TRANSIENT ANAYLSIS AND

MODELLING OF D.C SHUNT

GENERATOR WITH INTERPOLES

BY

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DEDICATION

This project work is dedicated to my only sister, Miss Ifeoma Queenet Sunday.

DECLARATION

I Sunday Kelechi Vincent declare that this work was done by me and has never been presented elsewhere for the award of a degree. I also hereby relinquish the copyright to the Federal University of Technology, Minna.

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ABSTRACT

A "Transient Analysis and modeling of D.C. Shunt Generator With Interpoles" is carried out with the aid of MATLAB/ SIMULINK®, a software package developed by MathWorks Inc. MATLAB programmes for the simulation are written, using the equations of the D.C. shunt generator. And the graphs which result from the simulation describe the transient behaviour of the D.C generator. The graphs also describe the behavior of D.C generator with interpoles and one without interpoles.

This project also reaffirms the importance of MATLAB/ SIMULINK as a very powerful and easy- to- use tool for system analysis.

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CHAPTER ONE

INTRODUCTION

1.1 BACKGROUND

An electrical generator is a machine which converts mechanical energy (or power) into electrical energy (or power). The direct current (D.C.) generator, therefore, is a machine that converts mechanical energy to electrical energy in D.C. form. [1].

The D.C. generator first appeared in 1833 when the first rotating machines embodying Ampere's commutator were produced. However, Michael Faraday had in 1832 demonstrated the first homopolar electromagnetic generator, which he soon followed with the heteropolar version. Despite over a century of technological progress, modern electromagnetic machines, the D.C. generator included, still exploit the basic Faraday principle of electromagnetic induction in 1831 which is stated as follows, whenever a conductor cuts a magnetic flux, an e.m.f is induced and the magnetitude of the induced e.m.f is equal to the rate of change of the flux linkage. [2].

The humanity has benefited a lot from the invention of the D.C. generator. For instance, the D.C. shunt generator with field regulators for which this project is concerned is used for ordinary lighting and power supply purposes. They are also used for charging batteries because their terminal voltages are almost constant or can be kept constant. [3].

1.2 AIMS/OBJECTIVES

1. To investigate the transient characteristics behaviour of D.C shunt generator with interpoles.

2. To improve the performance of D.C generators by incorporating an interpoles

3. To ascertain the roles played by MATLAB/SIMULINK for system analysis

1.3 METHODOLOGY

This project "Transient Analysis of D.C. Shunt Generator with Interpoles" will be done through digital computer simulation using MATLAB/SIMULINK®, which is simpler, compared to an analytical solution with differential equations which involve voltage, speed and current that may be functions of time thereby necessitating a step-by-step solution.

1.4.0 D.C. SHUNT GENERATOR

D.C. generators are broadly classified into separately excited and self-excited generators, based on the way their field windings are excited. The self-excited generators are further classified into shunt, series, and compound types based on the way their field windings are connected to the armature [3].

In the D.C. shunt generator, the field windings are connected across or in parallel with the armature conductors and have the full voltage of the generator applied across them, fig.1.1.
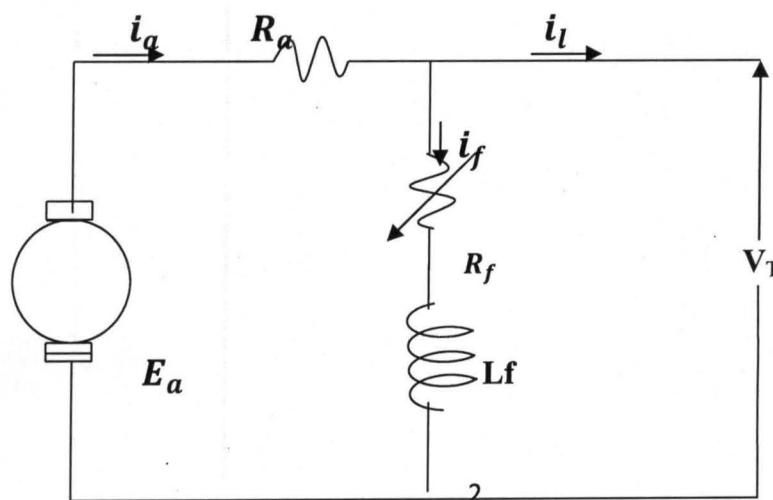
Fig.1.1 D.C. Shunt generator
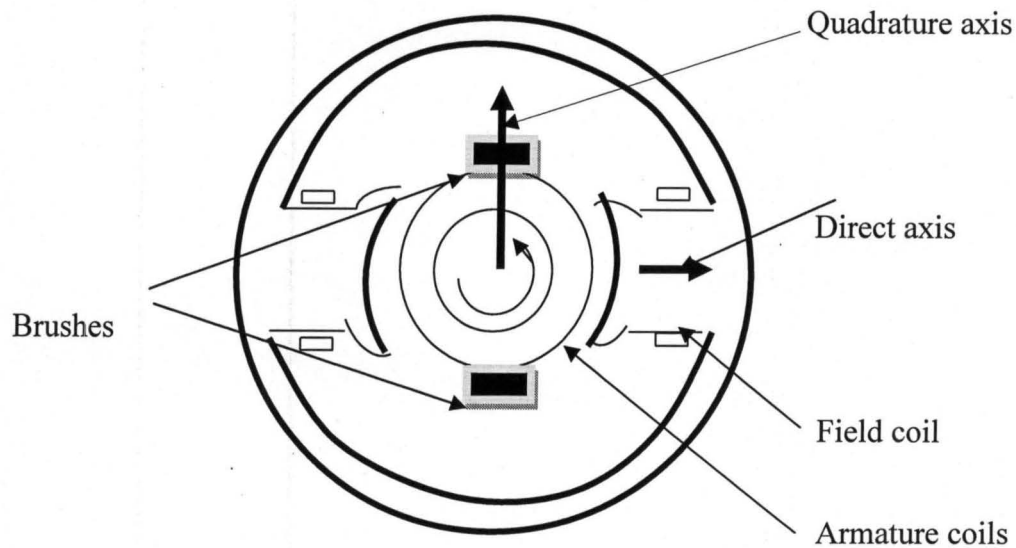
Fig.1.2 A schematic representation of the D.C. generator.

The characteristic of the D.C. shunt generator is that the voltage decreases with increase in load.

1.4.1 PROBLEM STATAMENT

The current induced in armature conductors of a D.C. generator are alternating, and to make it unidirectional, we need a commutator. The current flows in one direction under N-pole and opposite direction under S-pole during which there is reversal of current. This current reversal takes place along the brush axis (Q-axis). [4]

Commutation is the process by which current in the short-circuited coil is reversed while it crosses the brush axis. If the reversal (change from plus one to zero and to minus one) is

completed at the end of commutation period, then commutation is ideal. But if that is not the case some sparking is witnessed between brush and commutator, which results in progressive damage to both. This sparking is due to self-induced e.m.f. in the coil undergoing commutation. And this self-induced e.m.f. is known as reactance voltage, which hinders the reversal of currents, see

Fig 1.3 [3]

Fig.1.3
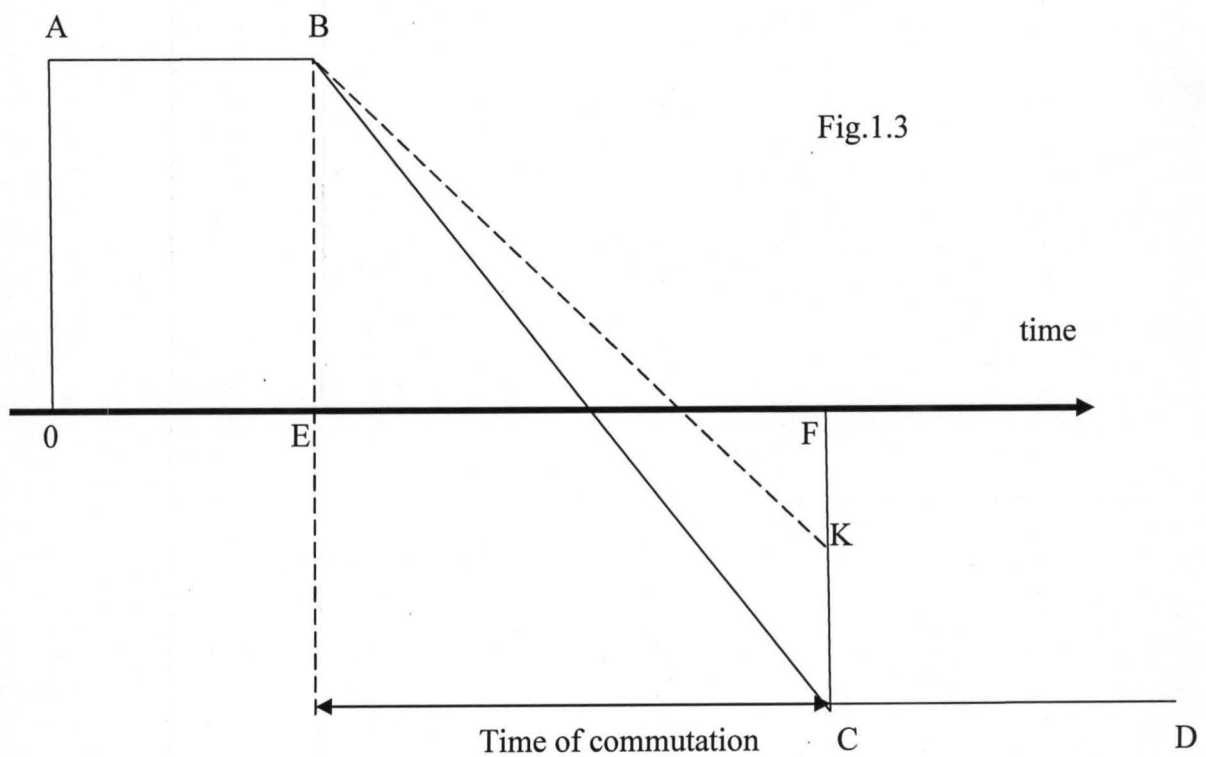
BK = Commutation due to self-induced e.m.f. leading to spark

BC = Linear commutation devoid of spark

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There are two methods of improving commutation available, namely, resistance commutation and e.m.f. commutation which employs either brush lead or interpoles [3]. And in this project the D.C. generator with interpoles will be considered.

1.4.2   INTERPOLES OR COMPOLES

These are small poles fixed to the yoke and spaced in-between the main poles. They are connected in series with the armature so that they carry full armature current. And they perform two functions: (1) since they have the same polarity as the main poles, they induce an e.m.f. in the coil (under commutation) which helps the reversal of current. This induced e.m.f. is known as reversing or commutating e.m.f. (2) they neutralize the X-magnetizing effect of armature reaction. Hence, brushes are not to be shifted from the original position. This cancellation of X-magnetization is automatic because both are produced by the same armature current.

CHAPTER TWO

LITERATURE REVIEW

In this chapter, the brief history of D.C generator is discussed; the chapter also discussed the review on previous works based on generator performance, types of experiment carried out by other researcher and contribution from the researcher. The chapter also discussed D.C machine analysis and classification, and also discussed on Matlab and Simulink.

There have been many advances in the development of electric motors/generators and there is an increasing need for new control technology, increased energy costs, public concern for unnecessary energy consumption and environmental impacts, and legislation requiring efficiency are among the forces behind development of motor controls. [6]

2.1 REVIEW OF PREVIOUS WORKS

2.1.1 BRIEF HISTORY OF D.C GENERATOR

The D.C. generator first appeared in 1833 when the first rotating machines embodying Ampere's commutator were produced. However, Michael Faraday had in 1832 demonstrated the first homopolar electromagnetic generator, which he soon followed with the heteropolar version. Despite over a century of technological progress, modern electromagnetic machines, the D.C. generator included, still exploit the basic Faraday principle of electromagnetic induction of 1831[2].

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2.2.0 REVEW ON IMPULSE VOLTAGE GENERATOR USING MATLAB.

In the design or use of impulse voltage generators for research or testing, it is required to evaluate the time variation of output voltage, the nominal front and tail times and the voltage efficiency for given circuit parameters. Also, it needs to predict circuit parameters for producing a given waveshape, with a given source and loading conditions. The loading can be inductive or capacitive. The waveshapes to be produced may be standard impulse, steep fronted impulse, short tailed impulse or steep front short tailed impulse. After performing an experimental investigation to evaluate the influence of tail resistance and tail inductance in the characteristic parameters of the impulses, provision of an analytical criterion is available for choosing the most suitable inductance value to be combined with a given resistance in order to generate the desired waves [7]

The analysis, design and practical implementation of impulse voltage generator without computer simulation is extremely laborious, time consuming and expensive. Various types of software like SPICE (Simulation Program with Integrated Circuit Emphasis) have been used to predict the performance of impulse voltage generator. Although SPICE can analyse generator circuits, it is less well suited for dynamic analysis and design, which SIMULINK can handle with ease. However SPICE is very slow and is not practical for the design purposes. [8]

In this paper, step-by-step modelling of an impulse voltage generator, used for the testing of high voltage power transmission and distribution equipments, has been carried out and the performance evaluated from the MATLAB package with its SIMULINK tool box suitable for dynamic system simulation. The system is first represented by a set of mathematical equations;

the derived equations are modelled with standard blocks available in SIMULINK and the complete system is then simulated. [7]
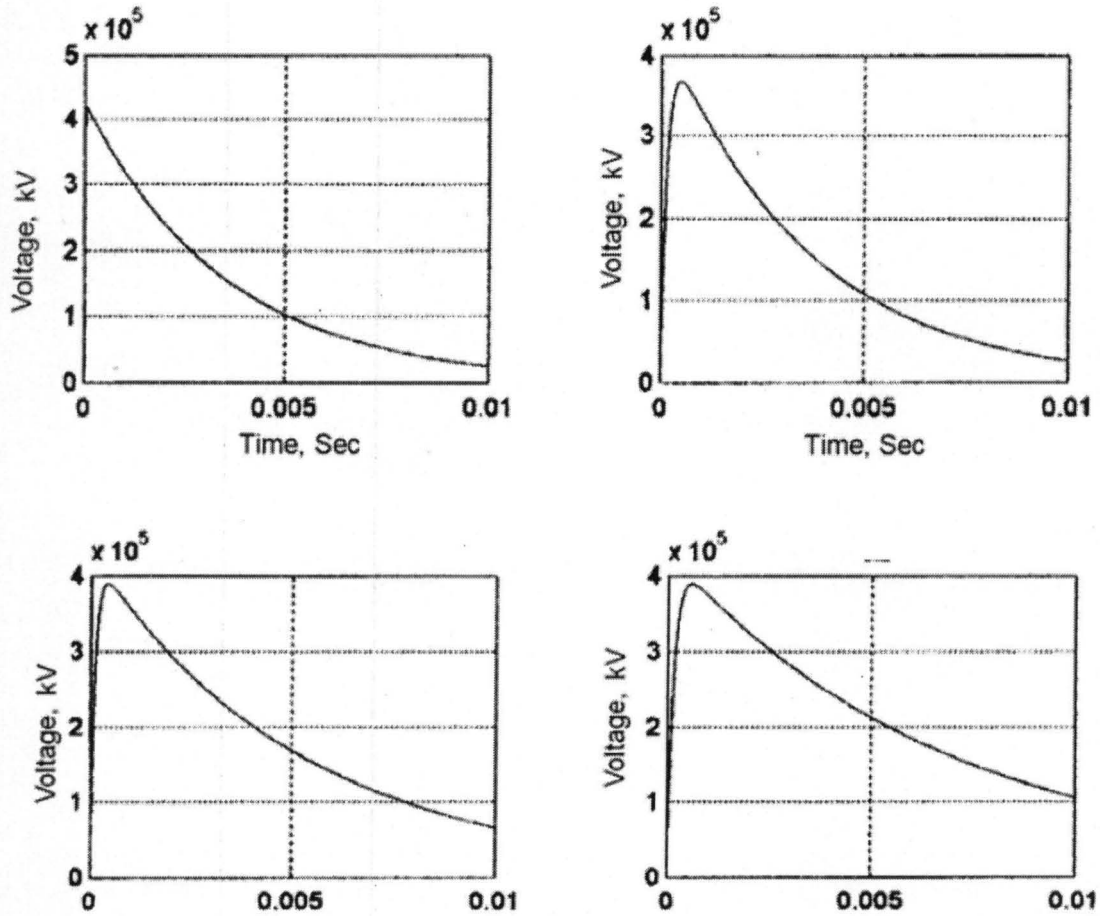
Fig 2.1 Simulated impulse waveform

An impulse generator essentially consists of a capacitor which is charged to the required voltage and discharged through a circuit. The circuit parameters can be adjusted to give an impulse voltage of the desired shape.

2.2.1 REVIEW ON DC MOTOR DRIVE FOR A DYNAMOMETER

The DC shunt motor on the existing dynamometer would benefit from a dedicated DC motor drive based on a full bridge DC-DC converter. The power obtained from the dynamometer is to be recirculated to a DC bus meaning that only the losses of the test bench need to be supplied by a mains power supply. The motor drive for the dynamometer will also need to be capable of simulating road load conditions provided by the user for the motor under consideration. This will be achieved using the microcontroller to introduce a control loop for the dynamometer. The development of this project involved modelling the dynamometer to help with the design of the controller that would be capable of controlling the torque of the dynamometer to the desired input level. This controller model was used to create coding for an Atmel microcontroller, which is required to run and interface with the power electronics, dynamometer and the user. The power electronics are based on a full bridge DC-DC converter constructed by David Finn for the Sunshark Solar Car and was slightly modified for the DC motor application. A number of tests were carried out to ensure the functionality of the torque controller including simple tests for the A/D converter and PWM switching with the microcontroller, simulating conditions for the controller when connected to a motor and initial testing with the power electronics connected to a motor. These were followed by the final testing of the torque controller on a motor using open and closed loop control configurations. [9]

The techniques involved with motor control, regeneration of mechanical power back to electrical power and the power electronics required to do so have been available for some time. DC motor control usually involves entering a desired speed/position to the controller, which will produce a

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control voltage for the power electronics to apply to the motor, with the actual speed/position being fed back to the controller. Power electronics in the form of a DC-DC converter is required to provide control for the motor as well as regulating the power from the generator back to the main supply, the DC bus.

However, this thesis will need to build on and modify these existing technologies to develop a motor controller that controls the torque of the dynamometer (hence creating a road load for the tested motor) and combine this with power electronics to produce a regenerative power loop for testing new motors and drivers. When completed, it will allow the testing of new motors to be used for the Sunshark and a hybrid electric car as well as being the basis of improved technology for larger power requirements. When completed, the test bench will become a relevant and useful piece of technology to be used by other studies. [7]

2.2 2. DC Motor/Generator Theory

DC machines are one of the most common used machines for electromechanical energy Conversion. The action of machines as being the conversion of energy from electrical to mechanical or vice versa results when a conductor moves in a magnetic field inducing voltage and a current-carrying conductor is placed in a magnetic field producing a mechanical force. These two effects occur simultaneously. In generating action, the rotating structure, the rotor, is driven by a prime mover of some sort. A voltage will be induced in the conductors that are rotating with the rotor. If an electrical load is connected to the winding formed by these conductors, a current will flow, delivering electrical power to the load. The current flowing through the conductor will interact with the magnetic field to produce a reaction torque, which

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will tend to oppose the torque applied by the prime mover. For motoring action, the process is reversed. [9]

2.3 .0 D.C. SHUNT GENERATOR ANALYSIS

2.3.1 DC MACHINE CLASSIFICATIONS

One significant advantage of a DC machine is the variety of performance characteristics that can be achieved by interconnecting the field and armature windings in various ways. The field windings can be excited by a DC source which is connected only to the field windings and not connected to the armature. Such a configuration is known as a separately excited DC machine.

The field excitation can also be provided by permanent magnets. This type of machine is also classified as a separately excited machine since the armature current is independent of the field excitation. If the field winding current is all or part of the armature current, the machine is known as a self-excited DC machine. There are three different configurations of self-excited DC machines. In a series self-excited DC machine, the field windings are connected in series with the armature. A shunt self-excited DC machine results when the field windings are connected in parallel with the armature. If a combination of series and parallel connections are made between the field and armature windings, this machine is classified as a compound self-excited DC machine. If a shunt connection of the field windings is implemented, a variable resistor is typically inserted in series with the field winding in order to easily control the field winding current. This variable resistor is known as the field rheostat. The field winding current in shunt-connected winding is typically low (approximately 1 to 2% of the rated armature current). Thus,

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copper losses in the shunt-connected winding are typically low. A series connected field winding must carry the much larger armature current. Thus, the resistance of a series field winding must be kept low in order that the copper losses of the field winding are not too severe. The various DC machine configurations can be analyzed by including the resistances of the field windings, the armature circuit (armature winding resistance and brush resistances) along with the load resistance and the field rheostat, if present. The analysis at DC steady-state does not require inclusion of the various winding inductances. For transient analysis, these inductances are important. [10]

The direct current generator comprises an electromagnet with one or more pairs of poles, an armature winding consisting of number of conductors in series-parallel, an armature core upon which the winding is mounted, and a commutator which has the effect of changing of changing the alternating e.m.f generated in the winding into a direct or continuous e.m.f.[4]

If a conductor be caused to move in a magnetic field so as to cut the line of force, an e.m.f is generated in the conductor, the value of which is proportional to the strength of the field, to the length of the conductor in the field, and to the speed of the conductor through the field. [10]

FLUX DIRECTION

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DIRECTION

OF E.M.F.   MOVEMENT OF CONDUCTOR
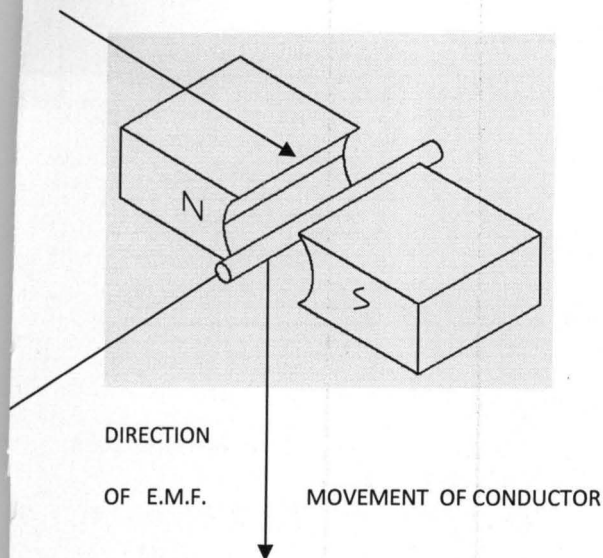
Figure 2.2 illustrating generation of e.m.f

Expressed in another way, the e.m.f is proportional to the cutting lines of force. The figure 2.0 above shows the direction of the e.m.f generated in a conductor moving across a magnetic field, in accordance with Fleming's Right-hand rule. The formula for the e.m.f of a direct generator is

$$E = \frac{QZNp}{60a}$$

Where Q is the flux per pole, in weber,

     Z is the total number of conductors on the armature winding

     N is the speed of rotation of the armature in revolutions per minute

     p is the number of pairs of poles

     a is the number of pairs of parallel paths in the winding

The electromagnet of a modern direct current generator consists of a circular yoke of cast steel, with a number of poles projecting inwardly, of polarity N and S alternately. The number of poles may be two, four, or any even number. The poles may be made of cast iron, cast steel, or

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wrought iron, and may be solid or built of laminations. A field winding carrying current is necessary to magnetize the poles, and this winding is wound on former and mounted on the pieces. The armature core consists of iron stampings bolted up together on the shaft of the machine and so arranged as to allow of efficient ventilation. Stampings are used instead of a solid core to cut down eddy current losses. The armature conductors, in which are generated the e.m.f. are wedge into the insulated slots in the surface of the armature core. The armature is caused to rotate by a mechanical power. The commutator, which is rotates with the armature, is built up of a number of copper commutator bars insulated from each other and from shaft. The ends of the armature conductors are soldered or welded to the risers of the commutator bars. Carbon brushes are arranged to bear upon the commutator. To assist in sparkless commutation, small auxillary poles called interpole or commutating poles are built into the machine. [10]

2.3 2. MATLAB/ SIMULINK

MATLAB stands for Matrix Laboratory. It is a software package developed by MathWorks Inc. for high performance computation and visualization. It is outstanding for its combination of analysis capabilities, flexibilities, reliability and powerful graphics [11].

MATLAB has built-in mathematical functions which solve problems in Matrix Algebra, Complex Arithmetic, Linear Systems, Differential equations, Signal processing, optimization, Non- linear systems, and other types of scientific computations. The most important feature of MATLAB is its programming capability, which is very easy to learn and use, and which allows user developed functions. FORTRAN algorithms and C codes can also be accessed through external interfaces [11].

MATLAB 7.1, is a newer version, which is used in this work. Other versions include MATLAB 6,MATLAB 7, MATLAB 7.1, and MATLAB 8, the latest version.

SIMULINK is one of the several optional toolboxes of MATLAB meant for special applications. It is a graphical mouse- driven program for the simulation of linear and non-linear dynamic systems. It can be accessed from the MATLAB command window; and when accessed the SIMULINK block library is displayed. The library consists of Continuous, Discrete, Functions and Tables, Math, Non- Linear, Signals and Systems, Sinks, and Sources categories. And double clicking on an icon in the library gives access to the category. [11]

CHAPTER THREE

3.0 TRANSIENT MODELLING AND SIMULATION OF D.C SHUNT GENERATOR

This chapter discussed the method used in carried out the project, which involved modelling of equations using D.C equations parameters, and the simulation using MATLAB program. MATLAB is specifically designed for simulating dynamic systems. The speed and capability of SIMULINK in simulating dynamic systems were the attraction to choose it for modeling D.C shunt generator in this work.

When changes in voltage or speed or load occur, a new condition is reached from the initial state through a transient period during which the stored field, inertial and elastic energies are redistributed, a process that cannot be instantaneous.

Transients occur in starting, braking, speed control, load changes, switching and faults. In this project, the analysis will focus on load changes, which is one of the ways transients are produced.

Before loading a shunt generator (on no-load characteristic), it is allowed to build up its voltage. The aim is to produce the residual magnetism in the poles. When a generator first starts to turn, an internal voltage will be generated. This voltage circulates a small current in the field circuit which increases the pole flux. The differential equation describing the build-up voltage of the field current is derived from Fig.3.1. [3]
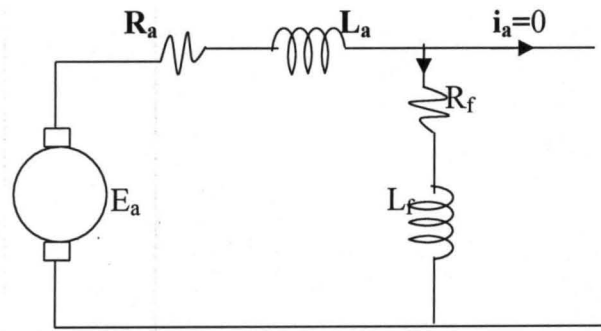
Fig. 3.1 Equivalent circuit for analysis of voltage build-up

$$(L_a + L_f)\frac{di_f}{dt} = E_a - (R_a + R_f)i_f -------------3.1$$

Where, E_a = generator voltage

R_a = armature resistance

R_f = field resistance

L_a = armature inductance

L_f = field inductance

i_f = field current

If the right hand side of equation (3.1) is positive, i_f and E_a will increase and hence build-up. The voltage build-up is higher when the field resistance is decreased and vice versa. If the shunt generator is loaded after the build-up, its terminal voltage drops with increase in load current. This drop is due to several reasons below:

Armature resistance drop: - as the load current increases, more and more voltage is consumed in the ohmic resistance of the armature current. The terminal voltage,

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$$V = E_a - i_a R_a \; - - - - - - - - - - - - - - - - - - - \; 3.2$$

1. Armature reaction drop: - this is due to the demagnetizing effect of armature reaction, pole flux is weakened and so the induced e.m.f in the armature under load condition.

2. The drop due to (1) and (2) result in a decreased field current which reduces the induced e.m.f.

This analysis is divided into two parts:-

- That without interpoles, and

- The one with interpoles

3.1.2 TRANSIENT MODELLING OF D.C. SHUNT GENERATOR WITHOUT INTERPOLES

D.C. machine which is separately excited (Fig. 3.2) has the voltage equations:
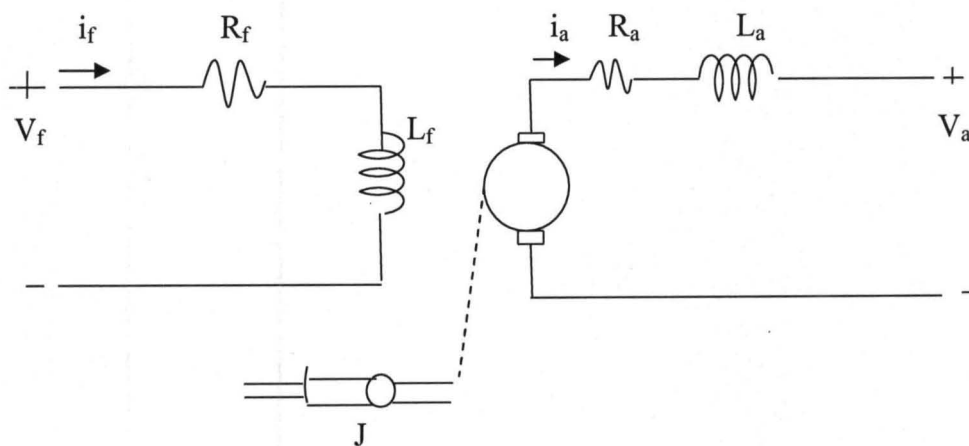
Fig. 3.2 Separately excited D.C machine

Field Voltage :
$$V_f(t) = R_f i_f(t) + L_f \rho i_f(t) \, ------------- 3.3$$

Armature Voltage:
$$V_a(t) = G\omega(t) i_f(t) + R_a i_a(t) + L_a \rho i_a(t) \, ---------- 3.4$$

Torque:
$$T(t) = T_m(t) + J\rho\omega(t) \, ----------------- 3.5$$

The machine torque is related to the armature current and field current as follows:

$$T(t) = G i_f i_a \, ------------------- 3.6$$

Where, T_m is mechanical torque (prime mover)   J is the moment of inertia

G is the torque constant ω is the machine speed

ρ is the derivative (d/dt)

Writing equations (3.3) and (3.4) as a matrix equation,

$$\begin{bmatrix} V_f(t) \\ V_a(t) \end{bmatrix} = \begin{bmatrix} R_f + L_f\rho & 0 \\ G\omega & R_a + L_a\rho \end{bmatrix} \cdot \begin{bmatrix} i_f(t) \\ i_a(t) \end{bmatrix} \, -------- 3.7$$

From equation (3.4)

$$V_a(t) = G\omega(t) i_f(t) + (R_a + L_a\rho) i_a(t) = K\omega(t) + (R_a + L_a\rho) i_a(t) \, --- 3.8$$

Where, $K = G i_f$

From equation (3.5) and (3.6)

$$K i_a(t) = T_m + J\rho\omega(t) \, ------------------- 3.9$$

Writing equations (3.8) and (3.9) in Laplace transformation, and matrix equation.

$$\frac{V_a(s)}{s} = K\omega(s) + (R_a + L_a s)i_a(s) - - - - - - - - - - - - - - - - 3.10$$

$$\frac{T_m(s)}{s} = -Js\omega(s) + Ki_a(s) - - - - - - - - - - - - - - - - - 3.11$$

$$\begin{pmatrix} \frac{V_a(s)}{s} \\ \frac{T_m(s)}{s} \end{pmatrix} = \begin{pmatrix} K & (R_a + L_a s) \\ -Js & K \end{pmatrix} \cdot \begin{pmatrix} \omega(s) \\ i_a(s) \end{pmatrix} - - - - - - - - - - - - - - - - 3.12$$

To determine the variation of speed and armature current, after the instant of application of the armature voltage,

The determinant of the equation (3.12),

$$\Delta = K^2 + Js(R_a + L_a s) = JL_a s^2 + JR_a s + K^2 - - - - - - - - - - - - - 3.13$$

The inverse matrix,

$$\begin{pmatrix} \omega(s) \\ i_a(s) \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} K & -(R_a + L_a s) \\ Js & K \end{pmatrix} \cdot \begin{pmatrix} \frac{V_a(s)}{s} \\ \frac{T_m(s)}{s} \end{pmatrix} - - - - - - - - - 3.14$$

The equation for speed $\omega(s)$ can be written as,

$$\omega(s) = \frac{[(KV_a(s) - R_a T_m(s)) - (L_a T_m)s]}{s(JL_a s^2 + JR_a s + K^2)}$$

$$= \frac{\frac{KV_a(s) - R_a T_m(s)}{JL_a} - \frac{T_m(s)}{J}s}{s\left[s^2 + \frac{R_a}{L_a}s + \frac{K^2}{JL_a}\right]}$$

$$= \frac{\frac{KV_a(s)-R_aT_m(s)}{JL_a} - \frac{T_m(s)}{J}s}{s(s+\alpha_1)(s+\alpha_2)}$$

$$= \frac{A-Bs}{s(s+\alpha_1)(s+\alpha_2)} \quad -------------- \ 3.15$$

Where, $-\alpha_1$ and $-\alpha_2$ are the roots of $s^2 + \frac{R_a}{L_a}s + \frac{K^2}{JL_a} = 0$

And $\quad A = \frac{KV_a(s)-R_aT_m(s)}{JL_a}$, $B = \frac{T_m(s)}{J}$

Using partial fraction on equation (3.15)

$$\frac{A-Bs}{s(s+\alpha_1)(s+\alpha_2)} \equiv \frac{K_0}{s} + \frac{K_1}{(s+\alpha_1)} + \frac{K_2}{(s+\alpha_2)} \quad ------------ \ 3.16$$

Multiplying both sides by s and put s = 0

$$K_0 = s\omega(s)\Big|s = 0 \quad = \frac{A}{\alpha_1\alpha_2} \quad ----------------3.17$$

Multiplying both sides by$(s + \alpha_1)$ and put $s = -\alpha_1$

$$K_1 = (s + \alpha_1)\omega(s)\Big|s = -\alpha_1 \quad = -\frac{A-B\alpha_1}{\alpha_1(\alpha_2-\alpha_1)} \quad ----------- \ 3.17$$

Multiplying both sides of equation (2.16) by$(s + \alpha_2)$ and put $s = -\alpha_2$

$$K_2 = (s + \alpha_2)\omega(s)\Big|s = -\alpha_2 \quad = -\frac{A-B\alpha_2}{\alpha_2(\alpha_1-\alpha_2)} \quad ----------- \ 3.18$$

Therefore, equation (3.16) becomes,

$$\omega(s) = \frac{K_0}{s} + \frac{K_1}{(s+\alpha_1)} + \frac{K_2}{(s+\alpha_2)} \quad ------------------ \ 3.19$$

Taking the inverse transformation

- $\omega(t) = K_0 + K_1 e^{-\alpha_1 t} + K_2 e^{-\alpha_2 t}$ $------------$ 3.20

This is the equation of the machine's speed with time.

From equation (3.9), expression for $i_a(t)$ can be obtain

$$Ki_a(t) = T_m(t) + J\rho\omega(t)$$

$$i_a(t) = \frac{T_m(t)}{K} + \frac{J\rho\omega(t)}{K}$$

Substitute for $\omega(t)$ from equation (3.20)

$$i_a(t) = \frac{T_m(t)}{K} + \frac{J(-\alpha_1 K_1 e^{-\alpha_1 t} - \alpha_2 K_2 e^{-\alpha_2 t})}{K}$$

- $i_a(t) = C_0 + C_1 e^{-\alpha_1 t} + C_2 e^{-\alpha_2 t}$ $--------------$ 3.21

Where, $C_0 = \frac{T_m(t)}{K}$, $C_1 = -\frac{JK_1\alpha_1}{K}$, $C_2 = -\frac{JK_2\alpha_2}{K}$

This is the equation of the armature current with time.

Also from equation (3.15) the expression for $T(t)$ can be obtain

$$T(t) = T_m(t) + J\rho\omega(t)$$

$$= T_m(t) + J(-\alpha_1 K_1 e^{-\alpha_1 t} - \alpha_2 K_2 e^{-\alpha_2 t})$$

$T(t) = D_0 + D_1 e^{-\alpha_1 t} + D_2 e^{-\alpha_2 t}$ $----------------$ 3.22

Where, $D_0 = T_m(t)$, $D_1 = -JK_1\alpha_1$, $D_2 = -JK_2\alpha_2$

But at t = 0, $T_m(t) = 0$, therefore,

- $T(t) = D_1 e^{-\alpha_1 t} + D_2 e^{-\alpha_2 t}$ $----------------$ 3.23

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This is the equation of the torque with time [5].

3.1.3 TRANSIENT MODELLING OF D.C. SHUNT GENERATOR WITH INTERPOLES

The presence of interpoles, the effective armature-current inductance L_a, increases very small. The effect of interpoles can be introduced by adding a q-axis field winding in series with the armature, that is, the interpoles winding is connected in series with the armature winding so that magnetizing forces of both windings oppose each other. The behaviour can be expressed as in fig.3.3 and the equations that follow:
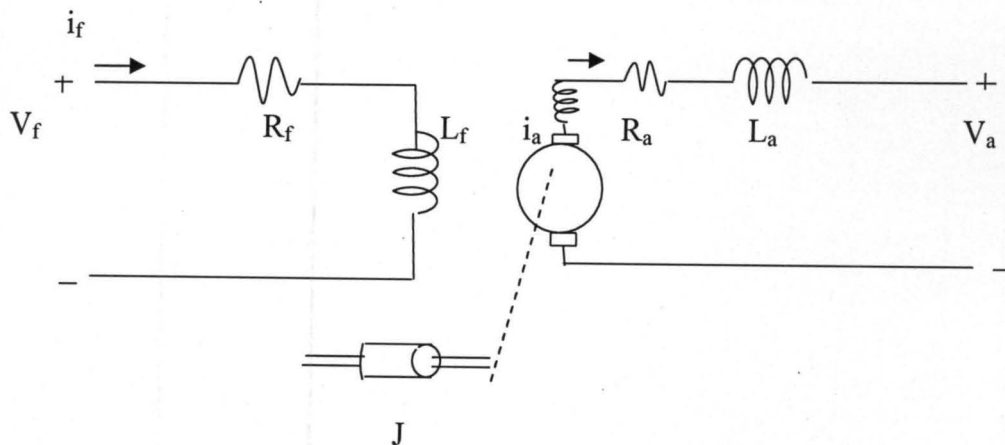
Fig. 3.3 separately excited generator with interpoles winding connected in series with armature.

Field: $V_f(t) = (R_f + L_f\rho)i_f(t) + L_{fs}\rho i_a(t) - - - - - - - - - - - - - 3.24$

Armature: $V_a(t) = (R_a + L_a\rho)i_a(t) + L_{fs}\rho i_f(t) + G\omega(i_f(t) + xi_a(t)) - -3.25$

Torque: $T(t) = T_m(t) + J\rho\omega(t) - - - - - - - - - - - - - - - - - -3.26$

The machine torque is related to the armature current and field current as follows:- $T(t) = G\left(i_f(t) + xi_a(t)\right)i_a(t) - - - - - - - - - - - - - - - - -3.27$

23

Where L_{fs} = mutual inductance between main pole and interpoles (series) field winding.

x= turn ratio N_s/N_f of interpole to main field.

Writing equations (3.23) and (3.24) in matrix form.

$$\begin{bmatrix} V_f(t) \\ V_a(t) \end{bmatrix} = \begin{bmatrix} (R_f + L_f\rho) & L_{fs}\rho \\ (L_{fs}\rho + G\omega) & (R_a + L_a\rho + G\omega x) \end{bmatrix} \cdot \begin{bmatrix} i_f(t) \\ i_a(t) \end{bmatrix} - - - - - - - 3.28$$

In view of the dominance of the armature current, the term $L_{fs}\rho i_f(t)$ is omitted, and for simplicity the generator speed $\omega(t)$ is assumed constant, hence $G\omega(t) = Q$ constant ----------- ---- 3.29

The transients are superposed on any current that exists at the instant of the short circuit. For main field winding it is i$_f$(t) while armature on no- load i$_L$ = 0. Therefore the transient component current i$_f$(t) and i$_a$(t) for the closed armature field circuit. An armature terminal at short circuit reduces the field as well as the armature voltage to zero.

$$0 = (R_f + L_f\rho)i_f(t) + L_{fs}\rho i_a(t) = R_f(1 + \tau_f\rho)i_f(t) + L_{fs}\rho i_a(t) - - -$$

3.30 $-E_a(t) = (R_a + L_a\rho)i_a(t) + Q(i_f(t) + xi_a(t)) = R_A(1 + \tau_a\rho)i_a(t) +$

$Qi_f(t) - - - - - 3.31$

Where E$_a$ = generated e.m.f.

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$$R_A = (R_a + Qx)$$

$$\tau_{f=}\frac{L_f}{R_f}, \text{ field time- constant}$$

$$\tau_a = \frac{L_a}{R_A}, \text{ armature time- constant}$$

-- 3.32

writing equations 3.27 and 3.30 in matrix form.

$$\begin{bmatrix} 0 \\ -E_a \end{bmatrix} = \begin{bmatrix} R_f(1+\tau_f\rho) & L_{fs}\rho \\ Q & R_A(1+\tau_a\rho) \end{bmatrix} \cdot \begin{bmatrix} i_f(t) \\ i_a(t) \end{bmatrix} - - - - - - - - - 3.33$$

To determine the variation of speed and armature current after the instant of application of the armature voltage.

The determinant, Δ of equation 3.33

$$\Delta = R_A R_f (1+\tau_f\rho)(1+\tau_a\rho) - QL_{fs}\rho - - - - -3.34$$

Thus the inverse matrix

$$\begin{bmatrix} i_f(t) \\ i_a(t) \end{bmatrix} = 1/\Delta \begin{bmatrix} R_A(1+\tau_a\rho) & -L_{fs}\rho \\ -Q & R_f(1+\tau_a\rho) \end{bmatrix} \begin{bmatrix} 0 \\ -E_a \end{bmatrix}$$

The expression for $i_a(t)$ can be written as

$$i_a(t) = E_a R_f(1+\tau_a\rho)/R_f(1+\tau_f\rho)(1+\tau_a\rho) - QL_{fs}\rho$$

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$$i_a(t) = E_a/R_A \, (1 + \tau_a\rho)/(1 + \tau_f\rho)(1 + \tau_a\rho) - (QL_{fs}/R_A R_f)\rho - - - 3.35$$

The denominator is

$$1 + (\tau_a + \tau_f)\rho + \tau_f\tau_a\rho^2 - (QL_{fs}/R_A R_f)\rho = 1 + (\tau_a + \tau_f - C)\rho$$

$$+ \tau_f\tau_a\rho^2 - -3.36$$

Where $C = QL_{fs}/R_A R_f$

If $D = C/\tau_f = QL_{fs}/R_A L_f,$ since $L_f = \tau_f R_f$

And $T_f = \tau_f(1 - D)$, $T_a = \tau_a/(1 - D)$

$$Then \; (1 + T_f\rho)(1 + T_a\rho) \equiv 1 + (\tau_a + \tau_f - C)\rho + \tau_f\tau_a\rho^2 - - - - 3.37$$

Since τ_a is small compared with τ_f

Therefore the transforms of the armature transient current for a sudden armature terminal short circuit is

$$I_a(t) = -\frac{E_a}{R_A} \cdot \frac{(1 + \tau_f\rho)}{(1 + T_f\rho)(1 + T_a\rho)}$$

$$I_a(s) = -\frac{E_a}{R_A} \cdot \frac{(1 + \tau_f s)}{s(1 + sT_f)(1 + sT_a)} - - - - - - - - - - - - - 3.38$$

Using partial fraction

$$\frac{(1 + \tau_f s)}{s(1 + sT_f)(1 + sT_a)} \equiv \frac{A_0}{S} + \frac{A_1}{1 + ST_f} + \frac{A_2}{1 + ST_a}$$

Multiplying both sides by s and put s =0

$$A_0 = (1 + sT_a)(1 + sT_f)I_a(s)\Big|s = 0 \qquad = 1 --------- 3.39$$

Multiplying both sides by $(1 + sT_f)$ and put $s = -\dfrac{1}{T_f}$

$$A_1 = s(1 + sT_f)I_a(s)\Big|s = -\frac{1}{T_f} \qquad = -T_f\left(\frac{T_f - \tau_f}{T_f - T_a}\right) ------- 3.40$$

Multiplying both sides by $(1 + sT_a)$ and put $s = -\dfrac{1}{T_a}$

$$A_2 = s(1 + sT_a)I_a(s)\Big|s = -\frac{1}{T_a} \qquad = -T_a\left(\frac{T_a - \tau_f}{T_a - T_f}\right) ------- 3.41$$

Therefore,

$$I_a(s) = -\frac{E_a}{R_A}\left(\frac{A_0}{S} + \frac{A_1}{1 + ST_f} + \frac{A_2}{1 + ST_a}\right)$$

$$i_a(s) = -\frac{E_a}{R_A}\left\{\frac{1}{s} - \frac{T_f - \tau_f}{T_f - T_a}\cdot\frac{1}{s + 1/T_f} - \frac{T_a - \tau_f}{T_a - T_f}\cdot\frac{1}{s + T_a}\right\}$$

The inverse transformation gives for the transient armature current is

$$i_a(t) = B_0 + B_1 e^{-t/T_f} + B_2 e^{-t/T_a} -------------- 3.42$$

Where, $B_0 = \dfrac{E_a}{R_A}, \quad B_1 = \dfrac{E_a}{R_A}\left(\dfrac{T_f - \tau_f}{T_f - T_a}\right), \quad B_2 = \dfrac{E_a}{R_A}\left(\dfrac{T_a - \tau_f}{T_a - T_f}\right),$

If T_a can be neglected in comparison with T_f or τ_f, then the armature current reduces to

$$i_a(t) = -\frac{E_a}{R_a}\left(1 + \frac{1}{1-D}\left\{De^{-t/T_f} - e^{-t/T_f}\right\}\right) - - - - - - - - - -3.43$$

Where $T_f = \tau_f(1 - D)$

The quality $R_a(1 - D)$ is the transient resistance. The armature short circuit current rises very rapidly at first (or instantaneously if $i_a = 0$) attaining a peak and then sinking more slowly to a steady short circuit value, $\frac{E_a}{R_a}$ [2].

From equation 3.26

$$T(t) = \left(Gi_f(t) + xi_a(t)\right)i_a(t) = \left(K + xi_a(t)\right)i_a(t)$$

Where $K = Gi_f(t)$

Substituting for $i_a(t)$ from equation 3.42

$$T(t) = -\left[K - x\left(\frac{E_a}{R_a}\left(1 + \frac{1}{1-D}\left\{De^{-t/T_f} - e^{-t/T_f}\right\}\right)\right)\right]\frac{E_a}{R_a}\left(1 + \frac{1}{1-D}\left\{De^{-t/T_f} - \right.\right.$$

$$\left. e^{-t/T_f}\right\}\right) -3.44$$

And from equations (3.26) and (3.27)

$$\left(K + xi_a(t)\right)i_a(t) = T_m(t) + J\rho\omega(t)$$

$$\rho\omega(t) = \frac{\left(K + xi_a(t)\right)i_a(t)}{J} - \frac{T_m(t)}{J}$$

$$\omega(t) = \frac{1}{J} \int \{K\, i_a(t) + x\, i_a(t)^2 - T_m(t)\}dt \ldots\ldots\ldots\ldots\ldots\ 3.45$$

3.2.0 SIMULATION OF D.C SHUNT GENERATOR USING MATLAB PROGRAM

3.2.1 SIMULATION OF D.C SHUNT GENERATOR WITHOUT INTERPOLES USING MATLAB PROGRAM

```
% M-file program of transient analysis
>> % of D.C. shunt generator without interpoles
>> t = 0: 0.02: 0.5;
>> Ra = 0.5;
>> La = 0.05;
>> J = 10;
>> Va = 250;
>> Tm = 100;
>> K = 1.2;
>> alpha1 = -9.71;
>> alpha2 = -0.29;
>> % Graph of speed against time
>> A = ((K*Va)-(Ra*Tm))/(J*La);
>> B = Tm/J;
>> K0 = A/(alpha1*alpha2);
>> K1 = -(A-(B*alpha1))/(alpha1*(alpha2-alpha1));
>> K2 = -(A-(B*alpha2))/(alpha2*(alpha1-alpha2));
>> w = K0+(K1*exp(-alpha1*t))+(K2*exp(-alpha2*t));
>> subplot(3,1,1)
>> plot(t, w, 'k')
>> plot(t, w, 'k')
>> % Graph of current against time
>> C0 = Tm/K;
>> C1 = -(J*alpha1*K1)/K;
>> C2 = -(J*alpha2*K2)/K;
>> ia = C0+(C1*exp(-alpha1*t))+(C2*exp(-alpha2*t));
>> subplot(3,1,2)
>> plot(t, ia, 'k')
>> plot(t, ia, 'k')
>> %Graph of Torque against time
>> D0 = Tm;
>> D1 = -J*alpha1*K1;
>> D2 = -J*alpha2*K2;
>> T = D0+(D1*exp(-alpha1*t))+(D2*exp(-alpha2*t));
>> subplot(3,1,3);
>> plot(t, T, 'k')
>> plot(t, T, 'k')
```

3.2.2 SIMULATION OF D.C SHUNT GENERATOR WITH INTERPOLES USING

MATLAB PROGRAM

```
% M-file program of transient analysis
% of D.C.shunt generator with interpoles
t = 0: 0.001: 0.2;
If = 6;
Ea = 240;
Rf = 40;
Lf = 10;
Ra = 0.05;
La = 0.0001;
Lfs = 0.0187;
x = 0.0019;
Q = 40;
J = 12;
K = 3.81;
Tm = 100;
RA = (Ra+(Q*x));
D = (Q*Lfs)/(RA*Lf);
tauf = Lf/Rf;
taua = La/RA;
Tf = tauf*(1-D);
Ta = taua/(1-D);
ia = (Ea/RA)*(1 + 1/(1 - D)*(D*exp(-t/Tf) - exp(-t/Ta)));
subplot(2,2,1)
plot(t, ia, 'r')
xlabel('time[s]')
ylabel('Armature current[amp]')
grid on
title('Graph of armature current against time[with interpole]')
H1 = 2*(Ea/RA)^2*(D/(1-D))*exp(-t/Tf);
H2 = 2*(Ea/RA)^2*(1/(1-D))*exp(-t/Ta);
H3 = 2*(Ea/(RA*(1-D)))^2*D*exp(-(t*(Tf+Ta))/(Tf*Ta));
H4 = ((D*Ea)/(RA*(1-D)))^2*exp(-(2*t)/(Tf));
H5 = (Ea/(RA*(1-D)))^2*exp(-(2*t)/Ta);
H6 = -((K*Ea)/RA)*(1 + 1/(1 - D)*(D*exp(-t/Tf) - exp(-t/Ta)));
T = H6+x*(((Ea/RA)^2)+H1-H2-H3+H4+H5);
subplot(2,2,2)
plot(t, T, 'b')
xlabel('Time[s]')
ylabel('Torque[Nm]')
grid on
title('Graph of Torque against Time[with interplot]')
P1 = ((Ea*K)/RA)*(t-((D*Tf)/(1-D))*exp(-t/Tf)+(Ta/(1-D))*exp(-t/Ta));
P2 = t-((2*Tf*D)/(1-D))*exp(-t/Tf)+((2*Ta)/(1-D))*exp(-t/Ta);
P3 = ((2*Tf*Ta*D)/((Tf+Ta)*(1-D)^2))*exp(-(t*(Tf+Ta))/(Ta*Tf));
P4      =       ((Tf/2)*((D/(1-D))^2))*exp(-(t*2)/Tf)+((Ta/2)*(1/(1-D))^2)*exp(-
(2*t)/Ta);
w = (1/J)*(-P1+(x*((Ea)/Ra)^2)*(P2 + P3 - P4) - (Tm*t));
subplot(2,2,3)
plot(t, w, 'g')
```

```
xlabel('Time[s]')
ylabel('speed[rad/s]')
grid on
title('Graph of speed against time[with interpole]')


EXP2
for i = 1:1:201
ira(i,1) = ia21(i)*1.0e+003;
ira(i,2) = ia21(i+201)*1.0e+003;
ira(i,3) = ia21(i+402)*1.0e+003;
for t = 1:1:201
Tr(t,1) = T21(t)*1.0e+004;
Tr(t,2) = T21(t+201)*1.0e+004;
Tr(t,3) = T21(t+402)*1.0e+004;
wr(t,1) = w21(t)*1.0e+003;
wr(t,2) = w21(t+201)*1.0e+003;
wr(t,3) = w21(t+402)*1.0e+003;
end
end
subplot(2,2,1)
ira(:,1), ira(:,2) ,ira(:,3)
plot(Time,ira(:,1),'k', Time,ira(:,2),'r' ,Time,ira(:,3),'b')
%plot(Time,irb,'k')
xlabel('Time[s]')
ylabel('Armature Current[A]')
grid on
title('Graph of varying armature resistance')
legend('Ra=0.03','Ra=0.05','Ra=0.07')
subplot(2,2,2)
Tr(:,1),Tr(:,2),Tr(:,3)
plot(Time,Tr(:,1),'k', Time,Tr(:,2),'r',Time,Tr(:,3),'b')
xlabel('Time[s]')
ylabel('Torque[N-m]')
grid on
legend('Ra=0.03','Ra=0.05','Ra=0.07')
subplot(2,2,3)
wr(:,1), wr(:,2), wr(:,3)
plot(Time,wr(:,1),'k',Time,wr(:,2),'r',Time,wr(:,3),'b')
xlabel('Time[s]')
ylabel('speed[m/s]')
grid on
legend('Ra=0.03','Ra=0.05','Ra=0.07')
subplot(2,2,4)
G = 0.4;
K = 3.81;
x = 0.0019;
i = 1:1:201;
ia1 = ia21(i)*1.0e+003;
ia2 = ia21(i+201)*1.0e+003;
ia3 = ia21(i+402)*1.0e+003;
w1 = w21(i)*1.0e+003;
w2 = w21(i+201)*1.0e+003;
w3 = w21(i+402)*1.0e+003;
E1 =[(K*w1)+(G*x*ia1)];
E2 =[(K*w2)+(G*x*ia2)];
```

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```
E3 = [(K*w3)+(G*x*ia3)];
plot(Time,E1,'k',Time,E2,'r',Time,E3,'b')
xlabel('Time[s]')
ylabel('Generated Voltage[V]')
grid on
legend('Ra=0.03','Ra=0.05','Ra=0.07')

EXP3
for i = 1:1:201
ira(i,1) = ia51(i)*1.0e+003;
ira(i,2) = ia51(i+201)*1.0e+003;
ira(i,3) = ia51(i+402)*1.0e+003;
for t = 1:1:201
Tr(t,1) = T51(t)*1.0e+004;
Tr(t,2) = T51(t+201)*1.0e+004;
Tr(t,3) = T51(t+402)*1.0e+004;
wr(t,1) = w51(t)*1.0e+003;
wr(t,2) = w51(t+201)*1.0e+003;
wr(t,3) = w51(t+402)*1.0e+003;
end
end
subplot(2,2,1)
ira(:,1), ira(:,2) ,ira(:,3)
plot(Time,ira(:,1),'k', Time,ira(:,2),'r' ,Time,ira(:,3),'b')
%plot(Time,irb,'k')
xlabel('Time[s]')
ylabel('Armature Current[A]')
grid on
title('Graph of varying armature inductance')
legend('La=0.00008','La=0.0001','La=0.0003')
subplot(2,2,2)
Tr(:,1),Tr(:,2),Tr(:,3)
plot(Time,Tr(:,1),'k', Time,Tr(:,2),'r',Time,Tr(:,3),'b')
xlabel('Time[s]')
ylabel('Torque[N-m]')
grid on
legend('La=0.00008','La=0.0001','La=0.0003')
subplot(2,2,3)
wr(:,1), wr(:,2), wr(:,3)
plot(Time,wr(:,1),'k',Time,wr(:,2),'r',Time,wr(:,3),'b')
xlabel('Time[s]')
ylabel('speed[m/s]')
grid on
legend('La=0.00008','La=0.0001','La=0.0003')
subplot(2,2,4)
G = 0.4;
K = 3.81;
x = 0.0019;
i = 1:1:201;
ia1 = ia51(i)*1.0e+003;
ia2 = ia51(i+201)*1.0e+003;
ia3 = ia51(i+402)*1.0e+003;
w1 = w51(i)*1.0e+003;
w2 = w51(i+201)*1.0e+003;
w3 = w51(i+402)*1.0e+003;
E1 = [(K*w1)+(G*x*ia1)];
E2 = [(K*w2)+(G*x*ia2)];
```

```
E3 =[(K*w3)+(G*x*ia3)];
plot(Time,E1,'k',Time,E2,'r',Time,E3,'b')
xlabel('Time[s]')
ylabel('Generated Voltage[V]')
grid on
legend('La=0.00008','La=0.0001','La=0.0003')
```

CHAPTER FOUR

4.0 RESULT OF SIMULATION

This chapter discussed the result obtained from the simulation, by comparing the graph of one without interpoles and one with interpoles

4.1.0 GRAPHS OF SIMULATION WITHOUT INTERPOLES

Graph of speed against time [without interpole]

Graph of current against time [without interpole]

Graph of Torque against time [without interpole]
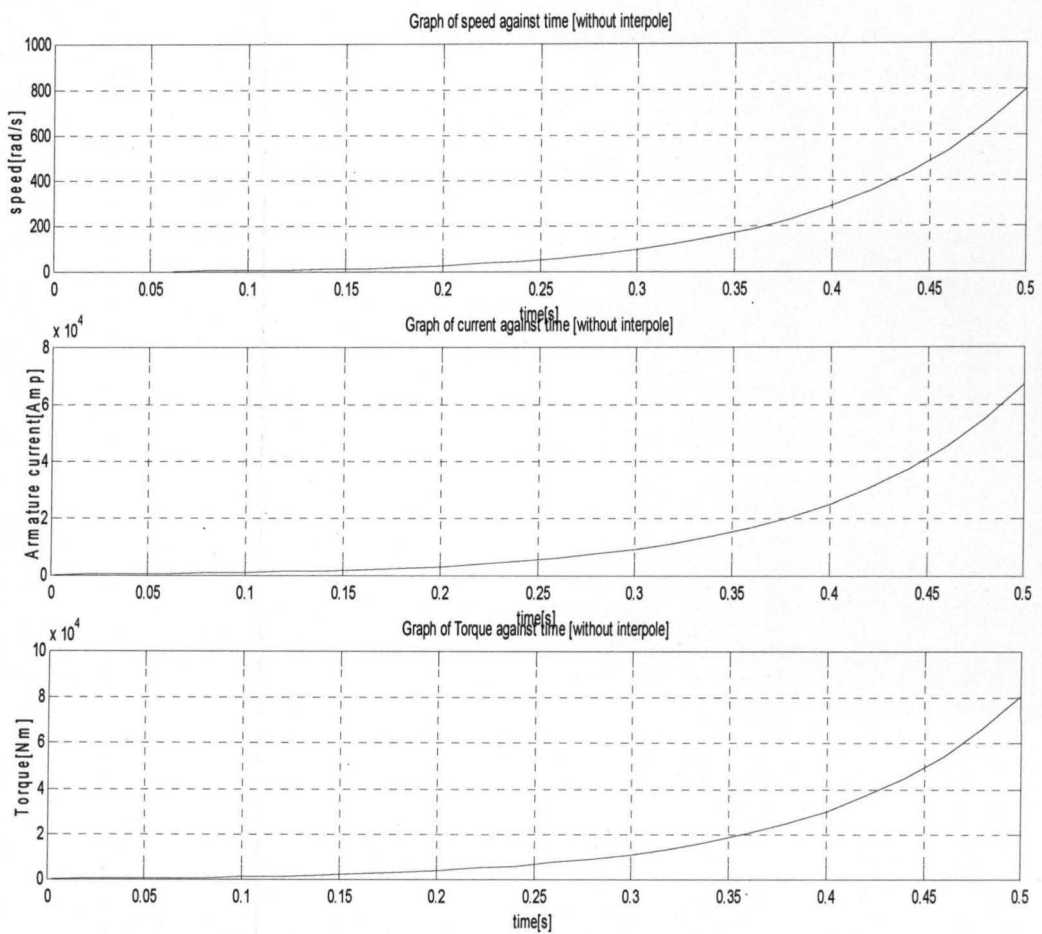
4.1.1 GRAPHS OF SIMULATION WITH INTERPOLES
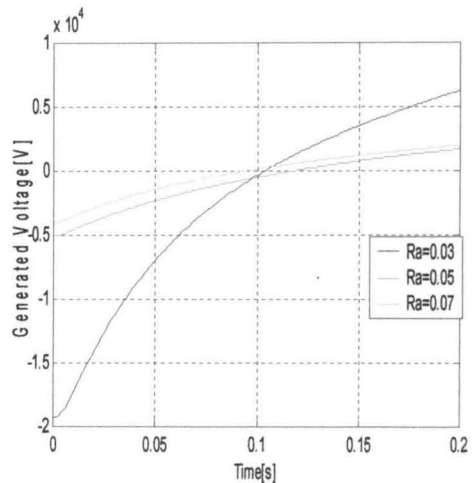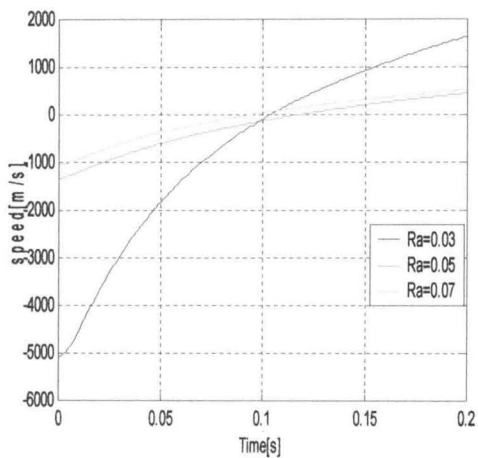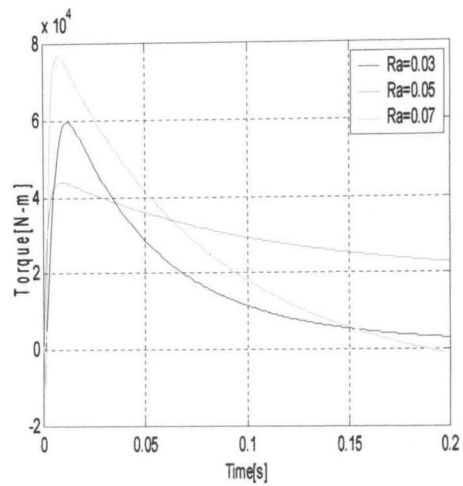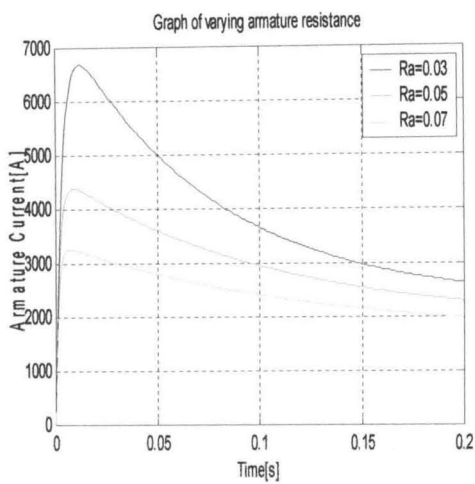
Graph of varying armature inductance
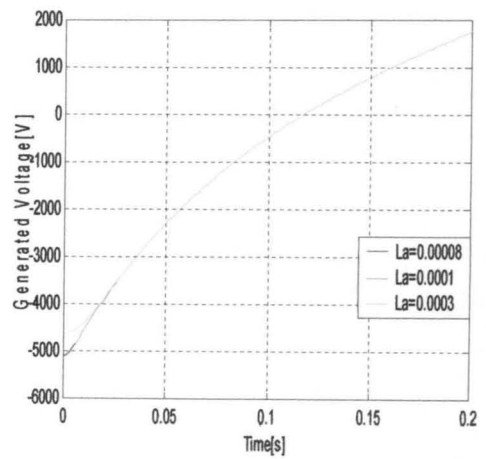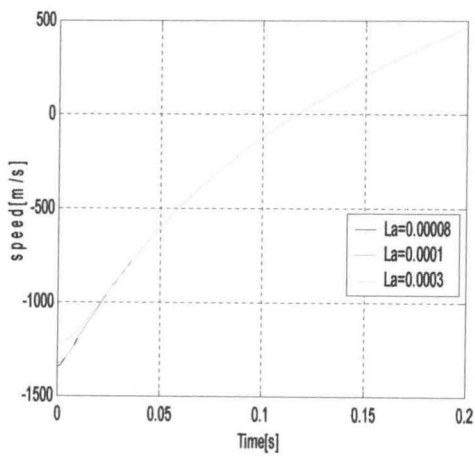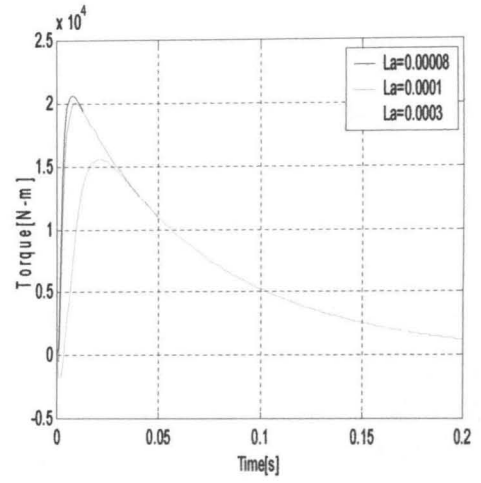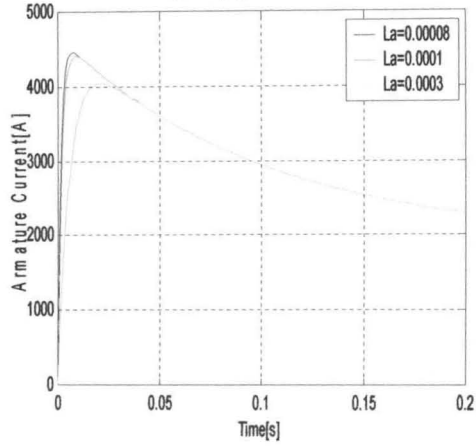
4.2 DISCUSSION OF RESULT

In this work, the transient behaviour of the D.C. shunt generator with interpoles and one without interpoles is simulated using MATLAB. It can be observed that the presence of interpoles has a marked effect on the transient behaviour of a D.C. generator

It can also be seen that a variation in the system parameters i.e. R_a and L_a has a significant effect on $i_a(t)$, $T(t)$, $\omega(t)$, and E_a.

With this analysis and simulation, the design engineer will know the transient behaviour of a D.C shunt generator and the effect of varying its system parameter with time when designing a D.C shunt generator. Thus, this will help in the optimization of a D.C generator in industries.

Therefore, D.C shunt generator with interpoles has mark significant effect and has improvement performance on D.C generator.

CHAPTER FIVE

5.0 CONCLUSION

5.1 SUMMARY

In this work, the transient behaviour of a D.C shunt generator with interpoles and one without interpoles is simulated using MATLAB. It can be observed that the presence of interpoles has marked effect on the transient behaviour of D.C shunt generator.

From the above analysis and the graph of simulations, it can be observed that incorporating interpoles on D.C generator will improve its performance, thus help to induce an e.m.f (under commutation), which helps the reversal of current. Also interpoles neutralize the X-magnetizing effect of armature reaction.

5.2 FUTURE WORK

There are number of topics for future work and development related to D.C generator/motor which include

1. Obtaining of transient behaviour of D.C generator by varying its field inductance, field resistance, and moment of inertia, e.t.c

2. Using Matlab/MathCAD to determine the desired torque of D.C motor

5.3 RECOMMENDATION

The knowledge and use of MATLAB/ SIMULINK and other technical software packages should be encouraged among students. This will prepare them for real- life situations in the industrial environment.

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APPENDIX

Results of simulation without interpoles

```
w =0.0000     0.3284    0.9504    1.9304    3.3463    5.2928    7.8852
   11.2631    15.5961   21.0905   27.9964   36.6178   47.3236   60.5622
   76.8778    96.9311  121.5249  151.6336  188.4408  233.3837  288.2075
  355.0315   436.4292  535.5258  656.1165  802.8095


ia =1.0e+004  *
    0.0167    0.0277    0.0412    0.0577    0.0777    0.1020    0.1316
    0.1677    0.2115    0.2648    0.3295    0.4082    0.5038    0.6199
    0.7610    0.9324    1.1405    1.3934    1.7005    2.0735    2.5265
    3.0766    3.7448    4.5562    5.5416    6.7382


T =1.0e+004  *
    0.0200    0.0333    0.0495    0.0692    0.0932    0.1224    0.1580
    0.2012    0.2538    0.3177    0.3954    0.4898    0.6045    0.7439
    0.9132    1.1188    1.3686    1.6721    2.0406    2.4882    3.0318
    3.6920    4.4937    5.4674    6.6499    8.0859
```

Results of Simulation with Interpoles

(Varying Armature Resistance, R_a)

```
Time= [0       0.0010    0.0020    0.0030    0.0040    0.0050    0.0060...
    0.0070    0.0080    0.0090    0.0100    0.0110    0.0120    0.0130...
    0.0140    0.0150    0.0160    0.0170    0.0180    0.0190    0.0200...
    0.0210    0.0220    0.0230    0.0240    0.0250    0.0260    0.0270...
    0.0280    0.0290    0.0300    0.0310    0.0320    0.0330    0.0340...
    0.0350    0.0360    0.0370    0.0380    0.0390    0.0400    0.0410...
    0.0420    0.0430    0.0440    0.0450    0.0460    0.0470    0.0480...
    0.0490    0.0500    0.0510    0.0520    0.0530    0.0540    0.0550...
    0.0560    0.0570    0.0580    0.0590    0.0600    0.0610    0.0620...
    0.0630    0.0640    0.0650    0.0660    0.0670    0.0680    0.0690...
    0.0700    0.0710    0.0720    0.0730    0.0740    0.0750    0.0760...
    0.0770    0.0780    0.0790    0.0800    0.0810    0.0820    0.0830...
    0.0840    0.0850    0.0860    0.0870    0.0880    0.0890    0.0900...
    0.0910    0.0920    0.0930    0.0940    0.0950    0.0960    0.0970...
    0.0980    0.0990    0.1000    0.1010    0.1020    0.1030    0.1040...
    0.1050    0.1060    0.1070    0.1080    0.1090    0.1100    0.1110...
    0.1120    0.1130    0.1140    0.1150    0.1160    0.1170    0.1180...
    0.1190    0.1200    0.1210    0.1220    0.1230    0.1240    0.1250...
    0.1260    0.1270    0.1280    0.1290    0.1300    0.1310    0.1320...
    0.1330    0.1340    0.1350    0.1360    0.1370    0.1380    0.1390...
    0.1400    0.1410    0.1420    0.1430    0.1440    0.1450    0.1460...
    0.1470    0.1480    0.1490    0.1500    0.1510    0.1520    0.1530...
    0.1540    0.1550    0.1560    0.1570    0.1580    0.1590    0.1600...
    0.1610    0.1620    0.1630    0.1640    0.1650    0.1660    0.1670...
    0.1680    0.1690    0.1700    0.1710    0.1720    0.1730    0.1740...
```

```
0.1750     0.1760     0.1770     0.1780     0.1790     0.1800     0.1810...
0.1820     0.1830     0.1840     0.1850     0.1860     0.1870     0.1880...
0.1890     0.1900     0.1910     0.1920     0.1930     0.1940     0.1950...
0.1960     0.1970     0.1980     0.1990     0.2000];
ia21 = [0      1.9884     3.4252     4.4586     5.1968     5.7193     6.0841...
6.3337     6.4992     6.6034     6.6629     6.6900     6.6935     6.6801...
6.6544     6.6199     6.5793     6.5343     6.4864     6.4366     6.3855...
6.3337     6.2815     6.2293     6.1773     6.1256     6.0743     6.0234...
5.9731     5.9234     5.8742     5.8256     5.7777     5.7304     5.6836...
5.6375     5.5920     5.5471     5.5029     5.4592     5.4160     5.3735...
5.3315     5.2901     5.2493     5.2090     5.1693     5.1300     5.0914...
5.0532     5.0156     4.9784     4.9418     4.9056     4.8700     4.8348...
4.8001     4.7659     4.7321     4.6988     4.6659     4.6335     4.6015...
4.5700     4.5389     4.5082     4.4779     4.4480     4.4185     4.3894...
4.3608     4.3325     4.3045     4.2770     4.2498     4.2230     4.1966...
4.1705     4.1448     4.1194     4.0943     4.0696     4.0453     4.0212...
3.9975     3.9741     3.9510     3.9283     3.9058     3.8836     3.8618...
3.8402     3.8189     3.7980     3.7773     3.7568     3.7367     3.7168...
3.6972     3.6779     3.6588     3.6400     3.6214     3.6031     3.5850...
3.5672     3.5496     3.5322     3.5151     3.4982     3.4816     3.4651...
3.4489     3.4329     3.4172     3.4016     3.3862     3.3711     3.3561...
3.3414     3.3269     3.3125     3.2984     3.2844     3.2706     3.2571...
3.2437     3.2304     3.2174     3.2045     3.1918     3.1793     3.1670...
3.1548     3.1428     3.1309     3.1192     3.1077     3.0963     3.0850...
3.0740     3.0630     3.0522     3.0416     3.0311     3.0208     3.0105...
3.0005     2.9905     2.9807     2.9711     2.9615     2.9521     2.9428...
2.9337     2.9246     2.9157     2.9069     2.8982     2.8897     2.8812...
2.8729     2.8647     2.8566     2.8486     2.8407     2.8329     2.8252...
2.8177     2.8102     2.8028     2.7955     2.7884     2.7813     2.7743...
2.7674     2.7606     2.7539     2.7473     2.7408     2.7344     2.7280...
2.7218     2.7156     2.7095     2.7035     2.6976     2.6917     2.6859...
2.6802     2.6746     2.6691     2.6636     2.6582     2.6529     2.6477...
2.6425     2.6374     2.6323     2.6274     2.6225...


0      1.8510     2.9497     3.5976     3.9754     4.1915     4.3107...
4.3721     4.3988     4.4048     4.3986     4.3851     4.3674     4.3472...
4.3256     4.3033     4.2807     4.2579     4.2352     4.2125     4.1900...
4.1677     4.1456     4.1237     4.1019     4.0804     4.0591     4.0380...
4.0171     3.9964     3.9759     3.9557     3.9356     3.9157     3.8960...
3.8765     3.8572     3.8380     3.8191     3.8003     3.7818     3.7634...
3.7452     3.7272     3.7093     3.6916     3.6741     3.6568     3.6396...
3.6226     3.6058     3.5892     3.5727     3.5563     3.5401     3.5241...
3.5083     3.4925     3.4770     3.4616     3.4463     3.4312     3.4163...
3.4015     3.3868     3.3723     3.3579     3.3437     3.3296     3.3156...
3.3018     3.2881     3.2746     3.2612     3.2479     3.2347     3.2217...
3.2088     3.1960     3.1834     3.1709     3.1585     3.1462     3.1340...
3.1220     3.1100     3.0982     3.0866     3.0750     3.0635     3.0522...
3.0409     3.0298     3.0188     3.0079     2.9971     2.9864     2.9758...
2.9653     2.9549     2.9446     2.9344     2.9243     2.9143     2.9044...
2.8947     2.8850     2.8754     2.8658     2.8564     2.8471     2.8379...
2.8287     2.8197     2.8107     2.8019     2.7931     2.7844     2.7757...
2.7672     2.7588     2.7504     2.7421     2.7339     2.7258     2.7178...
2.7098     2.7019     2.6941     2.6864     2.6787     2.6711     2.6636...
2.6562     2.6488     2.6415     2.6343     2.6272     2.6201     2.6131...
2.6062     2.5993     2.5925     2.5857     2.5791     2.5725     2.5659...
2.5595     2.5530     2.5467     2.5404     2.5342     2.5280     2.5219...
2.5159     2.5099     2.5039     2.4981     2.4923     2.4865     2.4808...
```

```
2.4752      2.4696      2.4640      2.4586      2.4531      2.4478      2.4425...
2.4372      2.4320      2.4268      2.4217      2.4166      2.4116      2.4067...
2.4017      2.3969      2.3920      2.3873      2.3825      2.3779      2.3732...
2.3686      2.3641      2.3596      2.3551      2.3507      2.3464      2.3420...
2.3378      2.3335      2.3293      2.3252      2.3210      2.3170      2.3129...
2.3089      2.3050      2.3010      2.2972      2.2933...
0.0000      1.7028      2.5312      2.9306      3.1197      3.2055      3.2408...
3.2513      3.2498      3.2423      3.2321      3.2205      3.2082      3.1958...
3.1833      3.1708      3.1584      3.1460      3.1337      3.1216      3.1095...
3.0975      3.0857      3.0739      3.0622      3.0506      3.0391      3.0277...
3.0164      3.0052      2.9941      2.9831      2.9721      2.9613      2.9505...
2.9398      2.9292      2.9187      2.9083      2.8980      2.8878      2.8776...
2.8675      2.8575      2.8476      2.8378      2.8280      2.8183      2.8088...
2.7992      2.7898      2.7804      2.7712      2.7619      2.7528      2.7438...
2.7348      2.7259      2.7170      2.7083      2.6996      2.6909      2.6824...
2.6739      2.6655      2.6571      2.6489      2.6406      2.6325      2.6244...
2.6164      2.6085      2.6006      2.5928      2.5850      2.5773      2.5697...
2.5621      2.5546      2.5472      2.5398      2.5325      2.5253      2.5181...
2.5109      2.5038      2.4968      2.4898      2.4829      2.4761      2.4693...
2.4625      2.4558      2.4492      2.4426      2.4361      2.4296      2.4232...
2.4168      2.4105      2.4043      2.3981      2.3919      2.3858      2.3797...
2.3737      2.3678      2.3618      2.3560      2.3502      2.3444      2.3387...
2.3330      2.3274      2.3218      2.3162      2.3107      2.3053      2.2999...
2.2945      2.2892      2.2839      2.2787      2.2735      2.2684      2.2633...
2.2582      2.2532      2.2482      2.2433      2.2384      2.2335      2.2287...
2.2239      2.2192      2.2145      2.2098      2.2052      2.2006      2.1961...
2.1916      2.1871      2.1827      2.1783      2.1739      2.1696      2.1653...
2.1610      2.1568      2.1526      2.1484      2.1443      2.1402      2.1362...
2.1322      2.1282      2.1242      2.1203      2.1164      2.1125      2.1087...
2.1049      2.1011      2.0974      2.0937      2.0900      2.0864      2.0828...
2.0792      2.0756      2.0721      2.0686      2.0651      2.0617      2.0583...
2.0549      2.0515      2.0482      2.0449      2.0416      2.0384      2.0351...
2.0320      2.0288      2.0256      2.0225      2.0194      2.0164      2.0133...
2.0103      2.0073      2.0043      2.0014      1.9985      1.9956      1.9927...
1.9898      1.9870      1.9842      1.9814      1.9787];
```

```
T21= [-0.0000     -0.0064      0.9241      2.0783      3.1513      4.0359      4.7150...

5.2088      5.5493      5.7689      5.8963      5.9547      5.9624      5.9333...
5.8780      5.8042      5.7178      5.6230      5.5227      5.4193      5.3142...
5.2088      5.1037      4.9995      4.8967      4.7955      4.6961      4.5986...
4.5031      4.4096      4.3181      4.2287      4.1412      4.0558      3.9723...
3.8906      3.8109      3.7330      3.6569      3.5825      3.5098      3.4389...
3.3695      3.3017      3.2355      3.1708      3.1075      3.0458      2.9854...
2.9264      2.8687      2.8123      2.7572      2.7034      2.6507      2.5993...
2.5490      2.4998      2.4517      2.4047      2.3588      2.3138      2.2699...
2.2270      2.1849      2.1439      2.1037      2.0644      2.0260      1.9884...
1.9516      1.9157      1.8805      1.8461      1.8124      1.7795      1.7473...
1.7157      1.6849      1.6547      1.6251      1.5962      1.5679      1.5403...
1.5132      1.4866      1.4607      1.4353      1.4104      1.3860      1.3622...
1.3389      1.3160      1.2936      1.2717      1.2503      1.2293      1.2087...
1.1885      1.1688      1.1495      1.1305      1.1120      1.0938      1.0760...
1.0586      1.0415      1.0248      1.0084      0.9923      0.9766      0.9611...
0.9460      0.9312      0.9167      0.9024      0.8885      0.8748      0.8614...
0.8483      0.8354      0.8228      0.8104      0.7982      0.7863      0.7747...
0.7632      0.7520      0.7410      0.7302      0.7196      0.7092      0.6990...
```

0.6890	0.6792	0.6696	0.6602	0.6509	0.6418	0.6329...
0.6242	0.6156	0.6072	0.5989	0.5908	0.5828	0.5750...
0.5674	0.5598	0.5524	0.5452	0.5381	0.5311	0.5242...
0.5175	0.5109	0.5044	0.4980	0.4917	0.4856	0.4795...
0.4736	0.4678	0.4621	0.4564	0.4509	0.4455	0.4402...
0.4349	0.4298	0.4247	0.4198	0.4149	0.4101	0.4054...
0.4008	0.3962	0.3917	0.3874	0.3830	0.3788	0.3746...
0.3705	0.3665	0.3625	0.3586	0.3548	0.3511	0.3474...
0.3437	0.3402	0.3366	0.3332	0.3298	0.3264	0.3232...
0.3199	0.3168	0.3136	0.3106	0.3075...		
0	1.8510	2.9497	3.5976	3.9754	4.1915	4.3107...
4.3721	4.3988	4.4048	4.3986	4.3851	4.3674	4.3472...
4.3256	4.3033	4.2807	4.2579	4.2352	4.2125	4.1900...
4.1677	4.1456	4.1237	4.1019	4.0804	4.0591	4.0380...
4.0171	3.9964	3.9759	3.9557	3.9356	3.9157	3.8960...
3.8765	3.8572	3.8380	3.8191	3.8003	3.7818	3.7634...
3.7452	3.7272	3.7093	3.6916	3.6741	3.6568	3.6396...
3.6226	3.6058	3.5892	3.5727	3.5563	3.5401	3.5241...
3.5083	3.4925	3.4770	3.4616	3.4463	3.4312	3.4163...
3.4015	3.3868	3.3723	3.3579	3.3437	3.3296	3.3156...
3.3018	3.2881	3.2746	3.2612	3.2479	3.2347	3.2217...
3.2088	3.1960	3.1834	3.1709	3.1585	3.1462	3.1340...
3.1220	3.1100	3.0982	3.0866	3.0750	3.0635	3.0522...
3.0409	3.0298	3.0188	3.0079	2.9971	2.9864	2.9758...
2.9653	2.9549	2.9446	2.9344	2.9243	2.9143	2.9044...
2.8947	2.8850	2.8754	2.8658	2.8564	2.8471	2.8379...
2.8287	2.8197	2.8107	2.8019	2.7931	2.7844	2.7757...
2.7672	2.7588	2.7504	2.7421	2.7339	2.7258	2.7178...
2.7098	2.7019	2.6941	2.6864	2.6787	2.6711	2.6636...
2.6562	2.6488	2.6415	2.6343	2.6272	2.6201	2.6131...
2.6062	2.5993	2.5925	2.5857	2.5791	2.5725	2.5659...
2.5595	2.5530	2.5467	2.5404	2.5342	2.5280	2.5219...
2.5159	2.5099	2.5039	2.4981	2.4923	2.4865	2.4808...
2.4752	2.4696	2.4640	2.4586	2.4531	2.4478	2.4425...
2.4372	2.4320	2.4268	2.4217	2.4166	2.4116	2.4067...
2.4017	2.3969	2.3920	2.3873	2.3825	2.3779	2.3732...
2.3686	2.3641	2.3596	2.3551	2.3507	2.3464	2.3420...
2.3378	2.3335	2.3293	2.3252	2.3210	2.3170	2.3129...
2.3089	2.3050	2.3010	2.2972	2.2933...		
0	-0.9787	2.5292	5.1526	6.6055	7.3103	7.6080...
7.6978	7.6844	7.6209	7.5337	7.4356	7.3330	7.2290...
7.1249	7.0217	6.9195	6.8186	6.7191	6.6208	6.5239...
6.4283	6.3341	6.2411	6.1494	6.0590	5.9698	5.8819...
5.7951	5.7096	5.6252	5.5419	5.4598	5.3789	5.2990...
5.2202	5.1425	5.0658	4.9902	4.9156	4.8420	4.7694...
4.6978	4.6271	4.5575	4.4887	4.4209	4.3539	4.2879...
4.2228	4.1585	4.0951	4.0326	3.9708	3.9099	3.8498...
3.7905	3.7320	3.6743	3.6173	3.5611	3.5057	3.4509...
3.3969	3.3436	3.2910	3.2391	3.1879	3.1373	3.0874...
3.0382	2.9896	2.9416	2.8943	2.8475	2.8014	2.7559...
2.7110	2.6666	2.6228	2.5796	2.5370	2.4949	2.4533...
2.4123	2.3718	2.3318	2.2924	2.2534	2.2149	2.1769...
2.1395	2.1024	2.0659	2.0298	1.9942	1.9590	1.9243...
1.8900	1.8561	1.8227	1.7897	1.7571	1.7249	1.6931...
1.6617	1.6307	1.6001	1.5699	1.5400	1.5106	1.4814...
1.4527	1.4243	1.3962	1.3685	1.3412	1.3142	1.2875...
1.2611	1.2351	1.2093	1.1839	1.1588	1.1340	1.1096...

```
    1.0854     1.0615     1.0379     1.0145     0.9915     0.9687     0.9462...
    0.9240     0.9021     0.8804     0.8590     0.8378     0.8169     0.7962...
    0.7758     0.7556     0.7357     0.7160     0.6965     0.6773     0.6583...
    0.6395     0.6209     0.6026     0.5845     0.5665     0.5488     0.5313...
    0.5140     0.4970     0.4801     0.4634     0.4469     0.4306     0.4144...
    0.3985     0.3828     0.3672     0.3518     0.3366     0.3216     0.3067...
    0.2920     0.2775     0.2631     0.2489     0.2349     0.2210     0.2073...
    0.1937     0.1803     0.1671     0.1540     0.1410     0.1282     0.1156...
    0.1030     0.0907     0.0784     0.0663     0.0543     0.0425     0.0308...
    0.0192     0.0078    -0.0035    -0.0147    -0.0258    -0.0368    -0.0476...
   -0.0583    -0.0689    -0.0793    -0.0897    -0.0999];

    w21= [-5.0590    -5.0565    -5.0422    -5.0121    -4.9671    -4.9096    -
4.8423...
   -4.7679    -4.6884    -4.6056    -4.5206    -4.4346    -4.3481    -4.2619...
   -4.1761    -4.0911    -4.0071    -3.9242    -3.8425    -3.7620    -3.6828...
   -3.6049    -3.5283    -3.4529    -3.3788    -3.3060    -3.2344    -3.1640...
   -3.0948    -3.0267    -2.9598    -2.8941    -2.8294    -2.7658    -2.7032...
   -2.6417    -2.5812    -2.5216    -2.4630    -2.4054    -2.3487    -2.2929...
   -2.2380    -2.1839    -2.1307    -2.0783    -2.0267    -1.9760    -1.9260...
   -1.8767    -1.8282    -1.7805    -1.7334    -1.6871    -1.6414    -1.5964...
   -1.5521    -1.5084    -1.4653    -1.4229    -1.3810    -1.3398    -1.2991...
   -1.2590    -1.2195    -1.1805    -1.1420    -1.1040    -1.0666    -1.0297...
   -0.9932    -0.9573    -0.9218    -0.8868    -0.8522    -0.8181    -0.7844...
   -0.7511    -0.7183    -0.6859    -0.6538    -0.6222    -0.5910    -0.5601...
   -0.5296    -0.4995    -0.4697    -0.4403    -0.4112    -0.3825    -0.3541...
   -0.3260    -0.2982    -0.2708    -0.2436    -0.2168    -0.1902    -0.1640...
   -0.1380    -0.1123    -0.0869    -0.0617    -0.0368    -0.0122     0.0122...
    0.0364     0.0602     0.0839     0.1073     0.1305     0.1535     0.1762...
    0.1987     0.2210     0.2431     0.2650     0.2867     0.3082     0.3294...
    0.3505     0.3715     0.3922     0.4127     0.4331     0.4533     0.4733...
    0.4931     0.5128     0.5323     0.5517     0.5708     0.5899     0.6088...
    0.6275     0.6461     0.6645     0.6829     0.7010     0.7190     0.7369...
    0.7547     0.7723     0.7898     0.8072     0.8245     0.8416     0.8586...
    0.8755     0.8923     0.9089     0.9255     0.9419     0.9583     0.9745...
    0.9906     1.0066     1.0226     1.0384     1.0541     1.0697     1.0853...
    1.1007     1.1160     1.1313     1.1465     1.1616     1.1766     1.1915...
    1.2063     1.2211     1.2357     1.2503     1.2648     1.2793     1.2936...
    1.3079     1.3221     1.3363     1.3503     1.3644     1.3783     1.3922...
    1.4060     1.4197     1.4334     1.4470     1.4605     1.4740     1.4875...
    1.5008     1.5141     1.5274     1.5406     1.5537     1.5668     1.5799...
    1.5928     1.6058     1.6186     1.6315     1.6443...
   -1.3277    -1.3267    -1.3214    -1.3114    -1.2981    -1.2825    -1.2657...
   -1.2481    -1.2301    -1.2120    -1.1940    -1.1760    -1.1581    -1.1404...
   -1.1229    -1.1055    -1.0884    -1.0714    -1.0547    -1.0381    -1.0217...
   -1.0054    -0.9894    -0.9735    -0.9578    -0.9423    -0.9270    -0.9118...
   -0.8967    -0.8819    -0.8672    -0.8526    -0.8382    -0.8240    -0.8099...
   -0.7960    -0.7822    -0.7685    -0.7550    -0.7416    -0.7284    -0.7153...
   -0.7023    -0.6895    -0.6768    -0.6642    -0.6517    -0.6394    -0.6272...
   -0.6151    -0.6031    -0.5912    -0.5795    -0.5678    -0.5563    -0.5449...
   -0.5336    -0.5224    -0.5113    -0.5003    -0.4894    -0.4786    -0.4679...
   -0.4573    -0.4468    -0.4364    -0.4261    -0.4159    -0.4058    -0.3958...
   -0.3858    -0.3759    -0.3662    -0.3565    -0.3469    -0.3373    -0.3279...
   -0.3185    -0.3092    -0.3000    -0.2909    -0.2818    -0.2729    -0.2640...
   -0.2551    -0.2464    -0.2377    -0.2290    -0.2205    -0.2120    -0.2036...
   -0.1952    -0.1869    -0.1787    -0.1705    -0.1624    -0.1544    -0.1464...
   -0.1385    -0.1306    -0.1228    -0.1151    -0.1074    -0.0997    -0.0922...
```

```
-0.0846   -0.0772   -0.0698   -0.0624   -0.0551   -0.0478   -0.0406...
-0.0334   -0.0263   -0.0192   -0.0122   -0.0053    0.0017    0.0085...
 0.0154    0.0222    0.0289    0.0356    0.0423    0.0489    0.0555...
 0.0620    0.0685    0.0750    0.0814    0.0878    0.0941    0.1004...
 0.1066    0.1129    0.1191    0.1252    0.1313    0.1374    0.1434...
 0.1495    0.1554    0.1614    0.1673    0.1732    0.1790    0.1848...
 0.1906    0.1964    0.2021    0.2078    0.2134    0.2191    0.2247...
 0.2302    0.2358    0.2413    0.2468    0.2522    0.2577    0.2631...
 0.2684    0.2738    0.2791    0.2844    0.2897    0.2950    0.3002...
 0.3054    0.3106    0.3157    0.3208    0.3260    0.3310    0.3361...
 0.3411    0.3462    0.3512    0.3561    0.3611    0.3660    0.3709...
 0.3758    0.3807    0.3855    0.3904    0.3952    0.4000    0.4047...
 0.4095    0.4142    0.4190    0.4237    0.4283    0.4330    0.4376...
 0.4423    0.4469    0.4515    0.4560    0.4606...
-1.0502   -1.0493   -1.0440   -1.0341   -1.0209   -1.0056   -0.9891...
-0.9718   -0.9543   -0.9366   -0.9190   -0.9016   -0.8843   -0.8672...
-0.8504   -0.8337   -0.8173   -0.8011   -0.7851   -0.7694   -0.7538...
-0.7385   -0.7234   -0.7085   -0.6937   -0.6792   -0.6649   -0.6507...
-0.6368   -0.6230   -0.6094   -0.5960   -0.5827   -0.5696   -0.5567...
-0.5439   -0.5313   -0.5189   -0.5066   -0.4945   -0.4825   -0.4706...
-0.4589   -0.4473   -0.4359   -0.4246   -0.4135   -0.4024   -0.3915...
-0.3808   -0.3701   -0.3596   -0.3492   -0.3389   -0.3287   -0.3186...
-0.3086   -0.2988   -0.2890   -0.2794   -0.2699   -0.2604   -0.2511...
-0.2419   -0.2327   -0.2237   -0.2147   -0.2059   -0.1971   -0.1884...
-0.1798   -0.1713   -0.1629   -0.1545   -0.1462   -0.1380   -0.1299...
-0.1219   -0.1139   -0.1061   -0.0982   -0.0905   -0.0828   -0.0752...
-0.0677   -0.0602   -0.0528   -0.0455   -0.0382   -0.0310   -0.0239...
-0.0168   -0.0098   -0.0028    0.0041    0.0109    0.0177    0.0245...
 0.0312    0.0378    0.0444    0.0509    0.0574    0.0638    0.0702...
 0.0765    0.0828    0.0891    0.0952    0.1014    0.1075    0.1136...
 0.1196    0.1255    0.1315    0.1374    0.1432    0.1490    0.1548...
 0.1605    0.1662    0.1719    0.1775    0.1831    0.1886    0.1941...
 0.1996    0.2050    0.2105    0.2158    0.2212    0.2265    0.2318...
 0.2370    0.2422    0.2474    0.2526    0.2577    0.2628    0.2679...
 0.2729    0.2779    0.2829    0.2879    0.2928    0.2977    0.3026...
 0.3075    0.3123    0.3171    0.3219    0.3267    0.3314    0.3361...
 0.3408    0.3455    0.3501    0.3547    0.3593    0.3639    0.3685...
 0.3730    0.3775    0.3820    0.3865    0.3910    0.3954    0.3999...
 0.4043    0.4087    0.4130    0.4174    0.4217    0.4260    0.4303...
 0.4346    0.4389    0.4431    0.4474    0.4516    0.4558    0.4600...
 0.4641    0.4683    0.4724    0.4766    0.4807    0.4848    0.4889...
 0.4929    0.4970    0.5010    0.5051    0.5091    0.5131    0.5171...
 0.5211    0.5250    0.5290    0.5329    0.5369];
```

Results of Simulation with Interpoles

(Varying armature inductance, L_a)

```
Time= [0      0.0010    0.0020    0.0030    0.0040    0.0050    0.0060...

 0.0070    0.0080    0.0090    0.0100    0.0110    0.0120    0.0130...
 0.0140    0.0150    0.0160    0.0170    0.0180    0.0190    0.0200...
 0.0210    0.0220    0.0230    0.0240    0.0250    0.0260    0.0270...
 0.0280    0.0290    0.0300    0.0310    0.0320    0.0330    0.0340...
```

```
0.0350    0.0360    0.0370    0.0380    0.0390    0.0400    0.0410...
0.0420    0.0430    0.0440    0.0450    0.0460    0.0470    0.0480...
0.0490    0.0500    0.0510    0.0520    0.0530    0.0540    0.0550...
0.0560    0.0570    0.0580    0.0590    0.0600    0.0610    0.0620...
0.0630    0.0640    0.0650    0.0660    0.0670    0.0680    0.0690...
0.0700    0.0710    0.0720    0.0730    0.0740    0.0750    0.0760...
0.0770    0.0780    0.0790    0.0800    0.0810    0.0820    0.0830...
0.0840    0.0850    0.0860    0.0870    0.0880    0.0890    0.0900...
0.0910    0.0920    0.0930    0.0940    0.0950    0.0960    0.0970...
0.0980    0.0990    0.1000    0.1010    0.1020    0.1030    0.1040...
0.1050    0.1060    0.1070    0.1080    0.1090    0.1100    0.1110...
0.1120    0.1130    0.1140    0.1150    0.1160    0.1170    0.1180...
0.1190    0.1200    0.1210    0.1220    0.1230    0.1240    0.1250...
0.1260    0.1270    0.1280    0.1290    0.1300    0.1310    0.1320...
0.1330    0.1340    0.1350    0.1360    0.1370    0.1380    0.1390...
0.1400    0.1410    0.1420    0.1430    0.1440    0.1450    0.1460...
0.1470    0.1480    0.1490    0.1500    0.1510    0.1520    0.1530...
0.1540    0.1550    0.1560    0.1570    0.1580    0.1590    0.1600...
0.1610    0.1620    0.1630    0.1640    0.1650    0.1660    0.1670...
0.1680    0.1690    0.1700    0.1710    0.1720    0.1730    0.1740...
0.1750    0.1760    0.1770    0.1780    0.1790    0.1800    0.1810...
0.1820    0.1830    0.1840    0.1850    0.1860    0.1870    0.1880...
0.1890    0.1900    0.1910    0.1920    0.1930    0.1940    0.1950...
0.1960    0.1970    0.1980    0.1990    0.2000];

ia51= [0       2.1886    3.3300    3.9193    4.2177    4.3628    4.4272...
4.4491    4.4488    4.4368    4.4188    4.3978    4.3753    4.3521...
4.3287    4.3052    4.2818    4.2586    4.2356    4.2128    4.1902...
4.1678    4.1456    4.1237    4.1020    4.0804    4.0591    4.0380...
4.0171    3.9964    3.9760    3.9557    3.9356    3.9157    3.8960...
3.8765    3.8572    3.8380    3.8191    3.8003    3.7818    3.7634...
3.7452    3.7272    3.7093    3.6916    3.6741    3.6568    3.6396...
3.6226    3.6058    3.5892    3.5727    3.5563    3.5401    3.5241...
3.5083    3.4925    3.4770    3.4616    3.4463    3.4312    3.4163...
3.4015    3.3868    3.3723    3.3579    3.3437    3.3296    3.3156...
3.3018    3.2881    3.2746    3.2612    3.2479    3.2347    3.2217...
3.2088    3.1960    3.1834    3.1709    3.1585    3.1462    3.1340...
3.1220    3.1100    3.0982    3.0866    3.0750    3.0635    3.0522...
3.0409    3.0298    3.0188    3.0079    2.9971    2.9864    2.9758...
2.9653    2.9549    2.9446    2.9344    2.9243    2.9143    2.9044...
2.8947    2.8850    2.8754    2.8658    2.8564    2.8471    2.8379...
2.8287    2.8197    2.8107    2.8019    2.7931    2.7844    2.7757...
2.7672    2.7588    2.7504    2.7421    2.7339    2.7258    2.7178...
2.7098    2.7019    2.6941    2.6864    2.6787    2.6711    2.6636...
2.6562    2.6488    2.6415    2.6343    2.6272    2.6201    2.6131...
2.6062    2.5993    2.5925    2.5857    2.5791    2.5725    2.5659...
2.5595    2.5530    2.5467    2.5404    2.5342    2.5280    2.5219...
2.5159    2.5099    2.5039    2.4981    2.4923    2.4865    2.4808...
2.4752    2.4696    2.4640    2.4586    2.4531    2.4478    2.4425...
2.4372    2.4320    2.4268    2.4217    2.4166    2.4116    2.4067...
2.4017    2.3969    2.3920    2.3873    2.3825    2.3779    2.3732...
2.3686    2.3641    2.3596    2.3551    2.3507    2.3464    2.3420...
2.3378    2.3335    2.3293    2.3252    2.3210    2.3170    2.3129...
2.3089    2.3050    2.3010    2.2972    2.2933...
     0    1.8510    2.9497    3.5976    3.9754    4.1915    4.3107...
4.3721    4.3988    4.4048    4.3986    4.3851    4.3674    4.3472...
4.3256    4.3033    4.2807    4.2579    4.2352    4.2125    4.1900...
```

```
4.1677    4.1456    4.1237    4.1019    4.0804    4.0591    4.0380...
4.0171    3.9964    3.9759    3.9557    3.9356    3.9157    3.8960...
3.8765    3.8572    3.8380    3.8191    3.8003    3.7818    3.7634...
3.7452    3.7272    3.7093    3.6916    3.6741    3.6568    3.6396...
3.6226    3.6058    3.5892    3.5727    3.5563    3.5401    3.5241...
3.5083    3.4925    3.4770    3.4616    3.4463    3.4312    3.4163...
3.4015    3.3868    3.3723    3.3579    3.3437    3.3296    3.3156...
3.3018    3.2881    3.2746    3.2612    3.2479    3.2347    3.2217...
3.2088    3.1960    3.1834    3.1709    3.1585    3.1462    3.1340...
3.1220    3.1100    3.0982    3.0866    3.0750    3.0635    3.0522...
3.0409    3.0298    3.0188    3.0079    2.9971    2.9864    2.9758...
2.9653    2.9549    2.9446    2.9344    2.9243    2.9143    2.9044...
2.8947    2.8850    2.8754    2.8658    2.8564    2.8471    2.8379...
2.8287    2.8197    2.8107    2.8019    2.7931    2.7844    2.7757...
2.7672    2.7588    2.7504    2.7421    2.7339    2.7258    2.7178...
2.7098    2.7019    2.6941    2.6864    2.6787    2.6711    2.6636...
2.6562    2.6488    2.6415    2.6343    2.6272    2.6201    2.6131...
2.6062    2.5993    2.5925    2.5857    2.5791    2.5725    2.5659...
2.5595    2.5530    2.5467    2.5404    2.5342    2.5280    2.5219...
2.5159    2.5099    2.5039    2.4981    2.4923    2.4865    2.4808...
2.4752    2.4696    2.4640    2.4586    2.4531    2.4478    2.4425...
2.4372    2.4320    2.4268    2.4217    2.4166    2.4116    2.4067...
2.4017    2.3969    2.3920    2.3873    2.3825    2.3779    2.3732...
2.3686    2.3641    2.3596    2.3551    2.3507    2.3464    2.3420...
2.3378    2.3335    2.3293    2.3252    2.3210    2.3170    2.3129...
2.3089    2.3050    2.3010    2.2972    2.2933...
     0    0.7082    1.3013    1.7973    2.2116    2.5570    2.8444...
3.0828    3.2801    3.4426    3.5760    3.6847    3.7728    3.8435...
3.8995    3.9431    3.9765    4.0011    4.0185    4.0297    4.0358...
4.0377    4.0359    4.0312    4.0240    4.0147    4.0037    3.9913...
3.9777    3.9632    3.9479    3.9320    3.9157    3.8989    3.8818...
3.8645    3.8471    3.8296    3.8120    3.7943    3.7767    3.7591...
3.7416    3.7241    3.7067    3.6895    3.6723    3.6553    3.6383...
3.6215    3.6049    3.5884    3.5720    3.5558    3.5397    3.5237...
3.5079    3.4923    3.4768    3.4614    3.4462    3.4311    3.4162...
3.4014    3.3867    3.3722    3.3579    3.3436    3.3296    3.3156...
3.3018    3.2881    3.2746    3.2612    3.2479    3.2347    3.2217...
3.2088    3.1960    3.1834    3.1708    3.1584    3.1462    3.1340...
3.1220    3.1100    3.0982    3.0865    3.0750    3.0635    3.0522...
3.0409    3.0298    3.0188    3.0079    2.9971    2.9864    2.9758...
2.9653    2.9549    2.9446    2.9344    2.9243    2.9143    2.9044...
2.8947    2.8850    2.8754    2.8658    2.8564    2.8471    2.8379...
2.8287    2.8197    2.8107    2.8019    2.7931    2.7844    2.7757...
2.7672    2.7588    2.7504    2.7421    2.7339    2.7258    2.7178...
2.7098    2.7019    2.6941    2.6864    2.6787    2.6711    2.6636...
2.6562    2.6488    2.6415    2.6343    2.6272    2.6201    2.6131...
2.6062    2.5993    2.5925    2.5857    2.5791    2.5725    2.5659...
2.5595    2.5530    2.5467    2.5404    2.5342    2.5280    2.5219...
2.5159    2.5099    2.5039    2.4981    2.4923    2.4865    2.4808...
2.4752    2.4696    2.4640    2.4586    2.4531    2.4478    2.4425...
2.4372    2.4320    2.4268    2.4217    2.4166    2.4116    2.4067...
2.4017    2.3969    2.3920    2.3873    2.3825    2.3779    2.3732...
2.3686    2.3641    2.3596    2.3551    2.3507    2.3464    2.3420...
2.3378    2.3335    2.3293    2.3252    2.3210    2.3170    2.3129...
2.3089    2.3050    2.3010    2.2972    2.2933];
```

```
T51= [0.0000    0.0762    0.8381    1.4253    1.7730    1.9542    2.0372...
      2.0658    2.0654    2.0498    2.0264    1.9992    1.9702    1.9406...
      1.9109    1.8813    1.8521    1.8233    1.7949    1.7670    1.7395...
      1.7125    1.6859    1.6598    1.6341    1.6089    1.5840    1.5596...
      1.5356    1.5119    1.4887    1.4659    1.4434    1.4213    1.3996...
      1.3782    1.3572    1.3365    1.3162    1.2962    1.2765    1.2571...
      1.2381    1.2194    1.2010    1.1828    1.1650    1.1475    1.1302...
      1.1132    1.0965    1.0801    1.0640    1.0480    1.0324    1.0170...
      1.0018    0.9869    0.9723    0.9578    0.9436    0.9296    0.9159...
      0.9024    0.8890    0.8759    0.8630    0.8503    0.8378    0.8255...
      0.8134    0.8015    0.7897    0.7782    0.7668    0.7556    0.7446...
      0.7338    0.7231    0.7126    0.7022    0.6920    0.6820    0.6721...
      0.6624    0.6528    0.6434    0.6341    0.6250    0.6160    0.6071...
      0.5984    0.5898    0.5813    0.5730    0.5648    0.5567    0.5487...
      0.5409    0.5331    0.5255    0.5180    0.5106    0.5034    0.4962...
      0.4892    0.4822    0.4753    0.4686    0.4620    0.4554    0.4489...
      0.4426    0.4363    0.4301    0.4241    0.4181    0.4122    0.4063...
      0.4006    0.3950    0.3894    0.3839    0.3785    0.3732    0.3679...
      0.3627    0.3576    0.3526    0.3476    0.3428    0.3379    0.3332...
      0.3285    0.3239    0.3193    0.3149    0.3104    0.3061    0.3018...
      0.2975    0.2934    0.2892    0.2852    0.2812    0.2772    0.2733...
      0.2695    0.2657    0.2620    0.2583    0.2547    0.2511    0.2476...
      0.2441    0.2406    0.2372    0.2339    0.2306    0.2274    0.2242...
      0.2210    0.2179    0.2148    0.2118    0.2088    0.2058    0.2029...
      0.2000    0.1972    0.1944    0.1916    0.1889    0.1862    0.1835...
      0.1809    0.1783    0.1758    0.1733    0.1708    0.1683    0.1659...
      0.1635    0.1612    0.1589    0.1566    0.1543    0.1521    0.1499...
      0.1477    0.1455    0.1434    0.1413    0.1393    0.1372    0.1352...
      0.1332    0.1313    0.1293    0.1274    0.1255...
      0.0000   -0.0542    0.5293    1.0884    1.4881    1.7411    1.8883...
      1.9661    2.0004    2.0082    2.0002    1.9829    1.9601    1.9344...
      1.9071    1.8790    1.8507    1.8224    1.7944    1.7667    1.7393...
      1.7124    1.6859    1.6598    1.6341    1.6088    1.5840    1.5596...
      1.5356    1.5119    1.4887    1.4659    1.4434    1.4213    1.3996...
      1.3782    1.3572    1.3365    1.3162    1.2962    1.2765    1.2571...
      1.2381    1.2194    1.2010    1.1828    1.1650    1.1475    1.1302...
      1.1132    1.0965    1.0801    1.0640    1.0480    1.0324    1.0170...
      1.0018    0.9869    0.9723    0.9578    0.9436    0.9296    0.9159...
      0.9024    0.8890    0.8759    0.8630    0.8503    0.8378    0.8255...
      0.8134    0.8015    0.7897    0.7782    0.7668    0.7556    0.7446...
      0.7338    0.7231    0.7126    0.7022    0.6920    0.6820    0.6721...
      0.6624    0.6528    0.6434    0.6341    0.6250    0.6160    0.6071...
      0.5984    0.5898    0.5813    0.5730    0.5648    0.5567    0.5487...
      0.5409    0.5331    0.5255    0.5180    0.5106    0.5034    0.4962...
      0.4892    0.4822    0.4753    0.4686    0.4620    0.4554    0.4489...
      0.4426    0.4363    0.4301    0.4241    0.4181    0.4122    0.4063...
      0.4006    0.3950    0.3894    0.3839    0.3785    0.3732    0.3679...
      0.3627    0.3576    0.3526    0.3476    0.3428    0.3379    0.3332...
      0.3285    0.3239    0.3193    0.3149    0.3104    0.3061    0.3018...
      0.2975    0.2934    0.2892    0.2852    0.2812    0.2772    0.2733...
      0.2695    0.2657    0.2620    0.2583    0.2547    0.2511    0.2476...
      0.2441    0.2406    0.2372    0.2339    0.2306    0.2274    0.2242...
      0.2210    0.2179    0.2148    0.2118    0.2088    0.2058    0.2029...
      0.2000    0.1972    0.1944    0.1916    0.1889    0.1862    0.1835...
      0.1809    0.1783    0.1758    0.1733    0.1708    0.1683    0.1659...
      0.1635    0.1612    0.1589    0.1566    0.1543    0.1521    0.1499...
      0.1477    0.1455    0.1434    0.1413    0.1393    0.1372    0.1352...
```

```
  0.1332      0.1313      0.1293      0.1274      0.1255...
  0.0000     -0.1745     -0.1741     -0.0710      0.0867      0.2681      0.4535...
  0.6312      0.7945      0.9402      1.0672      1.1758      1.2670      1.3424...
  1.4034      1.4519      1.4893      1.5173      1.5371      1.5500      1.5571...
  1.5592      1.5572      1.5517      1.5434      1.5328      1.5202      1.5061...
  1.4907      1.4744      1.4572      1.4395      1.4213      1.4028      1.3841...
  1.3652      1.3463      1.3274      1.3085      1.2898      1.2711      1.2527...
  1.2343      1.2162      1.1983      1.1806      1.1632      1.1459      1.1289...
  1.1122      1.0956      1.0794      1.0633      1.0475      1.0319      1.0166...
  1.0015      0.9867      0.9720      0.9576      0.9435      0.9295      0.9158...
  0.9023      0.8890      0.8759      0.8630      0.8503      0.8378      0.8255...
  0.8134      0.8015      0.7897      0.7782      0.7668      0.7556      0.7446...
  0.7338      0.7231      0.7126      0.7022      0.6920      0.6820      0.6721...
  0.6624      0.6528      0.6434      0.6341      0.6250      0.6160      0.6071...
  0.5984      0.5898      0.5813      0.5730      0.5648      0.5567      0.5487...
  0.5409      0.5331      0.5255      0.5180      0.5106      0.5034      0.4962...
  0.4892      0.4822      0.4753      0.4686      0.4620      0.4554      0.4489...
  0.4426      0.4363      0.4301      0.4241      0.4181      0.4122      0.4063...
  0.4006      0.3950      0.3894      0.3839      0.3785      0.3732      0.3679...
  0.3627      0.3576      0.3526      0.3476      0.3428      0.3379      0.3332...
  0.3285      0.3239      0.3193      0.3149      0.3104      0.3061      0.3018...
  0.2975      0.2934      0.2892      0.2852      0.2812      0.2772      0.2733...
  0.2695      0.2657      0.2620      0.2583      0.2547      0.2511      0.2476...
  0.2441      0.2406      0.2372      0.2339      0.2306      0.2274      0.2242...
  0.2210      0.2179      0.2148      0.2118      0.2088      0.2058      0.2029...
  0.2000      0.1972      0.1944      0.1916      0.1889      0.1862      0.1835...
  0.1809      0.1783      0.1758      0.1733      0.1708      0.1683      0.1659...
  0.1635      0.1612      0.1589      0.1566      0.1543      0.1521      0.1499...
  0.1477      0.1455      0.1434      0.1413      0.1393      0.1372      0.1352...
  0.1332      0.1313      0.1293      0.1274      0.1255];

w51= [-1.3397    -1.3382     -1.3310     -1.3187     -1.3032     -1.2860     -1.2680...
 -1.2495     -1.2311     -1.2126     -1.1943     -1.1762     -1.1582     -1.1405...
 -1.1229     -1.1056     -1.0884     -1.0714     -1.0547     -1.0381     -1.0217...
 -1.0054     -0.9894     -0.9735     -0.9578     -0.9423     -0.9270     -0.9118...
 -0.8967     -0.8819     -0.8672     -0.8526     -0.8382     -0.8240     -0.8099...
 -0.7960     -0.7822     -0.7685     -0.7550     -0.7416     -0.7284     -0.7153...
 -0.7023     -0.6895     -0.6768     -0.6642     -0.6517     -0.6394     -0.6272...
 -0.6151     -0.6031     -0.5912     -0.5795     -0.5678     -0.5563     -0.5449...
 -0.5336     -0.5224     -0.5113     -0.5003     -0.4894     -0.4786     -0.4679...
 -0.4573     -0.4468     -0.4364     -0.4261     -0.4159     -0.4058     -0.3958...
 -0.3858     -0.3759     -0.3662     -0.3565     -0.3469     -0.3373     -0.3279...
 -0.3185     -0.3092     -0.3000     -0.2909     -0.2818     -0.2729     -0.2640...
 -0.2551     -0.2464     -0.2377     -0.2290     -0.2205     -0.2120     -0.2036...
 -0.1952     -0.1869     -0.1787     -0.1705     -0.1624     -0.1544     -0.1464...
 -0.1385     -0.1306     -0.1228     -0.1151     -0.1074     -0.0997     -0.0922...
 -0.0846     -0.0772     -0.0698     -0.0624     -0.0551     -0.0478     -0.0406...
 -0.0334     -0.0263     -0.0192     -0.0122     -0.0053      0.0017      0.0085...
  0.0154      0.0222      0.0289      0.0356      0.0423      0.0489      0.0555...
  0.0620      0.0685      0.0750      0.0814      0.0878      0.0941      0.1004...
  0.1066      0.1129      0.1191      0.1252      0.1313      0.1374      0.1434...
  0.1495      0.1554      0.1614      0.1673      0.1732      0.1790      0.1848...
  0.1906      0.1964      0.2021      0.2078      0.2134      0.2191      0.2247...
  0.2302      0.2358      0.2413      0.2468      0.2522      0.2577      0.2631...
  0.2684      0.2738      0.2791      0.2844      0.2897      0.2950      0.3002...
  0.3054      0.3106      0.3157      0.3208      0.3260      0.3310      0.3361...
  0.3411      0.3462      0.3512      0.3561      0.3611      0.3660      0.3709...
```

```
 0.3758     0.3807     0.3855     0.3904     0.3952     0.4000     0.4047...
 0.4095     0.4142     0.4190     0.4237     0.4283     0.4330     0.4376...
 0.4423     0.4469     0.4515     0.4560     0.4606...
-1.3277    -1.3267    -1.3214    -1.3114    -1.2981    -1.2825    -1.2657...
-1.2481    -1.2301    -1.2120    -1.1940    -1.1760    -1.1581    -1.1404...
-1.1229    -1.1055    -1.0884    -1.0714    -1.0547    -1.0381    -1.0217...
-1.0054    -0.9894    -0.9735    -0.9578    -0.9423    -0.9270    -0.9118...
-0.8967    -0.8819    -0.8672    -0.8526    -0.8382    -0.8240    -0.8099...
-0.7960    -0.7822    -0.7685    -0.7550    -0.7416    -0.7284    -0.7153...
-0.7023    -0.6895    -0.6768    -0.6642    -0.6517    -0.6394    -0.6272...
-0.6151    -0.6031    -0.5912    -0.5795    -0.5678    -0.5563    -0.5449...
-0.5336    -0.5224    -0.5113    -0.5003    -0.4894    -0.4786    -0.4679...
-0.4573    -0.4468    -0.4364    -0.4261    -0.4159    -0.4058    -0.3958...
-0.3858    -0.3759    -0.3662    -0.3565    -0.3469    -0.3373    -0.3279...
-0.3185    -0.3092    -0.3000    -0.2909    -0.2818    -0.2729    -0.2640...
-0.2551    -0.2464    -0.2377    -0.2290    -0.2205    -0.2120    -0.2036...
-0.1952    -0.1869    -0.1787    -0.1705    -0.1624    -0.1544    -0.1464...
-0.1385    -0.1306    -0.1228    -0.1151    -0.1074    -0.0997    -0.0922...
-0.0846    -0.0772    -0.0698    -0.0624    -0.0551    -0.0478    -0.0406...
-0.0334    -0.0263    -0.0192    -0.0122    -0.0053     0.0017     0.0085...
 0.0154     0.0222     0.0289     0.0356     0.0423     0.0489     0.0555...
 0.0620     0.0685     0.0750     0.0814     0.0878     0.0941     0.1004...
 0.1066     0.1129     0.1191     0.1252     0.1313     0.1374     0.1434...
 0.1495     0.1554     0.1614     0.1673     0.1732     0.1790     0.1848...
 0.1906     0.1964     0.2021     0.2078     0.2134     0.2191     0.2247...
 0.2302     0.2358     0.2413     0.2468     0.2522     0.2577     0.2631...
 0.2684     0.2738     0.2791     0.2844     0.2897     0.2950     0.3002...
 0.3054     0.3106     0.3157     0.3208     0.3260     0.3310     0.3361...
 0.3411     0.3462     0.3512     0.3561     0.3611     0.3660     0.3709...
 0.3758     0.3807     0.3855     0.3904     0.3952     0.4000     0.4047...
 0.4095     0.4142     0.4190     0.4237     0.4283     0.4330     0.4376...
 0.4423     0.4469     0.4515     0.4560     0.4606...
-1.2115    -1.2114    -1.2107    -1.2087    -1.2053    -1.2003    -1.1938...
-1.1859    -1.1767    -1.1664    -1.1551    -1.1430    -1.1302    -1.1169...
-1.1030    -1.0888    -1.0743    -1.0596    -1.0447    -1.0297    -1.0146...
-0.9995    -0.9844    -0.9694    -0.9543    -0.9394    -0.9245    -0.9097...
-0.8950    -0.8804    -0.8660    -0.8516    -0.8374    -0.8233    -0.8093...
-0.7955    -0.7817    -0.7682    -0.7547    -0.7414    -0.7282    -0.7151...
-0.7022    -0.6893    -0.6767    -0.6641    -0.6516    -0.6393    -0.6271...
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-0.5336    -0.5224    -0.5113    -0.5003    -0.4894    -0.4786    -0.4679...
-0.4573    -0.4468    -0.4364    -0.4261    -0.4159    -0.4058    -0.3958...
-0.3858    -0.3759    -0.3662    -0.3565    -0.3469    -0.3373    -0.3279...
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-0.2551    -0.2464    -0.2377    -0.2290    -0.2205    -0.2120    -0.2036...
-0.1952    -0.1869    -0.1787    -0.1705    -0.1624    -0.1544    -0.1464...
-0.1385    -0.1306    -0.1228    -0.1151    -0.1074    -0.0997    -0.0922...
-0.0846    -0.0772    -0.0698    -0.0624    -0.0551    -0.0478    -0.0406...
-0.0334    -0.0263    -0.0192    -0.0122    -0.0053     0.0017     0.0085...
 0.0154     0.0222     0.0289     0.0356     0.0423     0.0489     0.0555...
 0.0620     0.0685     0.0750     0.0814     0.0878     0.0941     0.1004...
 0.1066     0.1129     0.1191     0.1252     0.1313     0.1374     0.1434...
 0.1495     0.1554     0.1614     0.1673     0.1732     0.1790     0.1848...
 0.1906     0.1964     0.2021     0.2078     0.2134     0.2191     0.2247...
 0.2302     0.2358     0.2413     0.2468     0.2522     0.2577     0.2631...
 0.2684     0.2738     0.2791     0.2844     0.2897     0.2950     0.3002...
 0.3054     0.3106     0.3157     0.3208     0.3260     0.3310     0.3361...
```

```
0.3411    0.3462    0.3512    0.3561    0.3611    0.3660    0.3709...
0.3758    0.3807    0.3855    0.3904    0.3952    0.4000    0.4047...
0.4095    0.4142    0.4190    0.4237    0.4283    0.4330    0.4376...
0.4423    0.4469    0.4515    0.4560    0.4606];
```