FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA
Department of Mathematics
B.Tech. Degree *First Semester Examination 2022/2023 Session*

Title: Functional Analysis | *Unit: 3* | *Time: 2hrs30mins*

Instruction*: *Answer only Four (4) | *Code: MAT513*

1(a). Define (i) metric space (ii) complete metric space (4, 3 marks).

1(b) Prove that (X, d) defines a metric space if d is defined by:

$$d(x,y) = \begin{cases} 1 & \text{if } x \neq y \\ 0 & \text{if } x = y \end{cases}$$ (8 marks)

2(a). Define (i) normed linear space (ii) Banach space; (3, 2 marks).

2(b) Prove that $\|\cdot\|$ defines a norm on $\mathbb{R}^n$ if $\|x\| = \{\sum_{j=1}^{n}|x_j|^p\}^{1/p}$ where $x = (x_1, x_2, \ldots\ldots\, x_n) \in X = R^n$; (10 marks).

3. State and prove the Contraction Mapping Principle; (5, 10 marks).

4. If $T : X \to Y$ is a linear map prove that

(i) $T(0) = 0$ (2 marks)

(ii) The Range of T
$R(T) = \{y \in Y : Tx = y, x \in X\}$ is a linear subspace of Y; (4 marks)

(iii) T is one – to - one if $Tx = 0 \Rightarrow x = 0$; (4 marks).

(iv) If T is one – to – one then T^{-1} exist on $R(T)$ and T^{-1} (5 marks)

5(a) Define the concepts: (i) inner product (ii) Hilbert space (iii) dual space; (2 marks each).

5(b) If B(., .) defines an inner product on a linear space the inequality
$|B(x, y)|^2 \leq B(x, x)\, B(y, y)$ (9 marks)

6(a) State the Riesz Representation theorem (3 marks)

6(b) Solve the integral equation $y(t) = \sin t + \lambda \int_0^{\pi} t\, s y(s) \cos s\, ds$

(12 marks)