

**USING SIMULATION APPROACH TO EVALUATE
THE PERFORMANCE OF DIGITAL MOBILE
WIRELESS SYSTEM**

by

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Bachelor of Engineering

IN

ELECTRICAL & COMPUTER ENGINEERING

FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA

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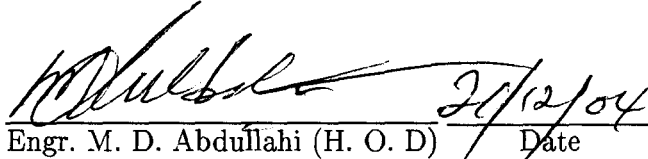
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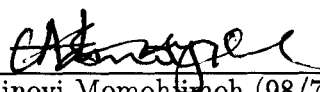
Certification

This is to certify that ADINOYI MOMOHJIMOH carried out the project work (research work) presented in this report under the supervision of his project supervisor.


Engr. S. N. Rumala (Supervisor) _____ Date


Engr. M. D. Abdullahi (H. O. D) _____ Date

External Examiner Date


Adinoyi Momohjimoh (98/7043EE) _____ Date

Dedication

I dedicate this to my mother's WISE response:

Whenever I asked her,

“What will be MY fate in this life? ” She effortlessly replied,

“You will surely be what you are, Just! Keep to rules”.

Acknowledgments

First and foremost I give thanks to Allah (swt) for his infinite mercy, who bestowed on me numerous favor, which I can not recount, to Him all praises and adoration, He is the absolute, deserving all my thanks and gratitude, Alhamdulillah.

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Project Abstract

In this project, the performance of digital mobile system operating in wireless communication environment is evaluated. The performance measure is the bit -error -rate (BER). BER represents how many bits will be in error, if we send certain amount of bits through the channel. Fading phenomena can drastically affect the performance of a terrestrial Communication system often caused by multipath conditions, this can degrade the BER, resulting in lost of data or dropped of call in a mobile Phone, We study the bit error rate performance using simulation. The channel model considered are the additive white Gaussian noise and Rayleigh fading. We shall attempt the following;

1. Adequately characterize the system and the channel. The channel is the medium in which the communication takes place.
2. Identify the problems posed by the channel and possibly identify some solutions.
3. Finally, we will conduct computer simulations of the system to evaluate its performance as function of the resources required for communication. One of such required resources is the transmitter power measured through signal to noise power ratio (SNR).

The MATLAB software has been used in the simulations.

Chapter 1

Introduction

1.1 Historical background of mobile wireless system

Since the birth of wireless systems, communication from anywhere at any time is a reality. In the earliest form of communications the transmission of human voice through the telephone line or channel was dominant. Radio-supported mobile communication has constantly grown in importance during the last few decades, and certainly, it shall continue to generate research interests.

In contrast to wire-line or conventional wired communication systems, mobile radio networks that comply with the wish for geographically unrestricted communication can be used anywhere where it is not economical or possible to install cables. I strongly believe that this is more urgent in places like Nigeria where the land-line system infrastructure is not adequately developed.

Although, wireless communication standards are being developed and tested in telecommunication advanced countries, this is far from being the case in Nigeria. For instance, the Global System for Mobile communication (GSM) was developed in the Europe. The digital advanced mobile and personal system (AMPS) is a North America system replacing the old analogue AMPS. Furthermore, code division multiple access system (CDMA) is another fast growing multiple access wireless technique [8]. To understand this system the schemes have to be modeled analytically or where it is impossible or too complex to do so, computer simulations are used to evaluate the system performance. This is vital since knowing how the system parameters behave with respect to the desired output can help system designers to have all the information required for designing optimum systems. Furthermore, such full knowledge of the system provides ways to improving the systems or in applying any system trade-off, which is always done in engineering design.

In this project, I intend to evaluate the performance of communication system operated in wireless environment. To that end, the following efforts were exerted in this project.

- Adequate system and channel models characterization performed. The channel is the medium in which the communication takes place.
- Problems posed by the wireless channel and possible solutions identified. The major problem is fading and solution to fight it is the diversity reception.
- Finally, computer simulations of the whole communication system were conducted. This effort helps to evaluate the performance of the system as a function of the resource required for the communication. One of such required resources is the transmitter power.

The performance of communication system is a measure of the amount of bit error committed in the cause of communicating by the mobile user for a given transmitter power. If we realize that the power required for transmission determines the size of the mobile terminal and hence the cost, we will appreciate why it is necessary to build system with using minimum power. This is more pressing as we move to miniaturize the handset. This suggest for a scheme that can help reduce channel error. Examples of such schemes are forward error correcting codes and diversity. Diversity technique will be implemented and the performance improvement will be discussed.

1.2 Literature Review

The problems engineers face in designing wireless communication system is dramatically different than in the fixed or wireline. This is due to the nature of the signal propagation in wireless communication. We have to remember that in wireless communication the signal is radiated into space with no wires guiding it. This unguided nature exposes the signal to many distortions that are not seen in wired communication, for example, the conventional telephone. Furthermore, there is problem of privacy which implies that the signal might not be saved from any mischievous eavesdropper. There is also the problem of intentional jamming common with military operations, where the military might want to fake enemy signal. Upon all this, there is the natural problem of fading, where, as a result reflections, diffraction of signals by human, trees, and man-made structure, a multiple of the same signal is received at the receiver. This

could be an advantage if properly harnessed but could impose a serious signal degradation in signal strength if not handle carefully. This phenomenon of fading is the consequence of these multiple received signal adding constructively or destructively [3].

All of the problems highlighted above require different sets of solution. For example, the military are the first to invest in spread spectrum application [8]. This is a whole research topic on its own but we just mention it in the passing. Spread spectrum (SS) is basically an expansion or spreading the signal over a frequency spectrum making the signal to essentially look like noise and hence becoming difficult for any one who wants to steal (eavesdrop) the signal. This spreading is achieved through a special code called spreading code assigned to each user. It is only the intended recipient, who of course knows the spreading code of the sender, that can understand or decode the signal. It is important to know that CDMA is one of the SS technology. The SS was military exclusive technology in the past due largely to the cost of the technology. In advanced countries, the military is heavily funded and hence can employ any technology regardless of its cost. However, in recent times, SS systems are now used commercially. Thanks to advances in technology that push their prices down. Any serious and extensive treatment of this topic will be well too advance for undergraduate research like my own, unless if restricted to only literature survey. But my hope in this project is to conduct simulation of wireless communication system since if successfully done would imply that a basic working knowledge of this system is acquired. This is the reason why the research is focused on the common enemy of all the wireless communication systems, and that is **fading**. In this project, therefore, we have presented and simulated the methods that are used to combat this enemy. These methods known as **diversity** help to combat the detrimental consequences of fading.

In classical diversity schemes multiple receive antennas are implemented to exploit the multipath phenomenon inherent in wireless systems in what is known as spatial diversity. Recently, interest has heightened in the area of transmit diversity where the burden of carrying multiple antennas in the mobile unit can be transferred to the base station. This area of research has witnessed a lot of interest in recent time. The transmit diversity scheme won the attention of wireless researchers for some simple reasons. Firstly the use of multiple antennas at the remote

units dictate large and expensive sets. This is not the dream of 3rd generation wireless systems and beyond. It is interesting to note that the task of carrying the multiple antennas can be assigned to the base station without much penalty. It can also be viewed that instead of a hundreds of mobile unit carrying multiple antennas few base stations do so. The economy in this cannot be overemphasized. A slightly different approach in presenting the case for transmit antenna diversity was given in the work by Alamouti [12]. He interestingly show a transmit diversity scheme which improves the signal quality at the receiver on one side of the link by simple processing across two transmit antennas on the opposite side. Among the benefits of this scheme is the fact that the bandwidth is not expanded as the redundancy is not in time or frequency but spatial (multiple antennas). The ease at which this scheme can be incorporated with the existing wireless system has made it a candidate for the 3-G system. It can be argued that the many cellular base station already have two antennas for the receive diversity [12]. There seems to be no technical reason stopping the use of these antennas for transmit diversity.

Diversity combining is a well-known and effective method for improving the performance of digital communication systems over fading channels [3]. The basic principle of M -fold diversity is to use M independent channels so that the probability of a “deep fade” on *all* channels is low. These independent channels can be created in a number of ways, including frequency, time, and/or polarization diversity; if multiple antennas are used to receive multiple versions of the received signal, the approach is called *spatial* diversity. A combining circuit is used to form a single resultant signal from the M different “branch” signals. The most common forms are *maximum ratio combining (MRC)*, *equal gain combining (EGC)* and *selection combining (SC)* [7].

- In MRC, the matched filter output of each diversity path is weighted by the fading attenuation of that path. The resultant SNR at the output of the combiner is the sum of the SNR’s of the M branches. If optimality is viewed as obtaining maximum SNR possible at the combiner output, MRC is optimal in this sense.
- In EGC, the resultant signal is simply the unweighted sum of the signals from the M branches. This project does not investigate this particular combining technique.

- In SC, the resultant signal is the one with highest SNR among the M received signals; in practice, the signal with the strongest received signal – i.e., signal plus noise – is selected.

The simplicity in SC is definitely not without a price, it is at the expense of system performance. In SC, $M - 1$ branches are rejected; hence, the receiver does not fully exploit the available diversity

Mobile communications have attracted attention since the early days of introducing wireless communications because it offers the ability to users to communicate with other users on the move. Each additional SNR gained through signal processing result in tremendous increase in system capacity (i.e, increase in the number of users that can communicate at the same time) or cell extension (increase the distance by which a user can still communicate with the base station). SNR gain implies that lower SNR is required to obtain the same error rate. To achieve this SNR gain we shall investigate the MRC and SC techniques as they are employed in wireless systems.

Chapter 2

Digital Transmission in Mobile Wireless System

2.1 Binary Signal Transmission

We consider in this chapter transmission of digital signal using baseband digital modulation technique through the simplest channel model known as additive white Gaussian noise (AWGN). This channel represent the electronic thermal noise (or amplifier noise) and is always present at the front end of receiver circuitry. Since amplifier (Low Noise Amplifier (LNA)) is an integral part of any receiver, AWGN is always part of communication receiver. The optimum receiver for detecting signal corrupted by AWGN will be described and we shall show the analytic performance of digital communication using binary phase shift keying (BPSK) and validate this analytical expression using simulation.

2.2 Baseband Modulation Techniques

In a binary communication system, the binary data consisting of 0's and 1's are transmitted by means of two signal waveforms, say, $s_0(t)$ and $s_1(t)$. In this case, the transmitter will do the following:

$$\text{if data bit is 0 send } s_0(t) \quad 0 \leq t \leq T_b$$

$$\text{if data bit is 1 send } s_1(t) \quad 0 \leq t \leq T_b$$

The commonly used pulses are the raised-cosine and rectangular. Rectangular pulse is shown in Figure 2.1. For the popular binary phase shift key modulation (BPSK), the two signals $s_0(t)$ and $s_1(t)$ are 180 degree out of phase. This implies that $s_1(t) = -s_0(t)$. Hence the modulation is also called antipodal. This 180 degree phase difference is a consequence of the fact that we need two phases to distinguish between the two signals. In higher signal sets, called quaternary phase shift

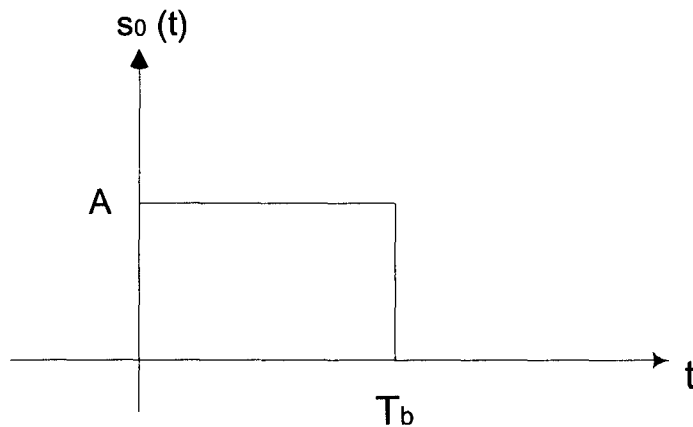


Figure 2.1: Signal waveform for binary communication.

keying (4-PSK or QPSK), 8 phase shift keying (8-PSK), this phase difference between the signal point will be 90 degree and 45 degree respectively. This is because we need distinct phases to uniquely identify the signals. Recall that in PSK modulation, the information is conveyed by the signal phase. In this case, the signal point will form a ring in a circle whose has radius measured from the reference point of the coordinate system. The square of this radius gives the energy of the signal point. Since each point is equal distance from the center, the PSK family is known as equal energy modulation. Figure 2.2 shows how the binary bit will be assigned to the phase of the modulation scheme. Note that Gray coding is applied in assigning those bits for the higher constellation, QPSK is shown as an example. Gray coding implies that two adjacent signals have bits mapping differing in ONLY one bit position. That is $(00) \rightarrow (10)$, $(10) \rightarrow (11)$, $(11) \rightarrow (01)$, and $(01) \rightarrow (00)$, all have one bit different from the adjacent signal point. Note that $(01) \rightarrow (10)$ is two bit position different. This is not allowed as it makes the performance worse. For the BPSK the signal mapping corresponding for the bits are shown in the figure, however if QPSK is to be implemented, the signal mapping for the two group of bits shown in Figure 2.2 are

- for (0 0), the signal mapping is 1,
- for (1 0), the signal mapping is j ,
- for (1 1), the signal mapping is -1 , and
- for (0 1), the signal mapping is $-j$

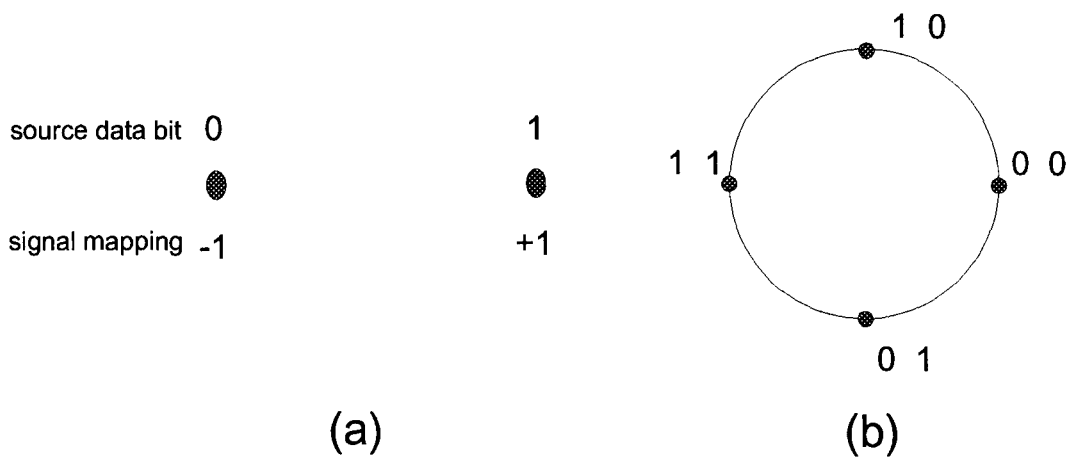


Figure 2.2: Signal set for family of phase shift keying (PSK) modulation: (a) BPSK, (b) 4-PSK or QPSK

Normally the signal pulse is transmitted in T_b second duration or bit interval and hence fixes the rate of transmission R . This rate is given as $R = 1/T_b$ bits/sec. R is the number of bits sent through the channel in 1 second.

We shall consider the channel model normally used to evaluate performance of wireless communication. They are additive white Gaussian noise channel and fading channel.

2.3 Wireless Channels Model: Fading Phenomenon

In mobile communication systems, the impulse response of the channels is in general, time varying and also there may be multiple of paths from the transmitter(base station) to the receiver (mobile unit or user). This multiple path phenomenon known as multi-path is due to sky matters and surrounding buildings and trees reflecting and bouncing signals. This causes multiple signals to arrive at the receiver. Since they traveled through different paths, they are arrived at differential time delays and therefore different phases, which may add up either constructively or destructively (i.e., sometimes the signal is strong and other, is weak). The consequence therefore, is the fluctuation in level of the received signal, thereafter, called *signal fading*.

Mathematical representation of the transmission through the channels is $r(t)$ [5]

$$r(t) = \alpha s(t) + n(t) \quad (2.1)$$

where $s(t)$ is the transmitted signal, $r(t)$ is the received signal, α is the fading gain and $n(t)$ is

the additive noise. Since they (α and $n(t)$) are random process, they can only be described by statistical characterizations. Random in the sense that their values are not known before hand.

The fade samples (α) is generally described by the Rayleigh distribution. Hence the fading is said to be Rayleigh fading. The Rayleigh probability density function (PDF) is given as [4]

$$p(\alpha) = \frac{\alpha}{\sigma^2} \exp\left(-\frac{\alpha^2}{2\sigma^2}\right), \quad \alpha \geq 0 \quad (2.2)$$

where σ is a measure of the strength of the fading. Observe that when $\alpha = 0$, the equation (2.1) implies that the signal is lost completely and you have only noise $n(t)$. This is normally referred to as deep fade. One can experience this when a mobile user drives through a tunnel. The signal is blocked completely. What you have is the hissing sound, which is the noise from the electronic parts of the phone. The PDF of this Rayleigh fading is shown in Figure 2.3 ¹

If α is constant, for example $\alpha = 1$, equation (2.1) becomes simply $r(t) = s(t) + n(t)$. This is the simplest channel you can ever get, no fading. If the pdf of $n(t)$ follows a normal distribution (Gaussian) it is called additive Gaussian noise. The normal pdf is given as in [9]

$$p_n(n) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(n-\mu)^2}{2\sigma^2}\right), \quad -\infty \leq n \leq \infty \quad (2.3)$$

where μ is the mean and σ^2 is the variance of the Gaussian process. Since in all the simulations, Gaussian noise is required we mention that this can be simulated by using the MATLAB command **randn**; All we need is to know the appropriate power of the noise to yield the signal to noise ratio simulated. The default power of the **randn** from MATLAB is 1 and mean is zero. In Figure 2.4 we show the PDF of the simulated Gaussian noise. The bell-shaped nature and its symmetry about the point zero confirm that it has a normal distribution and of zero mean.

We shall now consider transmission of binary data through additive Gaussian channel. However, before we proceed we have to define and describe the parameter which error performance is based. The parameter known as desired signal to noise power ratio.

¹See Appendix IV

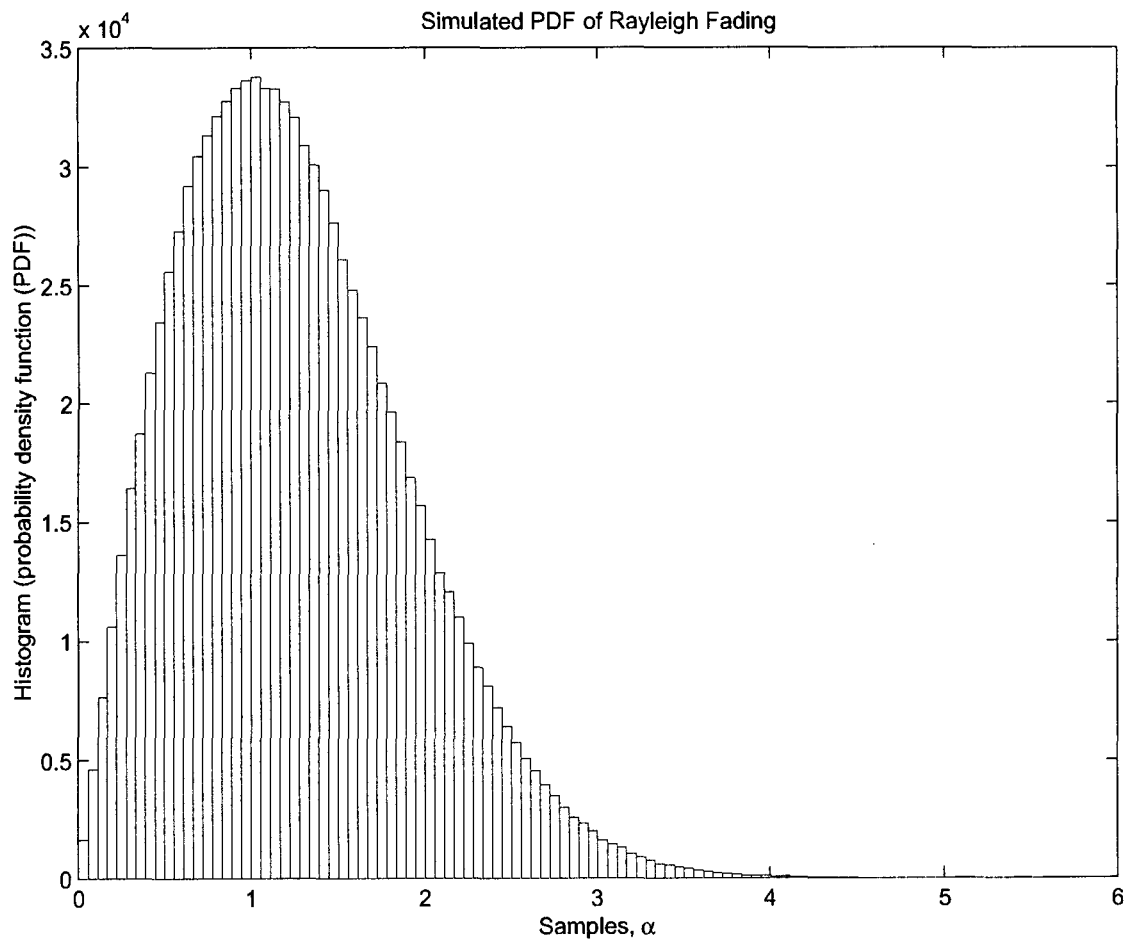


Figure 2.3: The probability density function of Rayleigh fading obtained from simulation (Appendix IV).

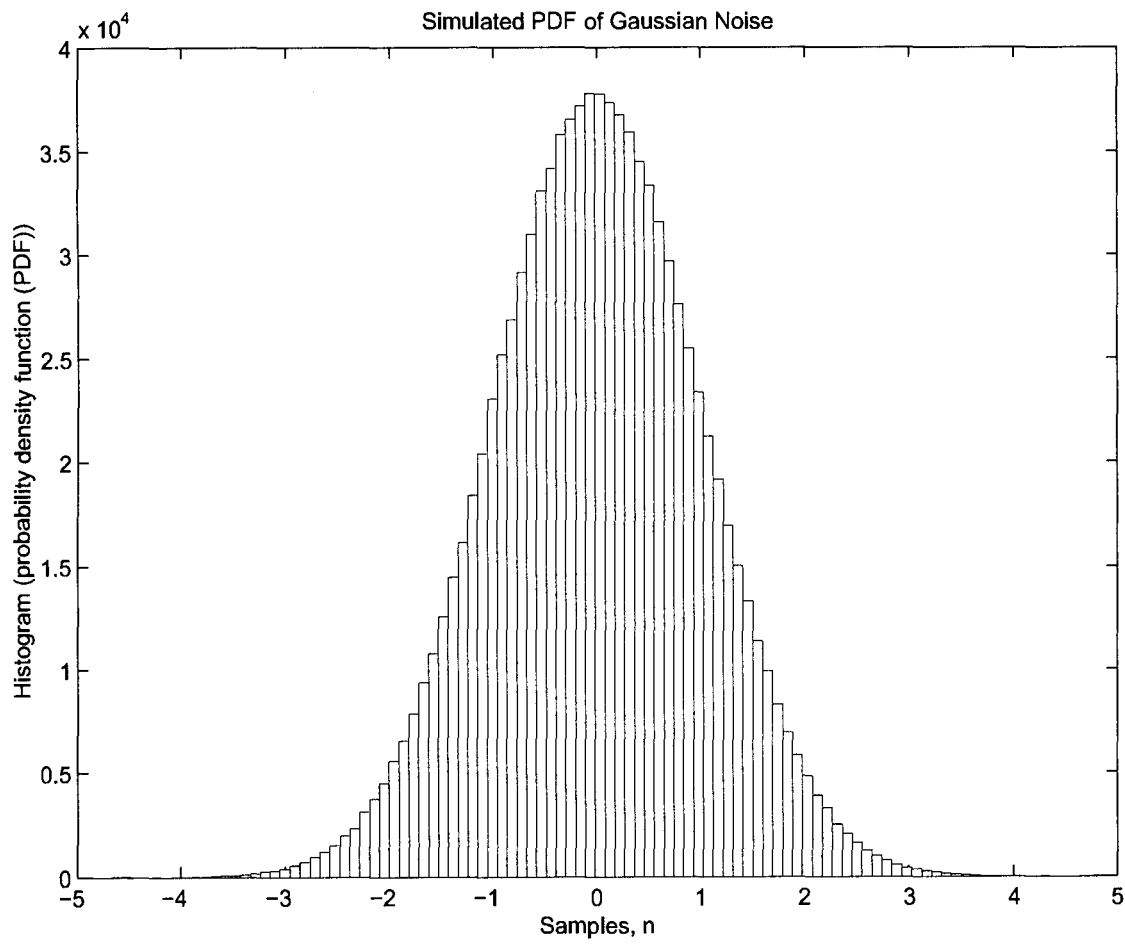


Figure 2.4: The probability density function of Gaussian noise obtained from simulation (Appendix IV).

2.4 Signal Energy to Noise Power Ratio (SNR)

Let $s(t)$ be the transmitted signal pulse $s(t)$, the energy in this pulse denoted as E_b . This is obtained by

$$E_b = \int_{t=0}^{T_b} |s(t)|^2 dt. \quad (2.4)$$

Since there is no fading i.e $\alpha = 1$, this is the signal energy received at the receiver. Now the variance of the noise after passing through the receiver filter can be denoted as N_0 , and it can be shown that $N_0 = 2\sigma^2$. Therefore, the ratio of the useful signal energy and the noise power is referred to as signal to noise ratio of E_b/N_0 . This is commonly expressed in deciBel (dB).

$$SNR = 10 * \log_{10}(E_b/N_0) dB \quad (2.5)$$

Example 1. Let us assume that $E_b = 1$ and the noise power $N_0 = 0.5$. Then $SNR = 10 \log_{10}(1/0.5) = 3.01$ dB.

Example 2. Assume that SNR is given as 10 dB. What is the E_b/N_0 in linear scale? $E_b/N_0 = 10^{SNR/10} = 10^{10/10} = 10$

Example 3. Assume that SNR is given as 5 dB. What is the E_b/N_0 in linear scale? $E_b/N_0 = 10^{SNR/10} = 10^{5/10} = \sqrt{10} = 3.1623$

2.5 BPSK in Additive White Gaussian Channel

From the name this channel distortion is additive in nature. That is, the transmitted signal is corrupted by the addition of noise, denoted as $n(t)$ which is a sample of a white Gaussian process. Such a channel is called additive white Gaussian noise (AWGN) channel. Consequently, the received signal $r(t)$ waveform is expressed as

$$r(t) = s_i(t) + n(t), \quad i = 0, 1, \quad 0 \leq t \leq T_b \quad (2.6)$$

The task of the receiver is to determine whether a 0 or a 1 was sent after observing signal $r(t)$ in the interval $0 \leq t \leq T_b$. The receiver is designed in such a way to minimize the probability of committing error in doing so. Such a receiver is called optimum receiver. Note: Optimality does

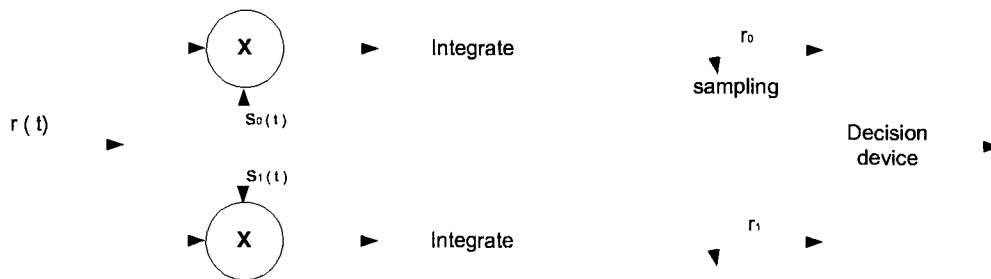


Figure 2.5: Received signal $r(t)$ cross-correlated with the transmitter signal $s_0(t)$ and $s_1(t)$.

not mean zero error probability. It simply implies this is the receiver which gives the minimum error probability possible. The statistics of $n(t)$ was already described in the channel models. It has normal distribution.

2.6 Optimum Receiver for Detecting BPSK in Additive Gaussian Noise Channel

The detection process consist of two parts. The **matched filter** (or sometime called **correlator**) and **slicing device** or **decision device**. In the following we shall describe the operation of these two parts.

2.6.1 Signal Correlator

The correlator essentially compares the received signal $r(t)$ with the two possible transmitted waveforms ($s_0(t)$ and $s_1(t)$) and selects which one of them is more close to the received signal. In mathematical terms, the signal correlator cross-correlates the received signal $r(t)$ with the possible signals. The block diagram of this operation is shown in Figure 2.5 The operation in the block diagram is the following. The signal correlator computes [2]

$$r_0(t) = \int_0^t r(\tau)s_0(\tau)d\tau \quad (2.7)$$

$$r_1(t) = \int_0^t r(\tau)s_1(\tau)d\tau \quad (2.8)$$

In fact, it is this correlation operation that earns such receiver the name correlator receiver. The two outputs are sample at kT_b , where $k = 1, 2, \dots$, That is, $r_k^{(0)} = r_0(t)|_{kT_b}$ and $r_k^{(1)} = r_1(t)|_{kT_b}$. The sampled output at $t = T_b$ is fed to the decision device or detector.

DETECTOR OF BINARY PHASE SHIFT SIGNAL

2.7 Detection for Binary Phase Shift Signals

The detector observes the correlator or matched filter outputs r_0 and r_1 and decides on whether the transmitted waveform is either $s_0(t)$ i.e (bit = 0) or $s_1(t)$ i.e (bit = 1) was transmitted. Normally, the decision is done in the following way

if $r_0 > r_1$ $s_0(t)$ and hence 0 was transmitted

else

$r_0 < r_1$ $s_1(t)$ and hence 1 was transmitted

In doing the detection above, it sometimes commits error. That is, there will be a time a 1 was transmitted and it will detect it as a 0 was transmitted. The rate at which bit error is committed is measured as probability of bit error. The mathematical derivation of bit error probability of BPSK in AWGN is beyond the scope of this project. But it has been given in most research papers and text book [2] as

$$P_b = Q\left(\sqrt{2E_b/N_0}\right) \quad (2.9)$$

where E_b and N_0 are defined above. A brief interpretation of this equation is in order. Q is the area under the tail of Gaussian distribution. In fact, $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-y^2/2} dy$. It is normally called Q -function and tabulated in most standard textbooks. This function is also related to the complementary error function (erfc). The erfc is available in MATLAB. Q -function is related to erfc by

$$P_b = Q\left(\sqrt{2E_b/N_0}\right) = \frac{1}{2} \text{erfc}\left(\sqrt{E_b/N_0}\right) \quad (2.10)$$

We can visualize easily and understand the impact of high E_b/N_0 on error performance if we plot equation (2.10). This is shown in Figure 2.6. It can be seen that larger E_b/N_0 gives lower error. Therefore a transmitter that can boast of high transmitted power would have a better SNR and hence low bit error.

In the previous discussion, we have assumed that there was no fading, i.e $\alpha = 1$. If α varies and the random behaviour is described by the probability density function of equation (2.2) then the analytical performance of BPSK in such channel is no longer given by the equation (2.10)

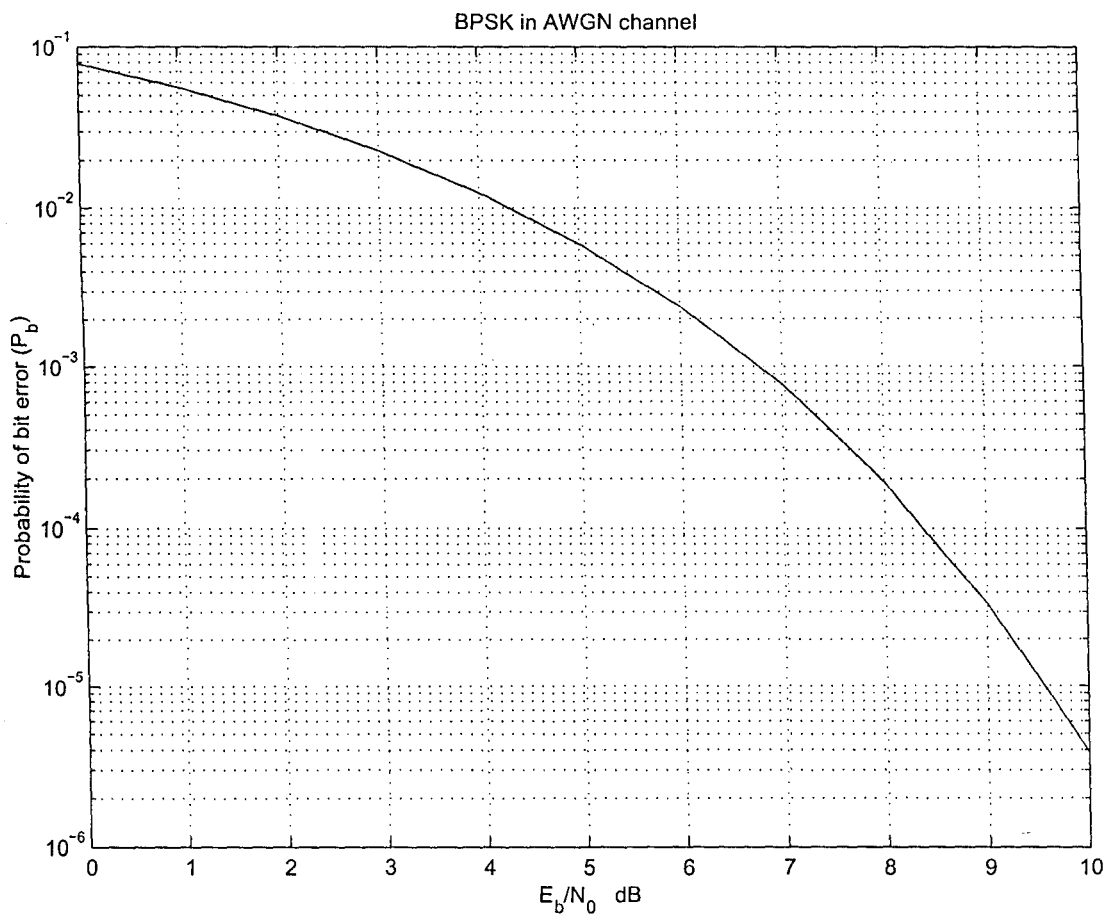


Figure 2.6: Theoretical performance of BPSK in AWGN channel (Appendix IV).

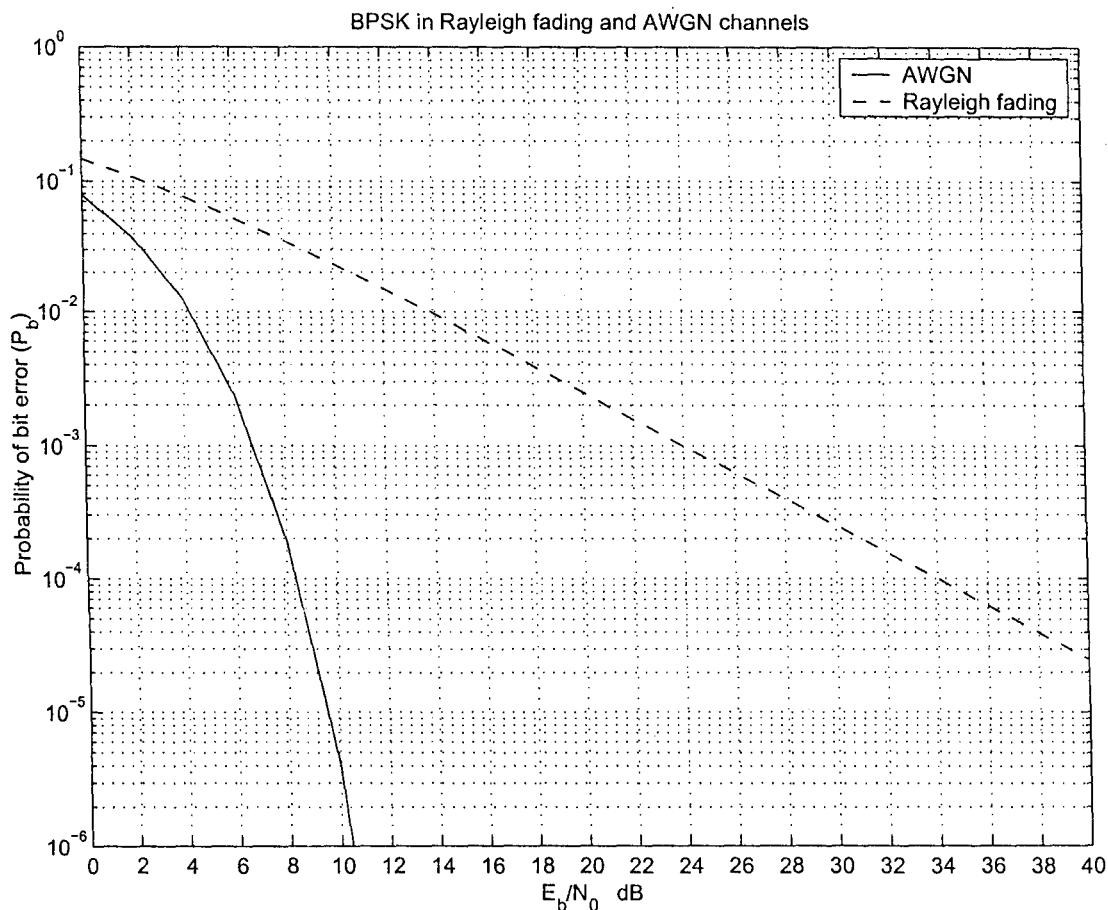


Figure 2.7: Theoretical performance of BPSK in fading and AWGN channel (Appendix IV).

but by the following expression found in [6].

$$P_{\text{error rate in fading}} = \frac{1}{2} \left(1 - \sqrt{\frac{E_b/N_0}{1 + E_b/N_0}} \right) \quad (2.11)$$

The derivation of this expression is also beyond the scope this report but is shown in [6]. In their formula E_b/N_0 is defined as ρ . This analytical expression is plotted in Figure 2.7 together with the performance in channel that is AWGN only. We do this to see that we require more power in fading channel than AWGN to achieve the same probability of error. For example consider system to be designed for an error rate of 10^{-4} (this implies that not more than 1 bit in 10000 bits can in error). You require to have E_b/N_0 of 8 dB and 34 dB when the channel is AWGN and Rayleigh fading, respectively. This can be seen from the curves in Figure 2.7. A

difference of about 26 dB!. To put this in perspective, this 26 dB implies that the transmitted signal power required in fading is $10^{26/10} \approx 400$ times that required in AWGN. This is a huge power requirement. This translates into high cost of systems and large size of terminals e.g mobile phone etc. In the next chapter, computer simulations of the system are conducted and we consider a technique that can be used such that smaller transmitter power can be used in Rayleigh fading. That is, the power difference between AWGN and Rayleigh for the same error performance will be reduced. Which translate into smaller cost and smaller size of mobile phones and other.

Chapter 3

Simulation and Results Discussion

3.1 Simulation Methodology

We begin this chapter by describing the simulation methodology. The general system block diagram of the system to simulate is shown in Figure 3.1. A representative simulation methodology has been presented in a compact form in the flow chart shown in Figure 3.2. In the appendices the flow chart for each program written is given. First let us discuss this general flow chart to give us a feeling of what the problem is all about. The function of each block is briefly described:

3.2 The Simulation Pseudocode in the Form of Flow Chart

- **Block I:** This is the signal source. It generates a uniform, independent, identically distributed (i.i.d) binary bits. Here also we initialize counter for the number bits simulated.
- **Block II:** At this stage, if coding is implemented, the generated binary bits are coded. Otherwise, the block is omitted.
- **Block III:** This the modulation stage where the binary bits are converted to modulation waveform. For an example, for binary phase shift keying (BPSK), a signal waveform of amplitude A can be transmitted for bit 1 and $-A$ is transmitted for a 0.
- **Block IV:** The modulated waveform is transmitted through the channel. The channel could be microwave, wireless, wired, coaxial, twisted pair. However, the channel considered will determine the statistical characteristics of the multiplicative distortion (α_k). For wireless channel, Rayleigh distribution is commonly assumed. But the white noise is same as it models the receiver amplifier noise.
- **Block V:** This block generates the noise and fading.

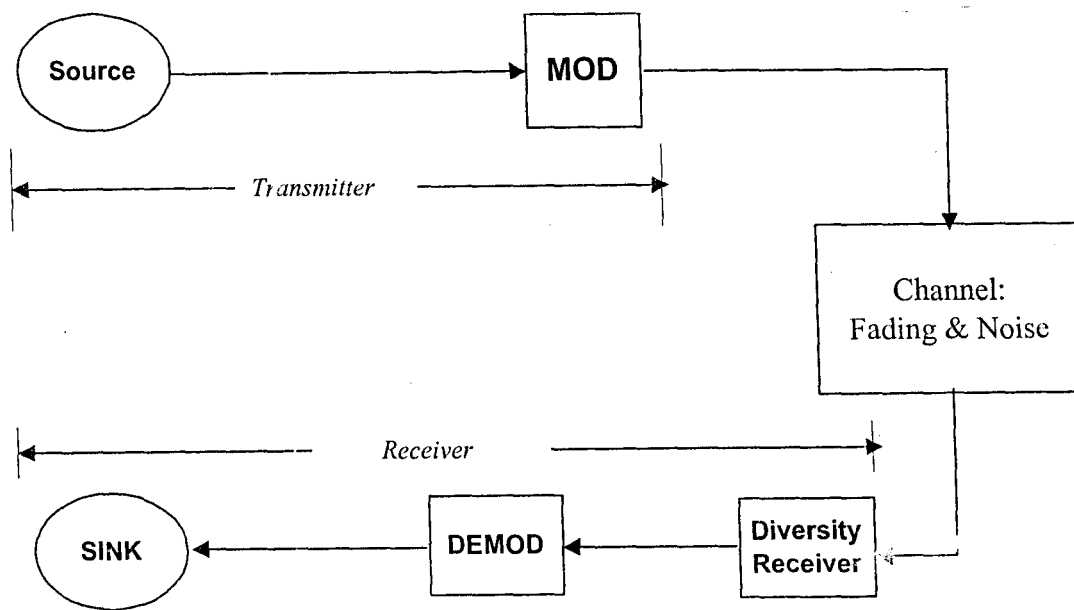


Figure 3.1: The basic system block diagram.

- **Block VI:** The receiver front end. The received signal is decoded if coding was used. Also if diversity channel exists, diversity combining is done.
- **Block VII:** The block makes decision as to what the transmitter sent.
- **Block VIII, IX, X, XI:** Are for comparing the transmitted bits with the detected, to indicate if channel error occurs or not. When a statistically sufficient number of bits are simulated, the simulation stops to calculate the fraction of bits in error during the process. This is the bit error rate (BER). All these steps are for a particular signal power to noise ratio (SNR). Different SNR can be specified to evaluate the system performance and the process restarts from block I

3.3 Performance of BPSK in Rayleigh Fading: Simulation Results I

The flow chart in Figure 3.2 can be implemented using any programming software like MATLAB, C, C++, BASIC, etc. We adopted MATLAB. In this section, simulated results for BPSK in Rayleigh fading as well as additive white Gaussian Noise (AWGN) are presented and compared with the analytical performance shown in chapter 2. No efforts or techniques are used to combat the effect of fading. Figure 3.3 shows the simulated results denoted with (*) for

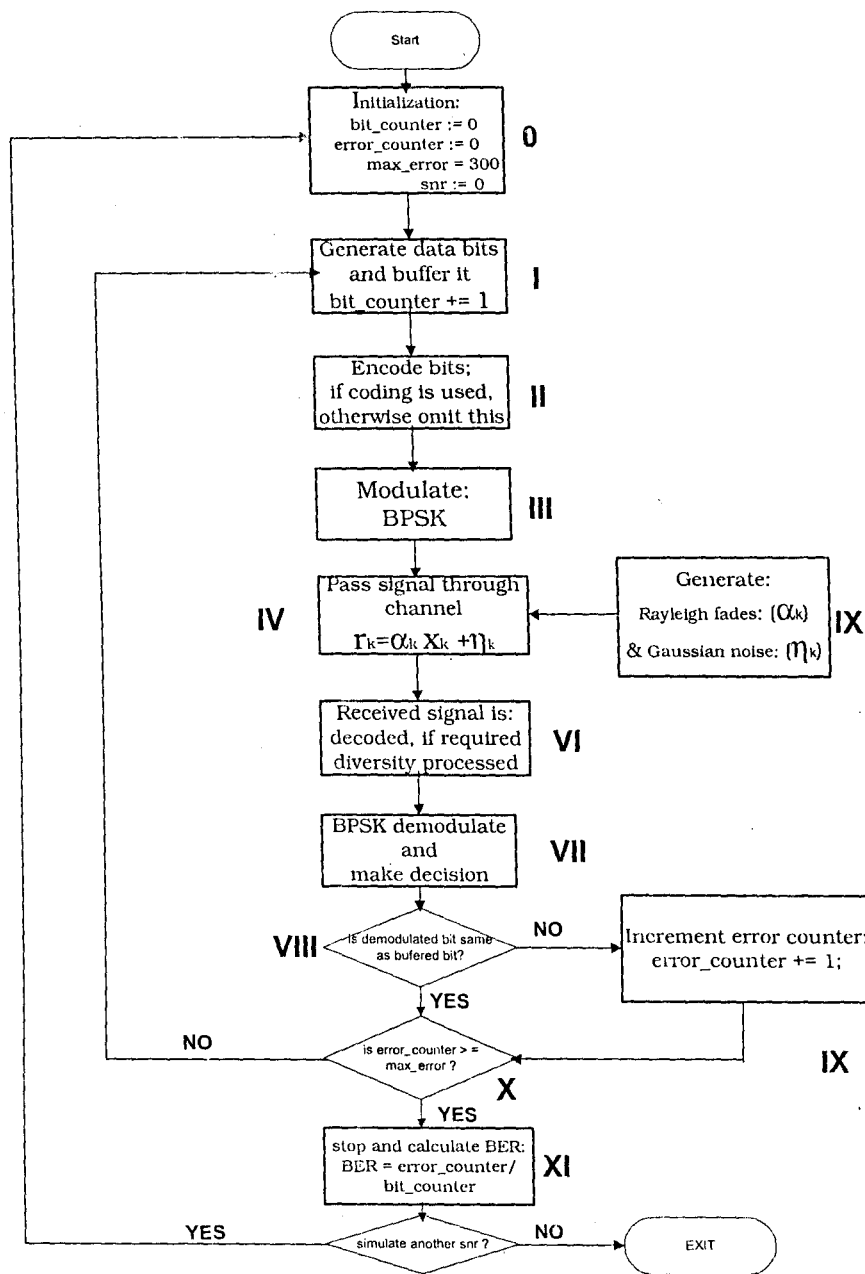


Figure 3.2: Simulation flow chart.

AWGN and (square) for fading channel. The analytical curves are shown in solid and broken lines respectively. One can observe that they match well with the analytical results. Therefore, validating our simulation efforts.

It should be observed that AWGN curve has waterfall nature [5]. The performance falls sharply with increasing signal-to-noise ratio. The implication is that the simulation requirement will be extremely demanding if one wants to simulate say for example the performance at SNR of over 10 dB. To put this in perspective, we set the maximum number of error committed to stop the simulation to 400 (we referred to this as being statistically sufficient). If the BER rate is 10^{-6} , the number of bits that need to be simulated is 400×10^6 . That is 400 Million! bits. This will take a very long computer time to complete. This is where simulation and analysis complement each other. In most cases (certainly not in all), at such high SNR (i.e., low error rate) region, some approximation can be invoked such that analytic expression can be obtained, hence, avoiding such lengthy simulation efforts.

As we mentioned before, high signal power required to obtain small error rate in fading channel is not desired. We introduce the technique called diversity that can be used to reduce this high signal power requirement. First, we discuss the idea behind diversity.

3.4 Diversity Technique

In the following discussions, we will expose the impact of spatial or antenna diversity on the performance of wireless communication systems. We shall try to explain this diversity technique using intuition.

Consider the model for the channel: $r(t) = \alpha s(t) + n(t)$. Remember equation (2.1)?, where $s(t)$ is the transmitted signal, α is the fading (multiplicative noise) that can assume any random value (even zero!), $n(t)$ is the noise, and $r(t)$ is the received signal. The lower the value of α the worse the channel is, as it makes the signal power to be very small compared to the noise's. Now, assume we can provide the receiver with two different paths such that each path experiences different α 's. Since they are random, we will expect that the two α 's will not be low at the same time. Therefore, if the receiver monitors these two independent channels and selects one with higher α , the performance can be better than the case we presented the results above, i.e., where

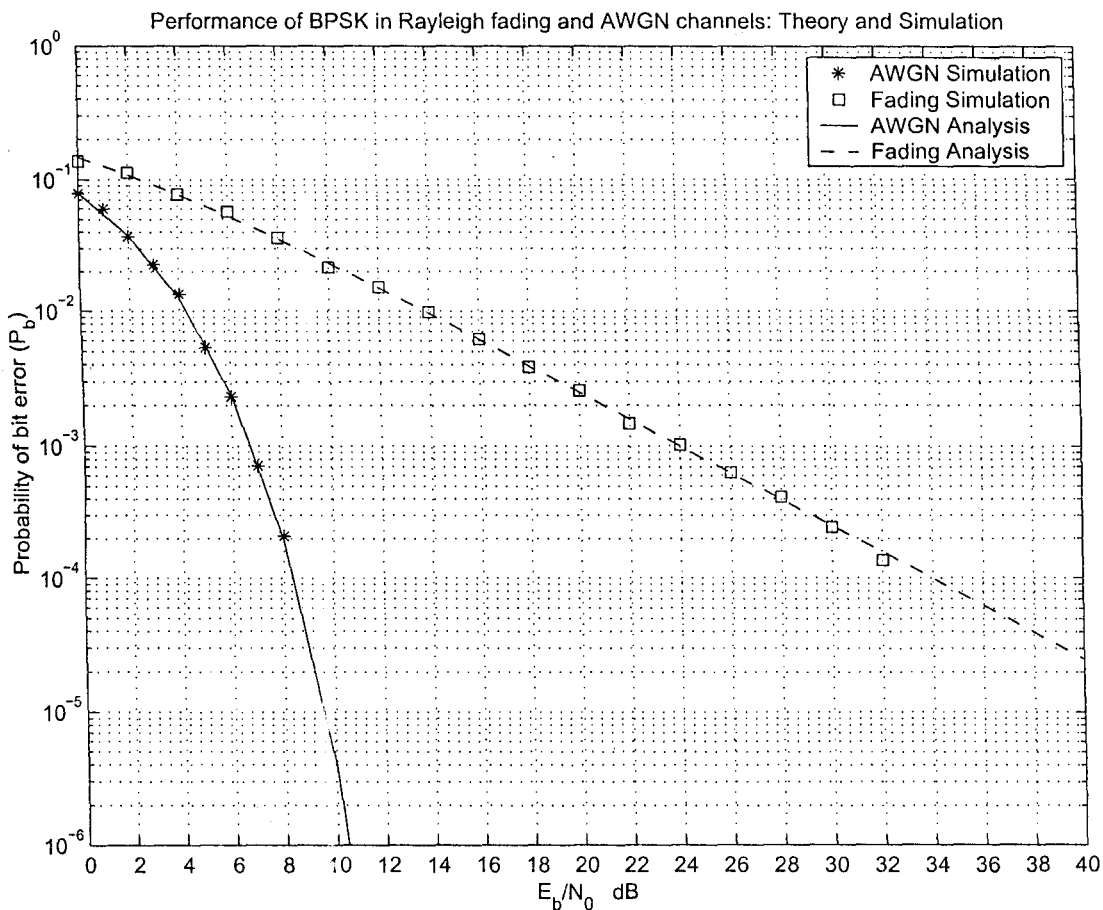


Figure 3.3: Performance of BPSK in Rayleigh fading: simulation vs analysis (Appendix I and III (where *no_of_antennas* = 1), IV).

the receiver sees only one path. The higher the number of independent paths we can provide the receiver the better the performance will be. Providing independent signal paths is done through using extra receiver antennas (multiple antennas). This is very cheap to do. This form of selecting from a number of paths is known as *selection combining diversity (SC)*. All you need to implement this technique is just a switch, switching from one antenna to another.

Let it be stated that we have used intuition to explain the concept and advantage of diversity, mathematical justification is possible but is certainly beyond the scope of this project.

3.5 Diversity Combining Techniques

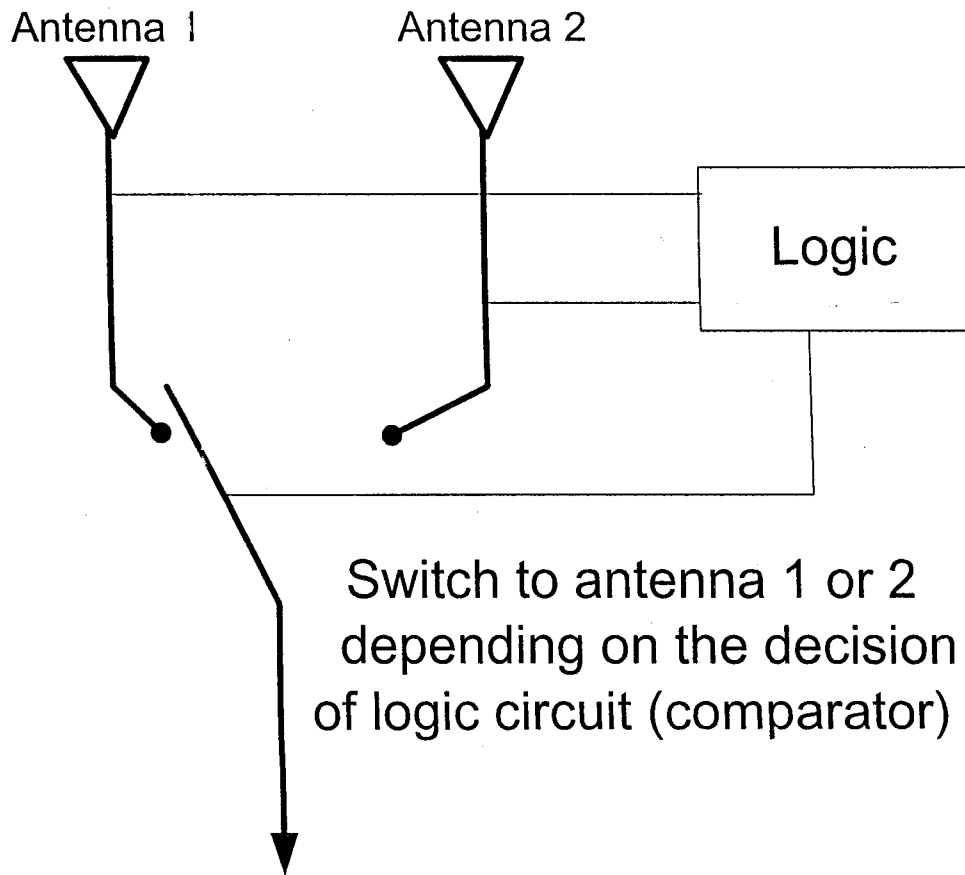
We have intuitively explained SC and MRC but the circuit shown in Figure 3.4 and Figure 3.5 could help to give a clearer picture to the discussions. We will not consider EGC since is suboptimal to MRC. In the following we show the performance of BPSK with MRC diversity in Rayleigh fading and compare it with BPSK in Rayleigh fading without diversity.

3.6 Performance of BPSK in Rayleigh Fading with Diversity Receiver: Simulation Results II

Simulations are performed for selection combining and maximal ratio combining receivers having M antennas. We have considered $M=2, 3,$ and 4 . The SC receiver simply select 1 of the M antennas for making decision while the MRC uses all the M -antennas to make its decision. In each case we will also show the case where no diversity is used. That is where there is one antenna ($M=1$) at the receiver. Such a scenario will help to show the advantage or gains of diversity.

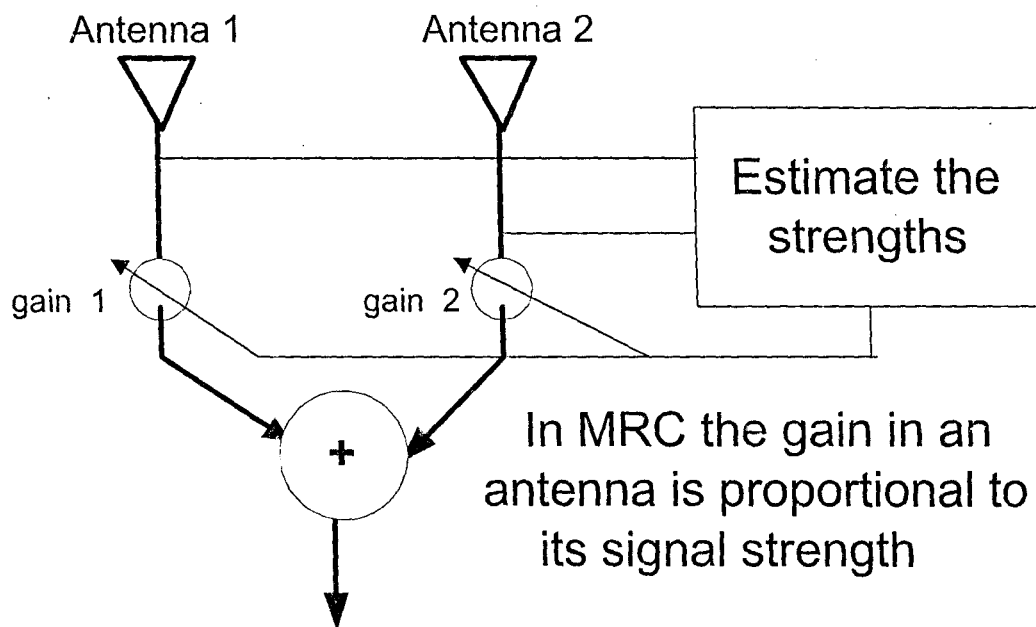
3.6.1 Results for Selection Combining

From Figure 3.6 it is observed that using multiple antennas and selecting 1 improves the system performance. Since the curves are moved close to the AWGN curve. Let us take BER of 10^{-4} as a case study. This error rate is good enough for many communication application. In fact $BER = 10^{-3}$ is considered suitable for voice applications. At $BER = 10^{-4}$ and without diversity ($M=1$, fading) you need about 34 dB as compared with 8 dB required for operating in a noise only environment. Now introducing two antennas ($M=2$) the SNR required drops to 18 dB for the case where one antenna is selected from 2. However, when the number of antenna is increased to 4 and we select 1 the SNR required drops to 10 dB. This is pretty close to the performance for



SELECTION COMBINING DIVERSITY

Figure 3.4: Operation of selection combining.



**MAXIMAL RATIO COMBINING
DIVERSITY**

Figure 3.5: Operation of maximal ratio combining.

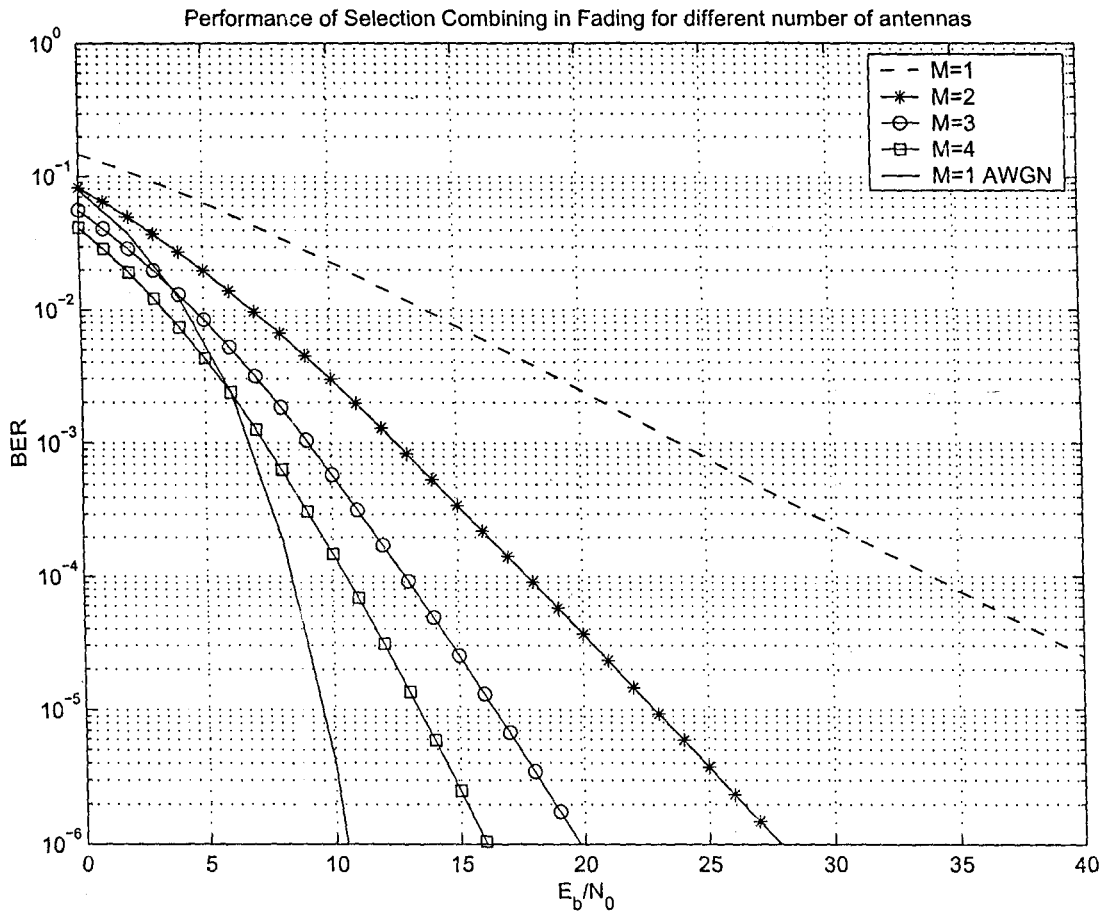


Figure 3.6: Performance of selection combining diversity in fading channel for different number of antennas (Appendix II).

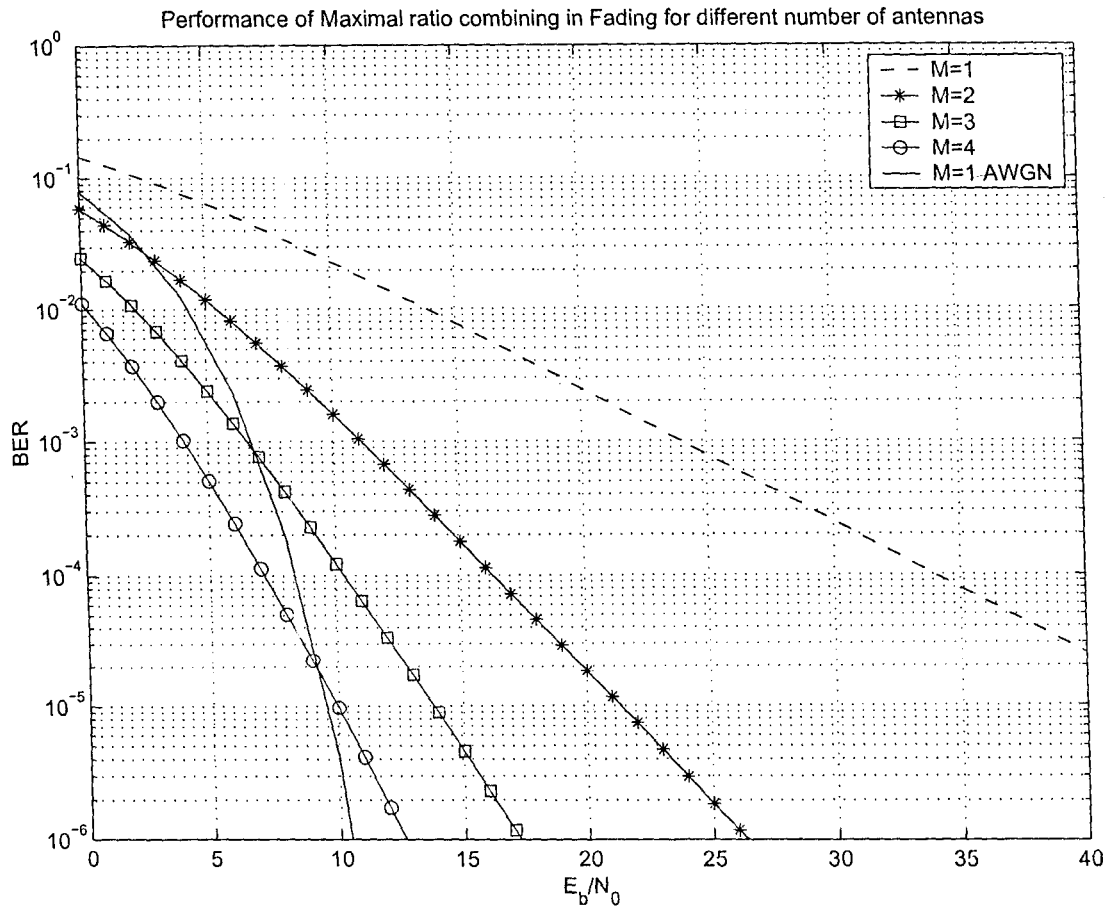


Figure 3.7: Performance of maximal ratio combining diversity in fading channel for different number of antennas (Appendix III).

no fading case (AWGN). Hence using $M=4$ and selecting one antenna amounts to a whopping SNR saving of about 24 dB (34 - 10 dB). The gain in using diversity is encouraging. As a final remark. Observe that selecting one signal path from M paths at the receiver can be seen as a waste of information. The discarded paths carry some useful information. If such information can be used in some way, it can help to improve the system performance. This is what MRC exploits to give a better BER performance than SC for the same number of antennas.

3.6.2 Results for Maximal ratio Combining

From Figure 3.7 it is observed that system performance is improved using the maximal ratio combining. The performance in Figure 3.7 is seen to be better than the selection combining given in Figure 3.6 for a given number of antennas. But observed that $M=1$ is the same in each

case. Interestingly, observe that MRC apparently gives a better performance than AWGN at $BER = 10^{-4}$. Since one requires less than 8 dB for this error performance when $M=4$. Hence for this error rate we do not need the 34 dB that is required if only one antenna is available at the receiver. Effectively, in this particular case, the MRC transforms the fading channel to the simplest AWGN channel. The simplest system distortion or noise you can ever have.

As a final remark, note that the law of diminishing returns set in as more antennas are being added. The largest incremental gain is observed when the second antenna is added. That is the gain in moving from 1 antenna to 2 is larger than the gain moving from 2 antennas to 3 antennas and so. This implies that one cannot continue to add more antennas because at a point, adding more antennas may not give any appreciable gains. This is true for the two diversity combining methods considered in the project, i.e., selection combining (see Figure 3.6) and maximal ratio combining (see Figure 3.7)

Chapter 4

Conclusions and Recommendation for Future Work

4.1 Summary and Conclusions

This project sets out to evaluate the performance of wireless communication. The performance measure is the bit error rate for a given transmitter power. The impediments to communication are noise and fading. These are as a result of the imperfections of the medium of communication known as the channel. The digital modulation technique considered is the Binary Phase Shift Keying (BPSK). We have considered the base-band simulation techniques. That is, carrier modulation is not explicitly simulated. This help to reduce the high sampling rate required and by extension the simulation efforts, if we have to carry out simulation at such high carrier frequency. After all, simulations obtained from base-band is not different from the band-pass simulations.

In the study carried in the project, we show the impact of fading and noise in the communication system. Fading in particular imposes a heavy transmit power on the transmitter to maintain acceptable error rate in the course of communication. Fading is peculiar to wireless systems, a consequence of the unguided nature of the wireless communication signals. The high transmitter power required for reliable communication is not a desirable thing for system designers. Efforts should be exerted to combating the fading phenomenon. We have considered diversity in this project.

The diversity we considered is basically putting more than one antennas (say M antennas) at the receiver. Each of these antennas receives different copy of the transmitted signal. Each signal copy will not experience the same fading. Since fading is a random process. The receiver could monitor all the antennas and select one with the best signal strength. This is what we referred

to as selection combining. Tremendous performance improvement is obtained over receiver that has only one antenna.

Furthermore, maximal ratio combining diversity was also considered. In this case, the receiver does not select one antenna from M , but uses all the M copies of the signal for making decision. The performance obtained here was seen to be better than the selection combining. Most importantly, when $M=4$ the fading channel is effectively transformed to the simplest additive white Gaussian noise channel. In fact, the performance obtained with MRC is better than AWGN noise! See Figure 3.7

In conclusion, the wireless system performance has been analyzed using simulation. Diversity schemes were also studied and shown to yield tremendous system performance improvement. Hence, diversity is effective in wireless systems.

4.2 Recommendations for Future Work

The following recommendation can be considered for any future extension of this work.

- Consider other forms of phase-shift keying modulation like 8-PSK, or 16-PSK.
- Consider other form of fading statistics like Rician or Nakagami fading.
- Consider Encoding the signal before transmitting it into the channel. Coding is also known to be effective in combating channel distortions.
- Consider building a prototype of the system described and analyzed in this project for the Electrical & Computer engineering department. This can be done in a SWEP (student work experience program) normally conducted at the end of 200 level. If that is too much, a group of final year students can undertake it as their final year project.

Chapter 5

Appendices

5.1 Appendix I: Matlab Simulation code for BPSK in Gaussian noise channel

```
%%==Simulate BPSK in AWGN channel
%%==Noise Only channel
%%==MATLAB version 5.2 was used, MATLAB is a product of:
%%                               www.mathworks.com
%%===== (c)adinoyi May 2004

clear all; close all; clc
max_snr = 8;
max_error=3;
k = 0;
for sn=0:2:max_snr
    k = k+1;
    snr(k) = sn
    sigma = sqrt(0.5/(10^(sn/10)));
    error_counter2 = 0;
    bit_counter = 0;

    while error_counter2 <= max_error
        temp = rand;
        if (temp < 0.5)
            dsource = 0;
        else
            dsource = 1;
        end

        %%BPSK Modulation a zero bit gives -1 and bit 1 gives 1.
        bpsk_signal = (2*dsource - 1);
        bit_counter = bit_counter + 1;

        alpha_awgn =1;
        r1 = alpha_awgn*bpsk_signal + sigma * randn ;%%The received signal
```

```

%%%%%%%%Detector makes decision when the channel is AWGN only

if (r1 < 0)
    decis1 = 0;
else
    decis1 = 1;
end

if (decis1 ~= dsource)
    error_counter2 = error_counter2+1;
else
    end

end %while end

BER = error_counter2/bit_counter
BER_awgn(k) = BER; %error_counter2/bit_counter;
fid = fopen('bpsk_awgn.dat','a');
fprintf(fid,'%d %12.8f\n',sn,BER);
fclose(fid);
end % snr

semilogy(snr,BER_awgn,'-');%Plots the BER vs SNR in log scale
hold on;

sn=0:2:max_snr
eb = 10.^(sn/10);
pe = 1/2 * erfc(sqrt(eb)); %Theoretical expression for BPSK in AWGN
semilogy(snr,pe,'--');
legend('simulation','theory');
title('Performance of BPSK in AWGN')
%legend('AWGN Simulation','Fading Simulation','AWGN ...
% Analytical','Fading Analytical')

```

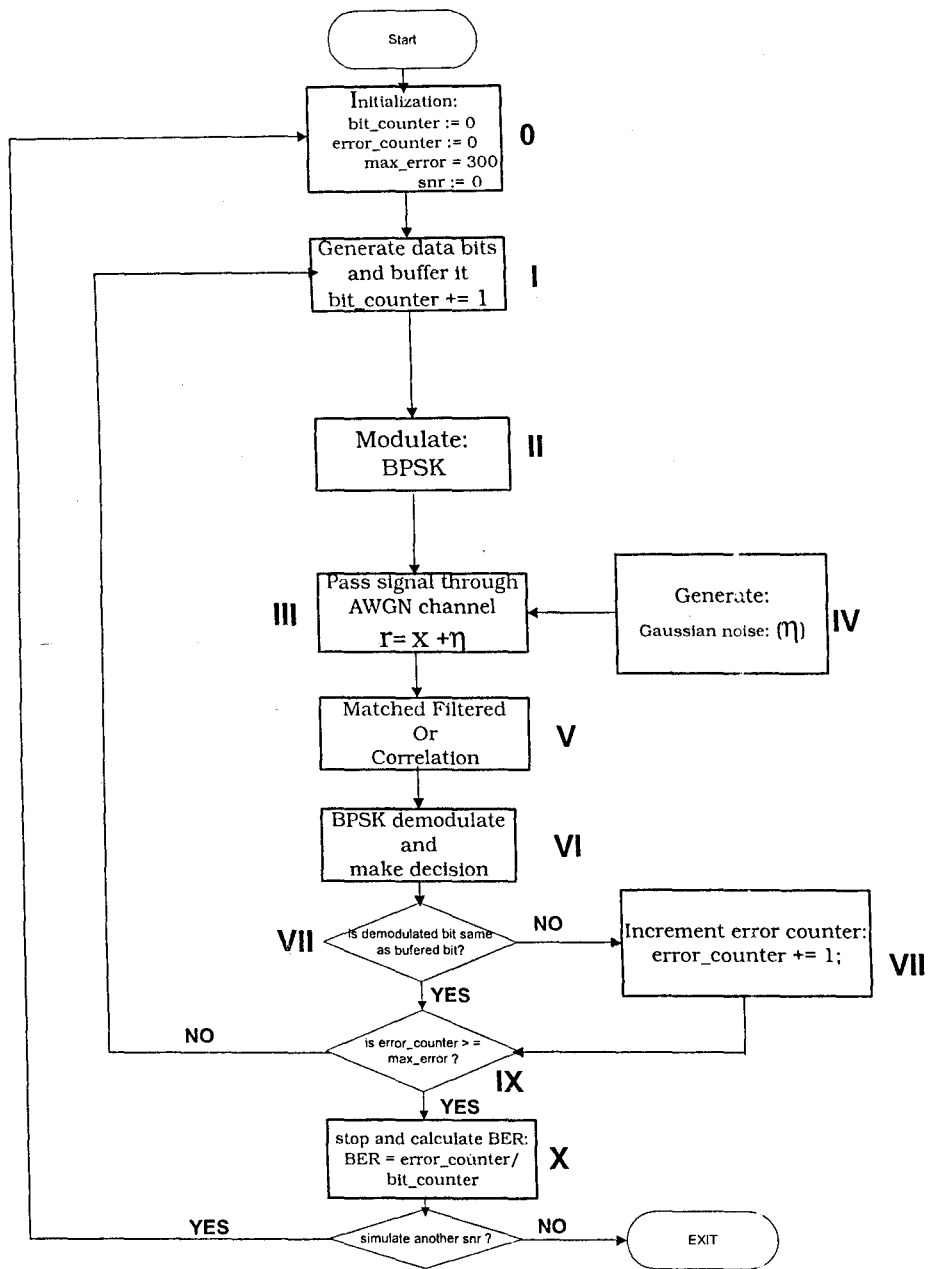


Figure 5.1: Simulation flow chart for AWGN channel.

5.2 Appendix II: Matlab Simulation code for BPSK with Selection Combining (SC) diversity in Rayleigh fading

```
%%%%==Simulate BPSK with selection combining in AWGN fading channel
%%%%==Where M antennas are provided at the receiver and the receive
%%%%==selects the antenna with the highest signal power
%%%%===== (c)adinoyi September 2004

clear all; close all
M = 2;          %%No of diversity antennas
max_snr = 32;
max_error=300;

k = 0;
for sn=0:2:max_snr
    k = k+1;
    snr(k) = sn
    sigma = sqrt(0.5/(10^(sn/10)));
    error_counter1 = 0;
    bit_counter = 0;

    while error_counter1 <= max_error
        temp = rand;
        if (temp < 0.5)
            dsource = 0;
        else
            dsource = 1;
        end

        bpsk_signal = (2*dsource - 1);%BPSK MOD.a zero bit gives -1 and bit 1 gives 1.
        bit_counter = bit_counter + 1;

        %%%===== Selection diversity in Rayleigh fading =====
        %%% There a M antennas, each with different fading; we select the one
        %%% with highest fade sample which is equivalent the one with highest SNR
        %%%=====
        alpha = 0;
        for mm = 1:M
            ray_comp = sqrt(1/2)*randn(1,2); %%generating Rayleigh fade sample
            aa(mm) = sum(ray_comp.^2)^0.5;

            if (aa(mm) > alpha)
                alpha = aa(mm);
            else;
        end
    end
end
```

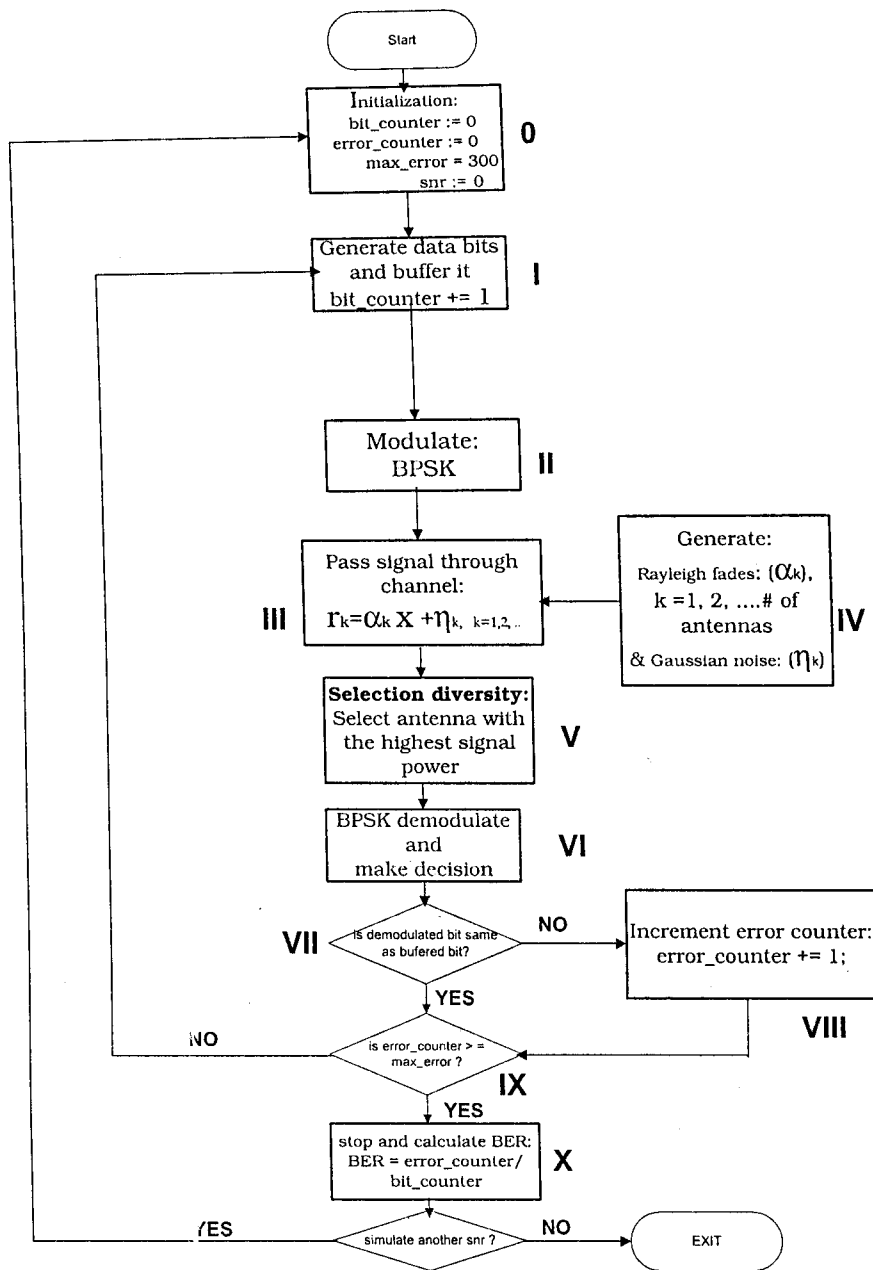


Figure 5.2: Simulation flow chart for selection combining diversity.

```

end
end
r = alpha*bpsk_signal + sigma * randn; %%Received signal

%%%%%%Detector makes decision

if (r < 0)
    decis = 0;
else
    decis = 1;
end

if (decis ~= dsource)
    error_counter1 = error_counter1+1;
else
    end

end %while end
BER = error_counter1/bit_counter;
BER_fade(k) = BER; %%Bit error rate for fading case
fid = fopen('bpsk_fade_sc_M2.dat','a');
fprintf(fid,'%d %12.8f\n',sn,BER);
fclose(fid);
end % snr
semilogy(snr,BER_fade,'--');

xlabel('E_b/N_0 (dB)');
ylabel('BER');
title ('Simulated Performance of selection diversity ...
Receiver technique in Rayleigh fading channels')

```

5.3 Appendix III: Matlab Simulation code for BPSK with Maximal Ratio Combining (MRC) diversity in Rayleigh fading

```
%%=====
%%==Simulate BPSK with Maximal ratio combining in fading and AWGN channel
%%==Where M antennas are provided at the receiver and maximizes
%%==the output SNR using all the antenna signals
%%===== (c)adinoyi September 2004
```

```
clear all; %close all
```

```
no_of_antennas = 4;           %%No of diversity antennas
```

```
max_snr = 20;
max_error=200;
```

```
SIGNAL_SET = 2;
```

```
BPSK =[-1 1];                %%Signal constellations
```

```
    k = 0;
    for sn=6:2:max_snr
        k = k+1;
        snr(k) = sn
        sigma = sqrt(0.5/(10^(sn/10)));
        error_counter1 = 0;
        bit_counter = 0;
```

```
        while error_counter1 <= max_error
            temp = rand;
            if (temp < 0.5)
                dsource = 0;
            else
                dsource = 1;
            end
```

```
%%BPSK Modulation a zero bit gives -1 and bit 1 gives 1.
```

```
    bpsk_signal = (2*dsource - 1);
    bit_counter = bit_counter + 1;
```

```
%%Rayleigh faded signal received via M different paths
```

```
%%=====
```

```
    alpha = 0;
    for mm = 1:no_of_antennas
        ray_comp = (1/2)^0.5 * randn(1,2);
```

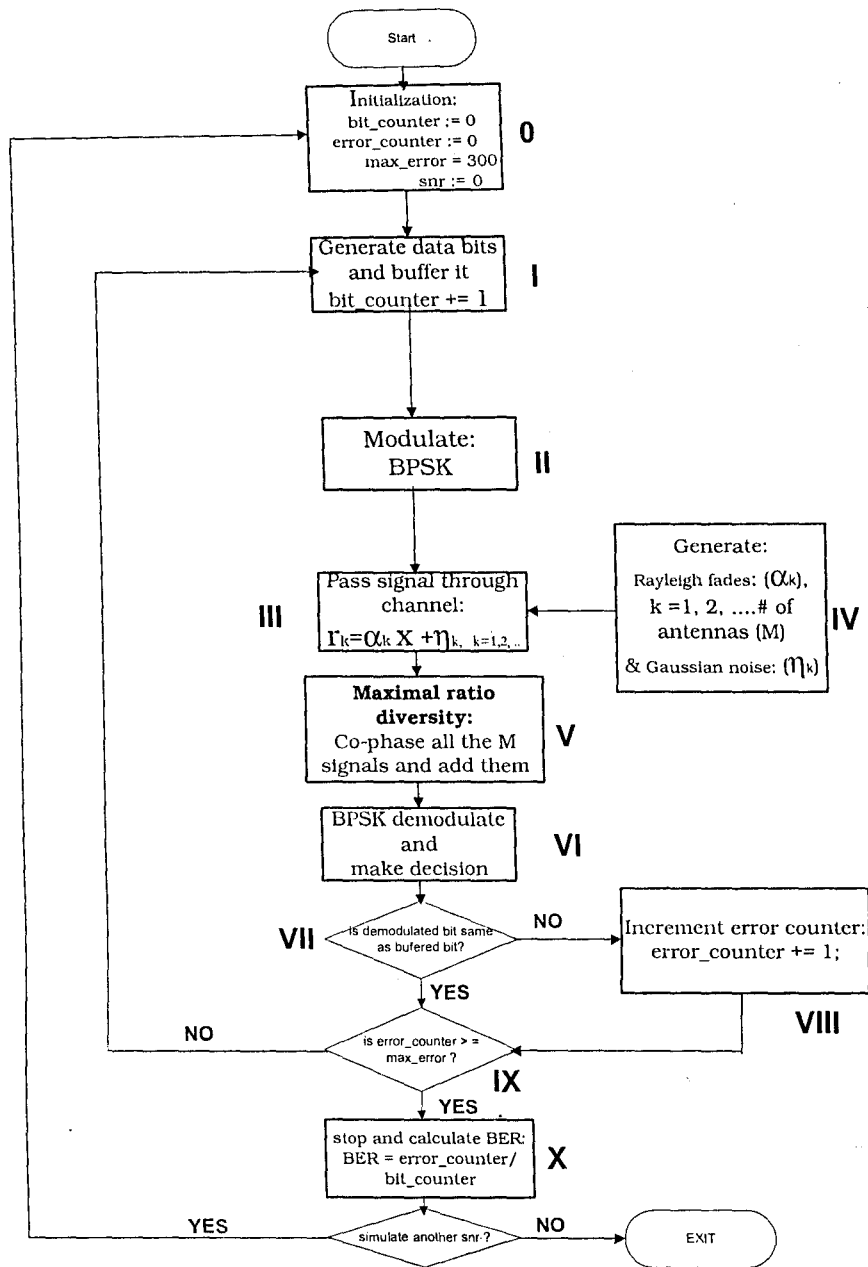


Figure 5.3: Simulation flow chart for maximal ratio combining diversity.


```

alpha(mm) = sum(ray_comp.^2)^0.5; %% Fading sample
rec(mm) = alpha(mm) * bpsk_signal + sigma*randn; %Received signal
end

```

```

%%%%%%=====
%%%%%% This is the begining of MRC combining

%%%%%% What is done here is equivalent to using all the
%%%%%% signal received on a number of receiver antennas
%%%%%% to maximize the received signal to noise ratio
%%%%%% Hence, improving the receiver performance.
%%%%%%=====

```

```

    for h=1:SIGNAL_SET;
        metric(h) = 0.0;
        for mm=1:no_of_antennas
            sum1 = rec(mm) - alpha(mm) * BPSK(h);
            metric(h) = metric(h)+ abs(sum1)^2;
        end
    end

    lowest = 1;
    MIN = metric(lowest);
    for p=1:SIGNAL_SET;
        if(metric(p) < MIN)
            MIN = metric(p);
            lowest = p;
        else
            end
    end

```

```

%%%%%% The end of MRC combining; Next to do detection
%% /**lowest is the demodulated symbol index **/

```

```

    if (lowest == 1)
        decis = 0;
    else
        decis = 1;
    end

```

```

    if (decis ~= dsource)
        error_counter1 = error_counter1+1;
    else
        end

```

```
end %while end
BER = error_counter1/bit_counter;
BER_fade_mrc(k) = BER; %%Bit error rate for fading case
fid = fopen('bpsk_mrc_no_antena4.dat','a');
fprintf(fid,'%d %12.8f\n',sn,BER);
fclose(fid);
end % snr
semilogy(snr,BER_fade_mrc,'--');
xlabel('E_b/N_0 (dB)'); ylabel('Bit Error Rate (BER)');
grid on;
title('Performance of maximal ratio combining diversity ...
in Rayleigh fading')
```

5.4 Appendix IV: Miscellaneous Matlab Codes

5.4.1 Histogram

The following codes are used to obtain the histogram of the fading and Gaussian noise shown in Figures 2.3 and 2.4 respectively

```
%%% This program generates and plot the histogram (probability density
%%% function) of the distribution discussed;
%%% 1. Gaussian Noise
%%% 2. Rayleigh Fading
```

```
%%%%%%%% 1. Gaussian
```

```
gaussian = randn(1000000,1);
hist(gaussian,100);
ylabel('Histogram (probability density function (PDF))');
xlabel('Samples, n');
title('Simulated PDF of Gaussian Noise')
```

```
%%%%%%%% 2. Rayleigh
figure;
```

```
rayleigh = sqrt(randn(1000000,1).^2 + randn(1000000,1).^2);
hist(rayleigh,100);
ylabel('Histogram (probability density function (PDF))');
xlabel('Samples, \alpha');
title('Simulated PDF of Rayleigh Fading')
```

5.4.2 The Analytical performance plot of BPSK for AWGN and fading

The following codes are used to obtain the theoretical results i.e., Equation (2.10) for AWGN and Equation (2.11) for fading channel and the two plots are shown in Figure 2.7

```
%%% This calculate and plot the theoretical bit error probability of BPSK
%%% in fading and Additive White Gaussian Noise channels.
```

```
snr=0:2:40;
eb = 10.^(snr./10);
pe = 1/2 * erfc(sqrt(eb)); %Theoretical expression for BPSK in AWGN
figure (1); semilogy(snr,pe,'-');
ylabel('Probability of bit error (P_b)');
xlabel('E_b/N_0 (dB)');
hold on;
```

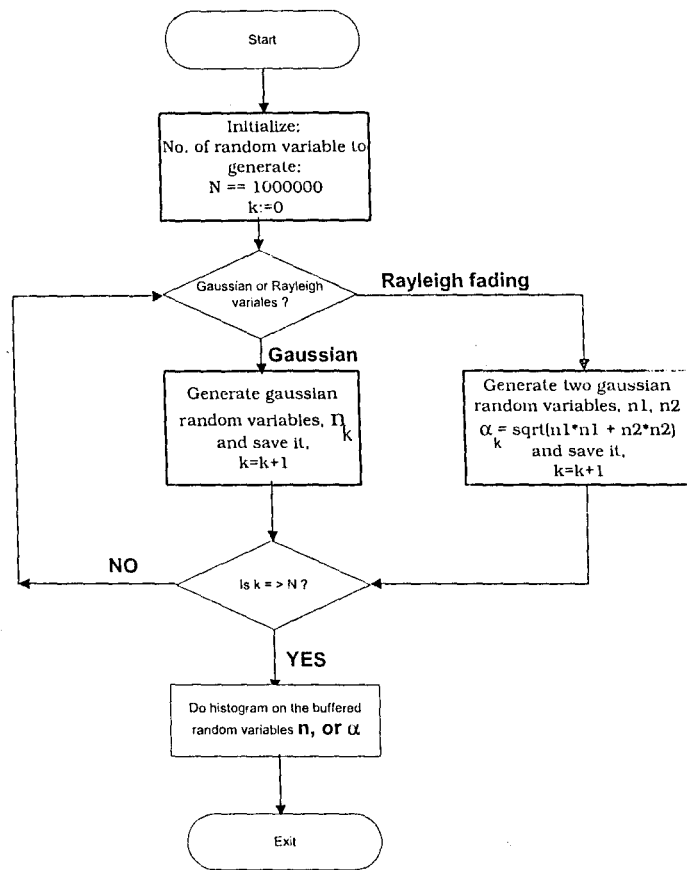


Figure 5.4: Simulation flow chart for generating the PDF of the noise and Rayleigh fading.

```
pe2 = 1/2 * (1-sqrt(eb./(1+eb))); %Theoretical BPSK in Rayleigh fading
semilogy(snr,pe2,'--');
grid on;
legend('AWGN Analysis','Fading Analysis')
axis([0 40 1e-6 1]);
title('Performance of BPSK in Rayleigh fading and AWGN channels:
... Theory and Simulation')
```

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