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SVG513: Adjustment Computations II

Examinations in First Semester of 2019/2020 Academic Year

Instructions

1. Answer any **three (3)** questions
2. Time allowed for the examination is **2.5 hours**

Question 1

a) For the linear model in equation (1), let the weight matrix of observations be W. Describe the properties of the equation

$$A^T WAx = A^T Wy \qquad (2)$$

and indicate how the best linear unbiased estimate (blue) as well as the biased estimate of x may be obtained.

b) Obtain the best linear unbiased estimate and a biased estimate of the parameters a, b, c in the parabolic equation

$$y(x) = ax^2 + bx + c$$

given the data

x	y
1	1±0.005
2	5±0.004
3	8±0.003
4	17±0.002
5	26±0.001

Note: For the biased solution of this problem, use the generalized solution

$$(A^T WA + 0.05I)x = A^T Wy$$

instead of equation (2).

Question 2

a) Compare the solution of the equations

$$\left.\begin{array}{rcl} 0.001 + y & = & 2 \\ x + y & = & 1 \end{array}\right\} \quad (3)$$

with the solutions of the equations

$$\left.\begin{array}{rcl} 0.001 + 0.997y & = & 2 \\ 0.998x + y & = & 1 \end{array}\right\} \quad (4)$$

and comment on the reasons for the similarity or difference between the solutions of the equations.

b) Discuss how the usual row reduction algorithm applied to equations (3) can lead to an ill-conditioned system of equations.

c) Illustrate how the possibility of ill-conditioning mentioned in question 3(b) can be avoided by the method of partial pivoting.

Question 3

a) Discuss the method and usefulness of equilibration (also called scaling) as applied to reconditioning of linear systems of equations. Use the following system of equations to illustrate your points:

$$(A|y) = \left(\begin{array}{ccc|c} 1 & -1 & 1 & 3 \\ 1000 & 1000 & 1000 & 2 \\ 0.9998 & 0.9998 & -0.9997 & 1 \end{array}\right)$$

b) How large does the condition number of a linear system of equations have to be before the system is considered ill-conditioned? Illustrate your points with a numerical example.

c) How many significant digits can you trust in the solution of the following system of equations?

$$\begin{pmatrix} 3 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$$

Question 4

In the following multiple regression model

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \cdots + \beta_m x_{mi} + e_i, \quad (5)$$

where $y_i, i = 1, \ldots, n$, are the dependent variables; x_{ki} is the i^{th} observation on the k^{th} independent/explanatory variable x_k; $(\beta_k)_{k=1}^{m}$ is the vector of unknown parameters; and e_i is the error that is not directly observed in the data.

a) Under what conditions does a least squares solution of the model (5) exist?
b) Illustrate the notion of multicollinearity for the multiple regression model (5).
c) Illustrate at least four methods for detecting multicollinearity in the regression model (5).
d) Illustrate at least four methods for remediating multicollinearity in the regression model (5).

Note: For questions (c) and (d) you do not have to actually provide numerical examples.

Question 5

a) Let A be an $n \times n$ regular matrix such that all upper-left submatrices are regular (i.e., their determinants are nonzero). Show that there exist a unit lower triangular matrix L and an upper triangular matrix U such that $A = LU$ and that this LU-factorization is unique.

b) Find the LU-factorization of the matrix

$$A = \begin{pmatrix} 2 & 1 & -2 \\ 2 & 3 & -1 \\ 2 & 5 & 1 \end{pmatrix}$$

and use it to solve the linear system

$$(A|y) = \left(\begin{array}{ccc|c} 2 & 1 & -2 & 3 \\ 2 & 3 & -1 & 2 \\ 2 & 5 & 1 & 1 \end{array}\right)$$