

**EFFECT OF COGNITIVE CONFLICT AND COLLABORATIVE  
INSTRUCTIONAL STRATEGIES ON GEOMETRY ACHIEVEMENT AND  
RETENTION AMONG SECONDARY SCHOOL STUDENTS IN MINNA,  
NIGER STATE**

**BY**

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MTech/SSTE/2018/8493**

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FEDERAL UNIVERSITY OF TECHNOLOGY  
MINNA, NIGER STATE**

**AUGUST, 2023**

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**A THESES SUBMITTED TO THE POSTGRADUATE SCHOOL FEDERAL  
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FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF THE  
DEGREE OF MASTER OF TECHNOLOGY (MTech) IN SCIENCE  
EDUCATION**

**AUGUST, 2023**

**DECLARATION**

I hereby declare that this thesis titled “**Effect of Cognitive Conflict and Collaborative Instructional Strategies on Geometry Achievement and Retention among Secondary School Students in Minna, Niger State**”, is a collection of my original work. It has not been presented for any other qualification anywhere. However, information from the works of other scholars (published or unpublished) and their contributions here have been duly acknowledged.

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MTech/SSTE/2018/8394

FEDERAL UNIVERSITY OF TECHNOLOGY  
MINNA, NIGERIA

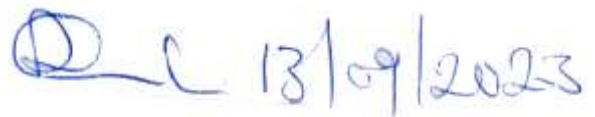
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**CERTIFICATION**

This theses titled: “**Effect of Cognitive Conflict and Collaborative Instructional Strategies on Geometry Achievement and Retention among Secondary School Students in Minna, Niger State**”, by MUSA, Yunusa Pyaske (MTech/SSTE/2018/8394) meets the regulations governing the award of the degree of Master of Technology in Science Education of the Federal University of Technology, Minna and it is approved for its contribution to scientific knowledge and literary presentation.

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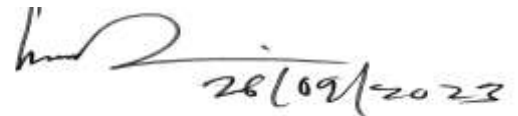
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## DEDICATION

This research is dedicated to my beloved parents for their guidance, prayers and inspiration throughout the period of this study.

## **ACKNOWLEDGEMENTS**

All praises are to Almighty Allah, Lord of the world, for His divine mercies and blessings, which have guided the researcher through this programme in this citadel of learning.

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Above all, the researcher wishes to express his sincere thanks to his late father (lifelong mentor), Alhaji Usman Turaki Gwada, for investing in him throughout his academic and moral ventures in life. The researcher remained grateful for the thoughts and persistent as well as fervent prayers from his immediate family, the Pyaskes', mother in person of Hajiya Salamatu Usman Turaki, wife, Mallama Khadija Ahmadu, children (Muhammad Yunusa Pyaske and Khadijat Yunusa Pyaske), sisters, brothers and friends.

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#### **ABSTRACT**

The study determined the Effect of Cognitive Conflict and Collaborative Instructional Strategies on Student's Achievement and Retention in Geometry among Secondary School Students in Minna, Niger State. The study employed quasi-experimental research design (pretest and posttest experimental-Control groups). Six (6) research questions were raised and six (6) research hypotheses were formulated and tested at 0.05 significant level. The population of the study comprised 8,251 mathematics students (2020/2021)

academic session in senior secondary schools in Minna, Niger State. Three (3) Senior Secondary Schools were randomly selected for the study. The researcher used an intact classes of 226 students (123 male and 103 female). The students were assigned into groups, experimental 1 and 2 groups and a control group. The experimental 1 and 2 groups were taught using cognitive conflict and collaborative instructional strategies respectively while the control group was taught using lecture method. Geometry Achievement Test (GAT) was used as research instrument for collecting data for the study. The instrument was validated by three experts and the reliability of the instrument was determined through test and retest method, the data collected were analyzed using Pearson's Product Moment Correlation Coefficient and reliability coefficient of 0.72 was obtained. After teaching, an achievement test (posttest) was administered on the three (3) groups and retention test was given after the period of two weeks. The data collected were analyzed using descriptive statistics of mean and standard deviation, analysis of variance (ANOVA) and independent sampled t-test. The findings revealed that both the cognitive conflict and collaborative strategies have significant impact on Mean achievement and retention of student in geometry. The study recommend that: Geometry is about identification, description of shapes and there properties, so teachers' should teach most of the concepts through cognitive conflict and collaborative strategies to improve students' achievement and retention in geometry.

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## CHAPTER ONE

### 1.0

### INTRODUCTION

#### 1.1 Background to the Study

The school mathematics resulted from the confluence of two traditions. The first rooted in Babylonia astronomy, Egyptian earth measurement and ancient commerce, is mathematics as a reckoning, as a tool required for everyday life. The second tradition is rooted in Greek geometry and medieval algebra, is mathematics as reasoning, as one of liberal art whose mastery marks an educated person. In this tradition, mathematics offers aesthetics satisfaction as well as a means of developing the mind capacity for abstract thought. Every society attempts to pass to its children the language and skills it has acquired or developed for dealing with numerical and spatial problems. When schools are organized to give children grounding in their culture or to achieve their desires, this practical sort of mathematics is what appears in the curriculum (Farhat, 2016).

Among the branches of mathematics is geometry. According to Farhat (2016) geometry is the branch of mathematics that deals with the properties of spaces. He maintained that, geometry in its most elementary form is concerned with metrical problems of determining the areas and diameters of two dimensional figures and surface areas and volumes of solids. He further added that other fields of geometry include descriptive geometry, analysis situs or topology, the geometry of spaces having four or more dimensions and non-Euclidean geometry. Furthermore, geometry is the mathematical study of shapes and sizes of figures termed plane geometry, when plane figures are involved and analytical geometry when algebra and coordinates (numbers) are applied to geometric problems. Additionally, geometry is designated Euclidean when axioms of Euclid form the basis of the system-particularly parallel postulates, namely that two

parallel lines do not intersect and non-Euclidean geometry when different sets of postulates are used to develop a consistent system.

According to Ahmad (2016) Mathematics is one of the science subjects taught in Nigeria's primary, secondary, and tertiary institutions. It logically consists of thinking, formulating and testing conjectures, making sense of things, forming and justifying judgment, inferences and conclusions. He further added that, geometry is one of the seven major areas in mathematics in content in curriculum and taught at all secondary schools in Nigeria, and it is an aspect of mathematics that deals with the study of different shapes and their properties. Furthermore, Geometry is often taught to students separated from their real life experiences. As a result, they often cannot apply the concepts they have been taught to solve real life problems and in some circumstances, gain no benefits from learning mathematics.

According to Abdel *et al.*, (2013) a basic knowledge of geometric concepts, their attributes and simple relations is fundamental for children to interact effectively with their environment as well as for them to enter into a formal study of geometry, itself, and other areas like science and engineering. Also Ahmad (2016) noted that all geometry instruction in Nigeria secondary school curriculum is to foster intellectual formation, that is, students should come to know what geometrical thinking is, what geometry is, what it studies and how it devises its method to do this study. He noted that geometrical thinking should not be identified as logical thinking for the latter is the domain of all mathematics. He further noted that the goal of geometry is to transmit important information about space that has been provided in the past and appear to be necessary in the years to come. That this necessity applies not only to preparing for further study of mathematics but for applying geometric knowledge to specific everyday affairs. He furthermore noted that knowledge of geometry is to develop skills in



geometric problem-solving that is techniques by which one may find answers to unknown situations through building of geometrical models of physical and behavioral theories or by using geometry as a means of explanation.

Teaching and learning mathematics is essential to students and important to teachers to impart knowledge more appropriately to students. Therefore, teachers need to use an instructional strategy that can facilitate teaching and learning mathematics more meaningful to ensure that the students pay attention to them so that they should not get lost in mathematics. It is essential to capture students' attention with appropriate methods to motivate students. Every geometry course taught calls on logical and cognitive reasoning and visualization ability. Cognitive conflict is related to the content of mathematics, such as geometry. Because of geometry's cognitive nature, the cognitive conflict has been linked with geometry achievement Mitchemore (2015).

Cognitive Conflict Instructional Strategy is one of the major independent variables to be used in this study. Cognitive Conflict occurs when students are presented with examples which cause them to question their incomplete understanding of geometry shapes (for example rectangle among others), in this strategy the teacher promote Cognitive Conflict in the classroom by having students predict which of the shapes drawn by the teacher matches their expectation. When a teacher causes discrepancy among students in the class or the teacher tried using an approach that will cause uncertainty/conflict on the students' previous ideas on how to solve geometry problems in order to encourage conceptual change, he/she will use model process cognitive conflict to anticipate how students may experience cognitive conflict. This model process help a teacher not to let students have conflict by guiding the student out of dissonance or confusion. Mufit *et al.* (2018) defined Cognitive Conflict as a conflict between structure Cognitive (that is, a structural Organized knowledge in the brain) with the environment (For example, an

experiment, demonstration, opinions of peers, books, or other), or conflict between concepts in the cognitive structure.

This instructional strategy has recently been receiving much attention in teaching and learning, particularly in the area of mathematics education. There is evidence in the mathematics education literature that significant number of students are often confronted with contradictions between their way of describing and explaining concepts and how such concepts are explained by their peers, teachers, or textbooks. As a result, mathematics educators have a great interest in conceptual change through the use of cognitive conflict, as it is instrumental in promoting deeper learning and conceptual understanding in mathematics (Adnyani, 2020). A counter example is an important way to create cognitive conflict that can support the development of specific knowledge on the students, is widely recognized as an important teaching strategy in conceptual change and can be utilized effectively as instructional approach to promote students' conceptual development (Mufit *et al.*, 2018).

Cognitive conflict refers to a situation where a student is confronted with a discrepancy between their existing cognitive elements (attitudes, perceptions, knowledge, and behaviours) to form new information or ideas (Zazkis & Chernoff, 2016). During mathematics learning, cognitive conflict occurs when students have a preconceived idea about how a mathematical problem should be solved, which differs from how it is being solved (Maumee & Mathews, 2017). Cognitive conflict can be viewed as an instructional strategy that is interactive, inspiring, fun, and challenging to students (Lee *et al.*, 2017). In mathematics education, several researchers have observed cognitive conflict as a situation that can play an important role in students' acquisition of mathematical concepts that can also act as evidence of mathematics learning (Susilawati *et al.*, 2017; Subanji & Maharani, 2018).

Utilizing Cognitive Conflict during Mathematics lessons can help promote the idea of Cognitively Guided Instruction (CGI) which underscores the need for teachers to pay more attention to students during mathematics learning to improve students' thinking and teacher skills in explaining concepts (Jacobs *et al.*, 2017). Furthermore, teachers need to appreciate and understand students' existing ideas and understandings and present students with situations that provoke cognitive conflict to reveal the inadequacy of the students' ideas and encourage the formation of new knowledge. Mathematics teaching based on cognitive conflict can improve students' ability to solve Mathematical problems, develop critical thinking and improve their communication skills (Widada *et al.*, 2018 and Gal., 2019). However, (Makonye & Khanyile, 2015) stressed that the use of cognitive conflict as an instructional strategy has not been sufficiently studied in the area of mathematics education, the few studies in this line have emphasized the need for mathematics teachers to possess knowledge about Cognitive Conflict and its role in mathematics instructions. With this knowledge, the teachers will be better placed to scaffold students' learning and improve students' critical reasoning skills.

As noted by some researchers, there is a need to extend and deepen teachers' professional understanding of teaching and learning practices and strategies that support students' conceptual understanding (O'Brien & Iannone, 2018). Furthermore, there is a need for research on instructional strategies that improve students' Achievement in Mathematics (Tinto, 2013). Indeed, researchers have also underscored the need for an investigation into how teachers modify their teaching methods towards responding to the needs of their students, as well as on how teachers can incorporate a variety of teaching methods to ensure students' active engagement during the learning process (Murphy *et al.*, 2019). In order to meet up with the above research needs, this study is aimed at using Cognitive Conflict as an Instructional Strategy. Cognitive conflict is a

conceptual scheme belonging to constructivism philosophy and it is based on the learner's positivity (Mady, 2017).

According to Abdel *et al.* (2013) teaching by Cognitive Conflict Instructional Strategy passes through the following stages:

- i. **Creating conflict:** students' attention is attracted through construction, misidentification and mislabeling or classification of geometric shapes and their properties. As a result, they are motivated to ask the teacher about these discrepant demonstrations.
- ii. **Searching for solution:** students become eager and start consulting books and peers to find a solution to these discrepant construction, identification, classification and labelling of geometry shapes and properties by their teacher. Activities necessary for solving this conflict are prepared, and students become active in observation, data registration, classification, prediction, and experimentation. They also learn a large part of the correct content of lessons.
- iii. **Solving the Conflict:** practicing activities such construction, identification and classification help the students to solve the conflict themselves and come up with answers to several questions, instead of listening to some theoretical explanations. As a result, they acquire several skills such as data collection, observation, cooperation, collaboration, teamwork and asking questions. Thereafter, students achieve cognitive balance by solving the cognitive conflict; a sound scientific one replaces a misconception.

Another instructional strategy in this study is Collaborative Instructional Strategy. "Collaborative learning" is an umbrella term for various educational approaches involving a joint intellectual effort by students or students and teachers together. Usually, students work in two or more groups, mutually searching for understanding,

solutions, or meanings or creating a product. Collaborative learning activities vary widely, but it is centred on students' exploration or application of the course material, not simply the teacher's presentation or explication of it. Collaborative learning represents a significant shift away from the typical teacher-centred or lecture-centred in college classrooms. In collaborative classrooms, the lecturing/listening/note-taking process may not disappear entirely but, it lives alongside other processes based on students' discussion and active work with the course material. Cooperation and collaboration could be treated as synonyms; teachers who use collaborative learning approaches tend to think of themselves less as expert transmitters of knowledge to students and more as expert designers of intellectual experiences for students as coaches or mid-wives of a more emergent learning process (Abdullahi, 2015).

Collaborative learning instructional strategy implies peers working together on a task, with the goal that all learners benefit from the social interaction component of collaboration. It is the most widespread strategy for teaching mathematics in Western countries. It promotes the teaching and learning process more than conventional teaching strategy. Collaborative learning teams are said to attain higher-level thinking and preserve information for longer times than students working individually. The groups tend to learn through discussion, clarification of ideas, and evaluation of other ideas. The information discussed is retained in long-term memory (Shafi, 2018).

Abdullahi (2015) suggested that students who work collaboratively on computational problems earn significantly higher scores than those who work alone. A student who demonstrated lower levels of achievement improved when working in diverse collaborating groups. In addition, students working in collaborative groups tend to be more intrinsically motivated, intellectually curious, caring for others and psychologically healthy. However, cognitive conflict instructional strategy and

collaborative instructional strategy, when used effectively, can assist a great deal to improve Achievement and Retention for quality mathematics education since group learning provides a source of support and create a more relaxed learning atmosphere that allows for positive learning experiences and retention. In addition, groups use some stress-reducing strategies as long as they remain together on a task (Jumaat *et al.*, 2014). This above strategies can be used to improve students' achievement and retention in geometry aspect of mathematics.

Achievement is one of the dependent variable in this study. According to Abdullahi (2015) achievement is a positive and optimal output as a result of successive solving of mathematics problem through effort, as a result of hard work. Despite the importance of geometry in specific and mathematics in general, the problem of students' poor achievement and retention capacity in the subject is undoubtedly worrisome and has been a major concern to the educationalist. Unfortunately, teachers and educators in different parts of the world are disappointed about the poor state of geometric skills in our mathematics classroom Royal Society / Joint Mathematical Council (JMC, 2020). Ajayi and Angura (2017) have revealed unimpressive students' academic achievement and retention at Nigeria's senior secondary school level, they also stated that, Students' overall achievement and retention in geometry lessons are consistently poor and in a declining state. Students are weak in understanding geometric principles, identifying geometry features in drawings, performing logical arguments in proofs, and solving multi-step problems. The overall effects of this weakness result in students' poor achievement and retention in mathematics.

Retention is another dependent variable to be use in this study; retention has been described by the Macmillan school dictionary as the ability to remember ideas and facts. Ajibola (2014) wrote that retention could be measured through verbal recall of learnt

materials. Retention of concepts learnt assist in influencing effective thinking, is one thing to be taught mathematics, specifically geometry, via appropriate strategy; it is another thing to remember after some reasonable period must have elapsed. Several studies have indicated that teaching methodology can improve learners' retention levels. For instance, Abdullahi (2015) pointed out that self-learning leads to better retention of information and the development of a favorable attitude toward science and technology. The condition of keeping something/facts you may be able to memorize in the short-term and long term, all depends on the method of learning, environment and gender.

Another important variable in this study is gender. Food and Agriculture Organization of the United Nations (FAO), 2013 define Gender in relation to man and woman, both perceptual and material. Gender is not determined biologically due to the sexual characteristics of either women or men but is constructed socially. Gender is a person's self-representation as male or female or how that person is responded to by social institutions based on the individual gender presentation. Yang and Chen (2013) state that among various human factors, spatial ability and gender differences are critical to geometric learning and gender differences play an important role in geometric learning because boys and girls show different outcomes in different learning environments when they learn geometry. It can be noted that the balance between spatial and logical ability can play a role in geometry performance in general (Battista, 2013). Bal (2014) argued that attitude is an important predictor in the context of success in geometry, and gender is an important factor affecting success because cultural factors are dominant over biological factors. Gender differences in mathematics achievement and ability have remained a source of concern as scientists seek to address the under-representation of women at the highest levels of mathematics, physical sciences and engineering (Asante, 2020).

West Africa Examination Council Chief Examiner, WAEC, (2017) reported that the general performance of candidates over the past years indicates poor quality of education at the senior secondary school level and also maintained that, the low level of performance calls for assessment and review of method of teaching and learning mathematics that encourage group work among students using geometrical figures to solve mensuration and geometry questions (**See Appendix H**). Thus, the present study will examine the effect of Cognitive Conflict Instructional Strategy and Collaborative Instructional Strategy on Achievement and Retention in the Geometry aspect of Mathematics among Senior Secondary School Students (SSII) in Niger State.

## **1.2 Statement of the Research Problem**

Geometry concepts teaching has been a topic of consideration in secondary schools in Nigeria as students continue to record poor performance in geometry as reported by the chief examiner, West African Examination council 2014 – 2017. (**See Appendix H**). Some major problems that contribute to consistently poor performance of students in senior secondary school certificate examination (SSCE) result in geometry concepts includes poor knowledge of geometry concepts by many mathematics teachers, adherence to old teaching methods despite exposure to more viable alternatives, teachers not using cognitive arousing strategies to improve students' cognitive reasoning in geometry and undue emphasis on syllabus coverage at the expense of meaningful learning of geometry concept.

Research studies have revealed that, many students in Nigeria secondary schools are not adequately prepared for geometry concepts and contents (Etsu & Ahmad, 2018). Ahmad and Idris (2017) opined that too much emphasis is often placed on formal symbolism and naming in the curriculum while relational understanding is underestimated. This therefore makes students in senior secondary schools to lack experience in reasoning



about geometry. They further stressed that, students would performed well and developed good reasoning about geometry situations if they had substantial experience in geometry during their junior secondary school classes. Therefore, they report that, the above problem is one of the reasons that made most of the secondary school students and the general public at large to erroneously believe and developed the idea that, geometry is difficult to study. No doubt, there is evidence of discontentment in the achievement of students in mathematics at the senior school certificate examination despite the positive roles played by science educators specially Mathematics educators.

It has been reported that teaching and learning of Mathematics, geometry in particular, has been unsatisfactory (Odetola & Salama, 2014). This was compounded by the conventional method of instructions adopted by most mathematics teachers, which led to poor students' comprehension of relevant mathematics concepts, especially in geometry (Abakpa & Iji 2013). Instructional methods of teaching mathematics have been identified to be one of the reasons why students perform poorly in mathematics, this should be a great reason mathematics educators to explore more in other modern instructional strategy since, conventional methods of teaching and have relative limitations on students' academic achievement and retention (Ahmad, 2016).

The identified problems does not create a conducive environment for geometry concepts learning, thereby debarring learners from having quality education and having tremendous consequences on achievement and retention. Thus, it has become necessary to search for an innovative pedagogy capable of improving the students' achievement and retention. Based on this, the study will investigate the effect of cognitive conflict instructional strategy and collaborative instructional strategy on achievement and retention in geometry aspect of Mathematics among secondary school students in Niger State.

### **1.3 Aim and Objectives of the Study**

The aim of the study is, to investigate the effect of cognitive conflict instructional strategy and collaborative instructional strategy on achievement and retention in geometry aspect of mathematics among secondary school students in Minna, Niger State. Specifically, the objectives of the study are:

1. Determine the achievement of students taught geometry concepts using cognitive conflict, collaborative instructional strategy and lecture method;
2. Find out the retention of students taught geometry concepts using cognitive conflict, collaborative instructional strategy and lecture method;
3. Determine the influence of gender on the achievement of students taught geometry concepts using cognitive conflict instructional strategy;
4. Find out the influence of gender on the achievement of students taught geometry concepts using collaborative instructional strategy;
5. Determine the influence of gender on the retention of students taught geometry concepts using Cognitive Conflict Instructional Strategy;
6. Find out the influence of gender on the retention of students taught geometry concepts using collaborative instructional strategy.

### **1.4 Research Questions**

The following research questions were raised to guide the study:

1. What is the difference in the mean achievement scores of students taught geometry concepts using cognitive conflict, collaborative instructional strategy and lecture method?
2. What is the difference in the mean retention scores of students taught geometry concepts using cognitive conflict, collaborative instructional strategy and lecture method?

3. What is the difference in the mean achievement scores of male and female students taught geometry concepts using cognitive conflict instructional strategies?
4. What is the difference in the mean achievement scores of male and female students taught geometry concepts using collaborative instructional strategies?
5. What is the difference in the mean retention scores of male and female students taught geometry concepts using cognitive conflict instructional strategies?
6. What is the difference in the mean retention scores of male and female students taught geometry concepts using collaborative instructional strategies?

### **1.5 Research Hypotheses**

The following null hypotheses were formulated and tested at  $P= 0.05$  level of significance:

**HO<sub>1</sub>:** There is no significant difference in the mean achievement scores of students taught geometry concepts using cognitive conflict, collaborative instructional strategy and lecture method;

**HO<sub>2</sub>:** There is no significant difference in the mean retention scores of students taught geometry concepts using cognitive conflict, collaborative instructional strategy and lecture method;

**HO<sub>3</sub>:** There is no significant difference in the mean achievement scores of male and female students taught geometry concepts using cognitive conflict instructional strategies;

**HO<sub>4</sub>:** There is no significant difference in the mean achievement scores of male and female students taught geometry concepts using collaborative instructional strategy;

**HO<sub>5</sub>:** There is no significant difference in the mean retention scores of male and female students taught geometry concepts using cognitive conflict instructional Strategy;

**HO<sub>6</sub>:** There is no significant difference in the mean retention scores of male and female students taught geometry concepts using collaborative instructional strategy.

### **1.6 Significance of the Study**

This study will contribute towards the improvement of teaching and learning Mathematics. The findings of this study would benefit the following bodies: Students, Teachers, School Administrators, Curriculum Planners and Developers and Learning Resource Centers.

To the Students, Cognitive Conflict and Collaborative Instructional Strategy would help students in brainstorming and help in making Geometry more interesting and improve students' Achievement in Mathematics and others science subjects. This would enable learners to develop Cognitive skills and a team working spirit in discovering new concepts and ideas in any discipline. It would bring the about improved achievement of students of mathematics in secondary schools.

This study finding would provide the teachers with easy and effective ways of teaching geometry and apply the method in other related topics of other science subjects. Through this research, it will be necessary for the teachers to refine their old ways of imparting the knowledge of geometry to the students. Adopting these research findings will bring confidence to the teachers and remove reluctance in teaching mathematics. This will promote effective teaching and learning, which will enhance the high performance of students in Mathematics. Furthermore, this study may help deepen Mathematics teachers' content knowledge while using the students' instructional materials, leading them to perform the activities by themselves as learners and analyze the materials to discuss the intended learning goals and progression of ideas for conceptual development.

This study would serve and promote the effort of educational administrators of Nigeria to abandon the long-standing practices of rote-learning, memorization and persistence teachers dominated instructional strategies. Through this study, the administrators will be stimulated to investigate and formulate constructive strategies to reduce or eliminate students' poor performance in the geometry aspect of mathematics. Furthermore, it will help the school planners and authorities in the state take corrective measures within their jurisdiction through adequate planning of resources to meet the demands and guide their actions on future projects on the education of students for successful teaching and learning processes in the school.

The findings of this study would help Curriculum planners and developers select and recommend appropriate learning experiences, knowledge, content and strategies that will enhance effective teaching and learning of mathematics and another science subject in our secondary schools.

The result of the study will help resource centers in evaluating processes through which achievement or change in learners' ability to do work or improvement of skills and growth of attitudes will be estimated. The resource centers can use the new teaching method as an avenue to provide in service training, workshops, and seminars to their teachers with a view of making use of these strategies as an approach to teaching geometry in secondary schools.

### **1.7 Scope of the Study**

The study focused on the effect of cognitive conflict and collaborative instructional strategies on academic achievement and retention of Mathematics students in senior secondary schools in Minna. Minna is the geographical area chosen for this research work. Minna is located on latitude  $9^{\circ} 37'N$  and longitude  $6^{\circ} 32'E$  respectively, and a total population of 321,687 according to the 2006 population census figure of the city.

Senior secondary school II classes was chosen because it is the foundation level for the Mathematics advanced topics career that begins in senior secondary school, and it requires a solid foundation to be laid. The strategies was used to teach some critical concepts in geometry aspect of mathematics. The independent variable for the study are cognitive conflict and collaborative instructional strategies and the dependent variables are achievement and retention while the moderating variable is gender. The research will last for Eight (8) weeks.

### **1.8 Operational Definition of Terms.**

The following terms were defined as used in the context of the work:

**Achievement:** Achievement refers to the students' correctly and successfully solving geometry problems.

**Cognitive-conflict:** Is a perceptual state in which student notices the discrepancy between his cognitive structure (previous idea) and external idea (new idea).

**Retention:** Is the ability to remember ideas and facts.

**Cognitive conflict instructional strategy:** a strategy that exposes students to a situation which is contrary to the concepts. Then the students are directed on experiments or demonstration to prove the concepts.

**Conventional method:** It is procedurally defined as a process in which the teacher presents geometry concepts, and students are made to listening without active participation.

**Collaborative instructional strategy:** A scaffolding enriched Strategy used by the teacher to involve students working and cooperating with each other to develop geometry learning skills and build new ideas.

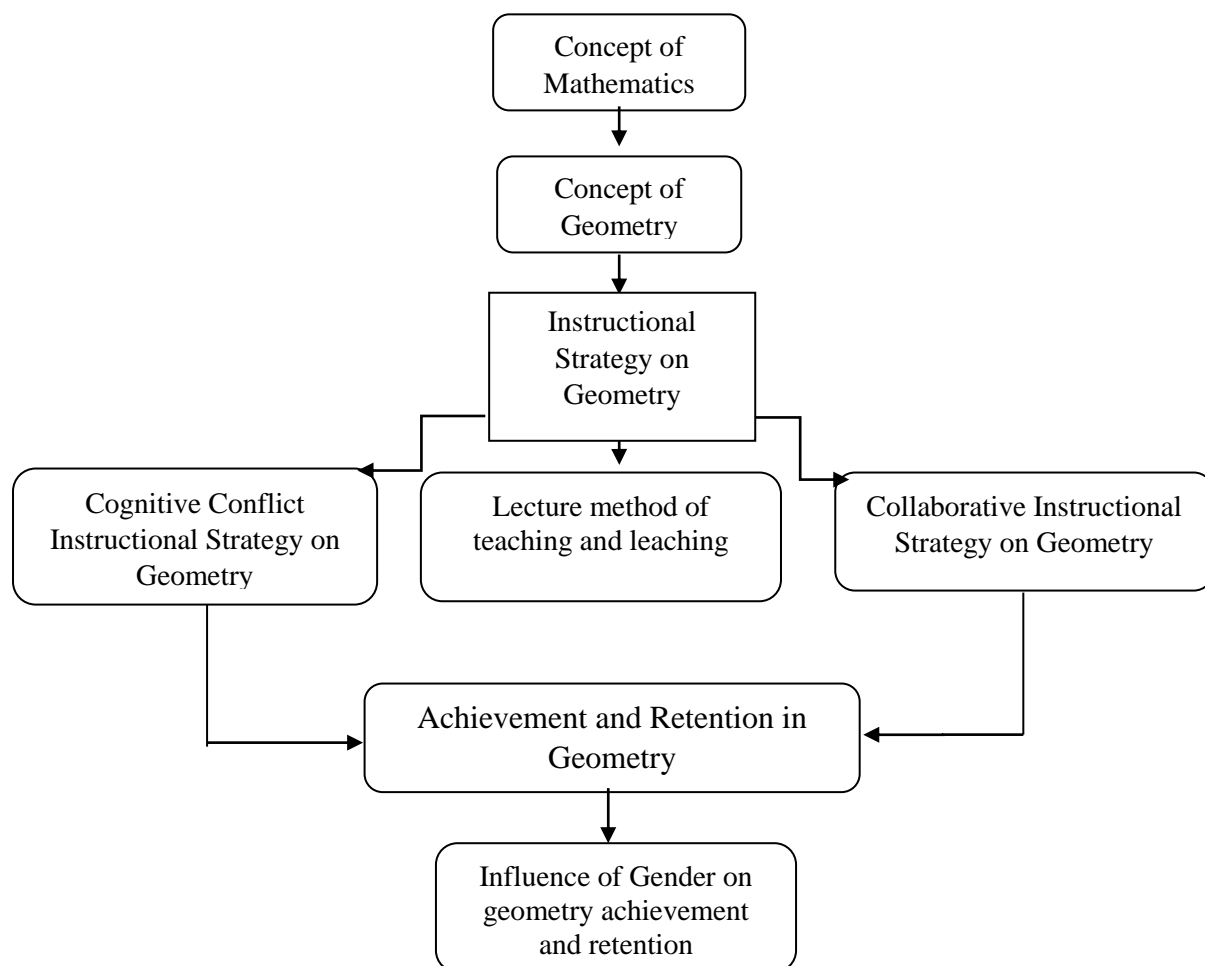
**Gender:** This refers to the description of students regarding their sex, male or female (boy or girl).

## CHAPTER TWO

### LITERATURE REVIEW

#### 2.0

#### 2.1 Conceptual Framework



**Figure 2.1:** Showing Conceptual Framework of the effect of cognitive conflict and Collaborative Instructional Strategies on Students' Achievement and Retention on Geometry.

#### 2.1.1 Concepts and nature of mathematics

Mathematics is a fundamental part of human thought and logic and integral to understanding the world and ourselves. Mathematics provides an effective way of building mental discipline and encourages logical reasoning and mental rigor. In addition, mathematical knowledge plays a crucial role in understanding the contents of



other school subjects such as science, social studies, and even music and art. Mathematics has a transversal nature. If we reflect on the history of curriculum in general, then mathematics (geometry and algebra) were two of the seven liberal arts in Greek and medieval times. This historical role supports the notion that mathematics has provided the mental discipline required for other disciplines (Kolawale & Olofin, 2020). According to Farhat (2016), Mathematical literacy is a crucial attribute of individuals living more effective lives as constructive, concerned and reflective citizens. Mathematical literacy includes basic computational skills, quantitative reasoning, spatial ability etc. Mathematics is applied in various fields and disciplines, i.e., mathematical concepts and procedures are used to solve science, engineering, and economics problems. (For example, understanding complex numbers is a prerequisite to learning many concepts in electronics.) The complexity of those problems often requires relatively sophisticated mathematical concepts and procedures compared to the aforementioned mathematical literacy.

According to Farhat (2016), Mathematics is very important because it is needed for all scientific training, it is a part of our human cultural heritage, and we have a responsibility to develop that heritage. Despite the existence of other subjects, Mathematics is one of the essential subjects that contribute to the nation-building of society. Mathematics has a pervasive influence on our everyday lives and contributes to the country. There is no other subject that has greater application than mathematics. However, most of the fields of knowledge in science are dependent on mathematics for solving problems. Mathematics application is universal to all learning and everyday living, from counting possession to measuring properties, predicting an event, planning budgets, providing models. All these are indications that Mathematics is useful in domestic and business deals, scientific discoveries and technological breakthroughs,

problem-solving and decision making in different situations in life (Kolawale & Olofin, 2020).

### **2.1.2 Geometry and its application**

According to Barnard and Cronje (2019), Geometry (Ancient Greek) Geo "Earth" Metria "Measurement" is a branch of mathematics concerned with the question of shapes, size and relative position of figures and properties of space. A mathematician who works in the field of geometry is called a geometer. They also reported that, geometry arose independently in several early cultures as a body of practical knowledge concerning length, areas, and volumes, with an element of formal mathematical science emerging in the west as early as Thales (6 Century BC). A major contributor to the field of geometry was Euclid 325BC, known as the father of geometry. They maintain that, geometry is a unifying theme for the entire Mathematics curriculum and, as such, is a rich source of visualization for arithmetical, algebraic and statistical concepts; For example, geometry region and shapes are useful for development work with the meaning of fractional numbers, equivalent fractions, ordering of fractions and computing fractions.

### **2.1.3 Influence of gender on geometry achievement.**

Yang and Chen (2013) state that among various human factors, spatial ability and gender differences are critical to geometric learning and gender differences play an important role in geometric learning because boys and girls show different outcomes in different learning environments when they learn geometry. It can be noted that the balance between spatial and logical ability can play a role in geometry performance in general (Battista, 2013). However, Bal (2014) argues that attitude is an important

predictor in the context of success in geometry, and gender is an important factor affecting success because cultural factors are dominant over biological factors.

Research findings show that gender differences in mathematics vary at the middle school level. However, evidence on when gender differences in perceptions of competence in Mathematics start are not entirely consistent. For example, Fennema and Sheman (2020) found no statistically significant gender-related differences in spatial visualization. Also, stated no differences in the achievement in the sixth grade level of boys and girls in the skills of measurement application, geometry application, and probability/statistics. Also posited that thirteen-year-old girls performed better at computation and spatial visualization than boys.

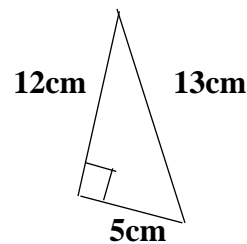
#### **2.1.4 Misconception on geometry**

According to Jean (2019), knowledge is not transferred from person to person. The individual does not passively receive knowledge from the environment, but is an active participant in the construction of his/her own mathematical knowledge. The construction activity involves the reception of new ideas and the interaction of these with the students' existing ideas. Also, students may not be able to perceive what the teacher sees in a geometric situation if they are at a particular level of the Van Hiele model and higher levels of understanding are required. It is impossible for learners to bypass or skip a level from the model. These situations result in misconceptions arising frequently. By discussion, a teacher can get pupils to explain how they came to their answers or rules and be able to analyze faulty interaction between the students' extant ideas and the new concept. When the teacher is able to understand the reason behind the misconception, it is corrected by challenging or contrasting it with the faithful conception. In geometry some common misconceptions arise:

**i. Identifying the Base and Height of a Triangle**

Invariably students are habituated by the standard triangle presented to them when the area of triangle algorithm is presented: one with horizontal base and height. When faced with any triangle they used the ‘bottom’ line as the height ‘upwards’ from the base. For example.

Question: Find the Area of the right angle triangle



Answer: Base = 5cm                      Height = 13cm

$$\text{Area} = \text{Base} \times \text{Height} / 2 = 5 \times 13 / 2 = 32.5 \text{ sq cm}$$

Suggested Remedies:

- i. Teacher can allow students to examine difference types of triangle, also with varying orientation to be better able to identify the base and the height in each. Students can be given the option of turning their books to analyses shapes in case of spatial problems
- ii. The relationship of between base and height must be explained clearly as well: their relationship of being perpendicular to each other.

**ii. Conservative misconception**

Students often believe that the rules of invariance that apply to algebra also apply to geometry shapes: there must be equality in all respect when A become B. This leads to the misconception that the perimeters are the same.

Suggested Remedy:

- i. Different shapes of pentominoes can be used to demonstrate that same areas don't imply same perimeter. Students see that for the same area, perimeter can vary when they investigate by checking the perimeters of the pentominoes.
- ii. Using a geoboard and rubber bands, students can construct different rectangles of varying dimensions but with the same perimeter and compare the resulting areas.

**iii. Shape properties**

When student develop a concept image (a mental image of a shape) without a concept definition (a specified definition of the shape or its properties, they often identify examples of shapes, but will also fail to identify examples of shapes that are not identical to their own mental image of the shape or the shape prototype, i.e., the figure does not "look like" the shape. Although characteristics such as orientation and proportions are irrelevant to the defining properties of a shape, they affect whether students recognize certain shapes. Some of the common misconceptions of triangles are as follows:

- i. Triangles have one point at the top and two points at the bottom
- ii. The bottom of a triangle is flat Some of the common misconceptions of rectangles are as follows:
- iii. Rectangles are always long
- iv. Rectangles have two long sides and two short sides

**2.1.5 Cognitive conflict instructional strategy**

Cognitive conflict refers to a situation where a student is confronted with a discrepancy between their existing cognitive elements (such as attitudes, perceptions, knowledge, and behaviors) and new information or idea (Subanji & Maharani, 2018). During geometry learning, cognitive conflict occurs when students have a preconceived idea about how a mathematical problem should be solved which differs from the way it is

being solved. Cognitive conflict can be viewed as a learning strategy that is interactive, inspiring, fun, and challenging to students. In mathematics education, cognitive conflict has been observed by several researchers as a situation that can play an important role in students' acquisition of mathematical concepts that can also act as evidence of mathematics teaching (Susilawati *et al.*, 2017).

Cognitive conflict is a widely recognized important factor in the process of conceptual change and can be effectively utilized as a teaching-learning strategy to promote the conceptual development of students (Mufit *et al.*, 2018). The notion of cognitive conflict has recently been receiving much attention in teaching and learning, particularly in the area of mathematics education. There is evidence in the mathematics education literature that significant numbers of students are often confronted with contradictions between their way of describing and explaining concepts and the way such concepts are explained by their peers, teachers, or textbooks. As a result, there is currently great interest among mathematics educators in the issue of conceptual change through the use of cognitive conflict, as it has been found to be instrumental in promoting deeper learning and conceptual understanding in mathematics (Adnyani, 2020).

Cognitive conflict can lead to students experiencing mathematics anxiety, which in turn can lead to low self-esteem and, ultimately, poor achievement in the subject (Devine *et al.*, 2018). In addition, mathematics teachers need to acquire competencies on how to utilize cognitive conflict moments to support students learning to guarantee conceptual understanding by students (Ashman & Conway, 2017; Mulungye *et al.*, 2016).

The utility of cognitive conflict in promoting reasoning skills can only be realized if students are supported to reconcile the conflicting ideas by examining, explaining, justifying, and questioning their preconceived ideas and beliefs (Peled and Shahbari, 2015; Putra *et al.*, 2019).

Geometry teaching based on cognitive conflict can as well improve students' ability to solve geometry problems, develop critical thinking, and improve their communication skills (Gal, 2019; Putra *et al.*, 2019). However, the use of cognitive conflict as a teaching-learning strategy has not been sufficiently studied in the area of mathematics education. The few studies in this line have emphasized the need for mathematics teachers to possess knowledge about cognitive conflict and its role in geometry teaching and learning. With this knowledge, the teachers will be better placed to scaffold students learning and to improve students' critical reasoning skills (Makonye & Khanyile, 2015).

Li (2019) synthesized the signs that students are likely to exhibit when confronted by cognitive conflict. The signs outlined include anxiety, hesitancy, uneasiness, tension, vacillation, doubt, perplexity, frustration, confusion, and reappraisal of the situation to try and resolve the conflict. Therefore, the teacher needs to check for these signs in students during mathematics teaching and learning with a view to assist them to overcome the cognitive conflict and there by gain a deeper understanding of the concepts involved (Wyrasti *et al.*, 2016). In the absence of such support, cognitive conflict can lead to students experiencing mathematics anxiety, which in turn can lead to low self-esteem and, ultimately, poor performance in the subject (Devine *et al.*, 2018). In addition, mathematics teachers need to acquire competencies on how to utilize cognitive conflict moments to support students learning to guarantee conceptual understanding by students (Ashman and Conway, 2017; Mulungye *et al.*, 2016).

The role of cognitive conflict in mathematics teaching and learning has been explored from different perspectives by different researchers. Some researchers have studied the phenomenon based on resolving it in a realistic situation with modeling characteristics as well as its role in developing students' understanding of specific mathematical

concepts (Peled and Shahbari, 2015). Some other researchers have examined the role of cognitive conflict in improving spatial visualization based on student ability, sources of students' errors, and misconception in algebra (Mulungye *et al.*, 2016; Susilawati *et al.*, 2017). Others have analyzed the characteristics of students' cognitive conflict when solving problems based on information processing theory (Pratiwi *et al.*, 2019). In addition, others have examined the effects of a mathematics cognitive acceleration program on student achievement and motivation (Finau *et al.*, 2018).

Cognitive Conflict Instructional Strategy will encourage high-level thinking, oral communication, self-management and leadership skills, promotion of students' faculty interaction. Increase in students' retention, self-esteem and responsibility.

The Cognitive Conflict Instructional Strategy involves:

- (a) Identify students' current state of knowledge;
- (b) Confronting students with contradictory information that is usually presented through texts and verbal, thus making explicit the contradiction, or guiding the debate with the student or among peers (small groups or the whole classroom); and
- (c) Evaluating the degree of conceptual change between the students' prior ideas or beliefs and a posttest after the instructional intervention.

Many studies have proved the effectiveness of applying Cognitive Conflict Instructional Strategy in modifying Mathematics misconceptions among students. A study by Dahlan (2016) Understanding in mathematics often occurs in significant jumps, accompanied by a clear sense of comprehension, rather than a smooth, steady process. On the other hand, lack of understanding may leave the individual in a general state of confusion, unable to pinpoint the difficulty. If we consider these phenomena to be a result of brain activity of the nature of a dynamical flow on a manifold, this suggests a model



encompassing these various aspects of understanding mathematics. As the learner restructures his mathematical schema to understand these ideas, cognitive conflict is bound to occur. It can give rise to path-dependent logic, in which the learner can give different answers to the same questions depending on the path of approach to that question. At this stage, the learner may restructure his ideas and rationalise them appropriately for a short term gain but inappropriate for long term schematic development.

Another study by Dahlan (2016) states that one way to reduce misconception is by directly experiencing the problem to cause accommodation disequilibrium, as suggested by Piaget's cognitive conflict. An interaction between cognition and environment used to stimulate conflict is also stated, which is the conflict between initial or entry concepts and new learning materials. In addition, also argues that the conflict between new learning material and environment can be explained by initial or entry concept. Meanwhile, Piaget believes that this conflict can be explained by the concept to be learned. In the learning process, the role of the teacher is of paramount importance. Programmed learning, work cards and so on may be an effective teaching substitute in certain circumstances, but the essential role of the teacher is helping in the schematic restructuring of the student.

The occurrence of conflict in the mind of a Mathematics learner will be apparent immediately to the sensitive teacher (Fraser, 2017). However Murphy *et al.* (2019) stressed that the simplest manifestations on students are confusion, annoyance, fear, or just a dull lost look in the eyes. It would be wrong to separate these emotional reactions from the cognitive side of learning. They are all signs of the state of the brain. In terms of a catastrophe interpretation, they may help us realize the nature of the mental blockage, an unsuitable line of thought, a catastrophic leap, or even path-dependent

decision making. Then it is the teacher's job to resolve the conflict suitably. Continual explanations along the same track to emphasize the required idea may not help because the student train of thought maybe lead elsewhere. The learner may not even know where the problem is, but the experienced mathematics teacher may be able to help by selecting the right approach. These ideas of the excellent teacher are as old as history itself. However, the history of science shows a continual reinterpretation and enrichment of earlier ideas. Perhaps a catastrophe interpretation will lead to placing the teacher's well-tried skills within a theory that describes the mechanism of the brain itself. In the meantime, by being sensitive to the possible conflicts in the learner's mind learning new mathematical ideas, we may find a practical way of understanding mental blockages in the learning process.

### **2.1.6 Cognitive conflict instructional strategy and cognitive structure**

The cognitive conflict term is related to the accommodation process, which Piaget introduced. In his theory, Piaget (Van De Walle, 2013) says that accommodation occurs when the new concept does not "fit" with the existing initial knowledge. This state is called disequilibrium by Piaget. If this state occurs, the brain replaces the existing schema with the new concept. Accommodation is a process of integrating a new stimulus into the cognitive structure formed unconsciously. Therefore, cognitive conflict is needed to accommodate a person's cognitive structure.

In a specific condition where people cannot adjust the initial cognitive structure with the new stimulus they get from learning activity, people call having a cognitive conflict. The cognitive conflict can also be caused by realizing the counter information with the prior information or idea. Moody (2020) states that cognitive conflict happens when there is a contradiction between fundamental knowledge and new information/knowledge of students. Fraser (2017) states that cognitive conflict is

formed by giving a counter-example or two contradicting examples, where the conventional method will fail to solve the problem. Sayce (2013) states that cognitive conflict means a condition where the students were given a new thing, a counter-example or a statement that makes students unsure about their prior comprehension/understanding of the fundamental knowledge they learn. We can do this by giving the wrong example/statement to the students with correct comprehension/understanding of a topic and vice versa. Then, the students are guided to recheck and compare the information given by their answers. It shows that mathematics learning with cognitive conflict is an excellent way to develop student's critical thinking ability.

Moody (2020) added by stating that even though adjustment examples or information given in learning are essential, the presence of inconsistency or a contradiction will help boost students' critical thinking rather than only giving a consistent matter commonly used in learning. This shows that giving a problem or material that can intrigue inconsistency or contradict can help the students think critically. He further states that learning includes an analysis process questioning a thing verifying a fact and a result of a problem, and redefining a concept are activities that require critical thinking ability. Mathematics critical thinking can be shown by students' steps or ways of thinking in solving a problem. The problems that can initiate critical thinking activity are the problems that cannot be quickly done by using a conventional way or only applying a formula without going through the analysis. Based on Bloom's taxonomy, analysis, synthesis, and evaluation stages are the stages that represent critical thinking ability.

Besides that, Sudiarta (2018) arranges the competencies of critical mathematical thinking, which include:

- i. Investigating the context and spectrum of the problem;

- ii. Formulating the problems;
- iii. Developing the answer's concept and reasonable argument;
- iv. Doing induction and deduction; and
- v. Evaluating.

From (i) until (iii) are the essential competencies the students should have for solving mathematical problems. Point (iv) represents the student's ability to give a hypothesis or prediction or other ideas to solve the problem given. The last point is point (v), which is the competence to give alternative ideas and use the ideas to solve the problems. This stage was done by giving other answers to the problem. Critical thinking ability can be seen in students' ways of solving problems. He also states that solving an open-ended problem can initiate critical thinking. Therefore, the indicators of critical thinking ability consist of the student's ability to give a correct answer, an alternative idea to solve the problems, and an alternative answer to the problem given.

By giving counter-information, students will compare it. The teacher will ask the student to think about other possibilities that can be the ways to solve the problems or other possibilities of the answers. They will evaluate their work and expect to know about their mistake by arguing why the counter information is correct or wrong. Indirectly, students train their critical thinking ability by solving problems to get the correct comprehension.

### **2.1.7 Cognitive conflict instructional strategy and students reasoning skills**

Every geometry course taught calls on logical and cognitive reasoning and visualization ability. Cognitive conflict is related to the content of mathematics, such as geometry. Because of geometry's cognitive nature, the cognitive conflict has been linked with geometry achievement (Mitchemore, 2015).

The signs expected to be exhibited by those who experience cognitive conflict include anxiety, hesitancy, uneasiness, tension, vacillation, doubt, perplexity, frustration, confusion, and reappraisal of the situation to try and resolve the conflict. Therefore, the teacher needs to check for these signs in students during mathematics teaching and to learn to assist them to overcome the cognitive conflict and gain a deeper understanding of the concepts involved (Wyrasti *et al.*, 2016). In the absence of such support, cognitive conflict can lead to students experiencing mathematics anxiety, which in turn can lead to low self-esteem and, ultimately, poor performance in the subject (Devine *et al.*, 2018). In addition, mathematics teachers need to acquire competencies on utilizing cognitive conflict moments to support students learning to guarantee conceptual understanding and achievement by students (Mulungye *et al.*, 2016; Ashman & Conway, 2017).

The utility of cognitive conflict in promoting reasoning skills which may result in better mathematics achievement can only be realized if students are supported to reconcile the conflicting ideas by examining, explaining, justifying, and questioning their preconceived ideas and beliefs. In the teaching-learning situation, cognitive conflict moments should be viewed as motivational moments to ensure knowledge transfer and skill development in problem-solving (Putra *et al.*, 2019).

The role of cognitive conflict in mathematics teaching and learning has been explored from different perspectives by different researchers. Some researchers have studied the phenomenon based on resolving it in a realistic situation with modeling characteristics and its role in developing students' understanding of specific mathematical concepts (Peled and Shahbari, 2015). Some other researchers have examined the role of cognitive conflict in improving spatial visualization based on student ability, sources of students' errors, and misconceptions in algebra (Mulungye *et al.*, 2016 and Susilawati *et al.*,

2017). Others have analyzed the characteristics of students' cognitive conflict when solving problems based on information processing theory (Pratiwi *et al.*, 2019). In addition, others have examined the effects of a mathematics cognitive acceleration program on student achievement and motivation (Finau *et al.*, 2018).

### **2.1.8 Cognitive conflict instructional strategy and retention ability on mathematics**

From an educational perspective, understanding improves the ability to retain ideas in mind. In a situation where a child cannot use mathematics knowledge that he acquired in the classroom or examination hall, he did not have a good grasp of mathematics concepts. Moreover, many students face some challenges in mathematics because of improper dissemination of the course and misconceptions of diverse ways of the subjects under study or use of proper 3-dimensional objects when necessary to be used. Therefore, the teaching of mathematics should involve Cognitive-driving or cognitive arousing practical activities such as accompanying teaching of length, width, area, and volume with the basic theme of determination of the effect of geometry objects in the measurement of area and volume and Collaborative learning methods (Garmendia *et al.*, 2017).

Assigning learners to teach others as a revision strategy is one effective method to reinforce repetition. Teaching others would require learners to organize the things they have learned to teach them to others, making it easier for them to recall better and apply knowledge during mathematics tests and examinations. The learner-teacher would be required to polish up on their skills before teaching something to others; this will demand that they learn something more than once or more, which will assist in binding the ideas onto the brain. When learners explain learned material to others, fading memories are reactivated, consolidated and strengthened. This practice improves retention and boosts active learning (Finau *et al.*, 2018).

Cognitive conflict instructional strategy help and encourage mathematics learners to develop a habit of keeping track of information, and paying more attention in class rather than studying or practicing at a later stage can be beneficial. Ordinarily, learners who keep track of information pay more attention in class and remember things better than those who do not. This is what calls 'Focus to remember'. To retain information, they have to concentrate and pay attention; if not, the information can be forgotten within the next few seconds (Sudiarta, 2018).

Efforts have been underway in the last two decades to increase sciences and math skills at an early age to increase technology and engineering expertise (Garmendia *et al.*, 2017). The Nigeria nation is experiencing a transition in its manufacturing areas, low skilled jobs are rapidly moving overseas. Unemployment has skyrocketed since the shutdown of many companies in recent history, and a state of recession lingers in the economy. The industry has been left desolate all over the country. In many cases, the problem is under-trained individuals in technology areas. The global market calls for highly innovative thinkers who bring creativity into the design process (Abdullahi, 2015).

Cognitive Conflict Instructional Strategy will encourage high-level thinking, oral communication, self-management and leadership skills, promotion of students' faculty interaction. Increase in students' retention, self-esteem and responsibility.

### **2.1.9 Collaborative instructional strategy: Concepts and its use in promoting effective learning.**

The teacher is to guide and direct the students in developing visualization skills in geometry. Thus, students must be encouraged and stimulated to collaborate and learn by themselves, and the job of the teacher is to ensure that the students accept responsibility for their learning and development. However, the most important aspect is to serve as a source or model in which sources of knowledge emanate. To fit into this changing world, students must acquire mathematics literacy. To create mathematically literate students is to engage students in mathematics teaching activities and learning through the use of Collaborative Instructional Strategy. This evidence for education is about helping students stay on track in mathematics building concept, open concept in steady progression skills. Perhaps, for the matter of fact, the essence of education is learners learning what they are made to learn. An individual has the ability, willingness, enthusiasm and capacity to discover, create, manipulate geometry shapes and learn for himself. Such emphasis on learning has enabled students to take a self-directed alternative where it is used as a tool to promote the exchange of ideas in the mathematics classroom (Abdullahi, 2015).

Collaborative Instructional learning will encourage high-level thinking, oral communication, and self-management and leadership skills. Promotion of students' faculty interaction. Increase in students' retention, self-esteem, teamwork spirit and responsibility. Collaborative learning in mathematics education popularity surged in the 1980s, but it has since continued to evolve (Garmendia *et al.*, 2017)

Collaborative Instructional Strategy involves:



- i. **Introduction:** it refers to assessing the previous knowledge or skills of the learners through discussion or questioning.
- ii. **Task discussion:** introducing the new task to arouse learners' interest and presentation of task through traditional method of teaching.
- iii. **Collaborative learning:** Students' are carefully divided into groups, and they are presented with a task to achieve a common goal.
- iv. **Number head:** each team member discusses their work to a large group.
- v. **Apprenticeship:** students work individually.
- vi. **Assessment:** students exchange their works to assess and discover their zone of proximal development (ZPD)

#### **2.1.10 Collaborative instructional strategy and retention in mathematics.**

Retention is a preservative factor of the mind (Abdullahi, 2015). The mind acquires the materials of knowledge through sensation and perception. These acquired materials in mind need to be preserved in the form of images for knowledge to develop. Retained images are revived or reproduced whenever a stimulating situation occurs to make memorization possible. Hence mathematics concepts need to be presented to the learners in a way or method that touches their sub consciousness which can trigger quick recalling of the concept being taught or learnt. Using such a teaching method as a collaborative Instructional Strategy, both high ability and low ability learners would be able to collaborate in terms of understanding, explaining and retaining the concept they have learnt in a mathematics class.

Many students become miserable and inattentive in a mathematics class after being taught a topic and discovering they could not memorize or recall such a concept with ease. The reason for this difficulty may vary, but this could sometimes be related to the teaching method being used to explain such topics. (Gal, 2019) asserted that the issue of

poor performance in mathematics examinations was due to the problem of teaching methods. There has also been an increasing awareness by those concerned with mathematics education that the conventional methods of teaching mathematics have not been very successful. For effective teaching to occur, the skilful mathematics teacher needs to use many different methods and techniques at his disposal. A carefully designed teaching method can make teaching and learning effective (Finau *et al.*, 2018).

### **2.1.11 Collaborative learning and cognitive load theory**

Similar to worked examples (as argued above), collaborative learning demonstrates another example of the borrowing and reorganizing principle. Knowledge can be borrowed from other group members and reorganized, linking new knowledge with old knowledge stored in long-term memory. Group interactions can help individuals make sense of the information and steer the reorganization of the information accordingly (Gal, 2019). Because humans have evolved to communicate, to share, and to obtain information from each other as biologically primary skills, collaborative learning may have an advantage over individual learning in that it involves sharing information and learning from each other, as occurs in everyday life (Sweller *et al.*, 2016).

Another advantage of collaborative learning is that it may assist in learning complex materials. Complex materials are difficult to learn because they impose a heavy working memory load (Sweller *et al.*, 2016). However, if the learning material is shared among several group members, an individual must process less task-relevant information, potentially reducing working memory load. Working memory resources can be allocated to learning about important aspects of the materials by processing relevant information communicated from other group members. Based on this view, collaboration should be effective by providing group members with the information

they otherwise need to search for themselves. This potential provision of information should reduce extraneous cognitive load. In this sense, a biologically primary activity, collaboration, may provide an advantage in acquiring biologically secondary knowledge such as mathematics. Combining the limited working memory resources of several individuals should increase the resources available to all in a manner that does not occur when students are engaged in individual learning and have to deal with the entire working memory load themselves. Hence, through collaboration, individuals may be better able to learn about complex materials.

#### **2.1.12 Cognitive conflict and collaborative strategy in geometrical shapes**

According to Ajayi and Angura (2017), resolving cognitive conflict in geometry is closely related to recognizing all figure units that can be mathematically relevant. There are two ways of looking at figures and recognizing what they stand for: the natural and the mathematical. One important issue in the learning of geometry in primary and secondary school is to identify the figural unit which can be discriminated in any constructed figure. Dimensional deconstruction describes the transition of a drawing seen as a tangible object to the figure conceived as generic and abstract objects. For example, a figure can be seen as a 2 D-object (a triangle as an area), a set of 1 D-objects (side) or 0 D-objects (vertices). While in a natural way, perception focuses exclusively on 1 D, 2D or 3D figural units, just like material objects, the mathematical way requires becoming aware of the different ways of looking at figures prior to knowing the basic classical figure.

#### **2.2 Theoretical Framework.**

This study is based on constructivism theories such as Jean Piaget's development theory, Van Hiele theory and Bruner's theory of cognitive learning. Constructivism theories are theories of knowledge that argues that humans generate knowledge and

meaning from an interaction between their experiences and their ideals. It has influenced several disciplines, including psychology, sociology, education and the history of science. The formalization of the theory of constructivism is generally attributed to Jean Piaget, who articulated mechanisms by which learners internalize knowledge. He suggested that through accommodation and assimilation, individuals construct new knowledge from their experiences. When individuals assimilate, they incorporate the new knowledge from their experiences in the already existing framework without changing that framework. According to the theory, accommodation is the process of reframing one mental representation of the external world to fit new experiences. However, constructivism is often associated with a pedagogy approach that promotes active learning.

The focus tends to shift from the teacher to the students in the constructivist classroom. A classroom is no longer a place where the teacher pours knowledge into passive students, who wait like empty vessels to be filled. In the constructivism model, the students are urged to be actively involved in their own learning process. However, this study is based on three constructivism theories

### **2.2.1. Jean Piaget's development theory**

Piaget's theory is based on the idea that the developing child actively and adaptively builds cognitive structures, for understanding and responding to physical experience within his or her environment. Through successive stages of intellectual development children develop intellectual structures that enable them to have a greater understanding not only of the world, but also themselves. Piaget considered intellectual activity to be a Cognitive function. In his theory; Piaget describes the development and adaptation of mental operations or thought structures for example thinking, counting, classification etc which progress through rich interactions with the world. Cognitive conflict and

collaborative experiments in mathematics help students to progress through such interactions. Piaget's theory of conceptual change involves four stages of intellectual development:

Sensory-motor stage (birth – 2 years old), pre-operational stage (age 2-7, Concrete operation (age 7-11) and formal operations (Beginning at age 11-15). Reasoning is freed from the concrete. Adolescents begin to construct whole systems of belief and can engage in more reflective reasoning such as thinking about other's thoughts or engaging in self-reflection. In scientific problems solving, formal thinking enables adolescents to systematically manipulate variables and reason about unknowns such as algebraic variables. This stage of intellectual development can be of use to the students in the development, create of new idea as cognitive conflict help learners to make evidence the key concepts or propositions to be learned and suggest connections between new and previous knowledge. Constructivist thinks that learners build knowledge actively through interactions with environmental stimuli. Piaget's theory of conceptual change and its four stages of intellectual development will be incorporated into this study.

### **2.2.2 Van Hiele description of learning.**

Van Hiele described how children learn geometry. According to him, students' progresses through the level of thought in geometry and these levels have some characteristics. Van Hiele proposed that learning is a discontinuous process implying that there are different quantitative levels of thinking, which are sequential and hierarchical. A student cannot function adequately at one level without mastering most of the previous levels. The progress from one level to the other is more dependent upon instruction than age or level become understood implicitly at another level, and each level has its own language.

During teaching and learning, two people who reason at different levels cannot understand each other. They cannot follow the thought processes of the other. Language is a critical factor in the movement through the levels. Van Hiele distinguished five levels of geometry thought. These levels of thought can be summarized as follows:

**Level 0 (Visualisation):** Students reason about basic geometry concepts, primarily using visual consideration of the concepts as a whole. In transformation geometry, students recognize transformation by changes in the figure and motion "visual approach" without explicit regard to the properties of its components, thereby creating cognitive conflict misconceptions.

**Level 1: (Analysis).** Students can distinguish the properties of figures. However, they are unable to interrelate these properties. Students also see each property in isolation from other properties.

**Level 2 (Abstraction).** A network of relations between the properties is formed. At this level, students perceived the implications and class inclusion of the properties. At this level, students try to resolve cognitive conflicts earlier created.

**Level 3: (Deduction).** Students distinguished the nature and significance of deduction. However, they do not understand the requirement for rigor and the relation between the deductive systems, which are achieved at level 4.

**Level 4: (Rigor).** The Students can compare systems based on different axioms and can study geometry in the absence of concrete models.

Just Like cognitive conflict and collaborative methods of teaching scaffolds through stages, Van Hiele's theory requires students to progress through the above five levels of thought in sequential order in understanding geometry. The above stages will be incorporated into this study.

### **2.2.3 Bruner's theory of cognitive learning**

Bruner's theory states; "To perceive is to categorize, to conceptualize, to learn is to form categories, to make decisions is to categorize. He maintained that people interpret the word in terms of similarities and differences and suggested a coding system in which people have a hierarchical arrangement of related categories. Each successively higher level of categories becomes more specific. The major variable in his theory of learning is the coding system into which learners organize this coding system. He believes that the system facilitates transfer; enhance retention and increase problem solving and motivation. He also advocated the discovery or collaborative oriented learning method in schools which he believed helped students discover the relationship between categories and peers.

Cognitive conflict and collaborative instructional strategies are conceptual schemes belonging to constructivism philosophy, it is based on the learner's positivity just like the theories explain above. The teacher uses them as an educational tool in the educational environment for replacing misconceptions with correct alternative conceptions. Recently, modern instructional strategies such as cognitive conflict and collaborative instructional strategies have aimed to correct and modify misconceptions among learners by applying the constructivism theory principles. Based on Piaget's theory of cognitive development; Cognitive Conflict instructional strategy means that conflict in a learner's mind can lead to learning. In the learner's cognition, the meaning is self-constructed through a mental effort and resistance to conceptual change and based on Van Hiele Description of learning, and collaborative instructional strategies are scaffolds while students can only proceed level by level learning with peers. A student cannot function adequately at one level without mastering most of the previous levels. The progress from one level to the other is more dependent upon instruction than

age or level become understood implicitly at another level, and each level has its own language. Cognitive conflict and collaborative instructional strategies belong to the constructivism theorem of learning which will be incorporated into the above theories in this study.

### **2.3 Empirical Studies**

Studies have been carried out to investigate the effect of cognitive conflict and collaborative instructional strategy among student in different field of studies. Empirical Study has been review respectively, on Cognitive Conflict, Collaborative instructional strategies, achievement and retention in mathematics and other related discipline and subjects.

#### **2.3.1 Empirical review on cognitive conflict instructional strategy**

Dickens *et al.*, (2020) carried out a study on the Manifestation and meaning of Cognitive Conflict among Mathematics students in Embu, Kenya. The study adopted a mixed-methods research design. The study populations were 2800 from two secondary school mathematics students and their teachers of twenty-five public secondary schools in Embu, Kenya. The study used purposive sampling techniques to select the sub-county and the participants for the study. The sample size of 350 was determined using the Yamane model. The study used surveys and one-on-one semi-structured interviews to collect research data. The validity of the instruments was prepared by information obtained from different articles online by the researcher in consultation with supervisors. The result from the piloting study gave a Cronbach's Alpha of 0.74, which indicated that the instruments were reliable to obtain valuable information for the study. The findings indicate that students experienced cognitive conflict in three significant ways: a moment to (co) construct one's mathematical meaning, confusion as a result of the teacher's behaviorist stance, and a fleeting moment of magic. The paper



recommends that teachers should take advantage of cognitive conflict as a strategy for scaffolding mathematics learning by giving students tasks that provoke critical thinking so that as students work on the tasks, their naïve understandings of the concepts are challenged. The study could not determine the effect of cognitive conflict strategy on students achievement and retention in the Geometry aspect of Mathematics but only investigated the manifestation of Cognitive conflict in General mathematics, hence the purpose of this current research.

Susilawati *et al*, (2017) investigate cognitive conflict strategy to improve students' lateral mathematic thinking ability. The researcher adopted Quasi-Experiment Non-equivalent Pretest-Posttest Control Group Design on two homogenous groups for the study. Randomly chosen, samples were 73 student teachers of medium qualification at a local public religion-based higher education of 257 population in Bandung, Indonesia. The instrument consists of 6 essay questions, each with two low, two middle, and two high opened ended questions. Five experts in mathematics education validated them through content and face validity. The instruments were then revised and tested to analyze the reliability coefficient (0.76). Data analysis was done through the significance test of t-test for normally distributed data (Gaussian) and the Mann-Whitney U for test if the data was not Gaussian.

Meanwhile, the F test was employed to see the interaction between dependent variables if normality was met; if not, then Adjusted Rank Transform was used. The findings show that: The overall and prior mathematical knowledge of students' average value of experimental group which belongs to a high category, higher than of control group which belongs to medium category. He recommends that cognitive conflict be used to challenge students' lateral thinking in basic geometrical concepts, which will provide adequate opportunities for students to make observations, explore, investigate, and

experiments to see and suspect the existence of truth and then test it before completing a geometry problem. However, this research could not establish a relationship between Cognitive Conflict instructional strategy and students' achievement and retention in the geometrical aspect of mathematics. Hence, the purpose of this current research work.

La *et al* (2018) carried out a study to investigate the Implementation of cognitive conflict strategy in a cooperative setting for improving students' mathematical understanding ability. The researcher adopted a quasi-experimental research design. The population in the study was the students of Secondary School II (SSII) in a Vocational High School in Negeri, Kendari, 2018/2019 academic session, while the samples were students of SS IIa and SS IIb, which was drawn using a purposive sampling technique. The instrument used in this study is a test of mathematical understanding ability in the form of a description that includes indicators of students' mathematical understanding ability, three experts validated the instrument, and the reliability coefficient was 0.78 analyzed after a pilot test. Data collected were analyzed using mean, standard deviation and bar graphs to answer the research questions and Analysis of Covariance (ANCOVA) to test the null hypotheses at a 0.05 level of confidence.

The results of this research are: there is a significant difference of mathematical understanding ability of Vocational High School in Negeri, who were taught with cognitive conflict strategy in a cooperative setting and those who were taught with expository strategy, where learning with cognitive conflict strategy in the collaborative setting is better than learning with expository strategy. However, this research could not establish a relationship between Cognitive Conflict instructional strategy and students' achievement and retention in the geometrical aspect of mathematics. Hence, this current research work will cover the gap.

Luh (2016) carried out a study on the use of cognitive conflict instructional strategy to reduce student misconceptions on the subject matter of rectilinear motion in Watopute. A quasi-experimental design was adopted for the study. The population in this study were all students of Senior Secondary School (SSS) I in who registered in the odd semester 2015/2016 academic year. Purposive sampling techniques were used to sample all students in SSS I, Documentation and Diagnostic Test (DDT) to understand the concept in the form of a multiple-choice test with open grounds for data collection. Data obtained from the documentation and diagnostic tests were analyzed using mean, the standard deviation to answer the research questions and Analysis of Covariance (ANCOVA) to test the null hypotheses at a 0.05 level of confidence. The study found out that there was a decrease misconception after learning with cognitive conflict strategy on the subject matter of rectilinear motion. Therefore, the researcher recommended, among others, that teachers should consider using the Cognitive conflict instructional strategy in teaching physics content. However, the study could not determine the relationship between Cognitive Conflict instructional strategy and students' achievement and retention in the geometrical aspect of mathematics; hence, this current research work will cover the gap.

Dewi and Widada (2017) investigate the Influence of Contextual Learning Models and the Cognitive Conflict to Understand Mathematical Concepts and Problems Solving Abilities. The research adopted a quasi-experiment and applied factorial design, 2x2 design. The population of this study was the students of junior high school students (SMP) Kota Bengkulu with the sample of 80 SMP Kota Bengkulu selected by purposive sampling technique. The concept comprehension ability test (CCAT), problem-solving test (PST), and cognitive conflict test as the covariate was used for data collection. The data collected were analyzed using covariate analysis. The findings of this study are: 1)

direct effect of cognitive conflict covariate on the mean of comprehension ability concept for students taught by Contextual Learning Model better than Model; and 2) the direct influence of cognitive conflict covariates on mean Problem-Solving Ability for students taught by Contextual Learning Model is better than Conventional Learning Model. However, this research paper could not address how cognitive conflict strategy could be used to improve and encourage retention in the geometry aspect of mathematics. Therefore, this current research will look into the effect of cognitive conflict in improving students' performance and retention in geometry.

Luh (2020) investigate the application of cognitive conflict strategy to develop students' mathematical critical thinking ability and character. The study adopted a quasi-experimental research-based method design to find the learning trajectory of junior high school students regarding the set topic. The study built a three-stage design to see the relationship between learning goals. The study was done at SMP Negeri 5 Kuta Selatan. Purposive sampling techniques were used to sample out five students from grade IIa for the first cycle thirty-two students from grade IIb for the second cycle. And thirty-two students form grade II for the third cycle. This study consisted of observations and interviews, instruction, posttest, and interviews. The instruments used were the Hypothetical Learning Trajectory (HLT), student's worksheet, lesson plan for teaching guide, and teacher's guidance book. Three (3) experts validated these instruments. 0.75 reliability coefficient was achieved using Alpha Cronbach's. The data collected were using mean, the standard deviation to answer the research questions and Analysis of Covariance (ANCOVA) to test the null hypotheses at a 0.05 level of confidence. The research finds out that cognitive conflict instructional strategy increases students' critical thinking. The study recommended that teachers or instructors should adopt a cognitive conflict instructional strategy for teaching sciences. However, the study could

not address how cognitive conflict strategy could improve and encourage retention in the geometry aspect of mathematics; this current research will look into the effect of cognitive conflict in improving students' performance and retention in geometry. Hence, the purpose of this current research work.

Zetriuslita *et al.* (2018) investigate the association among mathematical critical thinking skills, communication, and curiosity attitude as the impact of problem-based learning and cognitive conflict strategy (PBLCCS) in the number theory course. The research adopted a correlation research design. The study population was 11,265 grade II students of the 2018/2019 academic session. Stratified sampling and purposive sampling techniques were used to sample out 1226 students for the study. The instruments include a pretest to examine students' critical thinking ability and mathematical communication and the questionnaire to obtain the scores data of curiosity attitude. The reliability coefficient was calculated to be 0.65 after three experts validated the instruments. The Chi-Square test was used to process the data by using software SPSS version 21.00 version, and the association was measured with the Pearson correlation coefficient ( $r_{xy}$ ). The findings showed that: 1) there was no association between critical thinking skill and mathematical curiosity attitude as the impact of applying problem-based learning cognitive conflict strategy, 2) there was no association between mathematical communication and curiosity attitude as the impact of applying problem-based learning and cognitive conflict strategy, and 3) the impact of applying problem-based learning cognitive conflict strategy was more influential in developing critical thinking skill than communication skill and curiosity. This research study could not address how cognitive conflict strategy could improve and encourage retention in the geometry aspect of mathematics; this current research will look into the effect of

cognitive conflict in improving students' performance and retention in geometry. Hence, the purpose of this current research works.

Tatiane and Edmilson (2020) carried out a study on Cognitive conflict in the strategic decision of management teams in small enterprises. The descriptive methodological approach was adopted based on qualitative methods and multi-case studies. The population of the study consisted of small business enterprises of SE. Simple random sampling techniques were used to sample out 25 small business enterprises from SE. Data were collected with interviews and analyzed within and cross-case procedures, according to Miles and Huberman's recommendations. Four cases of the small enterprise were studied. Cognitive conflicts considerably determined the strategic decision processes. Finding and recommendations: Such conflicts questioned decision possibilities and highlighted aspects related to intuition and improvisation, both normally useful and present in the strategic decision processes of SE. Cognitive conflict inhibits improvisation because its occurrence creates useful questionings in decision-making preparation. Those questions generated deep discussion and analysis for decision making in the studied SEs. However, this paper could not determine the effect of cognitive conflict in improving students' achievement and retention in the geometry aspect of mathematics, hence, the purpose of this current paper.

Madu and Emma (2015) carried out a study on the Effects of Cognitive Conflict Instructional Strategy on Students' Conceptual Change in Temperature and Heat. The design of the study was quasi-experimental. The subjects were 249 senior secondary II students from 2 schools purposively sampled from 12 secondary schools. One of the intact classes in each school was assigned to the control group. In one school, there were 70 subjects for the experimental group and 60 for the control group, while in the other school, there were 60 for the experimental group and 59 for the control group. The

instrument for obtaining the data was thermal concept evaluation (TCE). Students in both groups were pretested using TCE to establish their level of the initial understanding of heat and temperature; the instrument was originally developed by Yeo and Zadnik and adapted by the researchers. The data generated from the TCE were analyzed using frequency and chi-square statistics. Based on the findings, it was recommended that the Cognitive Conflict instructional strategy should be encouraged among science teachers. However, this study could not determine the effect of cognitive conflict in improving students' achievement and retention in the geometry aspect of mathematics, hence, the purpose of this current paper.

Subanji and Maharani (2018) studied scaffolding based on the cognitive conflict in correcting the students' algebra errors. The researcher uses the Mix Method, which is a combination of quantitative and qualitative methods. The study population was 259 Second Grade Students of Junior High school in Malang. Twenty-five students were purposely sampled out and tested on Algebra material. The instrument used in this research was Algebra tests and the interview guidelines for data collection. The data collection procedures conducted by the research were determining the research subjects, delivering the Algebra tests, and conducting the interview. The quantitative data were collected through an essay test, while the qualitative data were collected through interviews and observation. Data analysis was done through the significance test of t-test for normally distributed data (Gaussian) and the Mann-Whitney U for test if the data was not Gaussian. The research found that, among others, Cognitive Conflict can increase the students' reasoning ability and recommended that cognitive conflict be improved in classroom learning. This study was carried out to determine student errors on algebra using the Cognitive Conflict instructional strategy; hence, this current

research aims to cover the gap and determine the effect of cognitive conflict to improve students' achievement and retention in the geometry aspect mathematics.

Susilawati *et al.* (2017)) carried out a study improvement of the mathematical spatial visualization ability of students through cognitive conflict. The study adopted a quasi-experimental design, specifically a pretest and posttest control group design. A simple random sampling technique was used to sample 73 student teachers at a university in Bandung, Indonesia. They were categorized into two groups; 35 students of class B as a control group who received individual expository (conventional) and 38 students of class C as experiment group who were exposed to cognitive conflict strategy cooperatively in a group. As for the instrument, an initial test was administered to identify students' prior Mathematical Knowledge, followed by a pretest and posttest on the Mathematical spatial visualization ability test (MSVAT). Before the instruments were employed, they were validated by experts in mathematics education through content and face validity, and reliability was obtained as 0.65. Data analysis was done through the significance test of t-test for normally distributed data (Gaussian) and the Mann-Whitney U for test if the data was not Gaussian. Study findings show that the mathematical spatial visualization ability of students exposed to cognitive conflict strategy has a higher improvement level than students exposed to expository teaching based on overall and prior mathematical knowledge.

This study adds into this repertoire of literature on the effect of Cognitive Conflict Instructional Strategy on mathematics Achievement and Retention among secondary school (SS-II) students in geometry aspect of mathematics learning by considering how students manifest cognitive conflict and what meanings are embedded in these manifestations, to inform how teachers can utilize cognitive conflict instructional



strategy to improve students' Achievement and Retention in Geometry aspect of mathematics.

### **2.3.2 Empirical review on collaborative instructional strategy**

Ochuenwike *et al.* (2019) investigate the effect of collaborative learning strategy on students' achievement in senior secondary school Geometry. The study adopted a quasi-experimental design with the non-equivalent pretest-posttest control group. A sample of two hundred and ninety-eight (298) SS II students participated in this study. They were randomly selected from a population of 1845 students. One hundred and forty-four (144) of them are males, and one hundred and fifty-four (154), are females. Geometry Achievement Test (GAT) was the instrument used for data collection. It was developed after content validation and exposed to item analysis. The reliability of GAT was established using Kuder-Richardson Formula, K-R20. Mean and standard deviation was used to answer the research questions, while the analysis of covariance (ANCOVA) was applied to test the hypotheses. The findings revealed a significant difference between students taught geometry with collaborative learning strategy and those taught with the conventional method. Recommendations were made, including, among others: that teachers of mathematics at senior secondary school level should adopt the collaborative learning strategy in teaching mathematical/geometry concepts as it facilitates students' achievement in mathematics. However, this research only determined the students' achievement on geometry and couldn't investigate the relationship between collaborative instructional strategy and students' retention ability on geometry aspect of mathematics. Hence, the reason for this current research works.

Muraina (2019) carried out a study to investigate the effects of collaborative learning techniques and Mathematics anxiety on the mathematics learning achievement of secondary school students in Gombe State, Nigeria. Pretest-posttest, control group

quasi-experimental design with a 2x2 factorial matrix was used in the study. The targeted population was 21,360. The multi-stage sampling technique was used in sampling 80 participants from four local government areas in the state. Mathematics Learning Achievement Test (MLAT) of thirty-one (31) multiple-choice items with four options was used for data collection. The reliability coefficient of the instrument was 0.79, which was determined using Kuder–Richardson formula 20 (KR20). Data collected from the field were analyzed using independent samples t-test statistical analysis. The results showed a significant difference in the Mathematics learning achievement of secondary school students exposed to collaborative learning techniques and those in the control group ( $t= 58.75$ ;  $p<0.05$ ). In view of these findings, the study recommends that educational stakeholders should intensify their effort to organize conferences on the implications of collaborative learning techniques for effective interventions towards enhancing Mathematics learning achievement among secondary school students. However, this study could not determine the retention ability of students after being taught geometry using collaborative instructional strategy; the current research work will investigate and determine the retention ability of students on the geometry aspect of mathematics in the Minna.

Timothy (2018) investigated enhanced collaborative teaching method on the Performance of Students in Essay Writing Task in Ado Ekiti, Nigeria. The study adopted a quasi-experimental design approach. The sample consisted of 50 Senior Secondary School final year students through multi-stage sampling techniques. Essay Writing Test (EWT) validated by three senior lecturers with a reliability coefficient of 0.76 was used for data collection. The data generated were subjected to statistical analysis, and the analysis results showed no significant difference between the pretest scores of both the control and experimental groups, which established the homogeneity

of both control and experimental groups before the application of the treatment on the experimental group. Therefore, it is recommended that English language teachers employ this method to improve students' performance. Also, teachers should encourage teamwork among students for the exchange of novel ideas. To this effect, educational institutions could organize and train teachers on how to secure better performance through collaborative pedagogical inputs. This current paper will investigate the effect of cognitive conflict and collaborative instructional strategy on geometry achievement among secondary school students in Minna.

Achufusi and Ngozi (2021) investigate the effect of Collaborative Learning Strategy on Students' Academic Achievement in Chemistry in Onitsha Education Zone, Anambra State". Quasi-experimental research design, specifically, pretest-posttest non-equivalent groups design, was adopted for the study. The population of this study comprised 3115 SS I Chemistry students. A sample size of 49 (18 males and 31 females) SS I Chemistry students in intact classes were selected for the study using the purposive sampling technique. The researcher constructed a Chemistry Achievement Test (CAT) of 30 multiple choice items with options A-D for data collection. The instrument was subjected to face and content validation by three experts. The reliability of CAT was determined using Kuder-Richardson Formula 20 (KR-20), which yielded a reliability coefficient of 0.87. The scores obtained from the pretest and posttest was analyzed using mean and standard deviation as well as ANCOVA at the level of significance (0.05). Data analysis was carried out using SPSS version 23.0. The research could not establish any relation between collaborative instructional strategy and students' achievement and retention in geometry, hence his current research.

Ali *et al.* (2021) carried out a study on collaborative instructional strategy (CIS) for Teaching Mathematics at the Primary level in Pakistan. Quasi-experimental design,

specifically pretest-posttest comparative group design, was adopted for the study. The sample size was 64 students of two groups (control and experimental). Mathematics Attainments Test (MAT) was developed to measure students' academic achievement. Collaborative mathematics instructional lesson plans (CMIL) were also developed to teach mathematics. The collected data were analyzed using the mean, standard deviation, pair sample t-test and independent-sample t-test. It was recommended that Collaborative Instructional Strategy (CIS) might be used to teach mathematics at the primary level. However, this research couldn't establish any relationship between collaborative instructional strategy and students' achievement and retention in geometry, hence his current research.

Roki (2019) investigate inquiry learning and collaborative learning strategies in teaching writing within a game-based learning framework. The design of the study was a mixed-method explanatory design. 37 students purposively selected at the higher education level of a population of 2753 students. Flow Condition Questionnaire (FCQ) was developed and used for data collection. Three experts validated the instrument, and the reliability coefficient, 0.76, was calculated using Chronbach Alpha. The statistical analysis used is ANOVA and post hoc analysis to compare the means between groups. Although the collaborative group is not significantly different compared to the control group, the students' learning experience was positive according to the Flow Condition Questionnaire and the structured interview. The students felt that learning with GBL was challenging, fun, serious, and easy to follow. However, the research study couldn't establish any relationship between collaborative instructional strategy and students' achievement and retention in geometry, hence the purpose of his current research.

Otiende *et al.*, (2021) carried out a study on the effect of collaborative teaching strategy on Students' academic achievement in physics in public secondary schools in Nyeri

County, Kenya. The quasi-experimental design was used. The target population was the entire form four Physics students (2021 academics session) in Nyeri County studying in Public secondary schools. A sample of 173 forms four students in four mixed day secondary schools in Nyeri County participated in the study. The sample was obtained through simple random and purposive sampling to obtain a list of mixed day secondary schools that offer the Physics subject at form four. The research instruments consisted of Physics Pre-test (PPT) and Physics Achievement Test (PAT). The Kuder Richardson test was used to determine the reliability of the PPT and PAT. A reliability coefficient of 0.7 and 0.8 was obtained for the PPT and PAT, respectively. Descriptive statistics (mean, standard deviation) and inferential statistics (t-test and ANOVA) were used in data analysis. The Statistical Package for Social Sciences (SPSS) version 22.0 was used for statistical analysis. The study established that Collaborative Teaching Strategy enhanced students' achievement in Physics. The study recommended that teachers should expose students to Collaborative Strategy more frequently and teacher training programs to equip teachers with skills for collaboration. However, this research could not establish any relationship between collaborative instructional strategy and students' achievement and retention in geometry, hence his current research.

Sweller *et al.* (2016) carried out a study on collaborative learning in improving the effectiveness of worked examples in learning mathematics. A quasi-experimental design was adopted for the study. A sample of One hundred twenty-two (122) students from four Year 7 classrooms in an Indonesian school in Kudus, Central Java, participated in the study. The mathematics achievement test (MAT) was used for data collection. Four Mathematics experts validated the instrument, and the reliability coefficient was calculated to be 0.78 using the Kuder Richardson test. The data collected were analyzed using Descriptive statistics (mean, standard deviation) and inferential statistics t-test and

(ANOVA). The Statistical Package for Social Sciences (SPSS) version 22.0 was used for statistical analysis. The result of the study revealed that collaborative instructional strategy had a significant positive effect on students' achievement in mathematics. Based on the results, it was recommended that mathematics teachers should adopt a collaborative strategy as a teaching technique to improve students' performance in schools. However, the research could not establish any relationship between collaborative instructional strategy and students' achievement and retention in geometry, hence his current research.

Al-kaabi (2016) carried out a study on the effects of collaborative learning on the Achievement of Students with Different Learning Styles at Qatar University (QU). The population of the study was 435 students. Multi-stage sampling techniques were used to select 81 students aged between 19 and 22. Achievement Test was developed and used for data collection. Three (3) experts validated the instrument, and the reliability coefficient of 0.75 was found using the Kuder Richardson test. Data were analyzed using SPSS. The study analysis involved first establishing whether the data followed a normal distribution. Then, one-way Analysis of Variance (ANOVA) was used to compare students' achievements due to the learning style, and III Independent Sample T-tests were utilized to discover if statistical evidence was significantly different. The findings revealed that collaborative learning had no beneficial effect on the students' exercises and poster skills or their midterm exam scores. However, collaborative learning significantly affected the students' proposal writing scores. Other than that, collaborative learning did not affect exam results, including pretest, posttest, midterm or final exam. The study could not establish any relationship between collaborative instructional strategy and students' achievement and retention in geometry, hence the purpose of his current research.

This paper adds into this repertoire of literature on the effect of Collaborative Instructional Strategy in mathematics Achievement and Retention among secondary school (SS-II) students in geometry aspect of mathematics learning, with a view to informing how teachers can utilize collaboration of the students to improve students' Achievement and Retention in Geometry aspect of mathematics.

### **2.3.3 Empirical review on achievement**

Onu *et al.* (2020) investigate Biology students' interest and Achievement through Collaborative Instructional Strategy in Obollo-A for education zone. The study adopted a quasi-experimental research design. The study population comprised of 1,691 SSI Biology students, from which a sample of 200 students from six (6) intact classes were sampled using a multi-stage sampling procedure to take part in the study. The biology achievement test and Biology interest inventory with a reliability coefficient of 0.78 were instruments used to collect data for the study. Data were analyzed using mean, standard deviation and ANCOVA. Findings revealed that students taught Biology using collaborative instructional strategy had better achievement and interest ratings than those taught with the conventional method, and female Biology students have slightly better interest and achievement than male Biology students when taught with collaborative instructional strategy and the interaction effect of gender and instructional method on achievement is significant. The study concludes that considering the ability of the collaborative instructional strategy to improve interest and achievement in Biology, it should be adopted as a method of teaching the subject in Nigerian secondary schools. Unlike Onu's paper, this present paper will cover Mathematics, especially the geometry aspect.

Nisreen (2019) carried out a study on the effectiveness of cognitive Conflict strategy in improving Academic achievement and modifying sex education misconceptions in

science courses among Intermediate Second-Grade Students. A quasi-experimental design was used for the study. The study population was represented by all the intermediate second-grade students in Jeddah. The sample was randomly selected and consisted of 56 intermediate second-grade students in Jeddah, equally distributed to two groups: an experimental group (28 students) and a control one (28 students). The instrument used for the research was an achievement test (AT) with a reliability coefficient of (0.83) which was achieved using Alpha Cronbach. Data collected were analyzed using mean, standard deviation and bar graphs to answer the research questions and Analysis of Covariance (ANCOVA) to test the null hypotheses at a 0.05 level of confidence. The study recommended holding training courses, workshops, conferences and synopses for Science teachers and supervisors on the use of cognitive conflict instructional strategy. The researcher concentration was on all science subjects. The research concentration is on geometry in this current work, and the scope is Minna.

Okeke *et al.* (2018) investigate the impact of collaborative learning strategy on senior secondary school chemistry students' academic achievement in the bio-akpor local government area of Rivers State. The study adopted a quasi-experimental pretest-posttest control group design. A total sample size of hundred (100) students participated in the study. The instrument used for data collection was a chemistry achievement test. The reliability coefficient was calculated using Person Product Moment Correlation with a reliability coefficient of 0.87. The research questions were analyzed with mean and standard deviation for data analysis. Analysis of covariance (ANCOVA) was used to test hypothesis 1, while t-test was used to test hypothesis 2 at a 0.05 level of significance. Findings showed that students who studied chemistry using a collaborative learning strategy achieved better than the lecture-based method. Some recommendations were made based on the findings. For example, science teachers should spice up their



teaching methods by using the collaborative method in their lesson delivery since it enhances achievement and active participation. However, the research could not establish any relationship between collaborative instructional strategy and students' retention in geometry, hence his current research.

Kumar (2017) carried out a study on the effect of Collaborative learning on enhancing student achievement in Montreal, Canada. A quasi-experimental design was adopted for the study. The study population was 2434 grade II students in Montreal, Canada. A simple random sampling technique was used to sample out 265 participants/students for the study. An achievement test was used to collect data for the study. Three experts validated the instrument, reliability of the instrument was found to be 0.75 alpha coefficient using the Cronbach-Alpha method. Data analysis of representative studies ( $k= 28$ ) of the instrument yielded a moderately weighted average effect size of 0.26. A mixed-effects model was used for the analysis of the moderators of effect size. The study found out that analyses of moderator variables were not significant or suffered from a lack of statistical power (i.e., grade levels). Implications for the use of collaborative learning are discussed, and recommendations for future researchers are suggested, along with the limitation and conclusions. This study couldn't determine the effect of collaborative instructional strategy as it is used to improve students' achievement and retention in the geometry aspect of mathematics, hence, the purpose of this current paper.

### **3.3.4 Empirical review on retention**

Joseph and Sule (2019) carried out a study on collaborative instructional strategy and secondary school student' chemistry retention tests in Adamawa State". The study adopted a quasi-experimental design which involves a pretest-posttest non-equivalent control group design. The sample for this study consisted of 114 subjects selected from

a population of 3543 SSII chemistry students in 118 public secondary schools. The sampling involved stratified purposeful and random sampling techniques. An instrument for this study was the tagged chemistry achievement test (CAT) developed by the researcher. Three experts validated the instrument, and a reliability coefficient of 0.85 using the K-R 20 formula was obtained. The data obtained were analyzed using Analysis of Covariance at 0.05 level of significance. The following major findings were obtained: there is significant retention of what has been learnt by the students in chemistry when taught using collaborative teaching strategy. Based on these findings, it was recommended that teachers use a collaborative teaching strategy to teach chemistry to increase retention of learned materials, thereby improving their Senior Secondary Certificate Examination results. However, this research could not establish any relationship between collaborative instructional strategy and students' achievement and retention in geometry, hence his current research.

Chianson *et al.* (2020) investigate the effect of collaborative learning strategy on students' retention in circle geometry in secondary schools in Benue State, Nigeria. The design adopted for the study was quasi-experimental. The sample for the study was 358 SSII students randomly selected from three education zones in Benue State, 2019-2020 academic session. Purposive sampling was used to select six government co-educational schools for the study. Simple random sampling was used to select two intact classes from each of the six schools. The study was conducted such that a Pre-GAT (Geometry Achievement Test) was administered, five weeks after circle geometry was taught to both groups of students, and Post-GAT was administered. Four weeks after the Post-GAT, the RET-GAT (Retention Geometry Achievement Test) was administered to the students. The GAT is 20 items with the 4-option multi-choice test. Three experts validated the instrument, and the instrument's reliability was found to be 0.73 alpha

coefficient using the Cronbach-Alpha method. Statistics of mean and standard deviation were used to test the hypotheses and determine the relationship of the data collected. The study's findings confirmed that students who were subjected to the collaborative learning strategy were able to retain the concepts of circle geometry more than those who were taught using the conventional learning approach. Hence there commendations were that students would be able to retain taught and learnt concepts in mathematics for a longer period if mathematics teachers applied the collaborative learning strategy in teaching. The content scope of this research is circle geometry and was carried out in Benue state, and the researcher could not cover the whole geometry aspect of mathematics hence the reasons for this current research.

Rita *et al.* (2019) investigate collaborative instructional strategy and students' retention in chemical reactions in Umuahia South Local Government Area, of Abia State. The study adopted a quasi-experimental pretest, posttest, non-randomized design. The sample size was one hundred and twenty (120) SS2 Chemistry students from three co-educational private secondary schools. An instrument titled Physical Chemistry Performance Test (PCPT) was used for data collection. The instrument's reliability was determined using Pearson Product Moment Correlation (PPMC), and the reliability coefficient obtained was 0.85. Mean and Standard Deviation were used in answering the research questions, while t-test and Analysis of Covariance (ANCOVA) were used to test the hypotheses using the SPSS package. The result of the study revealed that collaborative instructional strategy had a significant positive effect on students' retention in chemical reactions. Based on the results, it was recommended that Chemistry teachers should adopt a collaborative strategy as a teaching technique to improve students' retention of knowledge in schools. However, the study could not establish any

relation between collaborative instructional strategy and students' achievement in geometry, hence the purpose of his current research.

Emmanuel (2020) carried out a study to investigate the Utilization of Cognitive conflict instructional strategy and conceptual change pedagogy to enhance students' retention in thermal physics in Kogi East Education Zone, Kogi State. A quasi-experimental research design was adopted for the study. Specifically, pretest, posttest non-equivalent-control group design was used for the study. The study population was all the 7380 senior school two physics students in 153 co-educational secondary schools during the 2018/2019 academic session. The sample consisted of 294 SS two physics students drawn using a multi-stage sampling technique. Thermal Physics Attention Inventory (TPAI), validated by three experts with a reliability coefficient of 0.78, was used for data collection. Data collected were analyzed using mean, standard deviation and bar graphs to answer the eight research questions and Analysis of Covariance (ANCOVA) to test the null hypotheses at a 0.05 level of confidence. The study's findings revealed a significant difference in mean attention scores among students taught thermal physics using cognitive conflict instructional strategy. It was recommended that professional bodies and stakeholders in education encourage and support the teaching and learning of Physics using cognitive conflict instructional strategy and conceptual change pedagogy by organizing workshops, seminars, and conferences on its use by Physics teachers. This research work was on physics. While the current work will be done on mathematics, especially the geometry aspect, the work will investigate the effect of Cognitive conflict on students' achievement and retention of geometry aspects of mathematics.

#### **2.4 Summary of Literature Review**

This chapter explained the purpose of the study, concerning the above information from the literature reviewed, and shows that cognitive conflict and collaborative instructional

strategies strategy is highly recommended in learning and teaching mathematics and achievement, retention in geometry. A great amount of related literatures have been reviewed as provided in the conceptual framework and empirical study which together made the chapter two of this study. Several scholars investigated the effect of cognitive conflict and collaborative instructional strategy on students' achievement and retention in difference discipline, among others are: Emmanuel (2020), Susilawati *et al.* (2017), Madu and Emma (2015) and Chianson *et al.* (2020), Rita *et al.* (2019), Okeke *et al.* (2018) Just like the current study the researchers used quasi experimental research design. Their studies confirmed that students who were subjected to cognitive conflict and collaborative learning strategy performed well and retain the concepts more than those who were taught using the conventional learning approach. Furthermore, as confirmed by some scholars in the reviewed literature such as Mufit *et al.* (2018), Zazkis and Chernoff (2016), Abdullahi (20115), Shafi (2018) and many scholars, cognitive conflict and collaborative instructional strategies improve the ability of both teacher and students and facilitate teaching and learning geometry concepts which lead to greater achievement and retention in geometry among secondary students. Furthermore, cognitive conflict and collaborative instructional strategies expose students/learners to different ways of thinking about and solving problems and improve their ability to recognize mathematics relationships.

Differently, this research study compared the effect of cognitive conflict, collaborative instructional strategies and lecture method. In addition to other studies, this study investigated the influence of gender on the achievement and retention in geometry as all the reviewed research studies could not cover that. It is essential to further research and apply cognitive conflict and collaborative instructional strategies in learning geometry and its effect on geometry achievement and retention among secondary school students

in the Minna. In recent studies, less attention has been given to cognitive conflict and collaborative instructional strategies especially in Minna, Niger State. In this respect, the effect of cognitive conflict and collaborative instructional strategies on achievement and retention in geometry among secondary school students in Niger State becomes necessary.

## CHAPTER THREE

### 3.0 RESEARCH METHODOLOGY

#### 3.1 Research Design

The study used quasi-experimental research design (Non-equivalent, Non-randomized and Experimental-Control group design). A quasi-experimental design is a type of design that aims to establish a cause and effect relationship between an independent and dependent variable. However, the quasi-experimental design does not rely on random assignment. Instead, subjects are assigned to groups based on non-random criteria. Quasi-experimental is a valuable tool in situations where true experimental cannot be used for ethical or practical reasons. In a non-equivalent quasi-experimental research design, the researcher chooses existing groups that appear similar but where only one of the groups experiences the treatment.

**Table 3.1: Research design layout**

Groups	Pretest	Treatment	Post-test	Retention
Cognitive Conflict (Exp I)	O <sub>1</sub>	X <sub>1</sub>	O <sub>2</sub>	O <sub>3</sub>
Collaborative (Exp II)	O <sub>1</sub>	X <sub>2</sub>	O <sub>2</sub>	O <sub>3</sub>
Conventional (Control Group)	O <sub>1</sub>	X <sub>0</sub>	O <sub>2</sub>	O <sub>3</sub>

Where.

O<sub>1</sub>= Pretest Scores

O<sub>2</sub> = Post-test Scores

O<sub>3</sub> = Retention Scores

X<sub>1</sub>= Experimental Treatment (Cognitive Conflict Instructional Strategy)

X<sub>2</sub> = Experimental Treatment (Collaborative Instructional Strategy)

$X_0$  = No Treatment (Lecture method)

### **3.2 Population of the Study**

The population for the study consist all SSII mathematics students in senior secondary schools of 2020/2021 academic session in Minna. The target population is 8,251 (3,923 male and 4,328 female) senior secondary schools Mathematics students in SSII of 22 public secondary schools in Minna, Niger State. (Niger State Ministry of Education, 2020). (See Appendix L). The choice of SSII is based on the fact that the aspect of mathematics concepts to be taught, in terms of concept and content fall under SSII syllabus and scheme of work. In addition, the state public schools will be considered, mainly due to their common socio-economical background, admission and promotion policy, staffing and availability of instructional materials.

### **3.3 Sample and Sampling Techniques**

A simple random sampling techniques was used to select three senior secondary schools in Minna, Niger state. The researcher used three intact class which give a sample of 226 senior secondary mathematics students (123 male and 103 female) from the three selected senior secondary schools in Minna, Niger State. The three intact classes were designated as experimental 1 and 2 and control groups. An intact class is an already formed classroom of students that no further selection procedure is used, but the entire classroom is used to represent and serve as a sample of the population. The three selected senior secondary schools are:

1. Ahmadu Bahgo secondary school,
2. Zarumai Model Secondary school and
3. Hilltop Model Secondary School Minna



**Table 3.2: Distribution of sample size by schools, classes and gender**

<b>Groups</b>	<b>Schools</b>	<b>Male</b>	<b>Female</b>	<b>Total</b>
Experimental 1	Ahmadu Bahago Secondary School	46	25	71
Experimental 2	Hilltop Model Secondary School	38	36	74
Control Group	Zarumai Model	39	42	81
<b>Total</b>		<b>123</b>	<b>103</b>	<b>226</b>

(School Records)

### **3.4 Research Instruments**

The research instruments were used. They are:

(A) Treatment Instrument and (B) Test Instrument

**(A) Treatment instruments:** Three treatment instruments were used for the study. The three instrument are:

- i.** Cognitive Conflict Instructional Strategy Lesson Plan (Exp<sub>1</sub> Lesson Plan),
- ii.** Collaborative Instructional Strategy Lesson Plan (Exp<sub>2</sub> Lesson Plan) as experimental groups, which are the same in contents, objectives and evaluation and
- iii.** Conventional Instructional Strategy as a control group. Also, the two Instructional Strategies that will be used as an experimental group are constructivist teaching approaches and conceptual change approach.

**(i) Cognitive instructional strategy lesson plan (Exp<sub>1</sub> lesson plan):** In Cognitive Conflict Instructional Strategy (CCIS) lessons Plan, there are three stages, which are:

- a. Identifying students' current state of knowledge. This was done through the use of a pretest. In this case, Planning and Placement Team (PPT) will be used to determine students' preconceptions in Geometry.
- b. Confronting students with contradictory information: it is usually presented through texts, verbal questioning and debating, who make explicit the contradictions or only

guide the debate with the students or among peers or by the teacher and new techniques and;

- c. Evaluating the degree of change between students' prior ideas or beliefs and a posttest measure after the instructional intervention. The conflict is often induced by presenting information clearly for the experiment, or the teacher contradicts children's or students' ideas, beliefs, or theories.

For clarity, the researcher divided step two into four more steps and make the steps to be six steps. The six steps are:

**Step 1: Introduction:** Identifying students' alternative conception;

**Step 2: Presentation:** Presentation of the anomalous situation;

**Step 3: Arousing CC:** Creation of Cognitive Conflict with the anomalous situation.

**Step 4: Resolution of conflict:** The student resolves the cognitive conflict individually.

**Step 5:** Discussion and summary

**Step 6:** Evaluate the degree of change between students' initial idea and posttest measure. (See Appendix B)

In this method, students will be grouped into two pairs. First, the teacher will demonstrate an anomalous situation experiment to ascertain students' alternative conceptions. After this, students will be allowed to do the experiment and come up with the result that contradicts their previous conceptions and sets students in cognitive conflict. Finally, the students was asked to discuss the result of the experiment and their previous ideas with their peers.

Lastly, the teacher will collect the different ideas about the experiment, summarize them on the board and discuss them with the class through which correct ideas will be determined and explained in detail.

**(ii) Collaborative Instructional strategy lesson plan (Exp2 lesson plan):** In

Collaborative Instructional Strategy (CIS) Lesson Plan has the following steps:

**Step 1: Introduction:** it refers to assessing the previous knowledge or skills of the learners through discussion or questioning.

**Step 2: Task discussion:** introducing the new task to arouse learners' interest and presentation of task through traditional method of teaching.

**Step 3: Collaborative learning:** Students' are carefully divided into groups, and they are presented with a task to achieve a common goal.

**Step 4: Number head:** each team member discusses their work to a large group.

**Step 5: Apprenticeship:** students work individually.

**Step 6: Assessment:** students exchange their works to assess and discover their zone of proximal development (ZPD). (See Appendix C)

**(iii) Conventional Instructional strategy (TIS) lesson plan:** The teacher will use discussion teaching method. The Lesson was Plan in the following steps:

**Step 1:** Diagnosing students' misunderstanding

**Step 2:** Exploring the phenomena using the lecture and discussion method.

**Step 3:** Discussion of what the students read from the Geometry text books

**Step 4:** Development of dissatisfaction with the pre-existing conceptions

**Step 5:** Develop fruitfulness based on what they have read from the text, the teachers' explanation and their pre-existing conceptions. Here the teacher uses examples to explain the concept to the students. (See Appendix D)

**(B) Test instruments:** Two Test Instruments were used for this study. They are:

- i. Geometry Achievement Test (GAT)

ii. Geometry Retention Test (GRT)

**(i) Geometry achievement test (GAT):** Geometry Achievement Test (GAT) questions were drawn by the researcher through careful study of past examination questions of West Africa Examination Council and National Examination Council. This instrument consists of section A (Instructions and Bio-Data) and Section B (Multiple Choice Objectives Questions). Section A requires the students to read carefully the instructions on how to answer questions and fill in their details such as Gender, School Name, Class and Date, While Section B Contains forty (40) questions with four options (A-D) each. In addition, this instrument content geometry concepts of mathematics. Students were asked to choose the correct option from the four given options lettered (A-D) within the period of (30) minutes. GAT was used to measure Pre-test and Post-test scores of experimental and control groups. **(See Appendix E)**

**(ii) Geometry retention test (GRT):** Geometry Retention Test was developed through the randomization of Geometry Achievement Test (GAT) that is, the questions numbers and options were reshuffled. Just like GAT, GRT consist of two section A (Bio Data) and B (Multiple Choice Objectives Questions). Section A requires the students to fill in their details such as Gender, School Name, Class and Date, while section B consists of forty (40) questions with four options (A-D) each. Students were asked to choose the correct option from the four given options lettered (A-D) within the period of (30) minutes. In addition, this instrument content geometry concepts of mathematics. Geometry retention test (GRT) was used to measure retention scores of both the experimental and control groups.

### **3.5 Validity of research Instruments**

Three experts validated the face and content validity of the research instrument used for this study, they are; two senior lecturers in the Science Education Department, Federal

University of Technology, Minna and a senior lecturer in the Mathematics department, Federal University of Technology, Minna. The experts' observations, corrections and suggestions were used to produce the final copy of the instrument used for the study.

### **3.6 Reliability of Research Instrument**

In order to determine the reliability of the Geometry Achievement Test (GAT). Using test retest method, the GAT was subjected to pilot study on a sample of thirty (30) students (17 males and 13 females) in a school within the population of the study but not part of the schools to be used for the research study. The data obtained through test retest was used to assess the reliability of the research instrument using Pearson's product moment correlation coefficient (PPMCC). The reliability coefficient obtained was 0.72, this was considered an adequate reliable measure for the test instrument and was considered good enough for the research study.

### **3.7 Method of Data Collection**

With an authorized introductory letter (**See Appendix M**) collected from Science Education Department, Federal University Technology, Minna, Niger State. The researcher visited the three (3) selected senior secondary schools in Minna, and sought for the co-operation of the principals, staffs and even the students in those schools and their consent to participate. The mathematics teachers in those schools also assisted the researcher in guiding and controlling the students during the administration of both treatment and test instruments. The method of data collection is based on administering the test instrument to both groups (experimental and control groups). The teaching was carried out for the period of six (6) weeks by the researcher with the two experimental groups taught using cognitive conflict and collaborative instructional strategies while the control group was taught using lecture method. The Geometry Achievement Test (GAT) administered to the students served the purpose of pre-test in order to ascertain

the cognitive achievement of the students in geometry concepts before proper treatment was administered. Each lesson lasted for a period of (45 minutes). After finishing the experiment, a post-test was administered in order to measure the achievement of the sampled students and retention test was also administered after the period of one (1 week). The researcher also ensured that the instructions designed on the test instrument was clearly understood and followed by the students in order to be able to attempt to questions in the research instrument. The test was conducted at the same time and the scripts were collected immediately for marking. The pre-test score, post-test score and retention scores were subjected to data analysis.

**Table 3.3 Data collection steps, activities and duration**

<b>S/N</b>	<b>Steps</b>	<b>Activities</b>	<b>Duration</b>
1.	Visitation	Visitation of the Sampled Schools, tendering of introductory letter, sougning permission, Inspection of their facilities and training of mathematics teachers as research assistance	1 Week
2.	Administration of Pretest	Administration of pretest to the experimental and control groups	1 Week
3.	Teaching	Main Teaching/treatment	6 weeks
4.	Administration of Post-test	Administration of Post-Test to Assess Students Achievement on Geometry.	1 Week
5.	Break	Break for one week	1 Week
6.	Administration of retention-test	Administration of Retention-Test to assess Students Retention on Geometry	1 Week

### **3.8 Method of data Analysis.**

The data collected for the research study was analyzed using descriptive statistics of mean and standard deviation to answer research questions while inferential statistics of Analysis of Variance (ANOVA) was used to test hypothesis 1 and 2 while sample t-test was used to test hypotheses 3-6 at 0.05 level of significance. This level of significance forms the basis for the acceptance or the rejection of each of the hypotheses raised; the Statistics Package of Social Sciences (SPSS), version 23.0, was used for the data analysis.

## CHAPTER FOUR

### 4.0

### RESULTS AND DISCUSSIONS

#### 4.1 Pretest Analysis

**Table 4.1 Summary of ANOVA Analysis of Pre-test of Experimental Group 1 and 2 and Control group.**

<b>Group</b>	<b>Sum of Squares</b>	<b>Df</b>	<b>Mean Square</b>	<b>F</b>	<b>Sig</b>
Between Groups	128	2	.064	0.004	0.999
Within groups	3914.651	223	17.554		
Total	3914.779	225			

**Ns = Not significant at 0.05 level**

Table 4.1 presented the Analysis of Variance (ANOVA) result of pre-test scores of experimental group 1 and 2 (cognitive conflict and collaborative instructional strategy) and control group. The result indicate that there is no significant difference in the pretest scores between experimental 1 and 2 and control groups,  $F(0.004) = p\text{-value}(0.996)$  is greater than 0.05 alpha-level ( $p = 0.996 > 0.05$ ). With these findings the groups are comparable.

#### 4.2 Answers to Research Questions

**Research question one (1):** What is the difference in the mean achievement scores of students taught geometry concepts using cognitive conflict, collaborative instructional strategies and lecture method? To answer this research question, Mean and Standard Deviation were used as presented in Table 4.2



**Table 4.2: Mean Achievement Scores and Standard Deviations of Experimental Group 1 and 2 and Control Group.**

Groups	N	Pretest		Posttest		Main Gain	M.G.D
		Main ( $\bar{x}$ )	S.D	Main ( $\bar{x}$ )	S.D		
Experimental 1	71	15.66	3.28	27.03	5.44	11.37	0.28
Experimental 2	74	15.62	4.53	27.27	4.83	11.65	
Control Group	81	15.60	4.56	17.81	4.04	2.21	

Table 4.2 presented the Mean ( $\bar{x}$ ) and Standard Deviation of Achievement Scores for experimental group 1 and 2 (cognitive conflict and collaborative instructional strategies) and control groups. It is observed that both groups had improvement as indicated in their posttest. The Experimental group 1 had the mean ( $\bar{x}$ ) achievement gain scores of 11.37 with standard deviation 5.44 and Experimental group 2 had the mean ( $\bar{x}$ ) achievement gain of 11.65 with standard deviation of 4.83 while control group had the lowest mean ( $\bar{x}$ ) achievement gain scores of 2.21 with standard deviation 4.04. Experimental group 1 and 2 had an achievement Mean gain difference of 0.28 in favor of Experimental group 2.

**Research question two (2):** What is the difference in the mean retention scores of students taught geometry concepts using cognitive conflict, collaborative instructional strategy and lecture method? To answer this research question, Mean and Standard Deviation were used as presented in Table 4.3

**Table 4.3: Mean Retention Scores and Standard Deviations of Experimental Group 1 and 2 and Control Group.**

Groups	N	Posttest		Retention		Main Gain	M.G.D
		Main ( $\bar{x}$ )	S.D	Main ( $\bar{x}$ )	S.D		
Experimental 1	71	27.03	5.44	27.24	5.31	0.21	0.14
Experimental 2	74	27.27	4.83	27.34	4.79	0.07	
Control Group	81	17.81	4.04	18.01	4.04	0.20	0.01

Table 4.3 presented the Mean ( $\bar{x}$ ) and Standard Deviation of Retention Score for experimental group 1 and 2 (cognitive conflict and collaborative instructional strategies) and control groups. It is observed that both groups have high as indicated in their retention test. The Experimental group 1 had the highest mean ( $\bar{x}$ ) retention gain of 0.21 with standard deviation 5.31 and Experimental group 2 had the lowest mean ( $\bar{x}$ ) retention gain of 0.07 with standard deviation of 4.79 while control group had the mean ( $\bar{x}$ ) retention gain scores of 0.20 with standard deviation 4.04. Experimental group 1 and 2 had a retention Mean gain difference of 0.14 in favor of Experimental group 1. Control group had higher retention than Experimental group 2 and very close to Experimental group 1 with Mean gain difference of 0.01.

**Research question three (3):** What is the difference in the mean achievement scores of male and female students taught geometry concepts using cognitive conflict instructional strategy? To answer this research question, Mean and Standard Deviation were used as presented in Table 4.4

**Table 4.4: Mean Achievement Score and Standard Deviation of Male and Female in Experimental Group 1**

Gender	N	Pretest		Posttest		Main Gain	M.G.D
		Main ( $\bar{x}$ )	S.D	Main ( $\bar{x}$ )	S.D		
Male	46	16.11	3.49	25.85	5.29	9.74	4.62
Female	25	14.84	2.73	29.20	5.12	14.36	

Table 4.4 presented the Mean ( $\bar{x}$ ) Achievement Scores and Standard Deviation of Male and Female for experimental group 1 (cognitive conflict instructional strategy). It is observed that male had a mean ( $\bar{x}$ ) gain 9.74 with standard deviation of 5.29 while the female had a mean ( $\bar{x}$ ) gain of 14.36 with standard deviation of 5.12. The male and female students had achievement Mean gain difference of 4.62 in favor of female students.

**Research question four (4):** What is the difference in the mean achievement scores of male and female students taught geometry concepts using collaborative instructional strategy? To answer this research question, Mean and Standard Deviation were used as presented in Table 4.5

**Table 4.5: Mean Achievement Scores and Standard Deviation of Male and Female for Experimental Group 2**

		Pretest		Posttest			
Gender	N	Main ( $\bar{x}$ )	S.D	Main ( $\bar{x}$ )	S.D	Main Gain	M.G.D
Male	38	16.26	3.90	27.32	4.51	11.06	1.22
Female	36	14.94	5.08	27.22	5.21	12.28	

Table 4.5 presented the Mean ( $\bar{x}$ ) Achievement Scores and Standard Deviation of Male and Female for experimental group 2 (collaborative instructional strategy). It is observed that male had a mean ( $\bar{x}$ ) gain 11.06 with standard deviation of 4.51 while the female had a mean ( $\bar{x}$ ) gain of 12.28 with standard deviation of 5.21. The male and female students had achievement Mean gain difference of 1.22 in favor of female students.

**Research question five (5):** What is the difference in the mean retention scores of male and female students taught geometry concepts using cognitive conflict instructional strategy? To answer this research question, Mean and Standard Deviation were used as presented in Table 4.6

**Table 4.6: Mean Retention Scores and Standard Deviation of Male and Female for Experimental Group 1**

		Posttest		Retention test			
Gender	N	Main ( $\bar{x}$ )	S.D	Main ( $\bar{x}$ )	S.D	Main Gain	M.G.D
Male	46	25.85	5.29	26.20	5.23	0.35	0.11
Female	25	29.20	5.12	29.44	4.87	0.24	

Table 4.6 presented the Mean ( $\bar{x}$ ) Retention Scores and Standard Deviation of Male and Female for experimental group 1 (cognitive conflict instructional strategy). It is observed that male had a mean ( $\bar{x}$ ) gain 0.35 with standard deviation of 5.23 while the female had a mean ( $\bar{x}$ ) gain of 0.24 with standard deviation of 4.87. The male and female students had a retention Mean gain difference of 0.11 in favor of male students.

**Research question six (6):** What is the difference in the mean retention scores of male and female students taught geometry concepts using collaborative instructional strategies? To answer this research question, Mean and Standard Deviation were used as presented in Table 4.7

**Table 4.7: Mean Retention Scores and Standard Deviation of male and female for Experimental Group 2**

Gender	N	Posttest		Retention		Main loss	M.L.D
		Main ( $\bar{x}$ )	S.D	Main ( $\bar{x}$ )	S.D		
Male	38	27.32	4.51	27.29	4.42	-0.03	0.00
Female	36	27.22	5.21	27.19	5.20	-0.03	

Table 4.7 presented the Mean ( $\bar{x}$ ) Retention Scores and Standard Deviation of Male and Female for experimental group 1 (collaborative instructional strategy). It is observed that male had a mean ( $\bar{x}$ ) loss of -0.03 with standard deviation of 4.42 while the female had a mean ( $\bar{x}$ ) loss of -0.03 with standard deviation of 5.20. The male and female students had a retention Mean loss difference of 0.00, this implied that male and female students in Experimental group 2 had equal retention Mean loss.

### 4.3 Testing of Null hypotheses

**Hypothesis one (HO<sub>1</sub>):** There is no significant difference in the mean achievement scores of students taught geometry concepts using cognitive conflict, collaborative instructional strategy and lecture method.

**Table 4.8: Summary of ANOVA Analysis of Post-test Achievement Scores for Experimental Group 1 and 2 and Control Group.**

Group	Sum of Squares	Df	Mean Square	F	Sig
Between Groups	4532.673	2	2266.337	99.59	0.000
Within groups	5074.760	223	22.757		
Total	9607.434	225			

**Significant at 0.05 level**

Table 4.8 presented the Analysis of Variance (ANOVA) result of posttest Achievement scores of experimental group 1 and 2 (cognitive conflict and collaborative instructional strategy) and control group. The result indicate that there is significant difference in the Mean ( $\bar{x}$ ) Achievement scores of students taught geometry concepts using cognitive conflict, collaborative instructional strategies and those taught using lecture method, F (99.59) = p-value (0.000) is less than 0.05 alpha-level ( $p = 0.000 < 0.05$ ). Hence, Hypothesis One (**HO<sub>1</sub>**) is rejected. To determine the main source of difference, the data were subjected to Scheffe's Post-hoc Test as shown in Table 4.9

**Table 4.9 Scheffe Multiple Comparisons of Experimental 1 and 2 and Control**

(J) GROUP	(J) GROUP	Mean Diff.	Std Error	Sig	95% Confidence Interval	
					Lower Bound	Upper Bound
Experimental 1	Experimental 2	-.242	.792	.954	-2.20	1.71
	Control	9.213*	.776	.000	7.30	11.12
Experimental 2	Experimental 1	.242	.792	.954	-1.71	2.20
	Control	9.455*	.767	.000	7.57	11.35
Control Group	Experimental 1	-9.326*	.776	.000	-11.12	-7.30
	Experimental 2	-9.231*	.767	.000	-11.35	-7.57

**Significant at 0.05 level**

Table 4.9 presented the Scheff's analysis result of posttest Achievement scores of experimental group 1 and 2 (cognitive conflict and collaborative instructional strategy)

and control group. The result indicate that the observed significant difference was between experimental 1 and control groups with the highest Mean ( $\bar{\chi}$ ) difference of 9.46 and highest upper class boundary of 11.35 at 95% confidence level. Therefore, there exist a significant difference in achievement between students taught geometry concept using cognitive conflict instructional strategy and those taught using lecture method. The significant difference is between experimental 1 and control group.

**Hypothesis two (HO<sub>2</sub>):** There is no significant difference in the mean retention scores of students taught geometry concepts using cognitive conflict, collaborative instructional strategy and lecture method.

**Table 4.10 Summary of ANOVA Analysis of Retention-test of Experimental Group 1 and 2 and Control Group.**

<b>Group</b>	<b>Sum of Squares</b>	<b>Df</b>	<b>Mean Square</b>	<b>F</b>	<b>Sig</b>
Between Groups	4473.320	2	2236.610	100.832	0.000
Within groups	4946.497	223	22.182		
Total	9419.717	225			

**Significant at 0.05 level**

Table 4.10 presented the Analysis of Variance (ANOVA) result of Mean ( $\bar{\chi}$ ) Retention scores of experimental group 1 and 2 (cognitive conflict and collaborative instructional strategy) and control group. The result indicate that there is significant difference in the Mean ( $\bar{\chi}$ ) Retention scores of students taught geometry concepts using cognitive conflict, collaborative instructional strategies and those taught using lecture method, F (100.83) = p-value (0.000) is less than 0.05 alpha-level ( $p = 0.000 < 0.05$ ). Hence, Hypothesis two (HO<sub>2</sub>) is rejected. To determine the main source of difference, the data were subjected to Scheffe's Post-hoc Test as shown in Table 4.11

**Table 4.11 Scheffe Multiple Comparisons of Experimental 1 and 2 and Control**

(J) GROUP	(J) GROUP	Mean Diff.	Std Error	Sig	95% Confidence Interval	
					Lower Bound	Upper Bound
Experimental 1	Experimental 2	.095	.782	.993	-1.83	2.02
	Control	9.326*	.766	.000	7.44	11.21
Experimental 2	Experimental 1	-.095	.782	.993	-2.02	1.83
	Control	9.231*	.757	.000	7.36	11.10
Control Group	Experimental 1	-9.326*	.766	.000	-11.21	-7.44
	Experimental 2	-9.231*	.757	.000	-11.10	-7.36

**Significant at 0.05 level**

Table 4.9 presented the Scheff's analysis result of posttest Achievement scores of experimental group 1 and 2 (cognitive conflict and collaborative instructional strategy) and control group. The result indicate that the observed significant difference was between experimental 2 and control groups with the highest Mean ( $\bar{\chi}$ ) difference of 9.32 and highest upper class boundary of 11.21 at 95% confidence level. Therefore, there exist a significant difference in retention between students taught geometry concept using collaborative instructional strategy and those taught using lecture method. The significant difference is between experimental 2 and control group.

**Hypothesis three (HO<sub>3</sub>):** There is no significant difference in the mean achievement scores of male and female students taught geometry concepts using cognitive conflict instructional strategies.

**Table 4.12: Summary of Independent Sample t-test for Mean Achievement Score of Male and Female for Experimental Group 1.**

Gender	N	DF	Mean ( $\bar{\chi}$ )	SD	t- cal	P- Value	Remark
Male	46		25.85	5.29	2.58	0.875	Ns
		69					
Female	25		29.20	5.12			

**Ns = Not significant at 0.05 level**

Table 4.12 presented the Independent sample t-test result of mean ( $\bar{x}$ ) achievement scores of Male and Female students taught geometry concept using cognitive conflict instructional strategy. The result indicate that there is no significant difference in the mean ( $\bar{x}$ ) achievement scores of male and female student taught geometry concepts using cognitive conflict instructional strategy, t-cal (2.58) = p-value (0.875) is greater than 0.05 alpha-level ( $p = 0.996 > 0.05$ ). Hence Hypothesis three (**HO<sub>3</sub>**) is not rejected.

**Hypothesis four (HO<sub>4</sub>):** There is no significant difference in the mean achievement scores of male and female students taught geometry concepts using collaborative instructional strategy.

**Table 4.13: Summary of Independent t-test for Mean Achievement Score of Male and Female in Experimental Group 2.**

Gender	N	Df	Mean ( $\bar{x}$ )	SD	t-cal	P- Value	Remark
Male	38		27.32	4.51			
		72			0.083	0.487	Ns
Female	36		27.22	4.21			

**Ns = Not significant at 0.05 level**

Table 4.13 presented the Independent sample t-test result of mean ( $\bar{x}$ ) achievement scores of Male and Female students taught geometry concept using collaborative instructional strategy. The result indicate that there is no significant difference in the mean ( $\bar{x}$ ) achievement scores of male and female student taught geometry concepts using collaborative instructional strategy, t-cal (0.083) = p-value (0.487) is greater than 0.05 alpha-level ( $p = 0.996 > 0.05$ ). Hence Hypothesis four (**HO<sub>4</sub>**) is not rejected.



**Hypothesis five (HO<sub>5</sub>):** There is no significant difference in the mean retention scores of male and female students taught geometry concepts using cognitive conflict instructional Strategy.

**Table 4.14: Summary of Independent Sample t-test for Mean Retention Score of Male and Female in Experimental Group 1.**

Gender	N	Df	Mean ( $\bar{x}$ )	SD	t-cal	P-Value	Remark
Male	46		26.20	5.23			
		69			-2.56	0.931	Ns
Female	25		29.44	4.87			

**Ns = Not significant at 0.05 level**

Table 4.14 presented the Independent sample t-test result of mean ( $\bar{x}$ ) retention scores of Male and Female students taught geometry concept using cognitive conflict instructional strategy. The result indicate that there is no significant difference in the mean ( $\bar{x}$ ) retention scores of male and female student taught geometry concepts using cognitive conflict instructional strategy, t-cal (-2.56) = p-value (0.931) is greater than 0.05 alpha-level ( $p = 0.996 > 0.05$ ). Hence Hypothesis Five (**HO<sub>5</sub>**) is not rejected.

**Hypothesis six (HO<sub>6</sub>):** There is no significant difference in the mean retention scores of male and female students taught geometry concepts using collaborative instructional strategy.

**Table 4.15: Summary of Independent t-test for Mean Retention Score of Male and Female in Experimental Group 2.**

Gender	N	Df	Mean ( $\bar{x}$ )	SD	t-cal	P-Value	Remark
Male	38		27.29	4.42			
		72			0.085	0.483	Ns
Female	26		27.19	5.20			

**Ns = Not significant at 0.05 level**

Table 4.15 presented the Independent sample t-test result of mean ( $\bar{x}$ ) retention scores of Male and Female students taught geometry concept using collaborative instructional strategy. The result indicate that there is no significant difference in the mean ( $\bar{x}$ ) retention scores of male and female student taught geometry concepts using collaborative instructional strategy, t-cal (0.085) = p-value (0.483) is greater than 0.05 alpha-level ( $p = 0.996 > 0.05$ ). Hence Hypothesis six (**HO<sub>6</sub>**) is not rejected.

#### **4.4 Summary of Findings**

Based on the analysis of the data collected, the following major findings were made;

1. Cognitive conflict and collaborative instructional strategies have significant effect on geometry achievement among secondary school students.
2. Cognitive conflict and collaborative instructional strategies have significant effect on geometry retention among secondary school students.
3. There is no difference in mean achievement between male and female students taught geometry using cognitive conflict instructional strategies.
4. There is no difference in mean achievement between male and female students taught geometry using collaborative instructional strategies.
5. There is no difference in mean retention between male and female students taught geometry using cognitive conflict instructional strategies.

6. There is no difference in mean retention between male and female students taught geometry using collaborative instructional strategies.

#### **4.5 Discussion of Findings**

Findings from the study revealed that there is significant difference in the mean achievement of students taught geometry using cognitive conflict, collaborative instructional strategies and those taught with lecture method in favor of experimental group 2. This finding revealed the students taught geometry concepts using collaborative instructional strategy record higher Mean ( $\bar{x}$ ) achievement scores than the students taught geometry concepts using cognitive conflict and lecture methods. The findings is in line with that of Emmanuel (2020) who reported that knowledge gains were found to be significant with the experimental group using cognitive conflict teaching strategy more than their counter part that were strictly taught in the class using lecture method in Geometry concept. This finding is also in line with that of Susilawati and Didi, (2019) who reported that there is a significant difference between the learning experiences acquired from mathematics concepts taught using collaborative instructional strategy and student performance in geometry. Similar finding from Madu and Emma (2015) reported that students thought using cognitive conflict performed significantly better than those taught using expository method and suggested this method of teaching should be encourage among mathematics teachers.

The study also found that there is significant difference in the mean retention scores of students taught geometry using cognitive conflict, collaborative instructional strategies and those taught with conventional lecture method in favor of the students taught using cognitive conflict strategy of experimental group 1. The finding revealed that the students taught geometry concepts using cognitive conflict record higher Mean ( $\bar{x}$ ) retention scores than the students taught geometry using both collaborative strategy and

lecture method. The finding agree with the finding of Ahmad (2016) who reported that the significant difference is in favor of the experimental group in retention level exposed to cognitive conflict teaching strategy as indicated by the mean scores. The finding is also in line with the findings of Luh (2020) and Ochuenwike *et al.* (2019) which confirmed a significant differences in retention between students taught with cognitive conflict with the students taught using lecture method, and also recommended that cognitive conflict and collaborative teaching strategies should be adopted as the most effective teaching strategy in science because of its influence on retention, more than their counter part that were strictly taught in the class using conventional method.

Finding also shows that there is no significant difference in the mean achievement of male and female students taught geometry using cognitive conflict instructional strategy. This finding agreed with the finding of Susilawati *et al.* (2017) who reported that there was no significant difference in the performance of male and female students using collaborative instructional strategy in learning geometry. Tatiane and Edmilson (2020) and Madu and Emma (2015) also confirm that, there was not difference in the mean scores between male and female students exposed to cognitive conflict instructional strategy. This study disagree with Zetriuslita *et al.* (2018) who reported that, there is a significant difference between male and female students in mathematics curiosity attitude as the effect of applying problem based learning and cognitive conflict.

Finding also revealed that there is no significant difference in the mean achievement of male and female students taught geometry using collaborative strategy. The finding shown that both male and female maintain the same Mean ( $\bar{x}$ ) achievement scores. This finding agreed with the findings of Achufusi and Ngozi (2021) that there was no a significant difference in the performance of male and female students using

collaborative instructional strategy in learning geometry. Roki (2019) and Otiende *et al.*, (2021) also reported that, there was not difference in the mean scores between male and female students exposed to collaborative instructional strategy. This study disagree with Sweller *et al.* (2016) which revealed a significant difference between male and female students in learning and improving their effectiveness in work examples in mathematics. It was revealed from the study that, there is no significant difference in the retention scores of male and female students taught geometry using cognitive conflict instructional strategies. The result shows that both male and female Mean ( $\bar{\chi}$ ) retention score has no significant difference. This agreed with the findings of Abdullahi (2015) which pointed out that, strategies that encourages self-learning just like cognitive conflict strategy leads to better retention of information and the development of a favorable attitude toward science and technology and there is no gender influence. Tatiane and Edmilson (2020) and Madu and Emma (2015) also confirm that, there was not difference in the mean retention scores between male and female students exposed to cognitive conflict instructional strategy.

Finding also revealed that there is no significant difference in the Mean ( $\bar{\chi}$ ) retention of male and female students taught geometry using collaborative strategy. This findings agreed with the findings of Roki (2019) and Otiende (2021) who also reported that, there was not difference in the mean retention scores between male and female students exposed to collaborative instructional strategy. This study disagree with Sweller *et al.* (2016) who reported that, there is a significant difference between male and female students in learning and improving their effectiveness in work examples in mathematics using collaborative instructional strategy.

## **CHAPTER FIVE**

### **5.0 CONCLUSION AND RECOMMENDATIONS**

#### **5.1 Conclusion**

The following conclusions are made based on the finding of the study. The results of this study provide empirical evidence that the use of cognitive conflict and collaborative instructional strategies enhanced students' academic achievement and retention in geometry more than the use of conventional lecture method of teaching. There was significant difference in the mean achievement of students taught geometry using cognitive conflict and collaborative strategies and those taught with lecture method. The study revealed a significant difference in the retention scores of students taught geometry using cognitive conflict and collaborative strategies and those taught with lecture method. There was no mean difference between male and female students taught geometry using cognitive conflict and collaborative instructional strategies. There was no mean difference in retention scores between male and female students taught geometry using cognitive conflict and collaborative strategies. There was no significant difference in the retention scores of male and female students taught geometry using cognitive conflict and collaborative instructional Strategies. In addition, the used of conventional lecture method for teaching appear to be inferior to the use of cognitive conflict and collaborative strategies most especially in teaching geometry concepts.

#### **5.2 Recommendations**

From the results of this study, the following recommendations are made for the improvement of teaching and learning of geometry concepts.

1. Geometry teacher's awareness of cognitive conflict and collaborative strategies of teaching should be created through workshops, seminars and conferences on teaching strategies.

2. Geometry is about identification, description of shapes and their properties, so teachers should teach most of the concepts through cognitive conflict and collaborative strategies to capture and sustain student's retention of geometry concepts, this will improve their achievement and retention in geometry.
3. Curriculum developers should incorporate cognitive conflict and collaborative instructional strategies into mathematics curriculum, this might support mathematics teachers in teaching mathematics.
4. School administrators should encourage science and mathematics teachers to use cognitive conflict and collaborative instructional strategies for effective teaching and learning of science and mathematics courses.

### **5.3 Contributions to Knowledge**

The study has made useful contributions to knowledge in the following ways.

1. The study has helped in improving students' achievement and retention in Geometry through the use of cognitive conflict and collaborative strategies in teaching the students with highest Mean achievement gain of (11.65) and highest Mean retention gain of (0.21) between the groups.
2. The study has helped and provides opportunity for students with practical and better understanding of geometry concepts.
3. The study has assisted teachers in developing new learning experience to arouse retention skills of their learners.
4. This research work contribute to the existing scholarly literatures on the effect of cognitive conflict and collaborative strategies on students' achievement and retention.

#### **5.4 Suggestions for Further Study**

The following suggestion was given by the researchers for further studies.

1. A similar research work should be carried out in other state of Nigeria.
2. Studies can be carried out on the use of cognitive conflict and collaborative strategies for teaching and learning at tertiary level of education to see if similar results can be obtained.
3. The research covered only one concept of Mathematics, studies could be extended to other topics in Mathematics.
4. A similar research study should be carried out in subjects like Physics, Chemistry, Biology, Agricultural science etc.



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## Appendix A

### Research Instrument Validation Form



**FEDERAL UNIVERSITY OF TECHNOLOGY MINNA**  
**DEPARTMENT OF SCIENCE EDUCATION**

**POSTGRADUATE INSTRUMENT VALIDATION FORM**  
**(PIVAF)**

**NOTE:** This form must be attached to the instrument and objectives/hypotheses of the study to the validator.

1. NAME OF THE VALIDATOR: Dr. A. A. Hassan
2. SCHOOL/INSTITUTION/DEPARTMENT: SSCE / SC. EDUC.
3. TEL. PHONE NUMBER: 080 96276378
4. E-MAIL ADDRESS: hassan.achina@futu.edu.ng
5. NAME OF THE CANDIDATE: KINDA YUWALA P. O. K.
6. MATRIC/NO.: 19-TECH/SSCE/2018/8493
7. APPROVED THESIS TITLE: IMPACT OF COGNITIVE CONFLICT AND COLLABORATIVE INSTRUCTIONAL STRATEGIES ON GEOGRAPHY AND REGIONAL KNOWLEDGE SEC. SCHOOL STUDENTS IN MINNA, METRO
8. TITLE OF THE INSTRUMENT(S): GEN. ASSESS. TEST

**Action:** Please, you are requested to make input or corrections where necessary.

Categories	Appropriate	Not Appropriate
Is the outlook of the instrument appropriate to the respondents?	<u>Satisfactory</u>	
Are the word processing and layout of the instrument appropriate?	<u>✓</u>	
Are the items generated in the instrument appropriate?	<u>✓</u>	
Are the items enough and appropriate to generate a valid conclusion on the construct to be observed?	<u>✓</u>	
Are the items generated appropriate to the objectives of the study?	<u>✓</u>	
Are the items generated appropriate to provide valid responses or answers to the research and hypotheses raised in the study?	<u>✓</u>	
Do you consider the whole instrument appropriate for the study?	<u>✓</u>	

9. GENERAL REMARK: Satisfactory

10. NAME AND SIGNATURE: Dr. A. A. Hassan 11/02/2022






DEPARTMENT OF SCIENCE EDUCATION  
POST GRADUATE INSTRUMENT VALIDATION FORM  
(PIVAF)

NOTE: This form must be attached to the instrument and objectives/hypotheses of the study to the validator.

1. NAME OF THE VALIDATOR: Dr. Bashir A. U. Yankuzo.
2. SCHOOL/INSTITUTION/DEPARTMENT: SSTE/CE/EDU.
3. TEL. PHONE NUMBER: 09065542625
4. E-MAIL ADDRESS: bashir.a.u@funaab.edu.ng
5. APPROVED THESIS TITLE: IMPACT OF COGNITIVE CONFLICT AND COLLABORATIVE INSTRUCTIONAL STRATEGIES ON GEOM. ACQ. AND RETENTN. AMONG SEC. SCH. STDS IN MINNA MET. N/S.
6. TITLE OF THE INSTRUMENT: GEOMETRY ASSESSMENT TEST (GAT)

Instruction: Please, you are requested to make input or corrections where necessary.

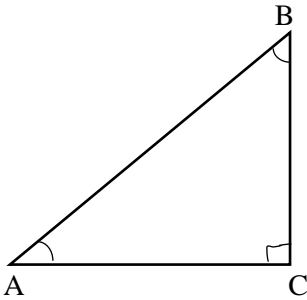
S/No.	Categories	Appropriate	Not Appropriate
1	Is the outlook of the instrument appropriate to the respondent?	<input checked="" type="checkbox"/>	<input type="checkbox"/>
2	Are the word processing and layout of the instrument appropriate?	<input checked="" type="checkbox"/>	<input type="checkbox"/>
3	Are the items generated in the instrument appropriate?	<input checked="" type="checkbox"/>	<input type="checkbox"/>
4	Are the items enough and appropriate to generate a valid conclusion on the construct to be observed?	<input checked="" type="checkbox"/>	<input type="checkbox"/>
5	Are the items generated appropriate to the objectives of the study?	<input checked="" type="checkbox"/>	<input type="checkbox"/>
6	Are the items generated appropriate to provide valid responses or answers to the research and hypotheses raised in the study?	<input checked="" type="checkbox"/>	<input type="checkbox"/>
7	Do you consider the whole instrument appropriate for the study?	<input checked="" type="checkbox"/>	<input type="checkbox"/>

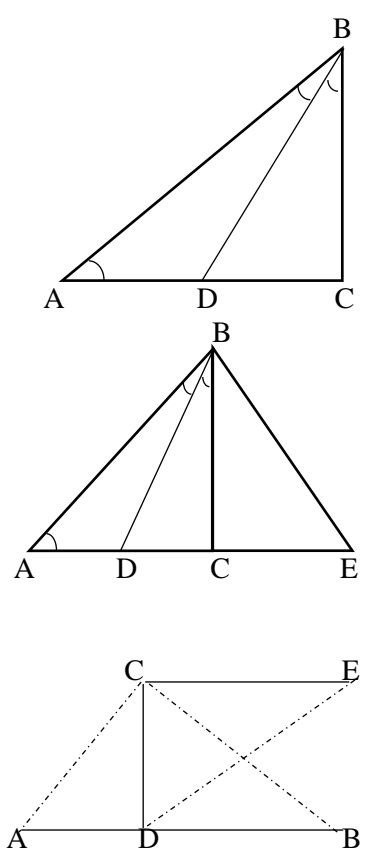
7. GENERAL REMARK: The Instrument is generally okay.
8. NAME AND SIGNATURE: Dr. Bashir A. U. Yankuzo 

## Appendix B

### Cognitive conflict instructional strategy

#### Lesson Plan

Subject	Mathematics		
Theme	Everyday Geometry		
Topic	Angle of Elevation and Depression		
Date			
Class	S.S Two (II)		
Average Age	16-18		
Duration	45 minutes		
Time	8:00 am – 8:45 am		
Number of Students			
Learning Objects	By the end of the lesson the students should be able to: <ol style="list-style-type: none"> <li>1. Identify Angles of Elevation and Angle of Depression</li> <li>2. Calculate Height and distance, given angles of elevation, depression</li> <li>3. Differentiate between angle of elevation, and angle of depression</li> </ol>		
Rationale	For the students to be able to clear misconception in identifying angles and apply the knowledge in solving other geometry questions		
Learning Materials	Commuter system, ruler, chalk, chalk board, textbook, cardboard papers.		
Pre-requisite knowledge	Students are used to stating trigonometric ratio such as Sine, Cosine and Tangent		
<b>LESSON DEVELOPMENT</b>			
<b>STEPS</b>	<b>TEACHER ACTIVITIES</b>	<b>STUDENTS ACTIVITIES</b>	<b>LEARNING POINT</b>
<b>STEP I</b> Identifying Students' Previous Knowledge	Teacher Introduced the lesson by asking the students to state/mention trigonometric Ratios and Geometric shapes	Students Stated/mentioned Such as (a) Sine (b) Cosine (c) Tangent And Geometric Shapes Such as (a) Triangle (b) Rectangle (c) Square (d) Polygon etc	Understanding the concept of geometry
<b>STEP II</b> Understanding students' alternative concepts	Teacher Write Geometry shapes on the board  	Students may give difference answers to the said such as:  <u>STUDENT (A)</u> On Question 1: Angle A is an angle of elevation while Angle B is an angle of depression  <u>STUDENT (B)</u> On question 1: both angle A and B are angles of elevation  <u>STUDENT C</u> On question 1: Both Angle A and B are all angles of	Understanding misconception and cognitive conflict.

	 <p>The teacher asked the students to state (1) whether Angle A and B are angles of elevation or depression</p> <p>(2) When a man standing at point A and B look at the top B, what angle will he formed with horizontal line AC and DC Respectively?</p> <p>The teacher guide the debate taking note of conflict and misconception in trying to state the correct answers to question 1 and 2 using available learning materials</p> <p>----After Students responses, teacher clear misconception and the conflict in the students while stating the correct answers.</p>	<p>depression</p> <p><u>STUDENT A</u> On question 2: A man standing at point A and D will formed angles of elevation with horizontal line AC and DC respectively, when he look at the top point B.</p> <p><u>STUDENT B</u> On question 2: A man standing at point A and D will formed angles of depression and elevation with horizontal line AC and DC respectively, when he look at the top point B.</p> <p><u>STUDENT A</u> On question 2: A man standing at point A and D will form angles of depression with horizontal line AC and DC respectively, when he look at the top point B.</p>	
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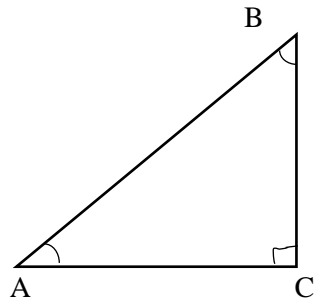
<p><b>STEP III</b> Creating Cognitive Conflict in the students</p>	<p>The teacher sketched geometric shapes and mislabeled the angles while asking the students to copy, identify and labelled the angles correctly.</p>	<p>Cognitive Conflict set in the students' cognitive domain. While students tried to resolve the conflict through textbooks and the previous steps and also discussing with peers. Finally the students settled and labelled the angles in the diagrams under the supervision of the teacher.</p>	<p>Resolving the earlier created cognitive conflict and encouraging self-learning and team work</p>
<p><b>STEP V</b> Evaluation</p>	<p>Teacher ask the students questions based on the lesson</p>	<p>Students responded to the questions</p>	<p>Confirming resolving of the conflict and the attainment of lessons objectives</p>
<p><b>STEP IV</b> Conclusion</p>	<p>Teacher summarized key point of the lesson and Mark their note book and give them Assignment</p>	<p>Students listen and ask question</p>	<p>To encourage retention of the lesson</p>

## Appendix C

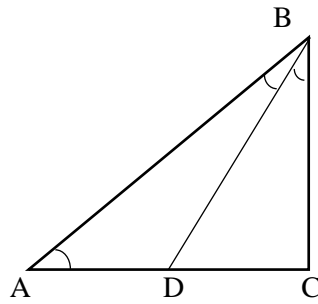
### Collaborative Instructional Strategy

#### Lesson Plan

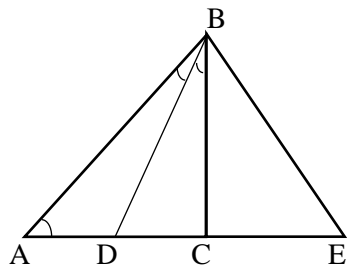
Subject	Mathematics		
Theme	Everyday Geometry		
Topic	Angle of Elevation and Depression		
Date			
Class	S.S Two (II)		
Average Age	16-18		
Duration	45 minutes		
Time	8:00 am – 8:45 am		
Number of Students			
Learning Objects	By the end of the lesson the students should be able to: 4. Identify Angles of Elevation and Angle of Depression 5. Calculate Height and distance, given angles of elevation, depression 6. Differentiate between angle of elevation, and angle of depression		
Rationale	For the students to be able to clear misconception in identifying angles and apply the knowledge in solving other geometry questions		
Learning Materials	Commuter system, ruler, chalk, chalk board, textbook, cardboard papers.		
Pre-requisite knowledge	Students are used to stating trigonometric ratio such as Sine, Cosine and Tangent		
<b>LESSON DEVELOPMENT</b>			
<b>STEPS</b>	<b>TEACHER ACTIVITIES</b>	<b>STUDENTS ACTIVITIES</b>	<b>LEARNING POINT</b>
<b>STEP I</b> Identifying Students' Previous Knowledge	Teacher Introduced the lesson by grouping the students and asking them to state/mention trigonometric Ratios and Geometric shapes	Students Stated/mentioned Such as (a) Sine (b) Cosine (c) Tangent And Geometric Shapes Such as (a) Triangle (b) Rectangle (c) Square (d) Polygon etc	Understanding the concept of geometry
<b>STEP II</b> Understanding students' alternative concepts	<p>“When a person stand and looks up at an object, the angle of elevation is the angle between the horizontal line of sight and the object. If a person stand and looks down at an object, the angle of depression is the angle between the horizontal line of sight and the object”.</p> <p>Teacher Write Geometry shapes on the board and asked each group to collaborate and state if angle A, B and C are angle of Elevation or depression</p>	<p>Students in groups may give difference answers to the said such as:</p> <p><u>Group (A)</u> On Question 1: Angle A is an angle of elevation while Angle B is an angle of depression</p> <p><u>Group (B)</u> On question 1: both angle A and B all angles of elevation</p> <p><u>Group C</u> On question 1: Both Angle A and B are all angles of depression</p>	Introducing new task to arouse learners' interest.



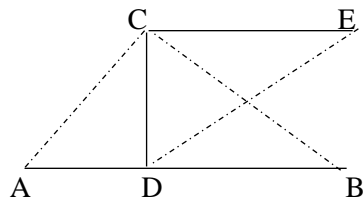
Group A On question 2:  
A man standing at point A and D will formed angles of elevation with horizontal line AC and DC respectively, when he look at the top point B.



Group B On question 2:  
A man standing at point A and D will formed angles of depression and elevation with horizontal line AC and DC respectively, when he look at the top point B.



Group A On question 2:  
A man standing at point A and D will formed angles of depression with horizontal line AC and DC respectively, when he look at the top point B.



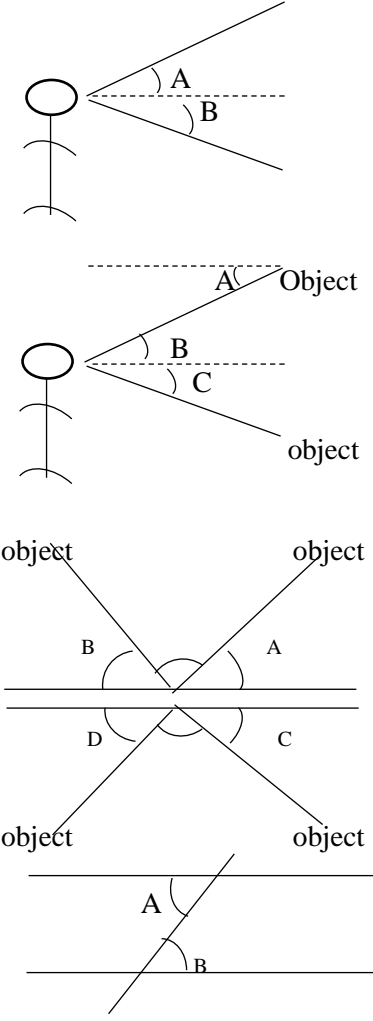
Each Group Collaborate with its members to share idea and complete the given tasks

The teacher asked the students to state (1) whether Angle A and B are angles of elevation or depression

(2) When a man standing at point A look at the top B, what angle will he formed with horizontal line AC and DC Respectively?

The teacher observes and guide the debate to state the correct to question 1 and 2 using available learning materials

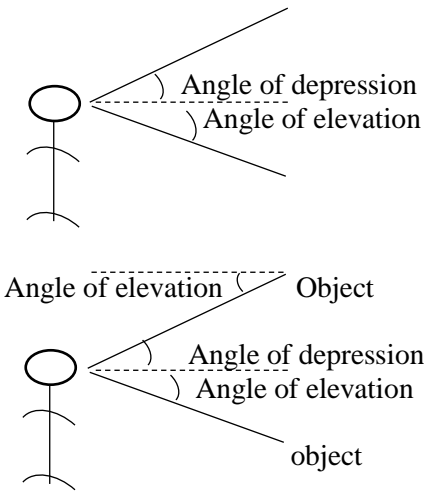
----After each group responses, teacher and student collaborate and state the correct answers.

<p><b>STEP III Collaborative Learning</b></p>	<p>The teacher carefully regroup the students into different groups and assigned one competent student to lead each group, thereafter, teacher give the students tasks to label all angles in the figures bellow.</p> 	<p>Each group collaborate using textbooks and the previous steps and also discussing with each member. Finally the students settled and labelled the angles in the diagrams under the supervision of the teacher.</p>	<p>To encourage ensure collaborative learning</p>
<p><b>STEP IV Number Head</b></p>	<p>The teacher observes each group as they solves or handle the tasks while identifying the non-contributing students. Thereafter, teacher ask each to discussed their solution to all class</p>	<p>each team member discusses their works to large class</p>	<p>To encourage collaborative learning</p>
<p><b>STEP V Apprenticeship</b></p>	<p>Teacher give them another task to solve individually.</p> <ol style="list-style-type: none"> <li>1. When a man standing at point A and look top B, what angle will he formed with horizontal line AC and DC</li> </ol>	<p>Students work individually.</p>	<p>To encourage retention and self-discovery</p>

	Respectively?		
<b>STEP VI</b> Assessment	The teacher ask students to exchange their work	students exchange their works to assess and discover their zone of proximal development (ZPD)	To encourage collaboration and self-discovery
<b>STEP VII</b> Assessment	Teacher summarized key point of the lesson and Mark their note book and give them Assignment	Students listen and ask question	To encourage retention of the lesson



**Appendix D**  
**Conventional Instructional Strategy**  
**Lesson Plan**

Date	
Class	SS II
Subject	Mathematics
Topic	Angle of Elevation and Depression
Duration	40 Minutes
Time	
Average Age	17-19
Sex	Mixed
Behavioral Objectives	By the end of the lesson the students should be able to: 1. Identify Angles of Elevation and Angle of Depression 2. Calculate Height and distance, given angles of elevation, depression 3. Differentiate between angle of elevation, and angle of depression
Instructional Materials	Improvised Graph board, board ruler, board compass and text book
Previous Knowledge	Students are used to stating trigonometric ratio such as Sine, Cosine and Tangent
Introduction	The teacher introduced the lesson by writing the topic on the chalk board after, and asking them questions on bearing such as 1: What are the four cardinal points?
Presentation	The teacher present the lesson based on the following steps
Step I	The teacher Define angle of elevation and depression: “‘If a person stand and looks up at an object, the angle of elevation is the angle between the horizontal line of sight and the object. and If a person stand and looks down at an object, the angle of depression is the angle between the horizontal line of sight and the object”.
STEP II	The teacher sketched geometric shapes and identify angles of elevation depression to the students. 

STEP III	<p>The teacher draw a shape and asked student to identify angle f elevation depression.</p> <p>When a man standing at point A looks at the top B, what angle will he formed with horizontal line AC and DC Respectively?</p>
Evaluation	<p>When a man standing at point B and look down A, what angle will he formed with horizontal line AC and DC Respectively?</p>
Conclusion	<p>Teacher summarized key point of the lesson and Mark their note book and give them Assignment</p>

## Appendix E

### Geometry Achievement Test [GAT]

#### SECTION A

1. **Introduction:** the purpose of this questions paper is to collect information which will be used in a research study on Effect of Cognitive Conflict and Collaborative Instructional Strategies on Geometry Achievement and Retention among Secondary School Students in Minna Metropolis. Every information you give is, therefore, strictly for academic purpose and will be treated confidentially. Your name is NOT required.

2. **Instructions:**

- i. Please read the questions or statements very carefully and respond appropriately.
- ii. Choose the most appropriate alternative from the options letter A to D in the given question.
- iii. Choose only one answer for each question
- iv. Attempt all the questions
- v. Respondents are free to ask questions in any of the items that need clarification
- vi. Erase any incorrect answer properly before choosing another option
- vii. Use pencil only
- viii. Time allowed: 1½ Hour

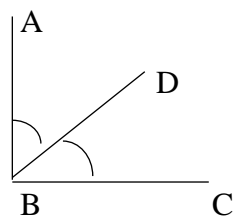
3. **Bio-Data** Male  Female

**Name of School:** \_\_\_\_\_

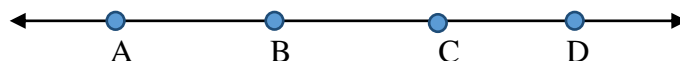
**Class:** \_\_\_\_\_ **Date:** \_\_\_\_\_

#### **SECTION B: Multiple Choice Objectives Questions**

1. If the measure of angle  $\hat{A}BD$  and  $\hat{C}BD$  are  $5x+18$  and  $7x$  respectively as shown below, which of the following could be the measure of angle  $\hat{A}BD$ ?

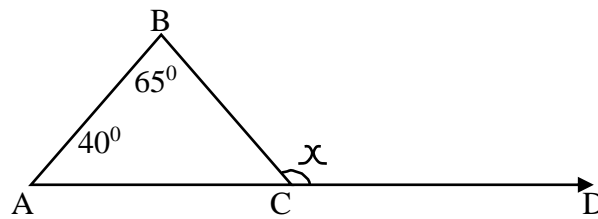


- (a)  $35^\circ$  (b)  $42^\circ$  (c)  $48^\circ$  (d)  $57^\circ$
2. C is a midpoint of line AD. If length BC is equal to 4 and AD is 24, what is the length of AB according to the figure shown below?



- (a) 4 (b) 6 (c) 8 (d) 12
3. One of the following solid shapes has a circular face and a curved surface.
- (a) A cube (b) Rectangle (c) Cylinder (d) Cone

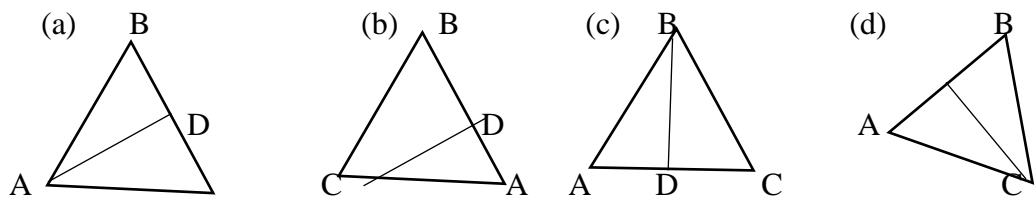
4. The Measure of the angles in triangle ABC have the ratio 5:7:8. What is the difference between the measures of the longest and smallest angle?  
 (a) 9 (b) 18 (c) 27 (d) 45 (e) 72
5. Which of the formulae below is correct in determining the volume of a cylinder?  
 (a)  $\pi rL$  (b)  $\pi r^2h$  (c)  $\pi rh$  (d)  $\pi rL^2$
6. What is the area of an equilateral triangle with a side length of 12?  
 (a)  $36\sqrt{3}$  (b)  $48\sqrt{2}$  (c) 72 (d)  $96\sqrt{3}$
7. A point or corner where three (3) or edges meet is called  
 (a) Sharp point (b) Plane (c) Vertex (d) Rectangle
8. How many diagonal does a regular hexagon have?  
 (a) 6 (b) 9 (c) 12 (d) 14 (e) 15
9. What is the value of  $x$  in the figure shown below?



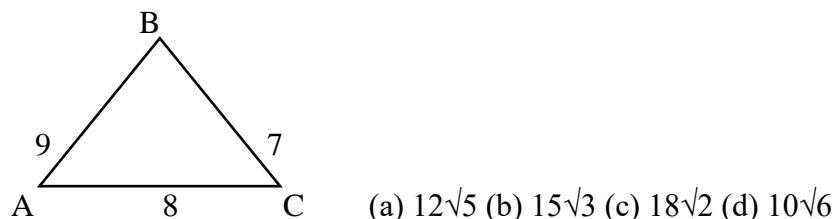
- (a) 45 (b) 55 (c) 60 (d) 75

10. What is the measure of each interior angle inside a regular hexagon?  
 (a) 45 (b) 72 (c) 90 (d) 120
11. What is the measure of exterior angle of a regular pentagon?  
 (a) 45 (b) 72 (c) 90 (d) 120

12. Which of the following triangles contain an altitude?



13. A typical example of a cube is -----  
 (a) Tin (b) Cylinder (c) An empty box of matches (d) A cube of Sugar
14. What is the area of the scalene triangle shown below?



15. A cube is a cuboid in which all faces are -----

- (a) Rhombus (b) Rectangle (c) Square (d) Cone

16. The length of the diagonal of a square is 10 inches. What is the area of the square?

- (a) 25 (b) 50 (c) 75 (d) 100

17. In rhombus ABCD. BC is 10 and BE is 6, what is the area of the rhombus?

- (a) 48 (b) 72 (c) 96 (d) 108

18. A kite ABCD. AD is 20, BE is 12, and BC is 13, what is the area of the kite?

- (a) 126 (b) 252 (c) 324 (d) 400

Use the figure below to answer questions 19-21



19. Find the Volume of the rectangular prism showed above

- (a) 122 (b) 420 (c) 240 (d) 250

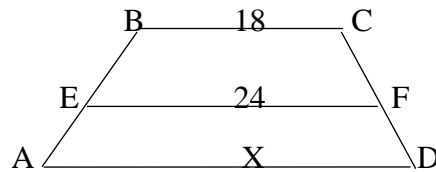
20. What is the Surface area of the rectangular prism showed above?

- (a)  $256^2$  (b)  $250^2$  (c)  $17^2$  (d)  $60^2$

21. What is the diagonal length of the rectangular prism showed above?

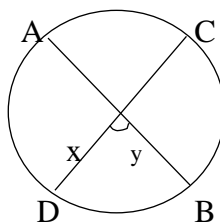
- (a) 14.5 (b) 13.6 (c) 17 (d) 60

22. What is the value of X in the trapezoid shown below?



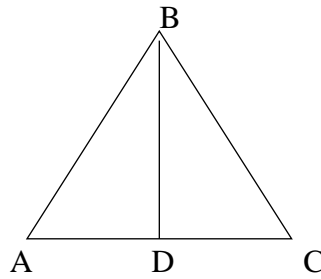
- (a) 28 (b) 29 (c) 30 (d) 31

23. What is the value of x in the figure shown below?



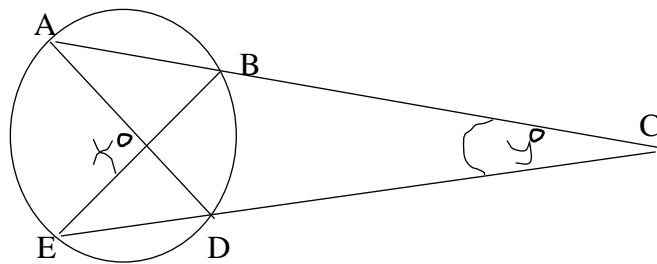
- (a) 6 (b) 8 (c) 9 (d) 12

24. Given the point A (2, 3), B (3, 20) and C (4, 9) on the triangle shown below, what is the slope of altitude BD?



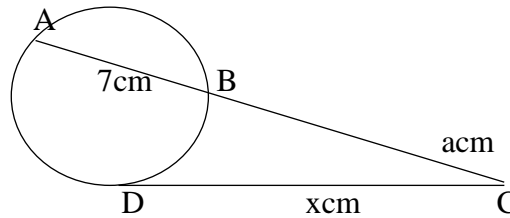
- (a)  $\frac{3}{4}$  (b)  $-\frac{5}{4}$  (c) 4 (d)  $-\frac{1}{3}$

25. What is the difference between the values of x and y in the figure below?



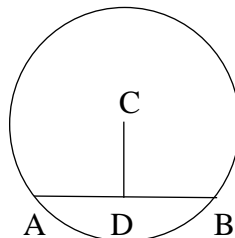
- (a) 30 (b) 40 (c) 50 (d) 70

26. What is the length of DC in the figure below?



- (a) 7 (b) 10 (c) 12 (d) 14

27. CD is perpendicular to AB. If  $AB = 7x - 5$  and  $AD = 3x - 1$ , what is the length of segment CD if the radius of circle C is 10?

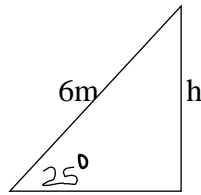


- (a) 4 (b) 6 (c) 7 (d) 9

28. The formulae used to calculate the volume of a cube is

- (a)  $L^2$  (b)  $L^3$  (c) L (d)  $L^4$

29. What is the value of h in the figure below?



- (a) 9cm (b) 7cm (c) 5cm (d) 12cm
30. The length, in cm, of the sides of a right angled triangle are  $x$ ,  $(x+2)$  and  $(x+1)$  where  $x > 0$ . Find, in cm, the length of its hypotenuse.
- (a) 6 (b) 5 (c) 13 (d) 17
31. The lengths of the parallel sides of a trapezium are 5cm and 7cm. if its area is  $120\text{cm}^2$ , find the perpendicular distance between the parallel sides.
- (a) 5.0 cm (b) 6.9 cm (c) 10.0 cm (d) 20.0 cm
32. The height of a pyramid on a square base is 15cm. if the volume is  $80\text{cm}^3$ , find the area of the square base.
- (a)  $8\text{cm}^2$  (b)  $9.6\text{cm}^2$  (c)  $16\text{cm}^2$  (d)  $25\text{cm}^2$
33. The arc of a circle 50cm long subtends an angle of  $75^\circ$  at the center of the circle. Find, correct to 3 significant figures, the radius of the circle. [Take  $\pi = \frac{22}{7}$ ]
- (a) 8.74cm (b) 38.2cm (c) 61.2cm (d) 76.4cm
34. Point P and Q are respectively 24m north and 7m east of point R, Calculate PQ in meters
- (a) 20 (b) 24 (c) 25 (d) 31
35. Point P and Q are respectively 24m north and 7m east of point R, what is the bearing of Q from P to the nearest whole degree?
- (a)  $16^\circ$  (b)  $17^\circ$  (c)  $73^\circ$  (d)  $106^\circ$
36. The value of  $\tan 315^\circ$  is (a) 1 (b) 2 (c) 0 (d) -1
- 37 The value of  $\sin 210^\circ$  is (a)  $-1/2$  (b) 2 (c) 5 (d)  $1/2$
38. In a given regular polygon, the ratio of the exterior angle to the interior angle is 1:3. How many sides is the polygon? (a) 4 (b) 5 (c) 6 (d) 8
- 39 One of the following is not a type of triangle (a) Right angle triangle (b) isosceles triangle (c) Rectangle triangle (d) equilateral triangle
40. The following are types of angles except (a) acute angle (b) obtuse angle (c) Straight angle (d) perfect angle

## Appendix F

### Pre-Test Marking Scheme (Key)

<b>1. C</b>	<b>11. B</b>	<b>21. B</b>	<b>31. D</b>
<b>2. C</b>	<b>12. A</b>	<b>22. C</b>	<b>32. C</b>
<b>3. C</b>	<b>13. D</b>	<b>23. D</b>	<b>33. B</b>
<b>4. C</b>	<b>14. A</b>	<b>24. D</b>	<b>34. C</b>
<b>5. B</b>	<b>15. C</b>	<b>25. C</b>	<b>35. D</b>
<b>6. A</b>	<b>16. B</b>	<b>26. C</b>	<b>36. D</b>
<b>7. C</b>	<b>17. C</b>	<b>27. B</b>	<b>37. A</b>
<b>8. B</b>	<b>18. B</b>	<b>28. B</b>	<b>38. D</b>
<b>9. D</b>	<b>19. C</b>	<b>29. A</b>	<b>39. C</b>
<b>10. D</b>	<b>20. A</b>	<b>30. B</b>	<b>40. C</b>



**Appendix G**  
**Instrument Reliability Result**

```

CORRELATIONS
/VARIABLES=PRETEST RETEST
/PRINT=TWOTAIL NOSIG
/STATISTICS DESCRIPTIVES
/MISSING=PAIRWISE.
    
```

### Correlations

Descriptive Statistics

	Mean	Std. Deviation	N
FIRST TEST	17.27	6.565	30
SECOND TEST	18.70	5.194	30

Correlations

		FIRST TEST	SECOND TEST
FIRST TEST	Pearson Correlation	1	.729**
	Sig. (2-tailed)		.000
	N	30	30
SECOND TEST	Pearson Correlation	.729**	1
	Sig. (2-tailed)	.000	
	N	30	30

\*\* . Correlation is significant at the 0.01 level (2-tailed).

## Appendix H

### 2017 WAEC Report (Evidence of Statement of Research Problem)

#### MATHEMATICS

#### RÉSUMÉ OF MATHEMATICS

##### **1. STANDARD OF THE PAPER**

The Chief Examiners for Mathematics (Core) 2 and Mathematics (Elective) 2 reported that the standard of the papers compared with that of the previous years.

##### **2. PERFORMANCE OF CANDIDATES**

The Chief Examiner for Mathematics (Core) 2 stated that there was an improvement in performance over that of last year's. The Chief Examiner for Mathematics (Elective) 2 performance of candidates was slightly higher than that of last year

##### **3. SUMMARY OF CANDIDATES' STRENGTHS**

(1) The Chief Examiner for Mathematics (Core) 2 stated that Candidates were able to:

- (a) apply Pythagoras theory in solving problems.
- (b) simplify and factorize algebraic expressions.
- (c) to draw trigonometric graphs and use it to solve relevant problems.
- (d) construct cumulative frequency tables and draw graphs of same distribution.
- (e) find the gradient of a line from a given equation.

(2) The Chief Examiner for Mathematics (Elective) 2 outlined that candidates exhibited an improvement in:

- (a) expressing a function as partial fractions.
- (b) finding the Spearman's rank correlation coefficient.
- (c) applying the Quotient rule to differentiate an algebraic fraction.
- (d) finding the magnitude of a resultant force with given magnitudes and directions.
- (e) finding identity element of a given binary operation and the inverse of the given elements.
- (f) using the general formula to find the equation of a circle which passes through three given points.

##### **4. SUMMARY OF CANDIDATES WEAKNESSES**

(1) The Chief Examiner for Mathematics (Core) 2 stated that candidates were unable to:

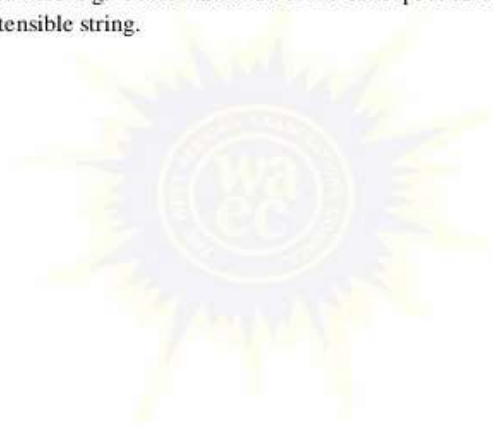
- (a) show evidence of reading values from graphs;
- (b) translate word problems into mathematical equations;
- (c) solve problems on mensuration, geometry and cyclic quadrilaterals.

(2) The Chief Examiner for mathematics (Elective) 2 mentioned that candidates exhibited lack of understanding in:

- (a) applying probability concepts to solve problems.
- (b) finding angles and tensions of an inextensible string fixed at two points.

**5. SUGGESTED REMEDIES**

- (1) The Chief Examiner for Mathematics (Core) 2 the following suggestions to be followed:
  - (a) In teaching, emphasis should be placed on showing evidence of reading from graphs.
  - (b) Algebraic concepts should be explained meticulously to help candidates translate word-problems into mathematical equations (statements).
  - (c) Teachers should encourage group work among candidates using geometrical figures to enable them solve questions on mensuration and geometry.
  
- (2) The Chief Examiner for Mathematics (Elective) 2 recommended the following to help candidates overcome their weaknesses;
  - (a) The candidates should be exposed to many exercises on probability.
  - (b) Teachers should give more attention to the concept of forces relating to tensions in an inextensible string.



---

## MATHEMATICS (CORE) 2

### **1. STANDARD OF THE PAPER**

The standard of this year's paper compared favourably with that of the previous year.

The performance of candidates was better compared to the previous year.

### **2. SUMMARY OF CANDIDATES' STRENGTHS**

The Chief Examiner for Mathematics (Core) listed the strengths of candidates as follows; ability to:

- (1) compute probabilities of given events;
- (2) construct a cumulative frequency table and draw graph of same distribution;
- (3) complete table of values of trigonometric relation in a given interval and draw graph of same;
- (4) determine the gradient of a given straight line and finding the equation of a straight line given the gradient and co-ordinates of a point through which the line passes;
- (5) draw a Venn diagram for a given information.

### **3. SUMMARY OF CANDIDATES' WEAKNESSES**

The Chief Examiner for Mathematics (Core) listed some of the weaknesses of candidates as follows; ability to:

- (1) translate word-problems into mathematical statements;
- (2) solve problems in circle theorems;
- (3) solve problems involving angles of elevation and depression;
- (4) solve problems involving ratio and proportions;
- (5) show evidence of reading from a graph.

### **4. SUGGESTED REMEDIES**

- (1) Teachers should teach students on how to translate word problems into mathematics statements.
- (2) The concept of circle theorems must be explained well in schools.
- (3) Teachers should stress on the need for candidates to read and understand the demands of the questions they attempt.
- (4) Candidates must be taught to show evidence of reading values on graphs.

## 5. DETAILED COMMENTS

### QUESTION 1

- (a) In a small town, 68% of the people owned Television, 72% owned Radio and 12% owned neither Television nor Radio.
- Represent the information on a Venn diagram.
  - What percentage of the population owned Television only?
- (b) Boadu and Ansah formed a company and agreed that their annual profit will be shared in the ratio 4:5 respectively. If at the end of the year, Ansah received GH¢5,000.00 more than Boadu, how much was Boadu's share?

Part (a) of this question required the representation of the given information on a Venn diagram. Most candidates answered it correctly. Others on the other hand, omitted essential requirements either for lack of knowledge or otherwise. Instead of  $n(U) = 100$ ,  $n(R) = 78$  and  $n(T) = 72$  candidates simply stated  $U = 100$ ,  $R = 78$  and  $T = 72$ .

Part (b) required that candidates shared a given sum in the stated ratio, 4:5. Most of the candidates were unable to form the relevant equation to enable them answer the question. A few of them were able to form the required equation. Some candidates omitted the essential units in the final answer.

Candidates were required to solve part (b) as follows:

$$\frac{5}{9}x - \frac{4}{9}x = 5000$$

$$\text{Boadu} = \frac{4}{9} \times 45,000$$

$$\text{Boadu's share} = \text{GH¢}20,000.00$$

### QUESTION 2

(a) Make  $y$  the subject of the relation:  $p = 2x \sqrt{\frac{q(1 + \frac{r^2}{y^2})}{s}}$

(b) Given that  $m = 3$ ,  $n = -2$  and  $x = -1$ , evaluate  $\frac{2mn^2x}{3m-n}$

In making  $y$  the subject of the relation given in 2(a), candidates were required to remove the radical sign by squaring both sides of the relation. Some of the candidates failed to square the factor ' $2x$ '. This resulted in error in their presentation.



The solution is as follows:

$$p^2 = 4x^2 \left( \frac{qy^2 + qr^2}{y^2s} \right)$$

$$p^2 y^2 s = 4x^2 q y^2 + 4x^2 q r^2$$

$$y^2 (p^2 s - 4x^2 q) = 4x^2 r^2 q$$

$$y^2 = \frac{4x^2 r^2 q}{p^2 s - 4x^2 q}$$

$$y = \pm \sqrt{\frac{4x^2 r^2 q}{p^2 s - 4x^2 q}}$$

Part (b) was quite easily done. The given values were substituted in the expression and same was evaluated. The solution is as follows:

$$\frac{2mn^2 x}{3m-n} = \frac{2(3)(-2)^2(-1)}{3(3)-(-2)}$$

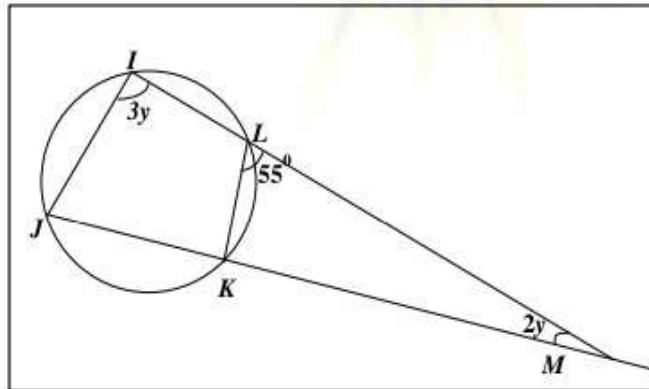
$$= \frac{6(4)(-1)}{9+2}$$

$$= -\frac{24}{11}$$

$$= -2\frac{2}{11}$$

### **QUESTION 3**

(a)



**NOT DRAWN TO SCALE**

In the diagram  $IJKL$  are points on a circle such that  $\angle JIL = 3y$  and  $\angle KML = 2y$   
 If  $\angle KLM = 55^\circ$ , find the value of  $y$ .

- (b) Given that  $\tan x = 1$ ,  $0^\circ \leq x \leq 90^\circ$ , evaluate  $\frac{1 - \sin^2 x}{\cos x}$

Part (a) was subject to various method aside the solution provided in the marking scheme. Some candidates demonstrated knowledge of circle theorems and geometric facts relating to the triangle. Some of the candidates on the other hand presented solutions that were not judicious.

The solution is as follows:

$$\angle LKM + 180 - 3y = 180$$

$$\angle LKM = 180 - 180 + 3y$$

$$\angle LKM = 3y$$

$$55 + 2y + 3y = 180$$

$$55 + 5y = 180$$

$$5y = 180 - 55$$

$$y = 25^\circ$$

Part (b) was quite easily answered. Given  $\tan x = 1$ , candidates obtained  $\cos x = \frac{1}{\sqrt{2}}$  and substituted these values in the expression and evaluated same. Other candidates worked out the value of  $x$  i.e.,  $\tan x = 1$ ,  $x = \tan^{-1}(1) = 45^\circ$  and put  $45^\circ$  in place of  $-2x$  to evaluate the expression. These attempts were commendable. Candidates were expected to solve it as follows:

$$\sin y = \frac{1}{\sqrt{2}} \quad \cos y = \frac{1}{\sqrt{2}}$$

$$\frac{1 - \sin^2 y}{\cos y} = \frac{1 - \left(\frac{1}{\sqrt{2}}\right)^2}{\frac{1}{\sqrt{2}}}$$

$$= \frac{\sqrt{2}}{2}$$

#### **QUESTION 4**

- (a) A cone and a pyramid have equal heights and volumes. If the base area of the pyramid is  $154 \text{ cm}^2$ , find the radius of the cone. [Take  $\pi = \frac{22}{7}$ ]
- (b) A spherical bowl of radius  $r \text{ cm}$  is a quarter full when 6 litres of water is poured into it. Calculate, correct to three significant figures, the diameter of the bowl. [Take  $\pi = \frac{22}{7}$ ]

In part (a), candidates were able to apply the appropriate formulae to evaluate the radius of the cone.

In part (b), some candidates could not quote the formula of sphere whilst others could not convert 6 liters to  $\text{cm}^3$ .

Candidates were required to solve (b) as follows:

$$\begin{aligned} \frac{1}{4} \times \frac{4}{3} \times \frac{22}{7} \times r^3 &= 6000 \\ r^3 &= \frac{6000 \times 3 \times 7}{22} \\ &= \frac{126000}{22} \\ r^3 &= 5727.272727 \\ r &= 17.8916 \\ \text{Diameter} &= 2(17.89160) \\ &= 35.8 \text{ cm} \end{aligned}$$

#### **QUESTION 5**

<i>Class</i>	<b>JHS 1</b>	<b>JHS 2</b>	<b>JHS 3</b>
<i>Boys</i>	<b>32</b>	<b>26</b>	<b>26</b>
<i>Girls</i>	<b>28</b>	<b>44</b>	<b>36</b>

The table above shows three classes: JHS 1, JHS 2 and JHS 3 in a school. The three classes were combined to select a prefect. What is the probability that the prefect will be:

- (a) a boy?  
 (b) a girl in JHS 2?

This question was popular and answered correctly by majority of candidates. Most of the candidates were able to find the total number of students in the school which enabled them to find the probabilities in (a) and (b) correctly.

#### **QUESTION 6**

- (a) Copy and complete the table of values for the relation  $y = 7\cos x - 3\sin x$ .

<i>x</i>	$0^\circ$	$30^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$150^\circ$
<i>y</i>	7.0			- 3.0		

- (b) Using a scale of 2 cm to  $30^\circ$  on the x-axis and a scale of 2 cm to 2 units on the y-axis, draw the graph of  $y = 7\cos x - 3\sin x$  for  $0^\circ \leq x \leq 150^\circ$
- (c) Use the graph to solve the equations:
- (i)  $7\cos x = 3\sin x$ ;  
 (ii)  $7\cos x = 3.2 + 3\sin x$ .

The table of values for the relation was completed correctly. The graph was also plotted. Some candidates however, plotted only the points. They did not join them to obtain a



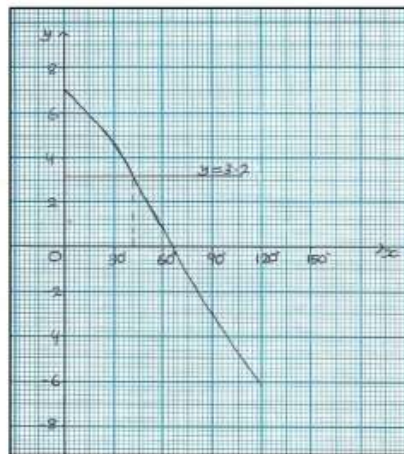
smooth curve. In some cases, the graph drawn by the candidates was like the line of best fit. In part (c)(i) and (c)(ii), candidates transformed the equations to the form  $y = 0$  and  $y = 3.2$  correctly. Some superimposed these lines on the graph, others did not. Again, some candidates did not use their graph to answer (c)(i) and (c)(ii) as the question demanded.

Candidates were expected to solve the question as follows:

(a)

$x$	$0^\circ$	$30^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$150^\circ$
$y$	7.0	4.9	0.9	-3.0	-6.1	-7.6

(b)



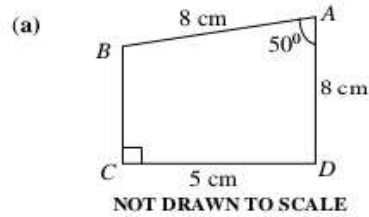
(c)

Plotting Graph

From Graph,  $y = 0$ ,  $x = 66^\circ \pm 3^\circ$

$y = 3.2$ ,  $x = 42^\circ \pm 3^\circ$

**QUESTION 7**



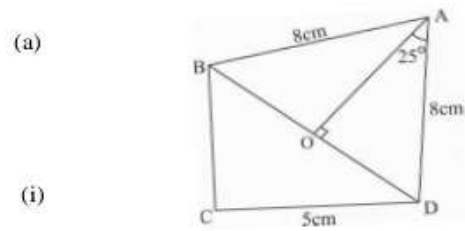
In the diagram  $|AB| = |AD| = 8\text{ cm}$  and  $|CD| = 5\text{ cm}$ . If  $\angle BCD = 90^\circ$  and  $\angle BAD = 50^\circ$ , calculate, correct to the nearest whole number:

- (i)  $|BD|$ ;
  - (ii) The area of  $\triangle BCD$ .
- (b) A man is five times as old as his son. In three years' time, the product of their ages will be 380. Find their present ages.

In part (a), most candidates assumed that lines  $CD$  and  $AD$  were orthogonal. On this premise they proceeded to calculate  $|BD|$  and the area of  $ABCD$  in (i) and (ii) respectively.

This assumption led to wrong answers. The candidates who used either the sine or cosine rules were able to find  $|BD|$  with ease, and the area of  $ABCD$ .

The solution is as follows:



(i)

$$|PD| = 8 \cos 65 = 3.3809$$

(ii)  $|BD| = 2 |PD| = 2(3.3809)$   
 $= 6.7618$   
 $= 7.0 \text{ cm}$

$$|BC| = \sqrt{6.7618^2 - 5^2}$$

$$= 4.5521 \text{ cm}$$

$$\text{Area} = \frac{1}{2} (5) (4.5521)$$

$$= 11.3803$$

$$= 11 \text{ cm}^2$$

Part (b) was quite easily done by most candidates. From the given information, they obtained the resulting quadratic equation and solved same to get the respective ages of the man and his son.

Candidates were required to solve the question as follows:

Let son's age =  $x$

Father's age =  $5x$

In three years, they will be  $(x+3)$  and  $(5x+3)$

$$(x+3)(5x+3) = 380$$

$$5x^2 + 18x + 9 = 380$$

$$5x^2 + 18x - 371 = 0$$

$$x = 7, \frac{-53}{5}$$

$$x = 7$$

Son's age = 7 years

Father's age = 35 years

### QUESTION 8

- (a) A market woman purchased a number of plates for GH¢150.00. Four of the plates got broken while transporting them to her shop. By selling the remaining plates at a profit of GH¢1.00 on each, she made a total profit of GH¢6.00. How many plates did she purchase?
- (b) If  $\frac{1}{32}, m, \frac{1}{8}, n$  is in Geometric Progression (G.P), find the values of  $m$  and  $n$ .

In part (a), most candidates were able to find the cost price of one plate, profit on each plate as GH¢1.00. Some however could not arrive at the profit equation and hence solve for plates bought.

Candidates were expected to solve the question as follows:

$$\begin{aligned}\text{Number of plates} &= x \\ \text{Cost of each plate} &= \frac{150}{x} \\ \text{Cost price} &= 150 + 6 = 156 \text{ cedis} \\ \text{Selling price of each} &= \frac{156}{x-4} \\ \text{Profit on each is GH¢1.00} \\ \frac{156}{x-4} - \frac{150}{x} &= 1 \\ 156x - 150(x-4) &= x(x-4) \\ x^2 - 10x - 600 &= 0 \\ (x-30)(x+20) &= 0 \\ x &= 30, x = -20 \\ \text{Plates bought} &= 30\end{aligned}$$

Part (b) was answered satisfactorily. Given the terms in the GP, candidates obtained the common ratio,  $r$ , using the relation  $r = \frac{u_2}{u_1} = \frac{u_3}{u_2} = \frac{u_4}{u_3}$  then continued to find the values of the other terms of the sequence.

Candidates were expected to solve the question as follows:

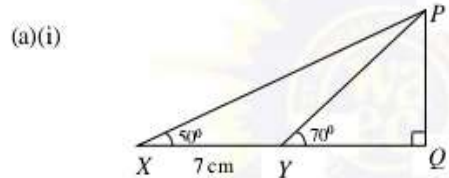
$$\begin{aligned}\frac{m}{\frac{1}{32}} &= \frac{\frac{1}{8}}{m} \\ m^2 &= \frac{1}{256} \\ m &= \frac{1}{16} \\ \text{common ratio} &= \frac{\frac{1}{16}}{\frac{1}{32}} = 2 \\ n &= \frac{1}{8} \times 2 = \frac{1}{4} \\ n &= \frac{1}{4}\end{aligned}$$

**QUESTION 9**

- (a) Two points  $X$  and  $Y$ , 7 meters apart are on the same horizontal ground. The angles of elevation of a point  $P$  from  $X$  and  $Y$  are  $50^\circ$  and  $70^\circ$  respectively.  $Q$  is a point on  $XY$  produced such that  $\angle YQP = 90^\circ$
- (i) Illustrate the information in a diagram
- (ii) Calculate, correct to two decimal places, the length:
- (a)  $\overline{XP}$ ;
- (b)  $\overline{YQ}$ .
- (b) solve the equation:  $\frac{3x}{1-x} + \frac{2x}{x+1} = 2$ .

In part (a) of the problem, most candidates could not illustrate the information in the appropriate diagram.  $Q$  is a point on  $XY$  produced such that  $\angle YQP = 90^\circ$  was misunderstood or ignored by some. Instead,  $Q$  was located between  $X$  and  $Y$ , resulting in wrong diagram and subsequently incorrect answers.

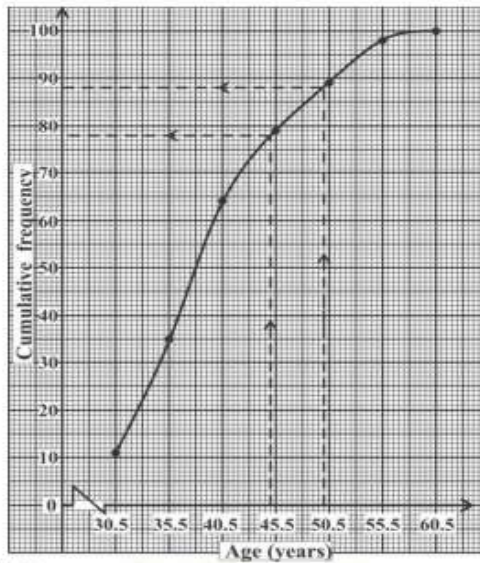
Candidates were expected to solve the question as follows:



(ii) (a)  $\angle XYP = 180^\circ - 70^\circ = 110^\circ$   
 $\angle XPY = 180^\circ - (110^\circ + 50^\circ) = 20^\circ$   
 $\frac{7}{\sin 20^\circ} = \frac{XP}{\sin 110^\circ}$   
 $|XP| = \frac{7 \sin 110^\circ}{\sin 20^\circ} = 19.2336$   
 $= 19.23 \text{ metres}$   
 $\angle XPQ = 180^\circ - (50^\circ + 90^\circ) = 40^\circ$

(b)  $\frac{XQ}{\sin 40^\circ} = \frac{19.23}{\sin 90^\circ}$   
 $|XQ| = \frac{19.23 \times \sin 40^\circ}{\sin 90^\circ} = 12.36$   
 $|YQ| = 12.36\text{m} - 7\text{m} = 5.36\text{m}$

Part (b) was solved with ease, candidates cleared the fractions, expanded the resulting expressions to obtain a quadratic equation which was solved easily.



Scale, Axes labelling

(b)(i)

Probability  $\frac{77}{100} \pm \frac{1}{100}$  or  $0.77 \pm 0.01$

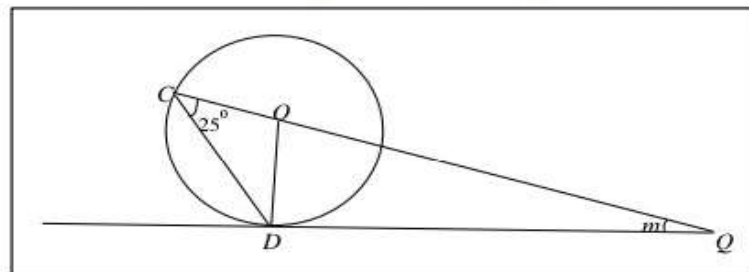
(ii)

$88 \pm 1$

$100 - 88 = 12 \pm 1$  worker

**QUESTION 11**

(a)



NOT DRAWN TO SCALE

The diagram shows a circle, centre  $O$ , with  $C$  and  $D$  as points on the circumference.  $\overline{DQ}$  is a tangent produced at  $Q$ . Find the value of  $m$ .

- (b) Find the equation of the line which has the same gradient (slope) as  $2y + x = 6$  and passes through the point  $(-2, 3)$ .
- (c) The ratio of the profit, cost of materials and labour in the production of an article is 5: 7: 13 respectively. If the cost of materials is Le 840 more than that of labour, find the total cost of producing the article.

Part (a) of the question was satisfactorily answered by only a few candidates who applied appropriate circle theorem relating an isosceles triangle, to find the value of  $m$  as indicated in the question.

The solution is as follows:

$$\begin{aligned}\angle DOQ &= 50^\circ \\ \angle CDQ &= 115^\circ \\ m + 115^\circ + 25^\circ &= 180^\circ \\ m &= 180^\circ - 140^\circ = 40^\circ\end{aligned}$$

Part (b) was quite easily solved. Candidates expressed the given equation in the form  $y = mx + c$  and deduced the gradient,  $m = \frac{-1}{2}$  and later found the equation of the line with gradient  $\frac{-1}{2}$  that passed through the point  $P(-2, 3)$ , using the equation  $y - y_1 = m(x - x_1)$ .

The solution is as follows:

$$\begin{aligned}y &= \frac{-1}{2}x + 3 \\ m &= \frac{-1}{2} \\ y - 3 &= \frac{-1}{2}(x + 2) \\ y &= \frac{-1}{2}x + 2 \\ 2y + x - 4 &= 0 \text{ or equivalent}\end{aligned}$$

Part (c) of this question was not a challenge to most candidates.

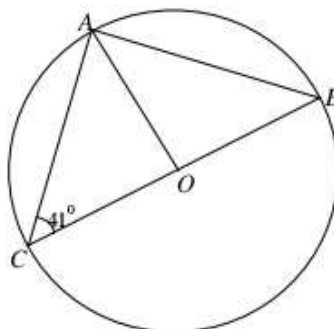
Candidates were required to answer the question as follows:

$$\begin{aligned}\text{Let } x &\text{ be total cost of producing the article.} \\ \frac{13}{25}x - 840 &= \frac{7}{25}x \\ 13x - 7x &= 21000 \\ x &= 3,500 \\ \text{Total cost of producing the article} &= \text{Le } 3,500.00\end{aligned}$$



**QUESTION 12**

(a)



**NOT DRAWN TO SCALE**

In the diagram, O is the centre of the circle ABC and  $\angle BCA = 41^\circ$ .

Find:

- (i)  $\angle BOA$ ;
  - (ii)  $\angle BAO$ .
- (b) The angle of depression of a point,  $P$ , on the ground from the top,  $T$ , of a building is  $23.6^\circ$ . If the horizontal distance from  $P$  to the base of the building is 50 m, calculate, correct to three significant figures, the height of the building.
- (c) A cow is tied to a post at the centre of a square grazing field of side 25 m by a rope 10 m long. Find, correct to two decimal places the percentage of the field the cow is able to graze on. [Take  $\pi = \frac{22}{7}$ ]

Part (a) was answered incorrectly, candidates failed to apply the appropriate theorems regarding the angle in a semi circle and angles in an isosceles triangle.

Some candidates assumed that  $\angle BOA = \angle AOC = 90^\circ$ , see question. This wrong assumption could not lead to the correct answer. Others who applied the appropriate theorems were able to answer the question with ease.

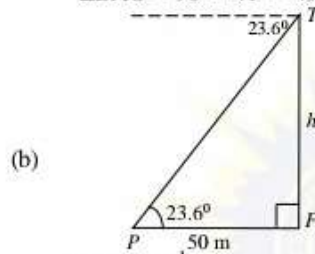
Part (b) was quite an easy question for those who were able to locate the angle of depression properly. Some candidates could not identify the angle of depression which affected their solution.



Part (c) was quite easily done. Candidates found the area of the grazing field as well as the portion grazeable by the cow and then computed the required percentage.

Candidates were required to solve question 12 (a), (b) and (c) as follows:

- (a)  $\angle BOA = 2\angle BCA$   
 (i)  $= 2(41) = 82^\circ$   
 $\angle AOC = 180^\circ - 82^\circ = 98^\circ$   
 $\angle OAC = 180^\circ - 41^\circ - 98^\circ$   
 (ii)  $= 41^\circ$   
 $\angle BAO = 90^\circ - 41^\circ = 49^\circ$



$$\tan 23.6 = \frac{h}{50}$$

$$h = 50 \tan 23.6 = 21.8469 = 21.8 \text{ m}$$


$$\text{Area of field} = 25 \times 25 = 625 \text{ m}^2$$

$$\begin{aligned} \text{Area of circle the rope makes} &= \frac{22}{7} \times 10^2 \\ &= 314.29 \end{aligned}$$

(c) 
$$\begin{aligned} \% \text{ Grazed} &= \frac{314.29}{625} \times 100\% \\ &= 50.29\% \end{aligned}$$

## Appendix I

### Major Supervisor Chapters Proceeding/Clearance Form



DEPARTMENT OF SCIENCE EDUCATION  
POSTGRADUATE PROGRESS REPORT (PPR) I

PHD  M.TECH

- NAME OF CANDIDATE: MUNISA PYASKE MUSA
- REGISTRATION NUMBER: M.Tech /SSTE/2018/8493
- YEAR ADMITTED: 2013
- APPROVED THESIS TITLE: IMPACT OF COGNITIVE CONFLICT AND COLLABORATIVE INSTRUCTIONAL STRATEGIES ON GEOMETRY REINFORCEMENT AND ATTENTION AMONG SS. STUDENTS IN MINNA
- COMPLETENESS OF CHAPTER ONE
 

S/N O.	Chapter One (Introduction)	Satisfactory (ST) (2)	Not satisfactory (NS) (1)
I	Background of the study	✓	
II	Statement of the problem	✓	
III	Aim/Objectives/Research Questions and Hypotheses	✓	
IV	Scope of the study	✓	
V	Significance of the study	✓	
VI	Definition of major terms and assumption of the study.	✓	

6. Score:  / 12 \* 100

- RECOMMENDATION: Approved for next level.
- ATTESTATION: Proceed to chapter two.
- SUPERVISOR: Major:  Co-Supervisor: 

NAME: Dr. (Mrs.) Lumba R. H.

SIGNATURE: [Signature]

DATE: 07/01/2022

POSTGRADUATE SEMINAR COORDINATOR

Name: ..... Signature/Date: .....



DEPARTMENT OF SCIENCE EDUCATION  
POSTGRADUATE PROGRESS REPORT (PPR) II

PBD

M.TECH

1. NAME OF CANDIDATE: YANUSA PYASTE MUSA
2. REGISTRATION NUMBER: M.Tech/SST/2018/8493
3. YEAR ADMITTED: 2018
4. APPROVED THESIS TITLE: IMPACT OF COGNITIVE CONFLICT AND COLLABORATIVE INSTRUCTIONAL STRATEGIES ON GEOMETRY ACHIEVEMENT AND RETENTION AMONG S.S. STUDENTS IN MINNA, NIGER STATE
5. COMPLETENESS OF CHAPTER TWO

S/NO.	Chapter Two (Literature Review)	Satisfactory (ST) (2)	Not satisfactory (NS) (1)
I	A Conceptual Framework Schema presentation	✓	
II	Review of Major Concepts and Variables in the study	✓	
III	Identification of Theoretical Framework Underlying the research	✓	
IV	Review of Empirical works related to the construct investigated	✓	
V	Relatedness of Papers Reviewed, Significance of the study	✓	

6. Total Score Obtained:  / 10 \* 100
7. RECOMMENDATION: Proceed to chapter three
8. ATTESTATION: Satisfactory
9. SUPERVISORS: Major:  Co-Supervisor:  Co-Supervisor:

NAME: Dr. (Mrs) Eunice R. W.  
SIGNATURE: [Signature]  
DATE: 07/07/2022

POSTGRADUATE SEMINAR COORDINATOR

Name: .....

Signature/Date: .....



DEPARTMENT OF SCIENCE EDUCATION  
POSTGRADUATE PROGRESS REPORT (PPR) III

PhD  M.Tech

1. NAME OF CANDIDATE: YUNUSU PYASTE MUSA
2. REGISTRATION NUMBER: M.Tech/SS/12/08/8493
3. YEAR ADMITTED: 2018
4. APPROVED THESIS TITLE: IMPACT OF COGNITIVE CONFLICT WITH COLLABORATIVE INSTRUCTIONAL STRATEGIES ON GEOMETRY ACHIEVEMENT AND ATTENTION AMONG SS STUDENTS IN MINNA, NIGER STATE
5. COMPLETENESS OF CHAPTER

S/N	Chapter Three: Research Method	Highly Satisfactory (4)	Satisfactory (3)	Fairly Satisfactory (2)	Not Satisfactory (1)
1	Appropriateness of Design Adopted for the study		✓		
2	Appropriateness of sample size	✓			
3	Clear Identification of all the elements in the population		✓		
4	Identification and Preparation of Instruments of the study	✓			
5	Adequateness of the Treatment of Instrument(s) for Experimental study (Where Applicable)		✓		

Total Score 12/20 \* 100

6. RECOMMENDATION: Ready for proposal Defense
7. ATTESTATION: Satisfactory
8. SUPERVISOR: Dr. (Mrs) Rumbi R. I. (Major)
- NAME: Dr. (Mrs) Rumbi R. I.
- Signature/Date: [Signature] 07/01/2022


POSTGRADUATE SEMINAR COORDINATOR

Signature/Date: .....

NOTE: Similar Copies for Co-Supervisors 1 and 2

## Appendix J

### Co-Supervisor Chapter Proceeding/Clearance Form



DEPARTMENT OF SCIENCE EDUCATION  
POSTGRADUATE PROGRESS REPORT (PPR) I

PhD  MTECH

1. NAME OF CANDIDATE: YUKIUSA PYASKE MUSA

2. REGISTRATION NUMBER: M Tech /SSTG/2018/6493

3. YEAR ADMITTED: 2018

4. APPROVED THESIS TITLE: IMPACT OF COGNITIVE COMPLEXITY AND COLLECTIVE METACOGNITIVE STRATEGIES AND GEOMETRY REINFORCEMENT AND PREFERENCE AMONG SS STUDENTS IN MINARA, ALBUSSA

5. COMPLETENESS OF CHAPTER ONE

S/NO.	Chapter One (Introduction)	Satisfactory (ST) (2)	Not satisfactory (NS) (1)
I	Background of the study	✓	
II	Statement of the problem	✓	
III	Aim/Objectives/Research Questions and Hypotheses	✓	
IV	Scope of the study	✓	
V	Significance of the study	✓	
VI	Definition of major terms and assumptions of the study	✓	

6. Score =  / (2 \* 100)

7. RECOMMENDATION: Approved for further necessary

8. ATTESTATION: Approved for further engagement

9. SUPERVISOR Major:  Co-supervisor:  Co-Supervisor:

NAME: Dr. Aminia Gigo Tafiala

SIGNATURE: *Aminia Gigo Tafiala*

DATE: 9/02/2022

POSTGRADUATE SEMINAR COORDINATOR

Signature/Date: .....





DEPARTMENT OF SCIENCE EDUCATION  
POSTGRADUATE PROGRESS REPORT (PPR) II

PED

MTECH

NAME OF CANDIDATE: MURRAY PYASLE NAUSA  
 REGISTRATION NUMBER: M.Tech/82TE/2018/2493  
 YEAR ADMITTED: 2018  
 APPROVED THESIS TITLE: IMPACT OF COGNITIVE AND COLLABORATIVE ABSTRACT  
 IONAL STRATEGIES ON GEOMETRY ACHIEVEMENT AND ATTITUDE  
 AMONG S.S STUDENTS IN MINNA, ALGERIA STATE  
 COMPLETENESS OF CHAPTER TWO

S/NO.	Chapter Two (Literature Review)	Satisfactory (ST) (2)	Not satisfactory (NS) (1)
I	A Conceptual Framework (Scheme- Representation)	✓	
II	Review of Major Concepts and Variables in the study	✓	
III	Identification of Theoretical Framework Underlying the research	✓	
IV	Review of Empirical works related to the construct investigated	✓	
V	Relatedness of Papers Reviewed. Significance of the study	✓	

Total Score Obtained =  / 10 \* 100  
 RECOMMENDATION: Approved for further action  
 ATTESTATION: Approved  
 SUPERVISORS: Major:  Co-Supervisor:  Co-Supervisor:   
 NAME: Dr. Aminu Gogo Tafida  
 SIGNATURE: Aminu Tafida  
 DATE: 8/02/2022  
 POSTGRADUATE SEMINAR COORDINATOR

Name: ..... Signature/Date: .....



DEPARTMENT OF SCIENCE EDUCATION  
POSTGRADUATE PROGRESS REPORT (PPR) III

Ph.D.  M.Tech

1. NAME OF CANDIDATE: James P. Nwaka  
 2. REGISTRATION NUMBER: M.Tech/SSTE/2018/8493  
 3. YEAR ADMITTED: 2019  
 4. APPROVED TITLE: Impact of Cognitive Conflict and Collaborative Instructional Strategies on Geometry Achievement and Retention Among SS Students in Minna, Niger State  
 5. COMPLETENESS OF CHAPTER:

S/N	Chapter Three Research Method	Highly Satisfactory (4)	Satisfactory (3)	Fairly Satisfactory (2)	Not Satisfactory (1)
1	Appropriateness of Design Adopted for the study		✓		
2	Appropriateness of sample size		✓		
3	Clear Identification of all the elements in the population		✓		
4	Identification and Preparation of Instruments of the study		✓		
5	Adequateness of the Treatment of Instrument(s) for Experimental study (Where Applicable).		✓		

Total Score: 20 / 20 \* 100

6. RECOMMENDATION: Recommended for further action  
 7. ATTESTATION: Approved for further action  
 8. SUPERVISOR: Dr. Amma Gogo Tafida (Co.)  
 NAME: .....  
 Signature/Date: Ala Eder 9/04/2022  
 POSTGRADUATE SEMINAR COORDINATOR

Signature/Date: .....

NOTE: Similar Copies for Co-supervisors 1 and 2

# Appendix K

## Grammarly Check Report Form

FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA  
DEPARTMENT OF SCIENCE EDUCATION  
PHD/M. TECH PROGRAMME  
GRAMMARLY CHECK REPORT FORM

1. Registration Number: M. Tech /SET/ 2018/8493

2. Candidate's Name: YUNUS PYASKO MUSA

3. Programme  
a. M.Tech:  b. PhD:

4. Area of Specialization: MATHEMATICS EDUCATION

5. Topic: IMPACT OF COGNITIVE CONFLICT AND COLLABORATIVE  
INSTRUCTIONAL STRATEGIES ON GEOMETRY ACHIEVEMENT  
AND RETENTION AMONG SS STUDENTS  
AND RETENTION AMONG SS STUDENTS IN MINNA, NIGERIA STATE

6. Number of Error Detected: 850

7. Attestation: Corrected:  Not Done:

Grammarly Checker:

Name: Mr. Moham I. U. M. Yusoff Signature/Date: [Signature] 20/02/22

Name: Dr. Rabiu M. Bello Signature/Date: [Signature] 08/02/22

(Head of Department)



## Appendix L

### Distribution of Research Population by Schools and Gender

S/N	Name of Schools	Gender		Total
		Male	Female	
1	Government Vocational Training Center	82	37	119
2	Ahmadu Bahago Secondary School Minna	420	148	568
3	Day Secondary School Barikin Sale	150	178	328
4	Day Secondary School Kwasau	109	94	203
5	Day Secondary School Limawa	411	258	669
6	Government Day Secondary School, Bosso Road	423	61	484
7	Government Girls Secondary School Minna	0	861	861
8	Women Day College	0	88	88
9	Zarumai Model School	131	62	193
10	FR. O'Connell Science College, Minna	474	0	474
11	Government Day Science College Tunga, Minna	312	350	662
12	Government Girls Science College Bosso Road, Minna	0	314	314
13	Bosso Secondary School Minna	135	188	323
14	Day Secondary School Gbada Gidan Mongoro	154	200	354
15	Day Secondary School Maitumbi Minna	301	453	754
16	Government Army Day Secondary School	83	70	153
17	Government Science College Chanchaga	59	37	96
18	Hilltop Model Secondary School	327	190	517
19	Sheikh Muhammad Sanbo College of Arts and Islamic Studies Tudun Fulani Minna	25	13	38
20	Government Technical College Minna	98	112	210
21	Maryam Babangida Girls Science College	0	370	370

22	Model Science College Tudun Fulani	229	244	473
	<b>Total</b>	<b>3923</b>	<b>4328</b>	<b>8251</b>

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Source: Ministry of Education, Niger State

Appendix M

Introductory letter to sampled schools

**FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA.**  
SCHOOL OF SCIENCE AND TECHNOLOGY EDUCATION  
DEPARTMENT SCIENCE EDUCATION

Vice Chancellor: PROF. ABDULLAH BALA, Ph.D FRSn  
Head of Department: DR. RABIU M. BELLO PhD, MStAN

Federal University of Technology,  
P.M.B. 65,  
Minna, Niger State,  
Nigeria.

Date: \_\_\_\_\_

Name of Student: YINUSA PIPAKE MUSA  
Matriculation No: MTech/15161/2018/2493  
Status: \_\_\_\_\_

TO WHOM IT MAY CONCERN

The bearer is a postgraduate student of the department. his/her humbly request for your assistance/support with information/data necessary for research towards improvement in knowledge skills and attitudes in line with international best practices for quality research output in the University.

The data/information given shall be used only for the purpose of the research and absolute confidentiality shall be maintained.

Please accept the assurances of my esteem regards.

Thank you.

  
Dr. Rabiu M. Bello  
HOD, Science Education.



  
PRINCIPAL  
HILLTOP MODEL SCHOOL  
MINNA  
Date 25/10/23

Contacts: +234-803-592-7009  
+234-802-635-8884  
E-mail: drabiu@fuminna.edu.ng

**FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA.**  
SCHOOL OF SCIENCE AND TECHNOLOGY EDUCATION  
DEPARTMENT SCIENCE EDUCATION

Vice-Chancellor: PROF. ABDULLAHI BALA, Ph.D FRS  
Head of Department: DR. RABIU M. BELLO PhD, MESTAN



Federal University of Technology  
P.M.B. 65,  
Minna, Niger State,  
Nigeria.

Date: \_\_\_\_\_

Name of Student: Yunusa Pyashe Must  
Matriculation No: MTECH/SSTE/2018/8493  
State: \_\_\_\_\_


TO WHOM IT MAY CONCERN

The bearer is a postgraduate student of the department. his/her humbly request for your assistance/support with information/data necessary for research towards improvement in knowledge skills and attitudes in line with international best practices for quality research output in the University.

The data/information given shall be used only for the purpose of the research and absolute confidentiality shall be maintained.

Please accept the assurances of my esteem regards.

Thank you.

  
Dr. Rabiu M. Bello  
HOD, Science Education.

Head of Department  
Science Education  
Fed. University of Technology  
- Minna -

Contact: +234-803-592-7009  
+234-802-635-6884  
E-mail: drabin@futmna.edu.ng

*Received*  
  
THE PRINCIPAL  
Ahmedu Bello College of Education  
MINNA  
Date: 10/02/2022