MATHEMATICAL MODEL FOR OPTIMAL SOLID WASTE MANAGEMENT IN MINNA METROPOLIS, NIGER STATE, NIGERIA

BY

MOHAMMED, Usman Nda
MTech /SPS/2018/ 8853

DEPARTMENT OF MATHEMATICS

FEDERAL UNIVERSITY OF TECHNOLOGY MINNA

JANUARY, 2022

MATHEMATICAL MODEL FOR OPTIMAL SOLID WASTE MANAGEMENT IN MINNA METROPOLIS, NIGER STATE, NIGERIA

BY

MOHAMMED, Usman Nda
MTech /SPS/2018/ 8853

A THESIS SUBMITTED TO THE POSTGRADUATE SCHOOL FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA, NIGERIAN IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE AWARD OF THE DEGREE OF MASTER OF TECHNOLOGY IN APPLIED MATHEMATICS

JANUARY, 2022

DECLARATION

I hereby declare that this thesis titled: "**Mathematical Model for Optimal Solid Waste Management in Minna Metropolis, Niger State, Nigeria''** is a collection of my original research work and it has not been presented for any other qualification anywhere. Information from other sources (published or unpublished) has been duly acknowledged.

MOHAMMED, Usman Nda
MTech/SPS/2018/8853
FEDERAL UNIVERSITY OF TECHNOLOGY
MINNA, NIGERIA

____
SIGNATURE/DATE

CERTIFICATION

The thesis titled "**Mathematical Model for Optimal Solid Waste Management in Minna Metropolis, Niger State, Nigeria'** by MOHAMMED, Usman Nda, MTech/SPS/2018/8853 meets the regulations governing the award of the degree of MTech of Federal University of Technology , Minna and it is approved for its contribution to scientific knowledge and literary presentation.

Dr. M. D Shehu Supervisor	________________________ Signature & Date
Dr. M. D. Shehu Head of Department	________________________ Signature & Date
Prof. J. Yisa Dean of School of physical Sciences	________________________ Signature & Date
Engr. Prof. O. K. Abubakre Dean of Postgraduate School	________________________ Signature & Date

DEDICATION

I, dedicate this thesis to Almighty Allah and my entire family.

ACKNOWLEDGEMENTS

My profound gratitude goes to Allah, the most beneficent and the most merciful for successful completion of this project work and for his grace in seeing me through my programme years.

I acknowledge the invaluable contribution of my project supervisor Dr. M.D. Shehu who is also Head of Department of Mathematics, Federal University of Technology, Minna for his constructive criticism, meaningful suggestion and guidance, encouragement, patience and perseverance exercised throughout the course of my project work.

I would also like to express my sincere appreciation to the entire lecturers; starting from Prof. N. I. Akinwande, Prof. Y. M. Aiyesimi, Prof. Y.A. Yahaya, Prof. U. Y. Abubakar, Prof. M. Jiya, Prof. D. Hakimi, Prof. A. A. Mohammed, Prof. R.O. Olayiwola, Dr. A. Ndanusa, Dr. A. I. Enagi, Dr. G.A. Bolarin, Dr (Mrs) .A.T.Cole, Dr. U. Mohammed, Dr. (Mrs). R. Muhammad, Dr. N. Nyor, Dr. S. I. Yusuf, Dr. S. A. Somma, Dr. O.R. Jimoh, Dr. F.A. Oguntolu, Dr. A. Yusuf, Dr. A. Lawal and Mr N.O. Salihu, Mrs. O. N. Abdurrahman, Dr. (Mrs). K. J. Audu and Mr. A. B. Zhiri, for all the knowledge imparted on me during the course of this program.

I also appreciate my family members particularly my late father- Alhaji Usman Ndagi, My mother's- Hajiya Aminatu Usman and Mallama Hauwa Usman, my precious wife – Mallama Salamatu Alhassan, uncles -Alh. Muhammad Tako, Alhassan Muhammad Tako, my Brothers-Late Baba Usman (Pele), Babandako Usman, Baba usman, Nma Ndagi, Danjuma. K. Mohammed, Danjuma Mohammed, my Sisters-Yaya Lanchikagi, Yamama, Ya'adiza Alfa and others.

I remain grateful to my mentor Al-musbahu Abdurahman, friends and associates particularly, Alh. Ibrahim Mohammed, Jamiu, Bala Mustapha, Jashua, Yahaya Ndagana, Alhassan Alhassan, Abraham Zhirin, the family of Mr and Mrs Alh. Ali-Baba Ishaq, the family of Mr. and Mrs. Alh. Alhassan Sokyara whose contributions in one way or the other have made the completion of this programme a successful one. I also acknowledge the effort of Adirab Education & IT Consultant; may Allah continue to lift you higher. Finally, kudos to all my course mates and friends on the campus, I appreciate you all for your support in one way or the other.

ABSTRACT

The existing solid waste management system in Minna municipal suffers from the control of residential waste generated and the vehicle routes. In this study, a mathematical model for municipal solid waste management system (MSWMS) is presented for Minna metropolis. The linear programming was used in formulating (MSWMS) model. The formulated was solved using Lingo Software, and the optimal solution provides the best control of waste generate and the least transportation cost. The result shows that the residential waste generated in Minna metropolis is control from 2, 489 tons to 1, 133 tons and the transportation cost is minimized from ₦ 7, 670, 000.00 to ₦ 3, 125,080.00 annually.

TABLE OF CONTENTS

Contents **Page**

LIST OF TABLES

Table **Page**

LIST OF FIGURES

Figure **Page**

CHAPTER ONE

1.0 INTRODUCTION

1.1 Background to the Study

Municipal Solid Waste (MSW) is a composition of both organic and inorganic materials generated from series of human activities in industrial site, domestic household, commercial centers, and other institutional workshops. The presence of MSW in a society is a great problem if not well managed due to its ability to induce environmental degradation (Suberu *et al*., 2012).

Municipal solid waste management (MSWM) is one of the critical environmental challenges of quick urban development that developing countries including Nigeria face (shamshiry *et al*., 2011). While the population densities in urbanized areas and per capita waste generation increased, the available land for waste disposal decreased proportionately. Solid waste management thus emerged as an essential, special sector for keeping cities healthy and livable.

Solid waste management refers to source separation, storage, collection, transportation and final disposal of waste in an environmentally sustainable manner (Puopiel, 2010). Solid waste management is an important environmental health service and an integral part of basic urban services. This is because the health implications of poor waste management can be very damaging to the people exposed to these unsanitary conditions. Diseases such as cholera, typhoid, dysentery and malaria are all related to the practice of poor waste management. The collection, transfer and disposal of waste have been generally assumed by metropolitan governments in both developed and developing worlds. This constitutes a basic and expected government function. The format varies in most urban areas where solid waste is collected either by a government agency or private contractor, despite the fact that developing countries do spend about 20 to 40 percentages of metropolitan revenues on waste management, they are unable to keep pace with the scope of the problem (Zerbock, 2003). In fact, when the governments of African countries were required by the World Health Organization (WHO) to prioritize their environmental health concerns, the result revealed that solid waste was identified as the second most important problem after water quality, (Zerbock, 2003).

Niger State is one of the States in Nigeria with a wider landmass, an attraction center for tourism, agricultural and commerce among other economic value (Maji, 2014). Minna, being the capital of Niger state has an estimated population of about 3,954,772 (population census 2006). Thus, the increasing number of population has resulted to an increase in the amount of municipal solid waste (MSW) generation. Owing to the structure of the society changes from agricultural with low-density and widespread population to urban, high-density population. Owing to this ever increasing waste, there is needed to be balanced with the provision of adequate waste collection and transportation system.

1.2 Historical Background of NISEPA

Niger State Environmental Protection Agency is established by the Niger state government towards ensuring a conducive and sustainable development of the environment for present and generations to come. The Niger State Government signed an edict cited as the Niger State Environmental Protection Agency in 1996. This edict was amended on 4th of May 2011 by the Niger State House of Assembly as Niger State Environmental Protection Law 2011. The executive General Manager with his team of qualified professionals based on directives from the Governor's office, prepare the Agency's programs towards achieving a target and expected completion time as well as their cost analysis.

1.3 Statement of the Research Problem

Minna**,** the capital city of Niger state has a population of more than one million people and it is estimated that more than one thousand tons of Municipal Solid Waste (MSW) is generated per year. The Niger State Environmental Protection Agency (NISEPA) is faced with the problem of how to control residential waste generated and to minimize the cost of transportation of the Solid waste from the collection point to the Landfill which arises in everyday living (NISEPA, 2020). Recyclable wastes were collected from the collection centres. The existing system is largely based on experience, which leads to high cost of spending by state government.

1.4 Aim and Objective of the Study

1.4.1 Aim of the study

The aim of this work is to formulate mathematical model for Municipal Solid Waste Management System (MSWMS) for control of solid waste in Minna metropolis as well as to minimize cost of transporting solid waste from collection point to landfill.

1.4.2 Objectives of the Study

The objectives of the study are to:

i. Use mathematical model for MSWMS to control solid waste in Minna metropolis
ii. Minimize the cost of transporting solid waste from collection point to land fill or dumping site
iii. Compare between North – west corner method, least cost method and Vogel's approximation method the most effectives approach in minimizing transportation cost.
iv. Use lingo software to determine optimal result

1.5 Justification of the Study

Solid waste management is an important environmental health service and an integral part of basic urban services. This is because the health implications of poor waste management can be very damaging to the people exposed to these unsanitary conditions. This is because recycling protects our health and environment when harmful substances are removed from the waste stream. Recycling also helps in conserving our natural resources because it reduces the need for raw material.

1.6 Significance of the Study

This study will be of significance in several ways. First and foremost, Proper solid-waste collection is important for the protection of public health, safety, and environmental quality in Minna metropolis. it will help government or Niger state Environmental Protection Agency to have firsthand information on the cost reduction, considering the fact that road transportation as a means of the movement of Solid waste material from the collection point to the Landfill in Minna Metropolis highly cost. Furthermore, the study will help Transportation department of the agency to acquire relevant skills towards improving their services. It will also be of help to the society to have serene and clean environment.

1.7 Definition of Terms

Incinerator: this is a disposal method that involves combustion of waste material. It converts waste materials into heat, gas, steam and ash.

Land-fill: A place where waste is buried under the ground.

Municipal Solid Waste (MSW): this is non-air and sewage emission created within and disposed of by municipality, including household garbage, commercial refuses, construction and demolition debris, dead animals and abandoned vehicles.

Municipal Solid Waste Management (MSWM): this involves the collection of waste from its sources and the transportation of waste to processing plants where it can either be converted into fuel (refuse derived fuel), electrical energy, compost (stabilized organic material) or recycle for reuse.

Recycle: is defined as the collection of materials separated from the waste

Solid Waste: Is any material which comes from domestic, commercial and industrial sources arising from human activities which has no value to people who possess it and is discarded as useless.

Solid Waste Management (SWM): refers to source separation, storage, collection, transportation and final disposal of waste in an environmentally sustainable manner.

Waste: it is defined as man's unwanted material that need to be discarded.

Waste Management: is the collection, transport, processing, recycling or disposal and monitoring of waste materials.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Solid Waste Management (SWM)

Solid waste is any material which comes from domestic, commercial and industrial sources arising from human activities which has no value to people who possess it and is discarded as useless. Waste disposal in the olden days where not difficult as habitations were sparse and land was plentiful. Waste disposal became problematic with the rise of towns and cities where large numbers of people started to congregate in relatively small areas in pursuit of livelihoods (Shafiul and Manoor, 2003).

Solid waste management is a global issue that is a growing source of concern in developed and developing countries due to increase urbanization; changes in consumer pattern and industrialization which all directly influence solid waste generally (Kadafa *et al*., 2013). Adedibu (1993) is of the view that the nature and composition of solid waste is a product of climatic and business activities in urban centers. He argue further that most of the agricultural produce such as maize, cassava, vegetable, millet are brought unprocessed during the rainy and harvesting seasons from the nearby farms.

The composition of refuse generated in an area determines the type of disposal method suitable for a particular form of waste and the effectiveness of a collection system depends on the cooperation of households and individuals in various sectors of the city in providing containers for storing refuse in accordance with the regulation and regularly placing the materials for collection (Afon, 2003).

Abumere (1983) links socio-cultural factors to land use pattern such as housing density and eating habits. He further states that solid waste accumulation is a product of chaotic land use pattern, the number of household living and that the eating habit in a house greatly determines the composition of refuse generated.

Abila and Kantola (2013) are of the view that municipal waste management problems in Nigeria cut across human health, air and water and land pollution among others. Adewole (2009) argue that continuous indiscriminate disposal of municipal solid waste is accelerating and is linked to poverty, poor governance, urbanization, population growth, poor standard of living and low level of environmental awareness.

Highfill and McAsey (2001) studies municipal waste management using optimal control model as:

$$\max_{c,x,z} \int_0^T U(C(t))e^{-pt}dt$$

Subject to

$$\frac{ds}{dt} = -z(t); \ \text{s(0) = so} \qquad (2.1)$$

$$c(t) = x(t) + z(t)$$

$$c(t) + \beta(t, x(t))x(t) \leq Y(t)$$

$$x(t), z(t) \geq 0$$

where

$C(t)$= the rate of consumption of the aggregate goods (food, shelters) by the representative consumer.

$x(t)$ denotes the rate at which waste is recycle and

$z(t)$ denotes the rate at which waste is deposited in a landfill.

The primary result of the study is the prediction that recycling, once begun, will always increase in a community whose income is increasing. This result holds even though recycling is not an argument of the utility function and the per unit cost of recycling increases with the amount of recycling (and the total recycling cost function is convex).

2.2 Municipal Solid Waste Management System (MSWMS)

According to Tchobanoglous and Kreith (2002) MSWMS is a complex process because it involves many technologies, associated with controlling generation, handling and storage, transportation, processing and final disposal of MSW under legal and economically acceptable social guidelines that protect the public health, and the environment. The main function of any MSWMS is the treatment of waste generated; where in addition, energy and recyclable materials can be recovered as by-products.

The complexity of a MSWMS can be described by the number of relationships between the components in the system. Due to the complexity of MSWMS, computer models should be built as supporting tools to explain, control or p redict the behavior of these systems, as well as to plan and assess waste management (Tanskanenn, 2000). Over the past decades, various evaluation techniques have been used to analyze MSWMS. These techniques include numerical solving methods, life cycle assessment (LCA), life cycle inventory (LCI), material flow analysis (MFA), input-output (IO) tables, and several optimization approaches, such as linear programming (LP), mixed integer programming (MIP), and dynamic programming (DP) (Nakata *et al.*, 2010; Diaz and Warith, 2006; Luoran and Horttanainen, 2008; Chang and Wang, 1996; Gielen,1998).

In order to design the optimal MSWMS, several characteristics of the target area are required along with the aspects relating to MSW and its management, such as the annual amount of generated waste, waste composition and density, and an existence of treatment technologies. The following aspects may also be included: urbanization rate, waste composition, climatic condition (Dayal *et al.,* 1993). It needs to be mentioned that the presence of a market for by-products, such as recycled materials, compost, and/or energy carriers produced from waste is another necessary aspect of the designed MSWMS (Rodionov & Nakata, 2011).

To be sustainable waste management needs to be appropriate to the local conditions of the target area with respect to economic, environmental and social perspectives. Social issues considered within waste management may include a variety of factors, such as household size, occupation, income, consumption patterns, willingness to separate at source, willingness and ability to pay and public acceptance of waste management plans (Nilsson-Djerf & Mcdougall, 2000; Desmond, 2006)

Rodionov and Nakata (2011), studies MSW utilization using linear programming optimization model. The main aim of their study was an effort to design an optimal waste management / utilization system and to assess the current waste management practices as one system. The mathematical formulation of the model is given below. The objectives function and the constrained are formulated as:

$$\min = \sum_j nc_j = \min \sum_j \left(tc_j - b_{kj}.p_k \right) \quad (2.2)$$

$$tc_j = \sum_j q_{ij}.\left(c_{capj}.CRF_j.LF/T_{ot} + C_{omj} + C_{trj}.\left(d_j + fr_j.dr_j \right) \right) \quad (2.3)$$

$$nc_j = tc_j - b_{kj}.p_k \quad (2.4)$$

$$CO_2 = \left(\sum_j \sum_i q_{ij}.emf_j + \sum_j \sum_i q_{ij}.\left(d_j + fr_j.dr_j \right) cap^{-1}{}_{tr}.em_{tr} \right) \quad (2.5)$$

$$En = \sum_i \sum_j q_{ij} \left(LHV_i + LHV_{RDF} + LFG_{gr}.eff_{coll}.LVH_{LFG} + gas_{gr}.eff_{coll}.LVH_{gas} \right) \eta_{eh} \quad (2.6)$$

Subject to the constraints

$$\sum_i q_i = \sum_j \sum_i q_{ij} \quad (2.7)$$

$$\sum_i q_{ij} \le T_{cap_j} \quad (2.8)$$

$$q_{ij} \le a_{ij}.\sum_i q_i \quad (2.9)$$

$$\sum_e \sum_j \sum_i q_{ije} = rf_{WTE}.\sum_i q_i \quad (2.10)$$

$$\sum_{m,c}\sum_{j}\sum_{i} q_{ijm,c} = f_{WTE} \cdot \sum_{i} q_i \tag{2.11}$$

$$\sum_{i} q_{i,landfill} + \sum_{j}\sum_{i} q_{ij} \cdot fr_j = fr_{LFD} \cdot \sum_{i} q_i \tag{2.12}$$

$$q_{ij} \leq 0 \tag{2.13}$$

Where, equation (2.2) and (2.3) is the objective function which is defined as the total cost between the total cost and revenue. The total cost of the MSW utilization system which is calculated from the summation of the costs for collection / transportation and treatment / landfill disposal. Equation (2.4) is the system net cost. Equation (2.5) is the environmental impact of the system. Equation (2.6) is the energy generated by the system. Equation (2.7) is the mass balance constraints which is the all wastes generated in the study area should be transported to treatment plants or disposal site. Equation (2.8) is the maximum capacity constraints which consider that planned capacity at each facility should be less than or equal to the maximum allowable capacity of the facility. Equation (2.9) is the waste availability constraint which is the waste flow use in the model is subject to the components of each waste material. Equation (2.10) and (2.11) is the waste utilization constraints which represent a relation between total amount of waste generated in the target area and amount of waste materials allocated for recycling, composting and energy production purposes. Equation (2.12) is the landfill disposal constraints which is the amount of waste allocated directly to landfill and untreated residues coming from other treatment facilities, which are to be transported to a final disposal site. Equation (2.13) is non-negativity constraint which assures that only positive amount of waste material are considered in the solution. In order to evaluate the impacts of different waste management option on the MSW utilization system, several scenarios were constructed in their study. A baseline or business as usual (BAU) scenarios and four alternative scenarios explain below. All scenarios are based on the input data given for the generated amount of in the target area. The study or target is St. Petersburg in Russia.

i. **BAU Scenario**; The BAU scenario represents the existing MSWMS in the target area, and is the baseline upon which the results from other scenarios are compared.

ii. **Low cost Scenario**; To achieve the optimal mix of treatment technologies with the minimal net cost of the proposed MSW utilization system, this scenario has been designed without considering regulation constraints, such as the desired amount of waste for recycling and/or for energy generation.

iii. **Max Energy Recovery Scenario (WTE);** In this scenario, the amount of waste allocated to energy production is maximized. In order to maximize the preference of energy produced from waste, the constraint on promoting the use of MSW for energy production is included in the model formulation.

iv. **Recycling Scenario (WTR);** The MSW utilization system presented in this scenario considers the maximum recycling capability of the proposed system. The constraint on the use of MSW for recycling purposes looking forward to increasing output of material recycled is considered in the model.

v. **WTE AND WTR;** In this scenario high priority has been given to the energy generated from waste and the recycling of waste materials. Regulation constraints used in the model formulation have been defined as a combination of the average maximal value for the amount of waste used for energy production (50%), and the average material recycling rates (30%).

Since the data of the waste material and the heating values for each type of MSW was not available, it was estimated using the average data from several existing literature.

In order to obtain the optimal solution, the following assumptions were made in their model.

i. The geographical boundary of St. Petersburg set the limits for the included type of waste streams, but there are no limits to location of the processing facilities.
ii. Current analysis is limited to the treatment of waste generated during a one year period. Constant residue rates have been set for all types of treatment technologies used in the present analysis.
iii. The waste source separation rate of 100% has been set for all types of waste materials.
iv. Another important assumption of their model is the omission of the scale effect for the waste treatment facilities.

Daskalopoulos *et al.* (1998) have presented a mixed integer linear programming model for the management of municipal solid waste streams, taking into account their rates and compositions, as well as their adverse environmental impacts. Using this model, they identify the optimal combination of technologies for handling, treatment and disposal of municipal solid waste in a better economical and more environmentally sustainable way. The single objective is composed of costs per tons of waste treated at the recycling, composting, incinerating plants and landfills.

Fiorucci *et al.* (2003) have presented a mixed integer nonlinear programming decision support model for assisting planners in decisions regarding the overall management of solid waste at a municipal level. By using the model, an optimal number of landfills and treatment plants, optimal quantities and the characteristics of refuse that have to be sent to treatment plants, to landfills and to recycling can be determined.

Nganda (2007) developed two mathematical models as tools for solid waste planners in the decisions concerning the overall management of solid waste in au municipality. The models have respectively been formulated as integer and mixed integer linear programming problems. The solid waste management models described above was developed using linear programming and mixed integer linear programming. Hasit and

Warner (1981) compared these two techniques when applied to the waste resource allocation programme model. In their scenarios, the number of cost combinations increased rapidly as the number of facilities increased, resulting in higher data requirements and programme handling.

Ogwueleka (2009) introduces new decision procedure for solid waste collection problem. The problem objective was to minimize the overall cost, which was essentially based on the distance travelled by vehicle. The study proposed heuristic method to generate feasible solution to an extended Capacitated Arc Routing Problem (CARP) on undirected network. The technique was compared with the existing schedule with respect to cost, time and distance traveled. The adoption of the proposed technique resulted in reduction of the number of existing vehicles by 25% and 16.31% reduction in vehicle distance traveled par day.

Daskalopoulos *et al*. (1998) have presented a mixed integer linear programming model for the management of MSW streams. This is similar to Kalu *et al*. (2017) studies mathematical model of MSWM. The cost in the objective function of their model caters for the environmental considerations related to the emission of greenhouse gases. This is also similar to Kalu *et al.* (2017) model. Unlike Kalu *et al.* (2017) model, Daskalopoulos *et al.* (1998) does not cover collection and transportation costs. Regulatory and technical constraints are not considered either.

Badran and El-Haggar (2005) studied optimization of solid waste management systems using operation research methodologies. A mixed integer linear programming model is a problem whose objective covers collection costs from the districts to collection stations, transportation costs from collection stations to their composting plants or to landfills. This is similar to Kalu *et al*. (2017) model. Unlike Kalu *et al*. (2017) model, their model did not cater for environmental considerations related to the emission of greenhouse gases.

The model of Chang and Chang (1998) minimizes overall cost through the solution of a nonlinear programming problem. Unlike Kalu *et al.* (2017) model their model does not cater for regulatory and environmental constraints. Kalu *et al*. (2017) present linear model and go at length in estimation of environmental hazard x cost and scavenged fraction.

Halidi (2011) has presented a mixed integer programming of municipal solid waste management in Ilala municipality. The mathematical formulation is given below;

$$\min .Z(q,x,y)=\sum_{j=1}^{15} f_j q_j + \sum_{i=1}^{j}\sum_{j=1}^{j} T_{IJ} x_{ij} + \sum_{j=1}^{j}\sum_{k=1}^{k} T_{jk} y_{jk} \tag{2.14}$$

Subject to constraints

$$\sum_{j=1}^{j} x_{ij} = W_{i,} i = 1,\ldots,I \tag{2.15}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} x_{ij} = \sum_{j=1}^{J}\sum_{k=1}^{K} y_{jk} \tag{2.16}$$

$$\sum_{j=1}^{j} x_{ij} \le C_j, \; j = 1,\ldots,J \tag{2.17}$$

$$\sum_{j=1}^{j} y_{ik} \le L_{k,} k = 1,\ldots,K \tag{2.18}$$

$$\sum_{j=1}^{j} Q_j \le N \tag{2.19}$$

$$\sum_{j=1}^{j} c_j q_j \ge \sum_{l=1}^{I} W_i \tag{2.20}$$

And non –negativity constraints

$$x_{ij} \ge 0, y_{jk} \ge 0, i = 1,\ldots,I, \; j = 1,\ldots,J, k = 1,\ldots,K \tag{2.21}$$

Equation (2.15) is the cost of objective function which is the sum of daily fixed cost of running the selected collection centers transportation cost from the collection centers and transportation cost from the collection centers to landfills respectively. Equation (2.16) ensures that all solid waste from each street are collected. Equation (2.17) guarantees that no solid waste remains at any collection centres. Inequality (2.18) ensures that the total waste sent to a collection centre does not exceed the centre capacity. Similarly, inequality (2.19) guarantees that waste sent to a landfill does not exceed the capacity of the landfill. Inequality (2.20) make sure that the total number of selected collection centres does not exceed the maximum number of collection centres required. Inequality (2.21) ensure that the selected collection centres are big enough to take all solid waste generated from all streets. There are several decision variables considered in their study and are define as fallowing;

x_{ij} = amount (in ton) of daily solid waste to be removed from source I to collection centre j(i=1,…,I and j=1,…,J).

y_{jk} = amount (in ton) of daily solid waste to be removed from collection centre j to landfill k(j=1,…,J and k=1,…,K).

q_j = a variable which can take a value of one or zero. It takes the value if a collection centre is to be set up at the location j and zero otherwise (j=1,…,J). Their model uses the following parameters; x_{ij} =

C_j = daily capacity of the collection centre j.

L_K = daily capacity of the landfill k.

W_i = amount of daily waste generated at source i.

f_j = fixed cost of the collection centre represented as daily fixed cost.

T_{jk} = transportation cost of one ton of waste from the source I to the collection centre j (i=1,…,I ; j=1,…,j)

N= the maximum number of collection centres

He formulated the model by taking into consideration the waste flow in Illala municipality.

The model results in a least transportation cost T_{sh} 10, 969,252 per day compared with the one given by Ilala municipality of T_{sh}14,000000per day. Furthermore, the study shows that any additional increase of the collection centre capacity up to500 tons will result in a decrease of the objective function value. The developed model was solved using GNU linear programming kit (GLPK). The GNU linear programming kit (GLPK) is a software package intended for solving large scale linear programming (LP), mixed integer programming (MIP) and other related problems.

Kalu *et al*. (2017) works on mathematical model of municipal solid waste management system (MSWMS) using operational research methodologist. The mixed integer programming problem was used in formulating the (MSWS) model. Mathematical formulation of the mixed integer model is given as follows, the objective function and the constrained are formulated as;

$$\text{Min. } Z(q,x,y) = \sum_{j=1}^{15}\left(F_j + hC_j\right)Qj + \sum_{i=1}^{90}\sum_{j=1}^{15} T_{IJ}\, x_{ij} + \sum_{j=1}^{15}\sum_{k=1}^{3} T_{jk}\, y_{jk}$$

Subject to:

$$\sum_{j=1}^{15} x_{ij} = W_{i,} i = 1,2,3,\ldots,90$$

$$(1-f)\ \sum_{i=1}^{90}\sum_{j=1}^{15} x_{ij} = \sum_{j=1}^{15}\sum_{kk=1}^{3} y_{jk}, i = 1,2,3,\ldots,90$$

$$(1-f)\ \sum_{j=1}^{90} x_{ij} \le C_j\ \ (j = 1,2,3,\ldots,15) \qquad (2.22)$$

$$\sum_{j=1}^{15} y_{jk} \le L_K, k = 1,2,3$$

$$\sum_{j=1}^{15} Q_j \le N$$

$$\sum_{j=1}^{15} Q_j C_j \ge \sum_{l=1}^{90} W_i$$

$$x_{ij} \ge 0, y_{jk} \ge 0, i = 1,2,3,\ldots,90; j = 1,2,3,\ldots,15; k = 1,2,3$$

Where the variables are define as following;

x_{ij} = amount (in ton) of daily solid waste to be removed from source I to collection centre j (i=1,…,m; j=1,…,n).

y_{jk} = amount (in ton) of daily solid waste to be removed from collection centre j to landfill k (j=1,…, n; k=1,…,k).

q_j = a variable which can take a value of one or zero. It takes the value if a collection centre is to be set up at the location j and zero otherwise (j=1,…,n). And parameters used in their model is also defined as following;

F_j=the fixed daily cost of maintaining the collection center

$Q(0,1)$=the binary integer, 1 if the collection center is open, and 0 if the collection centre is closed

T_{jk}=the transportation cost of waste from the collection centre to the Landfill k

Y_{jk} =quantity of waste at the collection centre j

L_k=the capacity of the landfill k

X_i=quantity of waste from source, i

C_j=the capacity of the collection centre j

N=the number of collection centre to be opened

W_i=all generated waste in tons per day from source, i

H_i=the total number of households in a given source i

I=1,2,3,…,90, j=1,2,3,…,15, k=1,2,3.

In order to obtain the optimal solution, the following assumptions were made in their model

i. It was assumed that all generated solid waste are collected from the source to the collection centre's and then transferred to the land.
ii. Industrial and institutional waste are transferred to the nearest collection centre at the expense of their generations.
iii. Transportation cost is proportional to both the distance traveled and the carried load.
iv. The collection of waste is done at the time when there is no traffic jam.
v. The distances are measured from the centroid of either the streets or the facilities.

2.3 Solid Waste Control

Moshen *et al.* (2016) used mathematical model for the control of the solid waste in Tehran metropolis. The results of the model shows the optimal status of the available system for Tehran's solid waste. Tehran has twenty two municipal region with eleven transfer stations, ten and six units respectively they concluded that the propose model is alternative method for improvement the (SWMS) by station and processing units.

Chinchodkar *et al.* (2017) formulated a mathematical transportation problem for the given MSW management; the y minimize the present transportation cost of waste transportation and provide optimal solution as bases for decision making. They formulated their model as:

Minimize $$\sum_{i=1}^{m}\sum_{j=i}^{n} c_{i,j} x_{i,j} \tag{2.23}$$

Subject to the constraints

$$\sum_{j=i}^{n} x_{i,j} \le s, \qquad i = 12,...,\text{m} \tag{2.24}$$

$$\sum_{i=1}^{m} x_{i,j} \le d, \qquad j = 12,...,\text{n} \tag{2.25}$$

$$\text{x}_{i,j} \ge 0, \qquad For \;\; all \;\; i, j$$.

Their result shows that much amount of cost reduction than existing one, they concluded that any additional increase in dumping site will result in a decrease of objective function value.

2.3.1 Basic feasible solution

Several practitioner have developed alternative methods for determining and initial basic feasible solution which takes costs into account their method are considered as the most popular in 1950's ad 1960's.Vogel's approximation method (VAM) which is one of well –known transportation methods in the literature was investigated to obtain more efficient initial solution.

Veldo *etal* (2018) used (VAM) methods to minimize transportation cost. They used the secondary data in their work, they argue that (VAM) is better than the (NWC) because the testing table VAM optimum solution immediately obtained the optimum solution .

2.4 Remarks

The following limitation were observed in Moshen *et al.* (2016)

MSWMS in Tehran metropolis is modeled with the concept of having collection centres.

i. The result of the presented model where based on assumptions in the absence of the available data and could change with more accurate data.
ii. The model was solved using linear programming toolbox of Matlab 2015a software for windows.

In addressing these limitations, the following observations were considered;

i. To formulate mathematical model for MSWMS to reduce residential waste generation.
ii. we use an effective transportation method by using transportation problem approach to minimize the cost of transporting solid waste from collection points to landfill
iii. Transportation cost for both Industrial and institutional waste are been taking care in our model.

CHAPTER THREE

3.0 MATERIALS AND METHOD

3.1 Formulation of Mathematical Model

Mathematical formulation, description of objective function and constraints of the model where presented.

3.2 Model Assumptions

The assumptions are intended to formulate the linear programming model which was used to model Municipal Solid Waste Management System for Minna metropolis. These assumptions are as follow:

i. It is assumed that all generated solid waste in Minna metropolis are collected from the sources to the collection centers and then transferred to the landfills.

ii. In Minna metropolis there is no waste separation at the sources. Waste separation is done at the collection centers where the recycling materials are separated from non-recycling materials

iii. Every ward is considered as a generation node.

iv. Transportation cost is proportional to both the distance traveled and the carried load.

v. The collection of waste is done at the time when there is no traffic jam.

3.3 Model Analysis

The proposed model is formulated taking into consideration the waste flow in Minna metropolis. The residential sources will dispose their waste in the collection bins located at designated areas. The various households are expected to take the collection bins to the nearest waste sources to them. The waste source is a point where a big waste container (about 7.1 tons and above) is placed for onward delivery to the collection centers. A transfer vehicle is responsible for waste transfer from the sources to the collection centers and then to the landfills. According to NISEPA the total solid waste generated by residential in Minna metropolis is 2489 tons

3.4 Mathematical Formulation of MSWM Model

To formulate mathematical model for MSWMS in Minna metropolitan to control residential waste by separating recycling materials from non-recycling materials at the collection point.

Let,

$$SW_T = SW_R \tag{3.1}$$

where,

SW_T = total number of waste generated

SW_R =daily waste generated in residential activities

then,

$$SW_R = P(SWr + SWn) \tag{3.2}$$

Where,

P=number of people that live in the area

SWr= recycling materials

SWn=non-recycling materials

Substituting eqn (3.2) into (3.1) we have

$$SW_T = P(SW_r + SW_n) \tag{3.3}$$

But

Time taken to transport from collection point to landfill is:

$$T = \frac{D}{S} \tag{3.4}$$

where,

T = time

D= total distance

S =Vehicle speed

Let $T_1 = \dfrac{d}{m}$ $\quad (3.5)$

Where,

T_1 =time required to travel from collection point 0 toward collection points 1

$d(0,1)$ =distance from NISEPA office to first collection point

m=capacity of the vehicle

$$T_i = \sum_{i=1}^{n} \frac{d(i,i+1)}{m} + Trq \quad (3.6)$$

Where,

T_i=time required to travel from collection point 1to the next point n(endpoint) added to the time required for loading /unloading at each point of the collection $d(i,i+1) =$ Distance between collection point i and $i+1$,for i =1…,n-1.

Trq = time taken to load / unload at the collection point i

$$T_n = \frac{d}{m} + Trq \quad (3.7)$$

Where,

T_n =time it takes to travel from collection points n(endpoint) to collection point 0

$d(n,0) =$ distance from point n (endpoint) back to NISEPA office

$$(T_1 + T_i + T_n) = \frac{fn}{m} \quad (3.8)$$

The objectives function is therefore given as;

$$\text{Min.} = \sum_{i=1}^{m} \sum_{j=1}^{n} \text{P}(\text{SW}_\text{r} + \text{SW}_\text{n}) + \sum_{i=1}^{k} \frac{fn}{m} \quad (3.9)$$

Subject to the following constraint;

$$\sum_{I=1}^{M} P(SW_r + SW_n) \leq j_k \tag{3.10}$$

$$\sum_{j=1}^{N} P(SW_r + SW_n) \leq m \tag{3.11}$$

Where

$S =$ volume of waste transported on k route

P= number of people that live in the area

SW_r =recycling materials

SW n = non-recycling materials

m =capacity of the vehicle

j_k =working hours

3.5 Data Description

NISEPA has five (5) trucks which are working currently. The transportation of solid waste from collection point to various dumping site or landfill are group into A,B and C by NISEPA and cost associated with each of the group is given in table 3.1 bellow

Table 3.1 Transportation cos

Collection Point (Groups)	Total Cost per Annual (₦)
A	2,800,000.00
B	2,450,000.00
C	2,420,000.00
Total	7,670,000.00

Source: NISEPA (2020).

The table above shows the total cost associated with each of the collection points grouped into A, B and C per annual. The total cost for group A is ₦2,800,000.00, B is ₦2,800,000.00 and C is ₦2,420,000.00. Therefore, the total cost spent per annual in

Minna municipality on transportation of solid waste from collection centre to landfill or dumping side is ₦7,670,000.00

3.6 Route for the Collection Points

The collections of solid wastes are grouped into SA, B and C from collection points to various dumping sites or landfills as shown in Table 3.2 – 3.4.

Table 3.2 Route for Group A

Sources	Maikukele d(km)	Chanchaga d(km)	Kpakungu d(km)	Maitunbi d(km)	Capacity of collection centre (ton)
Tunga B	20	10	26	20	2000
Shango	21	8	23	23	1000
NNPC MEGA	18	15	24	23	2500
Kure mkt	17	19	21	21	1000
123 Quqters	14	22	22	25	2000
Mi Wushishi	19	23	23	4	1600
Sabongari	11	25	25	20	500
Bahago Estate	15	22	22	11	800
Demand (ton)	3000	1500	2500	2000	

Table 3.2 shows the distance between the collection points and landfills for group A.

Table 3.3 Route for Group B

Sources	Maikukele d(km)	Chanchaga d(km)	Kpakungu d(km)	Maitunbi d(km)	Capacity of the Collection Points (ton)
Top Medical	21	10	14	18	1000
First Bank T	20	10	10	8	2100
London Street	16	22	8	12	2000
Tudun Fulani	10	21	12	14	500
Dutse Kura	15	18	10	15	1600
Gurara	25	20	5	25	800
NNPC Mega Station	24	18	8	20	2000
Bahago	17	14	12	15	1000
Demand (ton)	2500	3000	500	400	

Table 3.3 shows the distance between the collection points and landfills for group B.

Table 3.4 Route for Group C

Sources	Maikunkele d(km)	Chanchaga d(km)	Kpakungu d(km)	Maitunbi d(km)	Collection Point Capacity (ton)
Shiroro Junction	20	16	12	22	800
Flayout T	16	20	18	12	2000
Oduoye Q	22	22	12	14	500
General Hospital	24	18	14	11	1000
Nateco	25	16	12	14	2000
Nisepa Office	20	15	18	12	700
Kpakungu	24	23	3	24	2500
Maikunkele	4	26	24	20	1000
Demand (ton)	2000	1000	3000	1000	

Table 3.4 shows the distance between the collection points and landfills for group C.

3.7 Transportation Model

The mathematical formulation is a linear program with m, n decision variables, m + n functional constraints, and m, n non-negative constraints. The objective function and the constrained are formulated as:

Minimize $z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{i,j}x_{i,j}$ (3.12)

Subject to constraints

$$\sum_{j=1}^{n} x_{ij} \le s_i \qquad i = 1,2,\cdots,m$$

(3.13)

$$\sum_{i=1}^{m} x_{ij} \leq d_i \qquad j = 1,2,\cdots,n \tag{3.14}$$

$x_{ij} \geq 0$ *for* all i and j

Where,

$m =$ number of sources.

$n =$ number of destinations.

$s_i =$ capacity of i^{th} collection point (in tons)

$d_i =$ capacity of j^{th} Landfill (in tons)

$c_{i,j} =$ distance coefficients of solid waste material between i^{th} collection point and j^{th} Landfill (distance in kilometers)

$x_{i,j} =$ amount of solid waste material transported between i^{th} collection point and j^{th} Landfill (in tons)

A necessary and sufficient condition for the existence of a feasible solution to the transportation problem is that:

$$\sum_{i=1}^{m} s_i = \sum_{j=i}^{n} d_i \tag{3.15}$$

$$\sum_{i=1}^{m} s_i \neq \sum_{j=1}^{n} d_j \tag{3.16}$$

The transportation problem is known as an unbalanced transportation problem and contain two cases:

Case (1)

$$\sum_{i=1}^{m} s_i > \sum_{j=i}^{n} d_j \tag{3.17}$$

Case (2)

$$\sum_{i=1}^{m} s_i < \sum_{j=1}^{n} d_j \quad (3.18)$$

Introduce a dummy origin in the transportation table, the cost associated with this origin is set equal to zero. The availability at this origin is:

$$\sum_{i=1}^{m} s_i > \sum_{j=i}^{n} d_j \quad (3.19)$$

3.8 The Transportation Algorithms

The algorithms for solving a transportation problem may be summarized into the following steps

Step 1; **formulate the problem and arrange the data into matrix form:** the formulation of the transportation problem is the linear programming [LP] problem. In transporting problem, the objective function is the total transportation cost and the constraints are the amount of supply and demand available at each source and destination respectively

Step 2; obtain an initial basic feasible solution: there are three different methods to obtain an initial solution which are North-West corner method, Least cost method and Vogel's Approximation (or penalty) method. The initial solution obtained by any of the three methods must satisfy the following conditions;

i. The solution must be feasible, i.e. it must satisfy all the supply and demand constraints (also called rim condition)

ii. The number of positive allocation must equal to m+n-1, where m is the number rows and n is the number of column.

Step 3; test the initial solution for optimality: The solution obtained in step 2 is used to test the optimality. If the current solution is optimal, then stop. Otherwise determine a new improved solution.

Step 4; updating the solution: repeat step 3 until an optimal solution is reached

3.9 Analysis of Collection Points

Since the transportation of SWN in Minna metropolitan are grouped into A, B and C and there is cost associated with each of the group, we use the three method of minimizing the transportation cost namely; North-West corner method, least cost method and Vogel approximation method to solved for each of the group and compared the results with the NISEPA actual cost.

3.9.1 Analysis of group A collection point

The three methods namely, North – West Corner Method, Least Cost Method and the Vogel Approximation method are applied to group A to obtain the initial basic solution of minimizing transportation cost.

Table 3.5: Unbalance Transportation Tableau Collection Points Group A

Sources	**Maikukele d(km)**	**Chanchaga d(km)**	**Kpakungu d(km)**	**Maitunbi d (km)**	**Supply (ton)**
Tunga B	20	10	26	20	2000
Shango	21	8	23	23	1000
Bosso	18	15	24	23	2500
Kure mkt	17	19	21	21	1000
123 Quqters	14	22	22	25	2000
Mi Wushishi	19	23	23	4	1600
Sabongari	11	25	25	20	500
Bosso Estate	15	22	22	11	800
Demand (ton)	3000	1500	2500	2000	

Table 3.5 is unbalance tableau; we therefore introduce dummy column as shown in table 3.6 bellow

Table 3.6 Balance Transportation tableau group A

Sources	Maikunkeled (km)	Chanchagad (km)	Kpakungud (km)	Maitunbid (km)	Dummy	Supply (ton)
Tunga B	20	10	26	18	0	2000
Shango	21	8	23	23	0	1000
Bosso	18	15	24	23	0	2500
Kure mkt	17	19	21	21	0	1000
123 Quqters	14	22	22	25	0	2000
Mi Wushishi	19	23	23	4	0	1600
Sabongari	11	25	25	20	0	500
Bosso Estate	15	22	22	11	0	800
Capacity (demand)	3000	1500	2500	2000	2400	

3.9.1.1 North west corner method

This method is applying to solve table 3.6 as summarized as follows;

Step 1: Start with the cell at the upper left (north-west) corner of the transportation table (or matrix) and allocate commodity equal to the minimum of the rim values for the first row and first column, i.e. min (a_1, b_1).

Step 2: (a) If allocation made in Step 1 is equal to the supply available at first source (a_1, in first row), then move vertically down to the cell (2, 1), i.e., second row and first column. Apply Step 1 again, for next allocation.

(b) If allocation made in Step 1 is equal to the demand of the first destination (b_1 in first column), then move horizontally to the cell (1, 2), i.e., first row and second column. Apply Step 1 again for next allocation.

(c) If $a_1 = b_1$, allocate $x_{11} = a_1$ or b_1 and move diagonally to the cell (2, 2).

Step 3: Continue the procedure step by step till an allocation is made in the south-east corner cell of the transportation table.

Table 3.7: Result obtain as Basic Feasible Solution (IBFS) by applying the North-West Corner Method group A

Sources	Maikunkeled (km)	Chanchagad (km)	Kpakungud (km)	Maitunbid (km)	Dummy	Supply (ton)
Tunga B	20×2000	10	26	20	0	2000
Shango	21×1000	8	23	23	0	1000
Bosso	18	15×1500	24×1000	23	0	2500
Kure mkt	17	19	21×1000	21	0	1000
123 Quqters	14	22	22×500	25×1500	0	2000
Mi Wushishi	19	23	23	4×500	0×1100	1600
Sabongari	11	25	25	20	0×500	500
Bosso Estate	15	22	22	11	0×800	800
Demand (ton)	3000	1500	2500	2000	2400	

Table 3.7 shows how the initial basic feasible solution (IBFS) is obtained by applying the North-West Corner Method and the Monthly transportation cost $(20\times 2000) + (21 \times 1000) + (15 \times 1500) + (21 \times 1000) + (22 \times 500) + (25 \times 1500) + (4 \times 500) + (24 \times 1000) + (0 \times 1100) + (0 \times 500)$ +(0×800)=₦179,000.00

Annual Transportation cost (₦179,000$\times$12)=₦2,148,000.00

Optimality Test for NWC Group A

Modified distribution method also known as MODI method or U-V method is used to carry out the optimality test. Calculate the dual variables $+V_j = C_{ij}$ using $U_i + V_j = C_{ij}$ from table 3.7 we have

$U_1 = 0$, $U_2 = -1$, $U_3 = 0$, $U_4 = -3$, $U_5 = -2$, $U_6 = -23$, $U_7 = -32$, $U_{28} = -28$, $U_1 = 0$ and

$V_1 = 10$, $V_2 = 7$, $V_3 = 12$, $V_4 = 12$, $V_5 = 0$

Compute C_{ij} -U_i -V_j for unallocated cells

$A_{12} = 10 - 0 - 7 = 3$

$A_{13} = 26 - 0 - 12 = 14$

$A_{14} = 20 - 0 - 15 = 5$

$A_{15} = 0 - 0 - 0 = 0$

$A_{22} = 8 - (-1) - 7 = 2$

$A_{23} = 23 - (-1) - 12 = 12$

$A_{24} = 23 - (-1) - 15 = 9$

$A_{25} = 0 - (-1) - 0 = 0$

$A_{31} = 18 - 0 - 10 = 8$

$A_{34} = 23 - 0 - 15 = 8$

$A_{35} = 0 - 0 - 0 = 0$

$A_{41} = 17 - (-3) - 10 = 10$

$A_{42} = 19 - (-3) - 7 = 14$

$A_{44} = 21 - (-3) - 15 = 9$

$A_{45} = 0 - (-3) - 0 = 3$

$A_{51} = 14 - (-2) - 10 = 6$

$A_{52} = 22 - (-2) - 7 = 17$

$A_{55} = 0 - (-2) - 0 = 2$

$A_{61} = 19 - (-23) - 10 = 32$

$A_{62} = 23 - (-23) - 7 = 39$

$A_{63} = 23 - (-23) - 12 = 34$

$A_{71} = 11 - (-23) - 10 = 24$

$A_{72} = 25 - (-23) - 7 = 41$

$A_{73} = 25 - (-23) - 12 = 36$

$A_{74} = 20 - (-23) - 15 = 28$

$A_{81} = 15 - (-23) - 10 = 28$

$A_{82} = 22 - (-23) - 7 = 35$

$A_{83} = 22 - (-23) - 12 = 33$

$A_{84} = 11 - (-23) - 15 = 19$

Since all the values of C_{ij} -U_i -$V_j \geq 0$ hence solution is optimum and the transportation cost=$(20\times 2000) + (21 \times 1000) + (15 \times 1500) + (21 \times 1000) + (22 \times 500) + (25 \times 1500) + (4 \times 500) + (24 \times 1000) + (0 \times 1100) + (0 \times 500)$+$(0\times 800)$=₦179,000.00

3.9.1.2 The least cost method

Since the main objective is to minimize the total transportation cost, transport as much as possible through those routes (cells) where the unit transportation cost is lowest. This method is applying to solved table (3.6) and can be summarized as follows:

Step 1: Select the cell with the lowest unit cost in the entire transportation table and allocate as much as possible to this cell. Then eliminate (line out) that row or column in which either the supply or demand is fulfilled. If a row and a column are both satisfied simultaneously, then crossed off either a row or a column. In case the smallest unit cost cell is not unique, then select the cell where the maximum allocation can be made.

Step 2: After adjusting the supply and demand for all uncrossed rows and columns repeat the procedure to select a cell with the next lowest unit cost among the remaining rows and columns of the transportation table and allocate as much as possible to this cell. Then crossed off that row and column in which either supply or demand is exhausted.

Step 3: Repeat the procedure until the available supply at various sources and demand at various destinations is satisfied. The solution so obtained need not be non-degenerate.

The Table 3.8 below represents the result obtained as initial basic feasible solution (IBFS) by applying the Least cost Method.

Table 3.8: Result Obtain initial basic feasible solution (IBFS) by applying the Least Cost Method group A

Sources	**Maikunkeled (km)**	**Chanchagad (km)**	**Kpakungud (km)**	**Maitunbid (km)**	**Dummy**	**Supply (ton)**
Tunga B	20	10	26	20	0×2000	2000
Shango	21	8× 600	23	23	0× 400	1000
Bosso	18× 1600	15× 900	24	23	0	2500
Kure mkt	17×1000	19	21	21	0	1000
123 Quqters	14×400	22× 1600	22	25	0	2000
Mi Wushishi	19	23	23	4× 1600	0	1600
Sabongari	11× 500	25	25× 100	20×400	0	500
Bosso Estate	15	22	22× 800	11× 400	0	800
Demand (ton)	3000	1500	2500	2000	2400	

The table 3.8 shows how the initial basic feasible solution (IBFS) is obtained by applying the Least cost Method and the Monthly transportation cost is $(0\times 2000) + (8\times 600) +$

$(0 \times 400) + (18 \times 1600) + (15 \times 900) + (17 \times 1000)$+(14$\times$ $400) + (22 \times 1600) +$

$(4 \times 1600) + (25 \times 100) + (20 \times 400) + (22 \times 800) =$ ₦139,400.00

Annual transportation Cost (₦$139400 \times 12) =$ ₦1,6728,00.00

Optimality Test for LCM Group A

Calculate the dual variables $+V_j = C_{ij}$ using $U_i + V_j = C_{ij}$ from table 3.8 we have

$U_1 = 0, U_2 = 0, U_3 = -7, U_4 = -6, U_5 = -14, U_6 = -10, U_7 = 0, U_8 = -3$ and $V_1 = 11$,

$V_2 = 8, V_3 = 25, V_4 = 14, V_5 = 0$

Compute C_{ij} -U_i -V_j for unallocated cells

$A_{11} = 20 - 0 - 11 = 9$

$A_{12} = 10 - 0 - 8 = 2$

$A_{13} = 26 - 0 - 25 = 1$

$A_{14} = 18 - 0 - 14 = 4$

$A_{21} = 21 - 0 - 11 = 10$

$A_{23} = 27 - 0 - 25 = 2$

$A_{24} = 23 - 0 - 14 = 9$

$A_{33} = 24 - (7) - 25 = 6$

$A_{34} = 23 - (-7) - 14 = 1$

$A_{35} = 0 - (-7) - 0 = 7$

$A_{42} = 19 - (-6) - 8 = 17$

$A_{43} = 21 - (-6) - 25 = 2$

$A_{44} = 21 - (-6) - 14 = 13$

$A_{45} = 0 - (-6) - 0 = 6$

$A_{53} = 22 - (-14) - 25 = 11$

$A_{54} = 25-(-14)-14 = 25$

$A_{55} = 0-(-14)-0 = 14$

$A_{61} = 19-(-10)-11 = 18$

$A_{62} = 23-(-10)-(8) = 25$

$A_{63} = 23-(-10)-25 = 8$

$A_{72} = 25-0-8 = 17$

$A_{75} = 0-0-0 = 0$

$A_{81} = 15-(-3)-1 = 7$

$A_{85} = 0-(-3)-0 = 0$

Since all the values of $C_{ij} - U_i - V_j \geq 0$ hence solution is optimum and the transportation cost =($0\times 2000) + (8\times 600) + (0\times 400) + (18\times 1600) + (15\times 900) + (17\times 1000)$+($14\times 400) + (22\times 1600) + (4\times 1600) + (25\times 100) + (20\times 400) + (22\times 800) =$ ₦139,400.00

3.9.1.3 The vogel's approximation method

Vogel's approximation (penalty or regret) is preferred over NWCR and LCM methods. In this method, an allocation is made on the basis of the opportunity (or penalty or extra) cost that would have been incurred if the allocation in certain cells with minimum unit transportation cost were missed. Hence, allocations are made in such a way that the penalty cost is minimized. An initial solution obtained by using this method is nearer to an optimal solution or is the optimal solution itself. The steps of VAM are as follows:

Step 1: Calculate the penalties for each row (column) by taking the difference between the smallest and next smallest unit transportation cost in the same row (column). This difference indicates the penalty or extra cost that has to be paid if decision-maker fails to allocate to the cell with the minimum unit transportation cost.

Step 2: Select the row or column with the largest penalty and allocate as much as possible in the cell that has the least cost in the selected row or column and satisfies the rim conditions. If there is a tie in the values of penalties, it can be broken by selecting the cell where the maximum allocation can be made.

Step 3: Adjust the supply and demand and cross out the satisfied row or column. If a row and a column are satisfied simultaneously, only one of them is crossed out and the remaining row (column) is assigned a zero supply (demand). Any row or column with zero supply or demand should not be used in computing future penalties.

Step 4: Repeat Steps 1 to 3 until the available supply at various sources and demand at various destinations is satisfied.

Table 3.9 below represents the result obtained as initial basic feasible solution (IBFS) by applying the Vogel's Approximation Method.

Table 3.9: Result obtained as initial basic feasible solution (IBFS) by applying the Vogel's Approximation Method group A

Sources	**Maikunkele d(km)**	**Chanchaga d (km)**	**Kpakungu d (km)**	**Maitunbi d (km)**	**Dummy**	**Supply (ton)**	**Penalty**
Tunga B	20	10× 1500	26× 500	20	0	2000	10,6,0
Shango	21× 100	8	23× 900	23	0	1000	8,23,0
Bosso	18	15	24× 1100	23	0× 1400	2500	15,3,0
Kure mkt	17	19	21	21	0× 1000	1000	17,0
123 Quqters	14× 2000	22	22	25	0	2000	14,0
Mi Wushishi	19	23	23	4× 1600	0	1600	4,0
Sabongari	11× 500	25	25	20	0	500	11,0
Bosso Estate	15× 400	22	22× 400	11× 400	0	800	11,7,0
Demand (ton)	3000	1500	2500	2000	2400		
Penality	3 1 3 3	2 0 0 0	1 1 0 0	7 7 0 0			

The table 3.9 shows how the initial basic feasible solution (IBFS) is obtained by applying the Vogel's Approximation Method and the Monthly transportation cost = $(21\times 100) + (14\times 2000) + (11\times 500) + (15\times 400) + (10\times 1500) + (26\times 500) + (23\times 900) + (24\times 1100) + (4\times 1600) + (11\times 400) + (9\times 1400) + (0\times 1000) =$ ₦127,500.00

Annual transportation cost (₦127,500$\times$ 12) = ₦1,530,000.00

Optimality Test for VAM Group A

Calculate the dual variables $+V_j = C_{ij}$ using $U_i + V_j = C_{ij}$ from table 3.9 we have

$U_1 = 0, U_2 = -3, U_3 = -2, U_4 = -2, U_5 = -11, U_6 = -16, U_7 = -13, U_8 = -9$ and $V_1 = 18$ $, V_2 = 10, V_3 = 22, V_4 = 20, V_5 = 2$

Compute C_{ij} -U_i -V_j for unallocated cells from table 3.9

$A_{11} = 20 - 0 - 18 = 2$

$A_{14} = 10 - 0 - 20 = 0$

$A_{15} = 0 - 0 - (-2) = 2$

$A_{22} = 8 - (-3) - 10 = 1$

$A_{24} = 24 - (-3) - 20 = 7$

$A_{31} = 20 - (-2) - 28 = 2$

$A_{32} = 15 - (-2) - 10 = 7$

$A_{34} = 23 - (-2) - 20 = 5$

$A_{41} = 17 - (-2) - 18 = 1$

$A_{42} = 19 - (-2) - 10 = 11$

$A_{43} = 21 - (-2) - 22 = 1$

$A_{44} = 21 - (-2) - 20 = 3$

$A_{52} = 22 - (-11) - 10 = 23$

$A_{53} = 22 - (-11) - 22 = 11$

$A_{54} = 25 - (-11) - 20 = 16$

$A_{55} = 0 - (-11) - 2 = 9$

$A_{61} = 19 - (-16) - 18 = 17$

$A_{62} = 23 - (-16) - (10) = 29$

$A_{63} = 23 - (-16) - 22 = 17$

$A_{65} = 0 - (-16) - 2 = 14$

$A_{72} = 25 - (-13) - 10 = 28$

$A_{73} = 25 - (-13) - 22 = 16$

$A_{74} = 20 - (-13) - 20 = 13$

$A_{75} = 0 - (-13) - 2 = 11$

$A_{82} = 22 - (-9) - 10 = 21$

$A_{85} = 0 - (-9) - 2 = 7$

Since all the values of $C_{ij} - U_i - V_j \geq 0$ hence solution is optimum and the total transportation cost $=(21\times 100) + (14 \times 2000) + (11 \times 500) + (15 \times 400) + (10 \times 1500) + (26 \times 500) + (23 \times 900) + (24 \times 1100) + (4 \times 1600) + (11 \times 400) + (9 \times 1400) + (0 \times 1000) =$ ₦127,500.00

3.9.2 Analysis of group B collection point

The three methods namely, North – West Corner Method, Least Cost Method and the Vogel Approximation method are applied to group B to obtain the initial basic solution of minimizing transportation cost.

Table 3.10: Unbalance Transportation Tableau Group B

Sources	Maikunkele d(km)	Chanchaga d (km)	Kpakungu d (km)	Maitunbi d (km)	Supply (ton)
Top Medical	21	10	14	18	1000
First Bank T	20	10	10	8	2100
London Street	16	22	8	12	2000
Tudun Fulani	10	21	12	14	500
Dutse Kura	15	18	10	15	1600
Gurara	25	20	5	25	800
NNPC Mega Station	24	18	8	20	2000
Bahago	17	14	12	15	1000
Demand (ton)	2500	3000	500	400	

The table 3.10 is unbalance transportation tableau we therefore introduce dummy as shows in table 3.11 below.

Table 3.11: Balanced Transportation tableau group B

Sources	Maikunkele d (km)	Chanchaga d (km)	Kpakungu d (km)	Maitunbi d (km)	Dummy	Supply (ton)
Top Medical	21	10	14	18	0	1000
First Bank T	20	10	10	8	0	2100
London Street	16	22	8	12	0	2000
Tudun Fulani	10	21	12	14	0	500
Dutse Kura	15	18	10	15	0	1600
Gurara	25	20	5	25	0	800
NNPC Mega S	24	8	8	20	0	2000
Bahago	17	14	12	15	0	1000
Demand (ton)	2500	1600	3000	500	3400	

3.9.2.1 North west corner method

This method is use to solved table (3.11) and the result obtained as initial basic feasible solution (IBFS) is shown in table 3.12

Table 3.12: Result Obtained as Initial Basic Feasible Solution (IBFS) by applying the North-West Corner Method group B

Sources	**Maikukeled (km)**	**Chanchagad (km)**	**Kpakungud (km)**	**Maitunbid (km)**	**Dummy**	**Supply (ton)**
Top Medical	21×1000	10	14	18	0	1000
First Bank T	20×1500	10	10×100	8	0	2100
London Street	16	22	8×2000	12	0	2000
Tudun Fulani	10	21	12×500	14	0	500
Dutse Kura	15	18	10×400	15×500	0×700	1600
Gurara	25	20	5	25	0×800	800
NNPC Mega S	24	8	8	20	0×2000	2000
Bahago	17	14	12	15	0×1000	1000
Demand (ton)	2500	1600	3000	500	3400	

The table 3.12 shows how the initial basic feasible solution (IBFS) is obtained by applying the Least cost Method and the monthly transportation cost is $(21\times 100) + (20 \times 1500) + (10 \times 100) + (8 \times 2000) + (12 \times 500) + (10 \times 400) + (15 \times 500) + (0 \times 700) + (0 \times 800) + (0 \times 2000) + (0 \times 1000)$ =₦85,500.00

Annual transportation cost (₦85,500$\times$ 12) = ₦1,026,000.00

Optimality Test for NWC Group B

Calculate the dual variables $U_i + V_j = C_{ij}$ using $U_i + V_j = C_{ij}$ from table 3.12 we have

$U_1 = 0, U_2 = -1, U_3 = -3, U_4 = -1, U_5 = -1, U_6 = -1, U_7 = -1, U_8 = -1$ and $V_1 = 10$,

$V_2 = 0, V_3 = 5, V_4 = 8, V_5 = -1$

Compute $C_{ij} - U_i - V_j$ for unallocated cells from table 3.12

$A_{12} = 10 - 0 - 18 = 10$

$A_{13} = 14 - 0 - 11 = 3$

$A_{15} = 0 - 0 - (-1) = 1$

$A_{22} = 10 - (-1) - 0 = 11$

$A_{24} = 8 - (-1) - 8 = 1$

$A_{25} = 0 - (-1) - (-1) = 2$

$A_{31} = 16 - (-3) - 10 = 9$

$A_{32} = 22 - (-3) - 0 = 25$

$A_{34} = 12 - (-3) - 0 = 7$

$A_{35} = 0 - (-3) - (1) = 4$

$A_{41} = 10 - (-1) - 10 = 1$

$A_{42} = 21 - (-1) - 0 = 21$

$A_{44} = 14 + (-1) - 8 = 7$

$A_{45} = 0 - (-1) - (-1) = 2$

$A_{51} = 15 - (-1) - 10 = 6$

$A_{52} = 18 - (-1) - 0 = 7$

$A_{61} = 25 - (-1) - 10 = 16$

$A_{62} = 20 - (-1) - (0) = 21$

$A_{63} = 5 - (-1) - 5 = 1$

$A_{64} = 25 - (-1) - 8 = 18$

$A_{71} = 24 - (-1) - 10 = 5$

$A_{72} = 8 - (-1) - 0 = 9$

$A_{73} = 8 - (-1) - 4 = 4$

$A_{74} = 20 - (-1) - 8 = 13$

$A_{81} = 17 - (-1) - 10 = 8$

$A_{82} = 14 - (-1) - 0 = 15$

$A_{83} = 12 - (-1) - 5 = 6$

$A_{84} = 15 - (-1) - 8 = 8$

Since all the values of $C_{ij} - U_i - V_j \geq 0$ hence solution is optimum and the total transportation cost $(21\times 100) + (20 \times 1500) + (10 \times 100) + (8 \times 2000) + (12 \times 500) + (10 \times 400) + (15 \times 500) + (0 \times 700) + (0 \times 800) + (0 \times 2000) + (0 \times 1000) =$ ₦85,500.00

3.9.2.2 The least cost method

This is applied in table 3.11 and the result obtained as initial basic feasible solution (IBFS) is shown in table (3.13) below

Table 3.13: Result Obtained as Initial Basic Feasible Solution (IBFS) by applying the Least cost Method group B

Sources	Maikukeled (km)	Chanchaga d(km)	Kpakungud (km)	Maitunbid (km)	Dummy	Supply (ton)
Top Medical	21	10	14	18	0×1000	1000
First Bank Tunga	20	10	10	8	0×2100	2100
London Street	16	22	8×1700	12	0×300	2000
Tudun Fulani	10×500	21	12	14	0	500
Dutse Kura	15×1600	18	10	15	0	1600
Gurara	25	20	5×800	25	0	800
NNPC Mega S	24	8×1600	8×400	20	0	2000
Bahago	17×400	14	12×100	15×500	0	1000
Demand (ton)	2500	3000	500	400		

The table 3.13 shows how the initial basic feasible solution (IBFS) is obtained by applying the Least cost Method and the monthly transportation cost is $10\times 500 + 15\times 1600 + 17\times 400 + 8\times 1600 + 8\times 1700 + 5\times 800 + 8\times 400 + 12\times 100 + 15\times 500 + 0\times 1000 + 0\times 2100 + 0\times 300 =$ ₦73,300.00

Annual Transportation cost (₦73,300$\times 12$) $=$ ₦879,600.00.

Optimality Test for LCM Group B

Calculate the dual variables $U_i + V_j = C_{ij}$ using $U_i + V_j = C_{ij}$ from table 3.13 we have

$U_1 = 0, U_2 = -2, U_3 = 0, U_4 = -11, U_5 = -6, U_6 = -3, U_7 = 0, U_8 = -4$ and $V_1 = 13$,

$V_2 = 8, V_3 = 8, V_4 = 11, V_5 = 0$

Compute $C_{ij} - U_i - V_j$ for unallocated cells from table 3.13

$A_{11} = 21 - 0 - 13 = 8$

$A_{12} = 10 - 0 - 8 = 2$

$A_{13} = 14 - 0 - 8 = 2$

$A_{14} = 18 - 0 - 11 = 7$

$A_{21} = 20 - (-3) - 13 = 10$

$A_{22} = 10 - (-3) - 8 = 5$

$A_{23} = 10 - (-3) - 8 = 5$

$A_{24} = 8 - (-3) - 11 = 0$

$A_{31} = 16 - 0 - 13 = 3$

$A_{32} = 22 - 0 - 8 = 14$

$A_{34} = 12 - 0 - 11 = 1$

$A_{42} = 21 - (-11) - 8 = 24$

$A_{43} = 12 - (-11) - 8 = 15$

$A_{44} = 14 - (-11) - 11 = 14$

$A_{45} = 0 - (-11) - 0 = 11$

$A_{52} = 18 - (-6) - 8 = 16$

$A_{53} = 10 - (-6) - 8 = 8$

$A_{54} = 15 - (-6) - 11 = 10$

$A_{55} = 0 - (-6) - 0 = 6$

$A_{61} = 25 - (-3) - 13 = 15$

$A_{62} = 20 - (-3) - 8 = 15$

$A_{64} = 25 - (-3) - 11 = 17$

$A_{65} = 0 - (-3) - 0 = 3$

$A_{71} = 24 - 0 - 13 = 11$

$A_{74} = 20 - 0 - 11 = 9$

$A_{75} = 0 - 0 - 0 = 0$

$A_{82} = 14 - (-4) - 8 = 10$

$A_{85} = 0 - (-4) - 0 = 4$

Since all the values of $C_{ij} - U_i - V_j \geq 0$ hence solution is optimum and the total transportation cost is $10\times 500 + 15 \times 1600 + 17 \times 400 + 8 \times 1600 + 8 \times 1700 + 5 \times 800 + 8 \times 400 + 12 \times 100 + 15 \times 500 + 0 \times 1000 + 0 \times 2100 + 0 \times 300 =$ ₦73,300.00

3.9.2.3 The vogel's approximation method

This method is apply in table 3.11 and the result obtained as initial basic feasible solution (IBFS) is shown in table (3.14) below

Table 3.14: Result Obtained as Initial Basic Feasible Solution (IBFS) by applying the Vogel's Approximation Method group B

Sources	Maikukeled (km)	Chanchagad (km)	Kpakungud (km)	Maitunbid (km)	Dummy	Supply (ton)	Penalty
Top Medical	21	10	14	18	0× 800	1000	8,2,0
First Bank T	20	10	10× 1600	8× 500	0	2100	8,0
London Street	16× 2000	22	8× 2000	12	0	2000	8,0
Tudun Fulani	10× 500	21	12	14	0	500	10,0
Dutse Kura	15	18	10	15	0× 1600	1600	10,0
Gurara	25	20	5× 800	25	0	800	5,0
NNPC Mega S	24	8× 1600	8× 400	20	0	2000	8,0
Bahago	17	14	12	15	0× 1000	1000	15,0
Demand (ton)	2500	3000	500	400	3400		
Penality	5	2	3	0	0		
	1	0	0	0	0		
	0	0	0	0	0		
	0	0	14	0	0		
	0	0	0	0	0		

The table 3.14 shows how the initial basic feasible solution (IBFS) is obtained by applying the Vogel's Approximation Method and the monthly transportation cost is $16\times 2000 + 10 \times 500 + 8 \times 1600 + 10 \times 600 + 5 \times 800 + 8 \times 400 + 8 \times 500 =$ ₦ 67,000.00

Annual transportation cost (₦67000$\times$ 12) $=$ ₦804,000.00.

Optimality Test for VAM Group B

Calculate the dual variables $U_i + V_j = C_{ij}$ using $U_i + V_j = C_{ij}$ from table 3.14 we have

$U_1 = 0, U_2 = 0, U_3 = 0, U_4 = -6, U_5 = -1, U_6 = -5, U_7 = -2, U_8 = 0$ and $V_1 = 16, V_2 = 10$ $, V_3 = 10, V_4 = 8, V_5 = 0$

Compute $C_{ij} - U_i - V_j$ for unallocated cells from table 3.14

$A_{11} = 21 - 0 - 16 = 5$

$A_{12} = 10 - 0 - 10 = 0$

$A_{13} = 14 - 0 - 10 = 4$

$A_{14} = 18 - 0 - 8 = 10$

$A_{21} = 20 - 0 - 16 = 4$

$A_{22} = 10 - 0 - 10 = 0$

$A_{25} = 0 - 0 - 0 = 0$

$A_{32} = 22 - 0 - 10 = 12$

$A_{34} = 12 - 0 - 8 = 4$

$A_{35} = 0 - 0 - 0 = 0$

$A_{42} = 21 - (-6) - 10 = 17$

$A_{43} = 12 - (-6) - 10 = 8$

$A_{44} = 14 - (-6) - 8 = 12$

$A_{45} = 0 - (-6) - 0 = 6$

$A_{51} = 15 - (-1) - 16 = 0$

$A_{52} = 18 - (-1) - 10 = 9$

$A_{53} = 10 - (-1) - 10 = 1$

$A_{54} = 15 - (-1) - 8 = 8$

$A_{61} = 25 - (-5) - 16 = 14$

$A_{62} = 20 - (-5) - 10 = 15$

$A_{64} = 25 - (-5) - 8 = 22$

$A_{65} = 0 - (-5) - 0 = 5$

$A_{71} = 24 - (-2) - 16 = 10$

$A_{74} = 20 - (-2) - 4 = 14$

$A_{75} = 0 - (-2) - 0 = 2$

$A_{81} = 17 - 0 - 16 = 1$

$A_{82} = 14 - 0 - 10 = 4$

$A_{83} = 12 - 0 - 10 = 2$

$A_{84} = 15 - 0 - 10 = 5$

Since all the values of C_{ij} - U_i - $V_j \geq 0$ hence solution is optimum and the total transportation cost is $16 \times 2000 + 10 \times 500 + 8 \times 1600 + 10 \times 600 + 5 \times 800 + 8 \times 400 + 8 \times 500 =$ ₦ 67,000.00

3.9.3 Analysis of Group C Collection points

The three methods namely, North – West Corner Method, Least Cost Method and the Vogel Approximation method are applied to group C to obtain the initial basic solution of minimizing transportation cost.

Table 3.15: Unbalance Transportation Tableau Group C

Sources	Maikunkeled (km)	Chanchagad(km)	Kpakungud (km)	Maitunbid(km)	Supply (ton)
Shiroro Junction	20	16	12	22	800
Flayout T	16	20	18	12	2000
Oduoye Q	22	22	12	14	500
General Hospital	24	18	14	11	1000
Nateco	25	16	12	14	2000
Nisepa Office	20	15	18	12	700
Kpakungu	24	23	3	24	2500
Maikunkele	4	26	24	20	1000
Demand (ton)	2000	1000	3000	1000	

Table 3.15 is unbalance we therefore add dummy column as show in table 3.16 below

Table 3.16: Balance Transportation table group C

Source	Maikukeled (km)	Chanchagad (km)	Kpakungud (km)	Maitunbid (km)	Dummy	Supply (ton)
Shiroro Junction	20	16	12	22	0	800
Flayout T	16	20	18	12	0	2000
Oduoye Q	22	22	12	14	0	500
General Hospital	24	18	14	11	0	1000
Nateco	25	16	12	14	0	2000
NISEPA Office	20	15	18	12	0	700
Kpakungu	24	23	3	24	0	2500
Maikunkele	4	26	24	20	0	1000
Demand (ton)	2000	1000	3000	1000	3500	

3.9.3.1 The North West Corner Method

This method is apply in table 3.16 and the result obtained as initial basic feasible solution (IBFS) is shown in table (3.17) below

Table 3.17: Result Obtain as Initial Basic Feasible Solution (IBFS) by Applying the North-West Corner Method group C

Sources	Maikunkeled (km)	Chanchagad (km)	Kpakungud (km)	Maitunbid (km)	Dummy	Supply (ton)
Shiroro Junction	20×800	16	12	22	0	800
Flayout T	16×1200	20×800	18	12	0	2000
Oduoye Q	22	22×200	12×300	14	0	500
General Hospital	24	18	14×1000	11	0	1000
Nateco	25	16	12×1700	14×300	0	2000
Nisepa Office	20	15	18	12×700	0	700
Kpakungu	24	23	3	24	0×2500	2500
Maikunkele	4	26	24	20	0×1000	1000
Demand (ton)	2000	1000	3000	1000	3500	

The table 3.17 shows how the initial basic feasible solution (IBFS) is obtained by applying the North-west corner method and the monthly transportation cost is $(20\times 800) + (16\times 1200) + (20\times 800) + (22\times 200) + (12\times 300) + (14\times 1000) + (12\times 1700) + (14\times 300) + (12\times 700) + (0\times 2500) + (0\times 1000) =$ ₦106,200.00

Annual Transportation cost (₦106200$\times 12) =$ ₦1,1274,400.00

Optimality Test for NWC Group C

Calculate the dual variables $U_i + V_j = C_{ij}$ using $U_i + V_j = C_{ij}$ from table 3.17 we have

$U_1 = 0$, $U_2 = -4$, $U_3 = 0$, $U_4 = -1$, $U_5 = -3$, $U_6 = -5$, $U_7 = -6$, $U_8 = -6$ and $V_1 = 10$,

$V_2 = 12$, $V_3 = 7$, $V_4 = 8$, $V_5 = 0$

Compute C_{ij} - U_i - V_j for unallocated cells from table 3.17

$A_{12} = 16 - 0 - 12 = 4$

$A_{13} = 12 - 0 - 7 = 5$

$A_{14} = 22 - 0 - 8 = 16$

$A_{15} = 0 - 0 - 0 = 0$

$A_{23} = 18 - (-4) - 7 = 5$

$A_{24} = 12 - (-4) - 8 = 8$

$A_{25} = 0 - (-4) - 0 = 4$

$A_{31} = 22 - 0 - 10 = 12$

$A_{34} = 14 - 0 - 8 = 6$

$A_{35} = 0 - 0 - 0 = 0$

$A_{41} = 24 - (-1) - 10 = 15$

$A_{42} = 18 - (-1) - 12 = 7$

$A_{44} = 11 - (-1) - 8 = 3$

$A_{45} = 0 - (-1) - 0 = 1$

$A_{51} = 25 - (-3) - 10 = 18$

$A_{52} = 16 - (-3) - 12 = 7$

$A_{55} = 0 - (-3) - 0 = 3$

$A_{61} = 20-(-5)-10 = 15$

$A_{62} = 15-(-5)-12 = 8$

$A_{63} = 18-(-5)-7 = 16$

$A_{65} = 0-(-5)-0 = 5$

$A_{71} = 24-(-6)-10 = 16$

$A_{72} = 23-(-6)-12 = 17$

$A_{73} = 3-(-6)-7 = 2$

$A_{74} = 24-(-6)-8 = 22$

$A_{81} = 4-(-6)-10 = 0$

$A_{82} = 26-(-6)-12 = 10$

$A_{83} = 24-(-6)-7 = 23$

$A_{84} = 20-(-6)-8 = 18$

Since all the values of C_{ij} -U_i -$V_j \geq 0$ hence solution is optimum and the total transportation cost is $(20\times 800) + (16 \times 1200) + (20 \times 800) + (22 \times 200) + (12 \times 300) + (14 \times 1000) + (12 \times 1700) + (14 \times 300) + (12\times 700) + (0 \times 2500) + (0 \times 1000) =$ ₦106,200.00

3.9.3.2 The Least Cost Method

This method is apply in table 3.16 and the result obtained as initial basic feasible solution (IBFS) is shown in table (3.18) below

Table 3.18: Result obtained as Initial Basic Feasible Solution (IBFS) by Applying the Least Cost Method group C

Sources	**Maikunkele d (km)**	**Chanchagad (km)**	**Kpakungud (km)**	**Maitunbid (km)**	**Dummy**	**Supply (ton)**
Shiroro Junction	20	16	12	22	0× 800	800
Flayout T	16	20	8	12	0× 2000	2000
Oduoye Q	22	22	12	14	0× 500	500
General Hospital	24	18	14	11× 800	0× 200	1000
Nateco	25	16	12× 2000	14	0	2000
Nisepa Office	20	15× 500	18	12× 200	0	700
Kpakungu	24× 1000	23× 500	3× 1000	24	0	2500
Maikunkele	4	26	24	20	0	1000
Demand (ton)	2000	1000	3000	1000	3500	

The table 3.18 shows how the initial basic feasible solution (IBFS) is obtained by applying the Least cost method and the monthly transportation cost according to this route is given by $= (0\times 800) + (0 \times 2000) + (0 \times 500) + (11 \times 800) + (0 \times 200) + (12 \times 2000) + (15 \times 500) + (12 \times 200) + (24 \times 1000) + (23 \times 500) + (3 \times 1000) + (4 \times 1000) =$ ₦$85{,}200.00$

The Annual transportation Cost (₦$85200 \times 12) =$ ₦$1{,}022{,}400.00$

Optimality Test for LCM Group C

Calculate the dual variables $U_i + V_j = C_{ij}$ using $U_i + V_j = C_{ij}$ from table 3.18 we have

$U_1 = 0, U_2 = 0, U_3 = 0, U_4 = 0, U_5 = -15, U_6 = -1, U_7 = -9, U_8 = -15$ and $V_1 = 15$,

$V_2 = 14, V_3 = -3, V_4 = 11, V_5 = 0$.

Compute $C_{ij} - U_i - V_j$ for unallocated cells from table 3.18

$A_{11} = 20 - 0 - 15 = 5$

$A_{12} = 16 - 0 - 14 = 2$

$A_{13} = 12 - 0 - (-3) = 9$

$A_{14} = 22 - 0 - 11 = 11$

$A_{21} = 16 - 0 - 15 = 1$

$A_{22} = 20 - 0 - 14 = 6$

$A_{23} = 8 - 0 - (-3) = 5$

$A_{24} = 12 - 0 - 11 = 1$

$A_{31} = 22 - 0 - 15 = 7$

$A_{32} = 22 - 0 - 14 = 8$

$A_{33} = 12 - 0 - (-3) = 9$

$A_{34} = 14 - 0 - 11 = 3$

$A_{41} = 24 - 0 - 15 = 9$

$A_{42} = 18 - 0 - 14 = 4$

$A_{43} = 14 - 0 - (-3) = 11$

$A_{51} = 25 - (-15) - 15 = 25$

$A_{52} = 16 - (-15) - 14 = 17$

$A_{54} = 14 - (-15) - 11 = 18$

$A_{55} = 0 - (-15) - 0 = 15$

$A_{61} = 20 - (-1) - 15 = 6$

$A_{63} = 18 - (-1`) - (-3) = 16$

$A_{65} = 0 - (-1) - 0 = 1$

$A_{74} = 24 - (-9) - 11 = 22$

$A_{75} = 0 - (-9) - 0 = 9$

$A_{81} = 4 - (-15) - 15 = 4$

$A_{82} = 26 - (-15) - 14 = 27$

$A_{83} = 24 - (-15) - (-3) = 42$

$A_{84} = 20 - (-15) - 11 = 24$

$A_{85} = 0 - (-15) - 0 = 15$

Since all the values of C_{ij} - U_i - $V_j \geq 0$ hence solution is optimum and the total transportation cost is $(0\times 800) + (0 \times 2000) + (0 \times 500) + (11 \times 800) + (0 \times 200) + (12 \times 2000) + (15 \times 500) + (12 \times 200) + (24 \times 1000) + (23 \times 500) + (3 \times 1000) + (4 \times 1000) =$ ₦85,200.00.

3.9.3.3 The vogel's approximation method

This method is apply in table 3.16 and the result obtained as initial basic feasible solution (IBFS) is shown in table (3.19) below

Table 3.19: Result Obtained as Initial Basic Feasible Solution (IBFS) by applying the Vogel's Approximation Method group C

Sources	Maikunkeled (km)	Chanchagad (km)	Kpakungud (km)	Maitunbid (km)	Dummy	Supply (ton)	Penalty
Shiroro Junction	20	16	12	22	0× 800	800	12,0
Flayout T	16	20	18	12	0× 2000	2000	12,0
Oduoye Q	22	22	12	14	0× 500	500	12,0
General Hospital	24	18	14	11× 700	0	1000	11,7,0
Nateco	25× 300	16× 1000	12× 500	14× 300	0× 200	2000	12,2,4,14,0
Nisepa Office	20× 700	15	18	12	0	700	12,0
Kpakungu	24	23	3× 2500	24	0	2500	3,0
Maikunkele	4× 1000	26	24	20	0	1000	4,0
Demand (ton)	2000	1000	3000	1000	3500		
Penality	12 4 1 0	1 0 0 0	9 2 0 0	1 0 0 0	0 0 0 0		

The table 3.19 shows how the initial basic feasible solution (IBFS) is obtained by applying the Vogel's Approximation Method, the Monthly transportation cost according to this route is given as $0\times 800 + 4 \times 1000 + 0 \times 2000 + 3 \times 2500 + 0 \times 500 + 20 \times 700 + 0 \times 200 + 11 \times 1000 + 12 \times 500 + 16 \times 1000 + 25 \times 300 =$ ₦66,000.00

The Annual transportation cost according to this route is given by (₦66000 $\times$ 12) $=$ ₦792,000.00

Optimality Test for VAM Group C

Calculate the dual variables $U_i + V_j = C_{ij}$ using $U_i + V_j = C_{ij}$ from table 3.19 we have

$U_1 = 0$, $U_2 = 0$, $U_3 = 0$, $U_4 = -14$, $U_5 = 0$, $U_6 = -5$, $U_7 = -9$, $U_8 = -21$ and $V_1 = 12$,

$V_2 = 16$, $V_3 = 12$, $V_4 = 10$, $V_5 = 0$.

Compute $C_{ij} - U_i - V_j$ for unallocated cells from table 3.19

$A_{11} = 20 - 0 - 12 = 8$

$A_{12} = 16 - 0 - 16 = 0$

$A_{13} = 12 - 0 - 12 = 0$

$A_{14} = 22 - 0 - 10 = 12$

$A_{21} = 16 - 0 - 12 = 4$

$A_{22} = 20 - 0 - 16 = 4$

$A_{23} = 18 - 0 - 12 = 6$

$A_{24} = 12 - 0 - 10 = 2$

$A_{31} = 22 - 0 - 12 = 8$

$A_{32} = 22 - 0 - 16 = 6$

$A_{33} = 12 - 0 - 12 = 0$

$A_{34} = 14 - 0 - 10 = 4$

$A_{41} = 24 - (-14) - 12 = 26$

$A_{42} = 18 - (-14) - 16 = 16$

$A_{43} = 14 - (-14) - 12 = 16$

$A_{45} = 0 - (-14) - 0 = 14$

$A_{62} = 15 - (-5) - 16 = 4$

$A_{63} = 18 - (-5) - 12 = 11$

$A_{64} = 12 - (-5) - 10 = 7$

$A_{65} = 0 - (-5) - 0 = 5$

$A_{71} = 24 - (-9) - 12 = 21$

$A_{72} = 23 - (-9) - 16 = 16$

$A_{74} = 23 - (-9) - 10 = 23$

$A_{82} = 26 - (-21) - 12 = 35$

$A_{83} = 24 - (-21) - 16 = 13$

$A_{84} = 20 - (-21) - 10 = 31$

$A_{85} = 0 - (-21) - 0 = 21$

Since all the values of C_{ij} - U_i - $V_j \geq 0$ hence solution is optimum and the total transportation cost is $16\times 2000 + 10 \times 500 + 8 \times 1600 + 10 \times 600 + 5 \times 800 + 8 \times 400 + 8 \times 500 =$ ₦ 67,000.00

We use optimization in field of Waste management to improve the efficiency of the existing scenario in Niger State Environmental Protection Agency Minna. We applied the transportation problems for better transportation method, our intention is to minimize the cost reduction of Niger State Environmental Protection Agency (NISEPA) than the existing one and the results of the models are present in chapter four.

CHAPTER FOUR

4.0 RESULTS AND DISCUSSION

4.1 Analysis of Results

In this chapter, we presented the results of the model function and constraints for minimizing the annual cost of the transportation by Niger state Environments and protection Agency (NISEPA), and results of the model function are presented. The Computational results are performed using Intel® Pentium® Dual T3200@4.00GH 500MBMemory. The developed model was solved using LINGO 19.0.

Table 4.1 Results for solid waste control in Minna Metropolis

SW_t (tons)	**SWr (tons)**	**SWn (tons)**	**Optimal Solution (tons)**
2489	751.5	570.5	1322

Table 4.1 shows that SW_t (total solid waste generated in Minna Metropolis), The recycling solid waste control the total solid waste by 751.5 tons the objective bound is 1322(tons). The solution method used is branch-and-bound. The optimal solution is found to be 1322(tons) of the solid waste at the 58th iteration. This has control the volume of solid waste generated in Minna metropolis as against 2487 (tons) of the annual solid waste generated in Minna Metropolis.

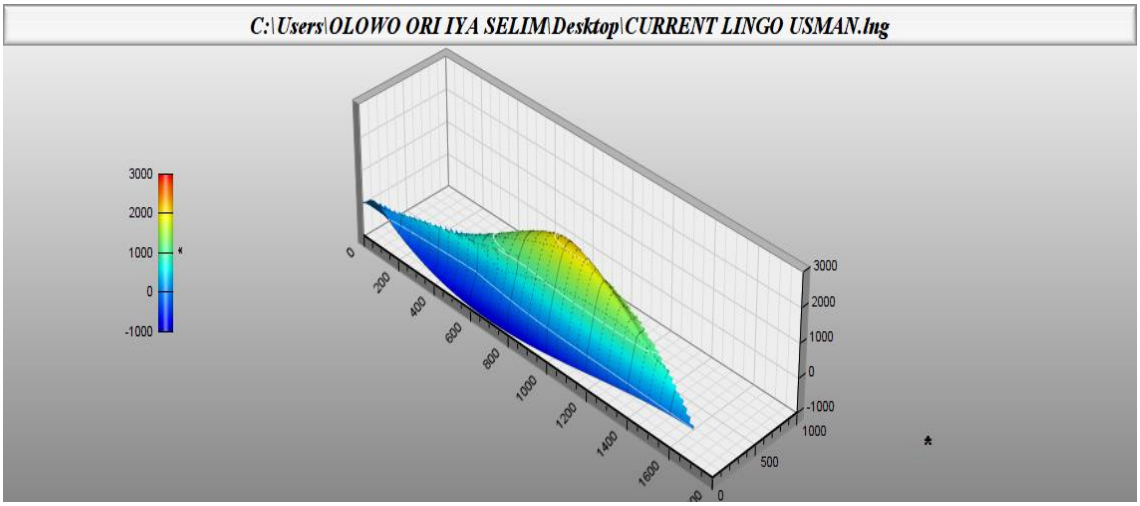

Figure 4.1: Graph for Solid Waste Control

Figure 4.1 shows the graph of residential solid waste reduction in Minna metropolitan. It shows that the developed model resulted in controlling the residential solid waste to 1322 tons from 2,487 tons.

Table 4.2: The Results for Minimizing the Cost of Transportation of the Solid Waste Group A

Time	North West-Corner Method	Least Cost Method	Vogel's Approximation Method	Lingo Software
Monthly cost(₦)	179,000.00	129,200.00	127,500.00	127,444.00
Total annual cost(₦)	2,148,000.00	1,550,400.00	1,530,000.00	1,529,326.00

Table 4.2 shows the results of the Monthly and Annual transportation cost according to the route from the collection points to the Landfill. the result obtained as initial basic feasible solution (IBFS) by applying the North-West Corner Method. Monthly Min Z =₦ 179,000.00 and Annual total transportation cost =₦ 2,148,000. The results obtained as initial basic feasible solution (IBFS) by applying the Least Cost Method is presented table 4.2 above. In Monthly Min Z = ₦129,200.00 and Annual total transportation cost = ₦1,550,400.00. Vogel Approximation Method is advanced version of least square method and most scholars believe VAM to be the most reliable Method in comparison with northwest corner method and Least cost method. Monthly transportation cost =₦ 127,500.00 and total transportation cost =₦ 1,530,000.00 The Lingo results is used to validate the Vogel Approximation Method, when compared with the tree method the, the lingo is closed to Vogel Approximation Method. The Monthly Transportation cost = ₦127,444.00 and Annual transportation cost = ₦1,529 336.00**.**

Table 4.3: Results for Minimizing the Cost of Transportation of the Solid Waste Group B

Time	North West-Corner Method	Least Cost Method	Vogel's Approximation Method	Lingo Software
Monthly cost(₦)	85,500.00	73,610.00	67,000.00	66,997.00
Total annual cost(₦)	1,026,000	883,320.00	804,000.00	803,972.00

Table 4.3 shows the results of the Monthly and Annual transportation cost according to the route from the collection points to the Landfill. The result obtained as initial basic feasible solution (IBFS) by applying the North-West Corner Method. Monthly transportation cost = ₦85,500 and Annual total transportation cost = ₦1026,000. The result obtained as initial basic feasible solution (IBFS) by applying the Least Cost Method. Monthly Transportation cost = ₦3,610,200.00 and Annual total transportation cost = ₦883,320.00. The Vogel Approximation Method is advanced version of least square method and most scholars believe VAM to be the most reliable Method in comparison with northwest corner method and Least cost method. Monthly transportation cost =₦ 67,000.00 and total transportation cost =₦804,000.00. The Lingo results is used to validate the Vogel Approximation Method, when compared with the tree method the, the lingo is closed to Vogel Approximation Method. The Monthly Transportation cost =₦ 66,997.00 and Annual transportation cost = ₦803,972.00

Table 4.4: Result for Minimizing the Cost of Transportation of the Solid Waste Group C

Time	North West-Corner Method	Least Cost Method	Vogel's Approximation Method	Lingo Software
Monthly cost(₦)	106,200.00	85,200.00	66,000.00	65,979.00
Total annual cost(₦)	1,123,200.00	1,022,400.00	792,000.00	791,752.00

Table 4.4 shows the results of the Monthly and Annual transportation cost according to the route from the collection points to the Landfill. The result obtained as initial basic feasible solution (IBFS) by applying the North-West Corner Method. Monthly transportation cost = ₦106,200.00 and Annual total transportation cost = ₦1,1232,200. The result obtained as initial basic feasible solution (IBFS) by applying the Least Cost Method. Monthly Transportation cost = ₦85,200.00 and Annual total transportation cost =₦ 1,022,400. Vogel Approximation Method is advanced version of least square method and most scholars believe VAM to be the most reliable Method in comparison with northwest corner method and Least cost method. Monthly transportation cost = ₦66,000.00 and total transportation cost = ₦792,000.00. The Lingo results is used to validate the Vogel Approximation Method, when compared with the three method, the lingo is closed to Vogel Approximation Method. The Monthly Transportation cost = ₦65,979.00 and Annual transportation cost = ₦791,752.00

Table 4.5: Summary of Comparison Between Actual Transportation Cost, Vogel's Approximation Method, Lingo Software Results and Optimal Solution Results

Group	Vogel's Approximation Method	Lingo Software results. (Validate)	Optimal Solution	NISEPA Actual Transportation cost in ton	Total saving in ton	Total saving Percent age
A(₦)	1,530,000.00	1,529,336.00	1,529,346.00	2,800,000.00	1,270,000.00	54.6%
B(₦)	804,000.00	803,972.00	803,982.00	2,450,000.00	1,646,000.00	32.8%
C(₦)	792,000.00	791,752.00	791,752.00	2,420,000.00	1,628,000.00	30.1%
Total(₦)	3,126,000.00	3,125,060.00	3,125,080.00	7,670,000.00	4,544,000.00	40.8%

Table 4.5 summarizes the comparison between the actual transportation cost, Vogel's approximation method, Lingo software results and optimal Solution results. Total solid waste transportation cost (ton) which is ₦7,670,000.00 is reduced to ₦3,126,000.00 which is a 40.8% in savings, while the total optimal result is ₦3,125,080.

CHAPTER FIVE

5.0 CONCLUTION AND RECOMENDATIONS

5.1 Conclusion

This research has succeeded in using the concept of Transportation model to solid waste management, the Niger State Environmental Protection Agency (NISEPA) is faced with the problem of how to reduce solid wastes generation and to minimize the cost of transportation of the Solid waste from the collection point to the Landfill which arises in everyday living.

Recyclable wastes were collected from the collection centres. We also used the three methods of solution namely: The North – west corner Method, the Least Cost Method and Vogel's Approximation Method to minimizes the cost of transportation from the residential collection point to the various Landfill. The Vogel's Approximation Method is bringing about the best results compare to North west corner method and the least cost Method. The result in three methods are different. The decision maker may choose the optimal result of the running of the three program (minimum) and determined the tons of solid waste transported from collection point i to Landfill j. Lingo software is also used to validate the results obtained from the Vogel's Approximation method. The Lingo Optimization Software could be used as a very reliable and effective tool for obtaining the optimal solution to diverse linear programming problems.

5.2 Recommendations

Based on the conclusion of the study the following recommendation were made:

i. researchers are encouraged to extend the present study by adding more parameters and obtain the result of the problem using a different approach other than basic feasible solution.

ii. Subsequent studies are encouraged to consider the recycling parts of the problems.

EFERENCES

Abila, B., & Kantola, J. (2013). Municipal Solid Waste Management Problems in Nigeria: Evolving Knowledge Management Solutions. *World Academy of Science, Engineering and Technology*, 7(8), 313-318.

Abumere, S. (1983) City Surface Waste in Nigeria Cities. *Environmental International*, 9(1), 391-396.

Adedibu, A. A. (1993). Development Control and Environment Protection: a case of Ilorin. A paper presented at the 20th Annual conference of the NITP Kano, Nigeria.

Adegoke, O. S. (1990). Waste Management within the context of Sustainable Development; In Aina Adedipe (Eds); The making of the Nigerian Environment Policy; FEPA Monograph.103-117.

Afon, A. O. (2003). Issues in Urban Residential Area Solid Waste Management Sustainability: Challenges of Environmental Sustainability in a Democratic Government; Proceedings of 11th National Conference of the Environment and Behaviour Association of Nigeria held in Akure, Nigeria.

Badran, M. F., & El-Haggar, S. M. (2005). Optimization of Municipal Solid Waste Management System in Port Said-Egypt"; Water Development, 26:534.

Chang, N. B., & Wang, S. F. (1996). Solid waste management system analysis by multiobjective mixed integer programming model. *J. Environ. Manag*, 48, 17-43.

Chang, Y.H., & Chang, N.B. (1998). Optimization Analysis for the Development of short-term Solid Waste Management Strategies using presorting process prior to Incinerators. *Resources, Conservation and Recycling*, 24, 7-32.

Chinchodkar, K. N., & Omprahkash, S. J. (2017). Development Mathematical Model for the Solid Waste Management on Dumping Ground at Mumbai for the Reduction of Existing Cost. *Journal of Statistics and System 12(1) 145-155.*

Cointreau, S. J. (1982). Environmental management of urban solid wastes in developing countries: a project guide." Urban Development Dept, World Bank. http://www.worldbank .org/html/f pd/urban//solid-wm/techpaper5.pdf.

Daskalopoulos, E., Badr, O. & Probert, S.D (1998) An Integrated Approach to Solid Waste Management; Resources; Conservation and Recycling. *Resour. Conserv. Recycl.*, 24, 33-50.

Desmond, M. (2006). Municipal solid waste management in Ireland: assesing for sustainability. *Ir. Geogr*, 39, 22-33.

Diaz, R. & Warith, M. (2006). Life-cycle assessment of municipal solid wastes: Development of the WASTED model. *Waste Manag*, 26, 886-901.

Fiorucci, P., Minciardi, R., Robba, M., & Sacile, R. (2003). Solid Waste Management in urban areas development and application of a decision support system. *Resources, Conservation and Recycling*, 37, 301-328.

Halidi, A. L. (2011) Optimization of Municipal Solid Waste Management System, An M.Sc. Dissertation, University of Dar es Salaam.

Hasit, Y. & Warner, B. D. (1981) Regional Solid Waste Planning with WRAP, Journal of Environmental Engineering Division. *ASCE*, 107, 511-525.

Kadafa, A.Y., Latifa, A., Abdullahi, H. S. & Suleiman, W. A. (2003) Comparative Assessment of the Municipal Solid Waste Services. *Nature and Science,* 11(6), 154-164

Kalu, E. O. (2017). Mathematical Model of Municipal Solid Waste Management System in Aba Metropolis of Abia State Nigeria. *Quest Journals Journal of Research in Applied Mathematics*, 3(7), 38–51.

Luoranen, M. & Horttanainen, M. (2008). Co-generation based energy recovery from municipal solid waste integrated with the existing energy supply system. *Waste Manag*, 28, 30-38.

Lyeme, H. A. (2011). *Optimization of municipal solid waste management system.*(M.Sc), Dar es salaam.

Maji, I. Y. (2014). *Effects of Infrastructural Facilities on Rental Values of Residential Properties in Minna: Nigeria*. Department of Building. Ahmadu Bello University Zaria. Zaria.

Mohsen, A.S.,Raza, S., Mohammed, A.A & Babak, O.(2016) Mathematical Modeling in municipal solid Waste Management:case Study of Tehran.*Journal of Environmental Health Science and Engineering.* 14(1), 543-556.

Nakata, T., Rodionov, M., Silva, D. & Jupesta, J. (2010). Shift to a low carbon society through energy systems design. *Sci. China Technol. Sci.*, 53, 134-143.

Nganda, M. K. (2007) Mathematical Models in Municipal Solid Waste Management; Thesis for the degree of Licentiate of Philosophy, Chalmers University of Technology and Goteborg University, Sweden.

Nilsson-Djerf, J. & Mcdougall, E. (2000). Social factors in sustainable waste management. *Warmer Bull*, 73, 18-20

Ogwueleka, T. C. (2009) Municipal Solid Waste Characteristics and Management in Nigeria. *Iranian Journal of Environmental Health Science and Engineering*, 6(3), 173-180

Puopiel, F. (2010). Solid Waste Management in Ghana: The case of Tamal Metropolitan Area, A Thesis submitted to the Department of Planning Kwame Nkrumah University of Science and Technology, 1- 10.

Shafiul, A. A. & Mansoor, A. (2003) Partnerships for Solid Waste Management in Developing Countries: Linking theories to realities in the Institute of Development Engineering, Water and Development Centre (W.E.D.C.) Loughborough University, U. K.

Shamshiry, E., Nadi, B. & Mahmud, A. R. (2011). *Optimization of municipal solid waste management*. Paper presented at the 2010 international conference on biology, environment and chemistry.

Suberu, M. Y., Mokhtar, A. S. & Bashir, N. (2012). Renewable Power Generation Opportunity from Municipal Solid Waste: A Case Study of Lagos Metropolis (Nigeria). *Journal of Energy Technologies and Policy, 2*(2), 15 - 24.

Tanskanenn, J. H. (2000). Strategic planning of municipal solid waste management. *Resour. Conserv. Recycl.*, 3(2), 111-133.

Tchobanoglous, G. & Kreith, F. *Handbook of Solid Waste Management*, 2nd ed.; McGraw-Hill: New York, NY, USA, 2002.

Zerbock, O. (2003). Urban Solid Waste Management: Waste Reduction in Developing Nation. Urban Development Dept, World Bank.

APPENDIX A

ALGORITHM FOR MSWM MODEL USING LINGO SOFTWARE

Step 1: specify all the sets in the model as;

```
MODEL:
SETS:
 TYPES/ A, B/;
 LANDFILLS/ P1, P2, P3,P4/;
 DISTCTR/ DC1, DC2, DC3, DC4/: F, Z;
 POP/ C1, C2, C3, C4, C5/;
 COLLECLINK( TYPES, POP): D;
 LANDLINK( TYPES, LANDFILLS): S;
 YLINK( DISTCTR, POP): Y;
 CLINK( TYPES, LANDFILLS, DISTCTR): C, X;
 GLINK( TYPES, DISTCTR, POP): G;
ENDSETS
```

Step 2: specify all the data as;

```
DATA:
S = 80, 40, 75,
 20, 60,21,34, 75;
C = 1, 3, 3, 5,
 4, 4.5, 1.5, 3.8,
 2, 3.3, 2.2, 3.2,
 1, 2, 2, 5,
 4, 4.6, 1.3, 3.5,
 1.8, 3, 2, 3.5
4, 4.6, 1.3, 3.5,
 1.8, 3, 2, 3.5;
F = 1000, 150, 1600, 159;
G = 5, 5, 3, 2, 4,
 5.1, 4.9, 3.3, 2.5, 2.7,
 3.5, 2, 1.9, 4, 4.3,
 1, 1.8, 4.9, 4.8, 2,
 5, 4.9, 3.3, 2.5, 4.1,
 5, 4.8, 3, 2.2, 2.5,
 3.2, 2, 1.7, 3.5, 4,
 1.5, 2, 5, 5, 2.3;
D = 25, 30, 50, 15, 35,
 25, 8, 0, 30, 30;
fN = 1000;
M = 10;
p = 4;
ENDDATA
```

Step 3: Specify all the variable in the mode as;

```
[OBJ] MIN = (SWr + SWn) + DIST+(fN/M);
SWr = @SUM( CLINK: C * X);
SWn =
 @SUM( GLINK( I, K, L):
 G( I, K, L) * D( I, L) * Y( K, L));
DIST = @SUM( DISTCTR: F * Z);
@FOR( TYPES( I):
 @FOR( LANDFILLS( J):
 @SUM( DISTCTR( K): X( I, J, K)) <= S( I, J))
);
@FOR( TYPES( I):
 @FOR( DISTCTR( K):
```

```
 @SUM( LANDFILLS( J): X( I, J, K)) =
 @SUM( POP( L): D( I, L)* Y( K, L)))
);
@FOR( POP( L):
 @SUM( DISTCTR( K): Y( K, L)) = 1
);
@FOR( POP( L):
 @FOR( DISTCTR( K): Y( K, L) <= Z( K))
);
@FOR( DISTCTR( K):
 @FOR( POP( L): @BIN( Y( K,L)))
)
```

Step 4: END Model