild	Water use efficiency
5	0	0
10	0	0
15	0	0
20	0.0001	0.0005
25	0.0005	0.0015
30	0.1217	0.00125
35	0.4127	16.3825
40	0.6587	34.32
45	0.8597	52.2575
50	1.0157	70.195
55	1.1267	88.1325
60	1.1927	106.07
65	1.2137	124.0075
70	1.1897	141.945
75	1.1207	159.8825
80	1.0067	177.82
85	0.8477	195.7575
90	0.6437	213.695
95	0.3947	231.6325
100	0.1007	249.57

The data used in this work are presented in Table 1. The data consist of twenty values of draw-down level (DDL), water-use efficiency (WUE) and crop yield (CY).

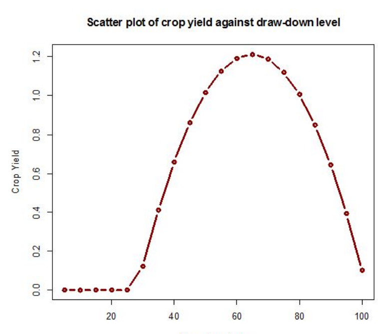

Figure 1: A scatter plot showing the relationship between the draw down values and the crop yield

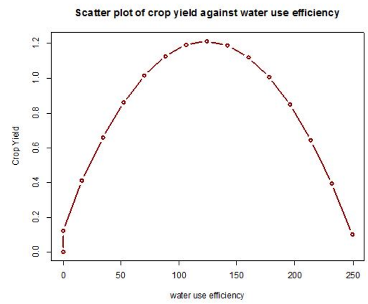

Figure 2: A scatter plot showing the relationship between the water-use efficiency and the crop yield

Fig. 1 and **Fig. 2** show that the relationship between crop yield and draw-down level or water-use efficiency is quadratic. This will lead us to fit a quadratic regression model to the data. The correlation between draw-down level and water-use efficiency given by **Table 2** is very high which suggests presence of multicollinearity problem.

Table 2:Correlation among the variables

	DD	CY	WUF
DD	1	0.539311	0.976068
CY		1	0.445312
WUF			1

Table 3:Results of the fitted regression model given by equation (2)

Coefficient	Estimate	Std. Error	t value	P value	Status
β_0	0.07491	0.02692	2.782	0.01395	Significant
β_1	-0.01440	0.00339	-4.243	0.00071	Significant
β_2	0.00053	0.00009	5.900	0.00003	Significant
β_3	0.01257	0.00075	16.821	< 0.00001	Significant
β_4	-0.00011	0.000007	-16.553	< 0.00001	Significant

Table 3 gives the results obtained by fitting the full model given by equation (2). The last column shows which coefficient is/is not significant based on the p values. The results indicate that all terms in the model are significant. Further examination of the model using the F test is presented in **Table 4**. The results from **Table 4** indicate that all the terms in the model contribute significantly to the fit of the model. A backward model selection was applied to the model and no term in the model was dropped.

Table 4:ANOVA table of the fitted regression model given by equation (2)

Source of Variation	Degrees of freedom	Sum Squares	Mean Squares	F value	P value	Status
DD	1	1.29452	1.29452	5357.306	< 0.00001	significant
DD^2	1	2.14671	2.14671	8884.052	< 0.00001	Significant
WUE	1	0.93965	0.93965	3888.691	< 0.00001	Significant
WUE ^{2}	1	0.06621	0.06621	274.0022	< 0.00001	Significant
Residuals	15	0.00363	0.00024			Significant
Model	4	4.44709	1.11177	4601	< 0.00001	
		Adjusted	R-squared:	0.9999		

The F-test for the adequacy of the model and the adjusted R-squared value indicate that the model is adequate. However, the high correlation among the predictors calls for further diagnosis of the model. **Table 5** gives the variance inflation factors of the predictors in the model.

Table 5:VIFs of the predictor terms in the model

Predictor	DD	DD^2	WUE	WUE^2
VIF	792.2204	6518.4397	340.6369	1461.0570

All the predictors have VIF values much greater than 10 which is a signal for serious multicollinearity (Rawlings et al; 1998). The presence of multicollieարity can have serious effects on the estimates of the regression coefficients and on the general applicability of the estimated model (Montgomery and Runger, 2003). Therefore, we proceed to solve the problem using ridge regression (Hoerl and Kennard, 1970; Hoerl, et. al., 1975).

Table 6:Ridge regression estimates of the model

Coefficient	β_0	β_1	β_2	β_3	β_4
Estimate	0.07138	-0.01390	0.00051	0.01266	-0.00011

The results obtained by fitting the model using the ridge regression technique are presented by **Table 6**. The ridge estimates estimates are not so different from that of OLS estimates. The model for the crop yield based on the ridge estimates is therefore,

$$\tilde{y} = 0.07138 - 0.01390x_1 + 0.00051x_1^2 + 0.01266x_2 - 0.00011x_2^2 \tag{5}$$

Figure 3 shows that both models fit the data well but one may use either model for prediction but the ridge estimates are more stable and reliable for evaluating marginal effects of the predictors.

The residuals of the model based on both OLS and Ridge estimates do not violate the normality assumption as shown by the Normal Probability (quantile-quantile)plots of the residuals given by **Figures 4 and 5**. A Kolmogorov-Smirnov normality test on the residuals of the model based on based on both OLS and Ridge gave P values of 0.05827 and 0.06255 respectively.

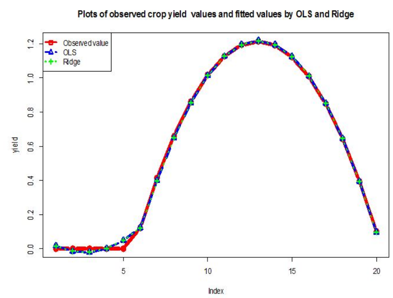

Figure 3: Fitted crop yield by the model using OLS and Ridge estimates superimposed on the actual

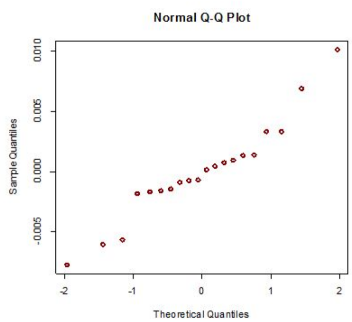

Figure 4: Qauntile-quantile plot of the residuals of the model using OLS

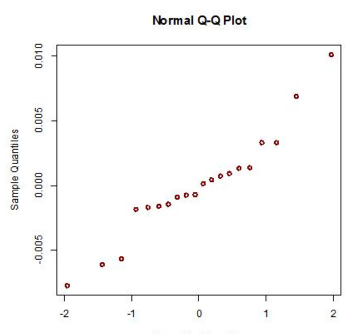

Figure 5: Qauntile-quantile plot of the residuals of the model using Ridge estimates

4. Conclusion

In conclusion, it was observed that draw-down level of an aquifer system has a significant effect on crop yield and this is in line with Adeboye and Shehu (2015). The results showed that crop yield has a negative and positive linear relationship with draw-down level and water use efficiency respectively. Contrastively, crop yield has a positive and negative quadratic relationship with draw-down level and water use efficiency respectively. The optimum maize yield is at the draw down level 65cm and the water use efficiency increase as the draw down level increases.The mathematical model for the crop yield using draw-down level and water-use efficiency as predictors is

$$\tilde{y}? = 0.07138 - 0.01390x_1 + 0.00051x_1^2 + 0.01266x_2 - 0.00011x_2^2.$$

It is generally advisable that a particular care be given to over withdrawer of groundwater in area where agricultural activities is taking place to reduce the effect on crop yield.

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Department of Mathematics and Statistics ,
Federal University of Technology,
Niger State, Nigeria.
m.shehu@futminna.edu.ng, patevansjj@gmail.com