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COMPUTERIZATION OF TWO–PERSON ZERO–SUM GAME

BY

USMAN, ABUBAKAR
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Usman, Abubakar

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CHAPTER ONE

1.0 INTRODUCTION:

Games are described by specifying behaviour within the rules of the game. It is concerned with deciding on course of action in situations where the decision makers has open to them a number of possible actions or strategies.

The term 'Games' does not only includes pleasurable activities, but also much more earnest competitive situation of war and peace.

There may be uncertainty for each participant because the action of others may not be known with certainty. This situation are not only found in games alone, but also in Business, politics, educational career, choice or war and other social activities.

Game is a branch of Mathematics that deals with situation of conflict involving decision-making process. In economic competition, political contests and many other situation decisions have to be made without their consequences being precisely known, because the outcome depends partly on circumstances that are beyond the control of the decision maker.

In any game, the participants always strive to maximise his benefit by studying his opponent, this can be related to a competitive Business situations, economics, administration and sociology as well as to military problems.

Two person Zero-sum games as the name implies are games involving two adversaries or players. It is a game where there are no penalties borne by either player so that the total resources owned by the two of them combined remain constant. They are called zero-sum games because one player wins whatever the other one losses, so that the sum of their net winnings is zero.

A lot of criterions are employed but the commonly used one to enable a player to determine his best strategy is the so-called maximum criterion, where the player looks at the minimum gain he stands to make from each of his different possible strategies. This criterion is based on the fact that whatever one player does his opponent will behave in a such way that success will be minimized.

The particular games that are subject of research are those active games that adults in different part of the world have enjoyed sufficiently in their early forms to develop for their code of rules and specifications, and to organise them for competitive play.

1.1 HISTORICAL BACKGROUND:

As early as the 17th century attempts were made to analyse games of chance and parlour games. A multitude of these games continue to be with us today, and in some (such as roulette) the outcome is purely accidental, in others (such as bridge) it depends on chance and the players behaviour, while a third group (such as chess) is complete controlled by skill.

In 1943, John VonNuemann and Oskar Morgenstern description of the links between Economic problems (competitive situations) and games, thus establishing the theory of games. Nowadays it is seen as a descriptive in the wider field of Mathematical operational research with describing and modeling these relations in Mathematical terms and finding the best possible strategy for a player (conflict situation relations).

1.2 GAME THEORY CONCEPT:

Game theory is the study of how players should rationally play games. Each player would like the game to end in an outcome, which gives him as large a play off as possible.

The player has some control over the outcome, since his choice of strategy will influence it. However, the outcome is not determine by his choice alone, but also depends upon the choices of all the other players and this is where the conflict and cooperation enter.

There may be conflict because different plays will, in general values outcomes differently.

Game are substantial by specifying bahviour within the rules of the games. The rules are in each case unambiguous, for instance, in a market, individuals are permitted to bargain to threaten with boycott, etc but they are not permitted to use physical force to acquire an article or to attempt to change its prices.

Rational play will involve complicated individual decisions about how to choose a strategy which will produce an outcome favourable to him, knowing that other players are trying to choose strategies which will produce an outcome favourable to them.

1.3 AIMS AND OBJECTIVES OF THE STUDY:

Todays world, uncertainty is an inescapable situation in every human activities. We live in business and communities whose operations influences our lives. In a situation where uncertainty plays a significant role, the process of decision making comes in.

The result of our decisions are not determine by our own choice alone, but by decisions of others and chance (in case of gambling). Therefore, game enable us to take decisions that have positive bearing on our own activities, for instance administrative policies, academic, war, business and other social activities.

1.4. SCOPE AND LIMITATIONS OF THE STUDY:

There are different games which are classified by the number of players they have, whether or not they are zero-sum and whether or not they have perfect information or recall.

These games includes:

- Games in extensive form
- Two-person zero-sum or matrix games
- Ore- person non zero sum games frequently called statistical games.
- Two-person non-zero sum games.
- Games involving three or more person (n-person games)

This research work is limited to two person zero-sum games often referred to as matrix games where there are at least two players. A player may be an individual, but it may also be a more general entity, the pay off of all participants at the end of the game sum to zero.

1.5 CLASSIFICATION OF GAMES

Although this research work has considered only two-person zero-sum games with a finite number of players, other games can be classified base on the numbers of players or strategies.

These include:

(a) **n – person game**:

Here more than two players can participate in the game. This is important particularly in competitive situations where there are more than two competitions involved, for instance in competition among business firms, in international diplomacy and so forth.

(b) **The non-zero-sum game**:

This involve a situation where the sum of the pay off to the players need not to be zero (or any other fixed constant). The situation could be seen where many competitive situations including non-competitive aspects that contribute to the mutual advantage or mutual disadvantage of the players. For example the adverting strategies of completing companies can affect not only how they will split the market but also the total size of the market for their competing products.

Non-zero sum games are further classified in terms of the degree to which the players are permitted to cooperate.

1.6 STRATEGY FOR PLAYING GAMES:

A strategy is a predetermined rule that specifies completely how one intends to respond to each possible circumstance at reach stage of the game.

The strategy for playing a game depends on the individual called player. Each player would like to take a strategy that would not be known to his opponent.

In the actual play of a game, a player instead of making his decision at each move, each player may formulate in advance of the play a plan (strategy) for playing the game from beginning to the end. A plan must be complete and cover all possible contingencies that may arise in the play. The player would incorporate in the plan the rules of the game, such a complete prescription for a play of a game by the player is called a strategy of that player and when using a strategy he losses no freedom of action since the strategy specifies the players actions in terms of the information that might become available.

1.7 <u>BASIC DEFINITION</u>:

(a) **MATRIX GAME**: Is a rectangular array of numbers with the entries arranged in rows and columns.

(b) **ZERO-SUM GAMES**: Is a game where at the end of play, one person gains everything that the other players losses .

(c) **GAMES**: A Mathematical activities of deciding on a course of action in situations.

(d) **TWO-PERSON ZERO-SUM GAMES**: Games involving only two adversaries or players, one player wins everything while the other one losses everything.

(e) **PAY OFF TABLE**: this is usually given only for player 1 because the table for play II is just the negative of this one.

(f) **STRATEGY**: Is a predetermined rule that specifies completely how one intends to respond to each possible circumstance at each stage of the game.

(g) **DOMINATED STRATEGY**: Is used to rule out a succession of inferior strategy until only one choice remains (i.e. if there is a strategy that is always at least as good regardless of what the opponent does).

(h) **DOMINANCE PRINCIPLE**: "A rational player should never play a dominated strategy".

CHAPTER TWO

2.0 REVIEW OF RELATED LITERATURE:

2.1 INTRODUCTION:

Life is full of conflict and competition. Situations involving adversaries in conflict include parlor games, military battles, political campaigns, advertising, and marketing campaigns by completing business firms and so forth are among the numerous examples.

This in general involves making decisions in a competitive environment which is the fundamental contribution of games in particular the two-person zero-sum games for it is the logical analysis of situation of conflict and cooperation.
School of anthropological thought known as functionalism holds that customs, institutions or behaviour patterns in a society can be interpreted as functional responses to problems which the society faces.
For instance Moore (1957) proposed that one could interpret divination as a societal mechanism for implementing mixed strategy solutions of a game.

2.2 HISTORICAL BACKGROUND OF COMPUTERS:

Computer is an electronic device which accepts and processes data by following a set of instructions (program) to provide an accurate and efficient result (information).

The first electronic computer was named ENIAC and was built in United States (US) in 1945. It used a large number of vacuum values and consumed as much as 20KW. It was prone to malfunctions due to short life span of the components and its actual capacity was not greater than that of a present day calculator.

A few years later the transistor was invented. This device took the place of the vacuum tubes and brought about a radical improvement in the size, reliability and power consumption of the second generation of computers.

The next generation of computers was consequence of the introduction of integrated circuits or clips, small pieces of silicon containing a large number of the electronic components needed to make the electronic circuits.

Within a few years integrated circuits became more and more complex through the development of a new fabrication technique called the very large-scale of integration (VLSI). Making use of this technology, computer were made powerful in terms of storage capacity and processing speed. It became possible to put all the circuits needed for the computer 'brain' (the processor) into a single integrated circuit not bigger than a match box. The new device was called the 'Micro processor' and it brought about a radical change in the computer word. In recent times, computers are found to be an assets, for it is realised that their application spread all over human endeavours, they are now used for house keeping chores, for word processing and business management, for teaching in class and at a distance, for the control of industrial systems, games, for probe and to inquire, to ask and to answer, to seek and to find, to summarise and to categorise and so forth. They are often represented as being much like super intelligent human beings.

2.3 <u>BRIEF REVIEW OF GAMES THEORY:</u>

Games represent the ultimate case of lack of information in which intelligent opponents are making in a conflicting environment.

Games have been with us for as long as there has been human consciousness. The desire to make believe to enter into self created micro world bound by certain rules and indisputable conventions is nothing new.

The primary objectives of game theory is the development of national criteria for selecting a strategy.

According to Fredrick S. K. and Gerald J. K (1986); the development is done under two key assumptions.

1 Both players are rational.

2 Both players choose their strategy solely to promote their own welfare (no compassion for the opponent).

It contracts with decision analysis, where the assumption is that the decision maker is playing a game with a passive opponent, nature which chooses it strategies in some random fashion.

It is usually impossible to delineate all conceivable strategies and say which outcomes they lead to and it is not easy to assign payoffs to any given outcome, since real – word game is enormously complex.

Game theory deals with rational player each player logically analyses the best way to achieve their ends, given that the other players are logically analyzing the best the best way to achieve their ends.

In other words, rational player assumes rational opponents. In the real world, it is quite doubtful that all players are rationally.

It does not have a unique prescription for play in games with two players whose interests are not completely opposed.

For game with more than two players what game theory offers is a vanity of interesting examples, analyses, suggestions and partial prescriptions of these situations

2.4 THE DEVELOPMENT OF TWO-- PERSON ZERO—SUM GAMES.

Zero- sum games represent conflict situations and our solution theory for them prescribes rational strategies for conflict. Since the most extreme form of conflict is war, it is therefore not surprising that some of the first proposed applications of game theory were to tactics in war. Hay wood (1954) and Beresford and Peston (1955) describe some application of game theory to situation from word war II.

Devenport (1960) applied the two- person game theory to an anthropological problem a classic and still controversial paper on Jamaica fishing. He studied a village of two hundred people on the south shore of Jamaica were inhabitants make their living by fishing. The fishing grounds extend out word from shore about 22 miles (35.2Kms). Stress twenty-six fishing areas in sailing, dug out canoes fish this areas by setting fish ports which are drawn and reset, weather and sea permitting on three regular fishing days each week.

Kozeika (1969) and Read and Read (1970) independently pointed out that there is a serious flaw in the analysis. The fishermen's opponent in this game is a natural phenomenon, the current. It is not a reasoning entity, and its behavior is not affected by what the fishermen do. In particular, it would not adjust its behavior to the advantage of non – optimal play by the fishermen. The fishermen's correct behavior in this context would be use to the expected value principle.

With this development there are huge reward system at work and computer games can be construed as satisfying or attempting to satisfy four central human needs:-

Firstly, there is the ever - present need for computer challenging fresh peaks to conquer. The computer game player is playing against his/her own or another person's score.

Secondly, the very human need to exert control over the unpredictable. In pre–electronic games days this was done by throw of the dice or a shuffle of the card peak; now the unpredictable is done by random digit generators.

Thirdly, the ever – present desire to transcend limitations, to enter into disguise, to indulge in fantasy.

Lastly, the most importantly is the human desire to alter everyday consciousness. By this we mean the desire to transcend normal humdrum everyday mode of conscious thought and instead enter another plane of awareness.

2.5 CHARACTERISTICS OF TWO – PERSON ZERO – SUM GAMES.

In situations where two or more decision makers simultaneously choose an action, and the action chosen by each affects the rewards earned by the other as the following characteristics:

1 There are two players (called the row player and the column player)

2 The row player must choose one of m – strategies simultaneous, the column player must choose one of n – strategies.

3 If the row player chooses her ith strategy and the column player choose her jth strategy, the row player receives a reward of aij and the column player loses an amount aij. Thus we may think of the row players reward of aij as coming from the column player such a game is known as Two – person Zero – sum game.

CHAPTER THREE

3.0 SYSTEM ANALYSIS AND DESIGN:

3.1 INTRODUCTION:

The conflicting situations today requires the best way to resolve situations that could be the affective and efficient for the needs and aspirations of an orgainsation and individual. The methods of determining how best to use the computers with other resources to perform tasks which meet the information needs of an organisation is often referred to as systems analysis.

The structure of a Two–person, constant or Zero–sum game is completely summarised in a single play off matrix, this however, restricts us to the final outcome of the game consequently, it is necessary to determine the behavioral patterns of the players, that is specify or find the manner which they will play the game. A player may be aggressive and even reckless, in that case he may aim at the highest possible payoff realizing that if his up with substantial loss.

This chapter discusses in detail the mathematical techniques for finding solution to two–person Zero–sum games problem, considerations would be given to the methods of graphical, simplex and approximation for solving two–person zero–sum games.

3.2 METHOD OF SOLUTION:

Uncertainty characterized human activities in the world we live. The situations that involve chance, choice and competition are frequently present in games. In view of the close association of the word game with situation involving chance, choice and competition.

Many competition problem concerned with optimizing (maximizing and minimizing) the utility or pay off subject to a system of inequality. Since competitive situations entail or involve two or more parties it becomes eminent that their expected. Pay off be represented by a function which is either maximizing or minimizing such a function is called objective function and its corresponding inequalities are called constraints.

3.3 <u>METHOD OF SOLVING GAMES USING GRAPHS</u>:

This method is applicable to a game with pay off matrix which is 2xN. The algorithms involve in using graph are outlined below:--

STEP 1: Confirm if the pay off matrix has a saddle point, if it does not have a saddle point, it means the game cannot be solve using this technique, otherwise proceed to step 2.

STEP 2: Denote an arbitrary strategy for the row player K(r) = (r, 1-r) and the expected pay off denoted by Fi (r), ii = 1,;2;..,n where K(r) played against column i

STEP 3: The function Fi (r), i = 1,2, …,n should be graphed for $0 \leq r \leq 1$

STEP 4: Find the maximum point on the polygonal line which bounds the graph of F_1, F_2, …………, Fn from below. Let x and y be defined by the property that the graph Fi (x) and Fi (Y) lie on this polygon and intersect at the highest point X<Y. Since the payoff matrix does not have a saddle point x and Y be certain to exists.

STEP 5: Solve for r* such that K_r (r*) = Ki (r*)

STEP 6: K* = (r*, 1-r*) represent an optimal strategy for the row player.

STEP 7: Let t (c) be the mixed strategy for the column player which plays column P with probability 1 – c, $o \leq c \leq 1$. Also let q_i (c) be the payoff when t(c) is played against row i. i = 1, 2.

STEP 8: Solve for the value of C* such that $q_1 (c^*) = q_2 (c)$. since the payoff matrix does not have a saddle point, C* will lie in the internal $0 \leq c \leq 1$.

STEP 9: An optimal strategy for the column player is given by

$t^* = (o, 0, \ldots.., 0, c^* 0, \ldots, 0, 1\text{-}c^*, 0, \ldots.0)$

Pth coordinate ith coordinate

STEP 10: The value of the game (the expected return to the row player is

$V = k^* (r^*) = k_1 (r^*) = q_1 (c^*) = q_2 (c^*)$

* Section 3.5 give an illustration of graphical example.

3.4 <u>SOLVING 2 x 2 GAMES</u>:

The solution of games with 2 x 2 playoff matrix can be given by a simple formular . However, the steps outlined in 3.3 would be used.

Consider the game with the payoff matrix.

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The matrix A has no saddle point if and only if a and d are both larger than b and c that is if and only if a > b, a > c, d > b, d > c, or a < c, a < b, d < b, and d < c.

Suppose A has no saddle point then

$R = a + d - b - c$, then the value V of the game is

$V = (ad - bc)/r$

and optimal strategies x and y for an optimal k1 and k2 are

$x = ((d - c)/r, (a - b)/r)$

$Y = ((d - b)/r, (a - c)/r)$ ______ (3.4.1)

Using the explanation above, let us consider a particular case: Solve a two-person zero-sum game with the following payoff matrix (3:4.2)

$$A = \begin{bmatrix} 10 & -8 \\ -7 & 5 \end{bmatrix}$$

This matrix has no saddle point because 10 and 5 are both greater that –8 and --7

Thus applying (3.4.1) above

R = 10-+5+8+7 = 30

det A = 50 – 56 = -6

x = ((5+7)/30, (10+8)/30) = (2/5, 3/5)

y = ((5+8)/30, (10+7)/30) = (13/30, 17/30)

The value of the game v = -6/30 = -1/5

Example: (3.4.3)

Consider the game with the payoff matrix

$$P = \begin{bmatrix} 5 & 7 \\ 6 & 2 \end{bmatrix}$$

This matrix P has no saddle point because 5 and 2 are both less that 6 and 7.

Thus by 3.4.1

r = 5 + 2 – 6 – 7 = -6

det A = 10 –42 = -32

X = ((-4)/-6, (-2)/-6) = (2/3, 1/3)

Y = ((-5)/-6, (-1)/-6) = (5/6, 1/6)

V= -32/-6 = 16/3

Example (3.4.4)

Two players have two cards P1 a red 5 and a black 4, P2 has a red 7 and a black 8. Each player selects one of his cards, with his choice unknown to his opponent, and the players compare the selected cards. If the selected cards are the same colour P1 wins the difference in face values from P2. If the selected cards are of different colour P2 wins the difference in face values from

P1. Denoting P1's two strategies by R5 and B4 and P2's by R7 and B8, the game tableaus is

P1\P2	R7	B8
R5	2	-3
B4	-3	4

This matrix has no saddle point because 2 and 4 are both greater than –3 and –3. Thus, let x and y be the optimal strategies for the matrix and r the value of the game.

i.e. $r = 2 + 4 + 3 + 3 = 12$

$x = ((4+3)/12, (2+3)/12) = (7/12, 5/12)$

$y = ((4+3)/12, (2+3)/12) = (7/12, 5/12)$

$v = (8-9)/12 = -1/12$

The fact that the components in the optimal strategies are equal in magnitude it means that both players must vary their options from one game to the next in an unsystematic manner.

3.5 GRAPHICAL SOLUTION OF (2xN) AND (Mx2) GAMES:

This is applicable to games in which at least one of the players has two strategies only.

Consider the following (2xn) games

	y_1	y_2	....	y_n
x_1	a_{11}	a_{12}	....	a_{in}
$x_2 = 1-x_1$	a_{21}	a_{22}	....	a_{2n}

___ (3..5.1)

It is assured that the game does not has a saddle point. Since A has two strategies, it follows that $X_2 = 1 - X_1$, $X_1 \geq 0$ $X_2 \geq 0$.

His expected payoffs corresponding to the pure strategies of B are given by

B'S PURE STATEGY	A'S EXPECTED PAYOFF
1	$(a_{11}-a_{21})\, x_1 + a_{21}$
2	$(a_{12}-a_{22})\, x_1 + a_{22}$
3	$(a_{13}-a_{23})\, x_1 + a_{23}$
:	:
:	:
n	$(a_{ln}\ a_{2n})\, x_1 + a_{2n}$

This shows that A"S average payoff varies linearly with x_1

According to the minimax creterion for mixed strategy games, player A should select the value of x_1 that maximizes his minium expected payoffs. This may be done by plotting the straight line as function of x_1

Example (3. 5. 2)

Consider the following (2 x 4) games.

		1	2	3	4
	1	2	2	3	-1
A	2	4	3	2	6

This game does not have a saddle point. Thus, A's expected payoff's corresponding to B's pure strategies are given as follows

B's Pure Strategies	A's expected payoffs
1	$-2x_1 + 4$
2	$-x_1 + 3$
3	$x_1 + 2$
4	$-7\, x_1 + 6$

The table is constructed using the example 3. 5. 1. above.

The function A_1, A_2, A_3 and A_4 are linear in x_1, their graph are shown below in figure 1.

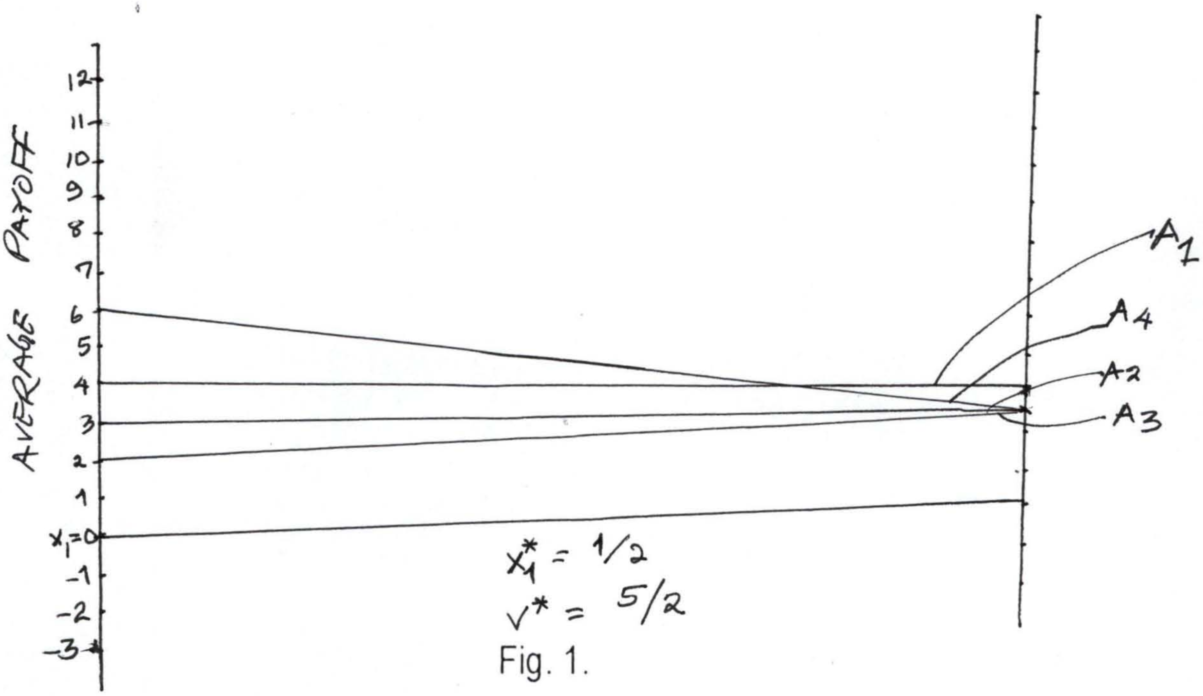


Fig. 1.

The horizontal axis represents the x_1-axis and the payoffs to the row players is represented by the vertical axis.

Taking the point of view of the row player for each choice of x_1 the value $A_1(x_1)$, $A_2(x_1)$ $A_3(x_1)$ and $A_4(x_1)$ represent respectively, the expected payoffs when the strategy $B(x_1) = (x_1, 1 - x_1)$ is played against each of the four pure strategies of the column player. If we take $x_1 = ½$, then $A_1(½) = 3$ $A_2(½) = 2½$, $A_3(½) = 2½$, $A_4(½) = 2½$ as seen in graph above (fig 1)

If the row player uses strategy $B(½) = (½,½)$ by using strategy A_2, A_3 and A_4 i.e. the combinations (2, 3) , (2, 4) and (3, 4) every time column player can retain or hold the expected payoff to the player.

The combination (2, 4) must be excluded as non-optimal. Also the value of the a game $V^* = A_2(x_1^*) = A_3(x_1^*) = A_4(x_1^*) = 5/2$

i.e. $A_2(½) = A_3(½)$ $A_4(½) = 5/2$.

The method discussed here can only work for any two–person zero-sum game in which one player has only two pure strategies. The discussion can be use to serve as guide for solving game problems with playoff matrices which are 2 x n.

To solve a game which has an m x 2 playoff matrix, we simply interchanges the rows and columns of the playoff matrix (finding the transpose of the original payoff matrix), next multiply each entry by –1 and solves the new game by using the method discussed above. The new game is now 2 x m. if S and t are optimal for the new game and if the value V,
Then t* = S and S = t and V = -V give a solution of the original game.

Let us consider a particular game to illustrate the above assertion:
Example 3. 5. 3. find a solution for two – person Zero – sum game with the payoff matrix

$$P = \begin{pmatrix} -1 & 2 \\ 3 & -2 \\ 2 & -1 \end{pmatrix}$$

Solution:

We begin the solution by multiplying each entry in the payoff matrix P by –1 after interchanging the rows and columns or finding the transpose of the payoff matrix. This gives a new payoff matrix.

$$P = \begin{bmatrix} 1 & -3 & -2 \\ -2 & 2 & 1 \end{bmatrix}$$

Using the method in 3. 5. 1.

$P_1(x_1) = 3x_1 - 2$, $P_2(x_1) = -5x_1 + 2$, $P_3(x_1) = -3x_1 + 1$

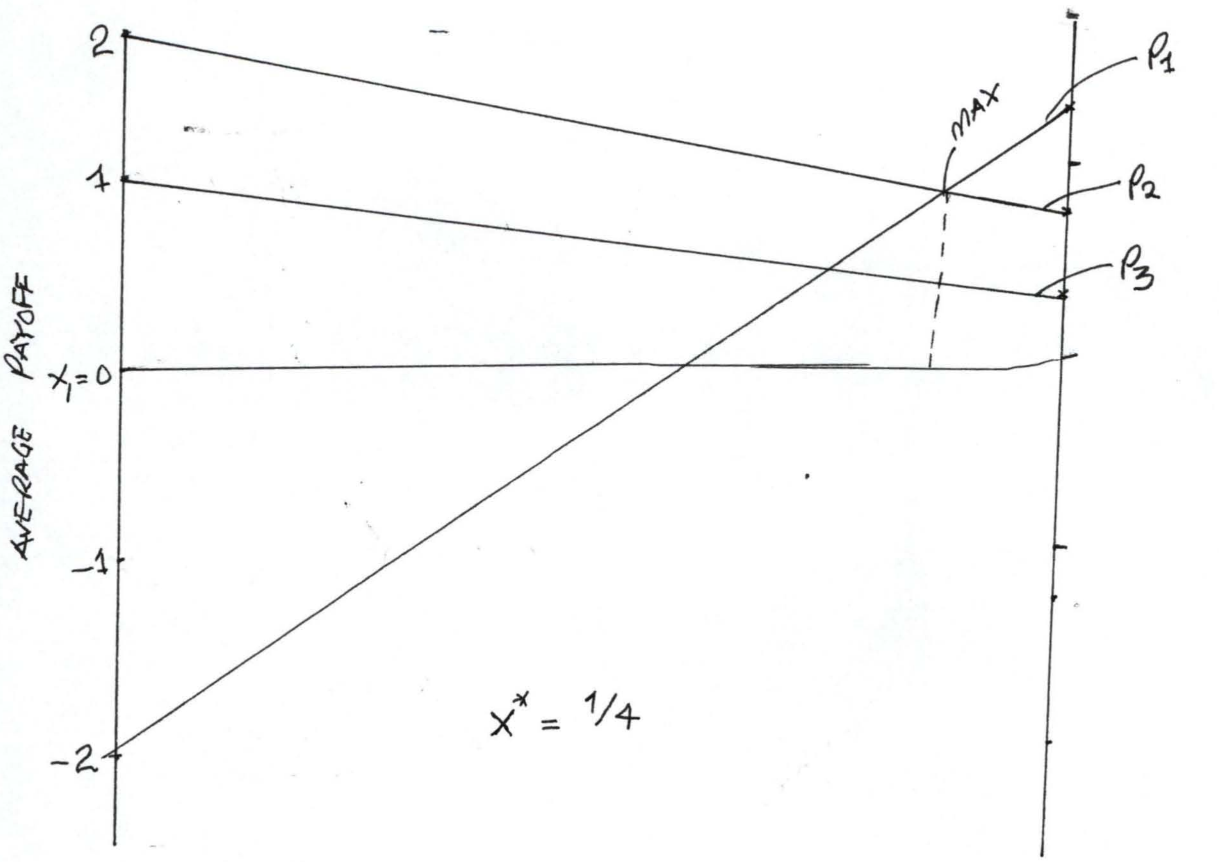


Fig. 2

The lines in the graph meet at the point (¼, ¼) for which x_1 = ¼.

An optimal strategy for the row player in the new game S = (¼, ¾). The line which bounds the graph in fig 2 from below corresponds to strategies 1and 2 and the method outlined in fig 1 are used and optional strategy for the column player is given by t = (5/8, 3/8, 0)

i.e. from matrix. $\begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix}$

$x_1 = 5/8 \Rightarrow t = (x_1, (1 - x_1), 0)$

i.e. t =(5/8, 3/8, 0).

The value of the new game is V = - V = - P_1 (x_1) = 5/4.

A solution of the original 3 x 2 game is given by

$S^* = (5/8, 3/8, 0)$, $t^* = (¼, ¾)$

$V = 5/4$.

Example 3. 5. 4.

Consider the following (4 x 2) games.

		B 1	2
	1	2	4
A-	2	2	3
	3	3	2
	4	-2	6

The game does not have a saddle point.

Let y_1 and $y_2 = 1 - y_2$ be B's mixed strategy.

Thus:

A's Pure strategy	B's expected payoff
1	$-2y, + 4$
2	$-y, + 3$
3	$y, + 2$
4	$-8y, + 6$.

At $y^*, = 2/3$ => $A_1 (2/3) = 8/3$ $A_2 (2/3) = 7/3$, $A_3 = 8/3$ and $A_4 (2/3) = 2/3$

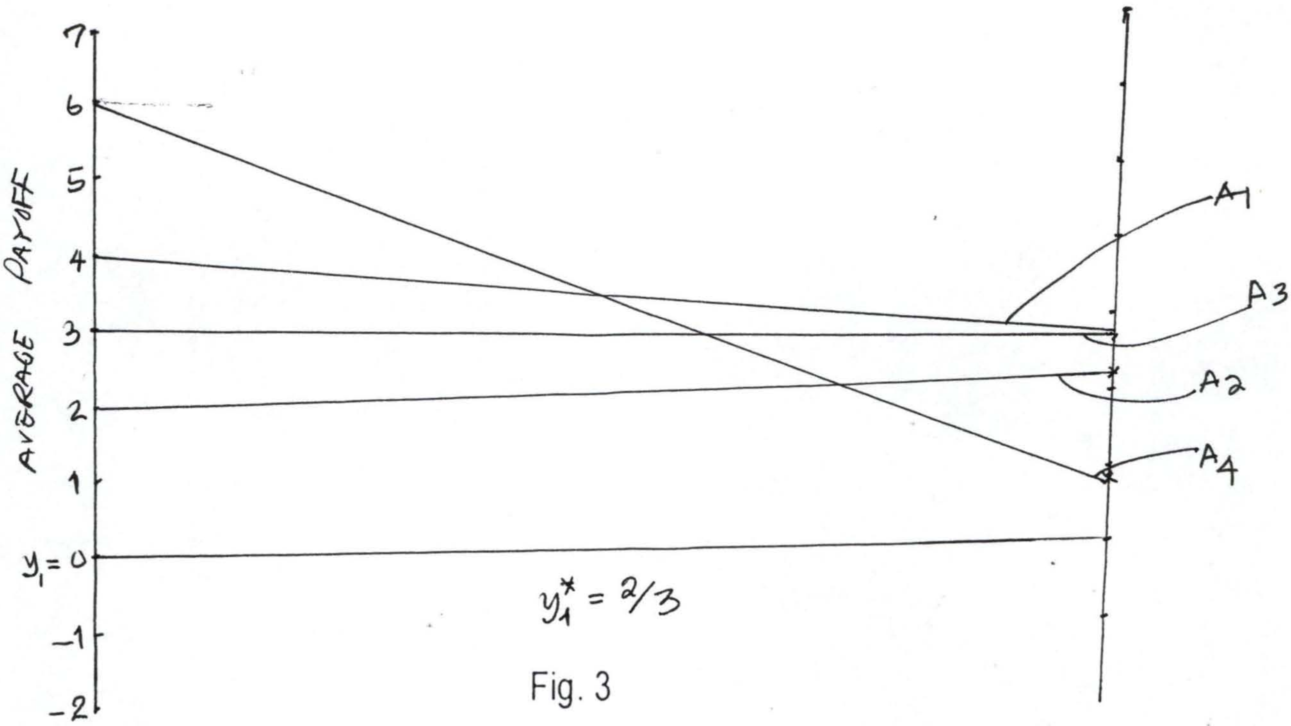


Fig. 3

The lines intersecting at minmax point correspond to A's pure strategy 1 and 3. This indicates $x^*_2 = x^*_4 = 0$ consequently $x_1 = 1 - x_3$ and A's average payoffs corresponding to B's pure strategies are

B's Pure strategy	A's Expected Payoff
1	$-x_1 + 3$
2	$2x_1 + 2$

i.e. $(a_{11} - a_{31})\, x_1 + a_{31}$ and $(a_{12} - a_{32})\, x1 + a_{32}$

The point x^*_1 is determined by solving

$$-x^* + 3 = 2x^*_1 + 2$$

This gives $x^* = 1/3$.

Thus, A's optional strategies are:

$$x^*_1 = 1/3,\ x^*_2 = 0,\ x^*_3 = 2/3,\ \ x^*_4 = 0$$

This yields $V^* = 8/3$.

3.6 SOLUTION OF (M x N) GAMES BY LINEAR PROGRAMMING:

Game theory bears a strong relationship to linear programming, since every finite two-person Zero-sum game can be expressed as a linear programming problem and conversely, every linear programming problem can be represented as a game.

In fact, G. Dantzig states (1963), P.24) that J. Von Neunann, father of game theory when first introduced to the simplex method of linear programming (1947), immediately recognized his relationship and further pin pointed and stressed the concept of durability in linear programming. If the maximum value of the game is non-negative the value of the game is greater than Zero (provided that the game has no saddle point).

Thus, assuming that $V > 0$ the constraints of the linear programming become:

$$a_{11} x_1/v + a_{21} x_2/v + \dots + am_1 x_m/v > 1$$

$$a_{12} x_1/v + a_{22} x_2/v + \dots + am_2 x_m/v > 1$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$a_{1n} x_1/v + a_{2n} x_2/v + \dots + a_{mn} x_m/v \geq 1$$

$$x_1/v + x_2/v + \dots + x_m/v = 1/v$$

Let $x_i = x_i/v$, $i = 1, 2, \dots m$

Since $\max v = \min 1/v = \min (x_1 + x2 + \dots + xm)$ the problems becomes

Minimize $Z = x_1 + x_2 + \dots + xn$

Subject to

$$a_{11} x_1 + a_{21} x_2/v + \dots + am_1 x_m \geq 1$$

$$a_{12} x_1/v + a_{22} x_2/v + \dots + am_2 x_m/v \geq 1$$

$$\vdots \quad \vdots \quad \vdots \quad \vdots$$

$$a_{in} x_1/v + a_{2n} x_2/v + \dots + amn\, x_m/v \geq 1$$

$$x_1/v + x_2/v + \dots + x_m/v = 1/v$$

Let $xi = xi/v$, $i = 1,. 2, \ldots m$

Since $\text{Max } v = \min 1/v = \min \{x_1 + x_2 + \ldots + x_m\}$

The problem becomes

$$\text{Minimize } z = x_1 + x_2 + \ldots + x_n$$

Subject to

$$a_{11}x_1 + a_{21} x_2 + \ldots + a_m x_m \geq 1$$

$$a_{12} x_1 + a_{22} x_2 + \ldots + am\, xm \geq 1$$

$$a_{1n} x_1 + a_{2n} x_2 + \ldots + a_{mn} xm \geq 1$$

$$x_1, \; x_2 \quad \ldots, x_m \geq 0$$

Player B's problem is given by

$$\min \{\max \sum_{j=1}^{n} aij\, yj, \; \sum_{j=1}^{n} a2j\, yj, \ldots \sum_{j=1}^{n} amj\, yj)$$

subject to

$$y_1 + y_2 + \ldots + y_n = 1$$

This can also be expressed as a linear programming as follows:

$$\text{Maximize } w = y_1 + y_2 + \ldots + y_n$$

Subject to

$$a_{11}y_1 + a_{21} y_2 + \ldots \quad + a_{1n} y_n \leq 1$$

$$a_{21} y_1 + a_{22}y_2 + \ldots \quad + a_{2n} y_n \leq 1$$

:

:

$$a_{1m} y_1 + a_{2m} y_2 + \ldots \quad + a_{mn} y_n \leq 1$$

$$y_1, y_2, \ldots, y_n \geq 0$$

where

$w = 1/v$, $yi = yi/v$, $i = 1,2, \ldots n$

Thus B's problem is actually the dual of A's problem. Thus the optimal solution of one problem automatically yields the optimal solution to the other.

3.7 SIMPLEX METHOD:

simplex method can be used to solve MxN game as earlier mentioned. The basic idea is simply the first step, is the conversion to two-person zero-sum game into a Linear programming problem (LLP), written as a standard maximum problem (SMP).

The standard maximum problem (SMP) and its dual provides the optimal strategies for the column and row player respectively. Simplex method also yields the value of the game, the steps in using simplex method are outlined below.

1. Convert the payoff matrix with all positive entries adding the same positive constant C to each. Denote the new payoff matrix say Pc.
2. Use the simplex method to solve the standard maximum problem SMP (Pc, c, r) where s, r are vectors with each coordinate having value 1. The number of coordinates of S is the same as the number of columns of Pc.
3. Let the optimal vector obtained for the SMP (Pc, S, r) be $x = (x_1, x_2, \ldots x_n)$ and the optimal vector for the dual be
   $Z = (Z_1, Z_2, \ldots\ldots Z_m)$ and let $M = x_1 + x_2 \ldots + x_n$
   Since SMP (Pc, S, v) maximizes $x_1 + x_2 + \ldots + x_n$
   and the dual problem minimizes $Z_1 + Z_2 + \ldots + Z_m$
   i.e. $M = Z_1 + Z_2 + \ldots\ldots + Z_m$
4. An optimal strategy for the row player is the game with payoff matrix P optimal strategy for the column player is $(X_1/m, x_2/m, \ldots\ldots x_n/m)$

The value of the game is (1/m) - K

Let us consider an example 3.6.1 to illustrate the above algorithms.

Solve the two-person zero-sum game with payoff matrix.

$$P = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Solution:

From observation the payoff matrix P of the game does not have a saddle point so it cannot be reduced in size by dominance.

Using simplex method, we have to convert the payoff matrix with all positive entries by adding the same suitable constant to every entry.

If we add 2 to every entry in P1, this is because the least entry is –1. So we obtain P2 which has all positive entries.

$$P2 = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

the next step, we consider the SMP (Pc, S,r) where S and r are vectors with all coordinates equal to 1. The number of coordinates for s, also equals the number of coordinates for r P2 is 3x3 matrix, we have

$$S = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad r = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

the coordinate of s and r is obtained from the algorithm (2) section 3.7.

The 3rd step is to solve the SMP (Pc, s,r) using the simplex method.

Tab. 1

Basic	X	Y	Z	W1	W2	W3	Basic Solution
W1	3	2	2	1	0	0	1
W2	1	3	1	0	1	0	1
W3	2	1	3	0	0	1	1
P	**-1**	**-1**	**-1**	**0**	**0**	**0**	**0**

The row which contain the number inscribe in circle is known as the main row. While the number 3 is called unit PIVOT (after dividing row through by 3).

Tab 2

Basic	X	Y	Z	W1	W2	W3	Basic Solution
X	1	2/3	2/3	1/3	0	0	1/3
W2	0	7/3	1/3	-1/3	1	0	2/3
W3	0	-1/3	5/3	-2/3	0	1	1/3
P	**0**	**-1/3**	**-1/3**	**1/3**	**0**	**0**	**1/3**

The least row is the index row with the column that contain 7/3 as the key column.

Tab. 3

Basic	X	Y	Z	W1	W2	W3	Basic Solution
X	1	0	4/7	9/21	-2/7	0	1/7
Y	0	1	1/7	-1/7	3/7	0	2/7
W3	0	0	-12/7	-5/7	1/7	1	3/7
P	**0**	**0**	**-2/7**	**2/7**	**1/7**	**0**	**3/7**

Table 4:

Basic	X	Y	Z	W1	W2	W3	Basic Solution
X	1	1	0	8/7	-1/3	1/3	0
Y	0	0	0	-1/12	5/12	-1/12	¼
Z	0	1	1	-5/12	1/12	-7/12	¼
P	0	0	0	1/6	1/6	1/6	½

Since there is no negative entry in the last row of the tableans 4. The SMP (P2, S,r) and its dual have been solved.

An optimal vector for the SMP (P2, S, r) is (x, y, z) = (0, ¼, ¼) and an optimal vector for the dual problem is (W1, W2, W3) = (1/6, 1/6. 1/6).

The relationship between these optimal vectors and optimal strategies for the player of the game with the payoff matrix A is as follows:

1. Taking M to be the sum of the coordinates of the optimal vector for the SMP (P2, S, r).

   M = 0 + ¼ + ¼ = ½

   An optimal strategy for the column player is

   t = (0/m, 1/4/m, 1/4/m) = (0, ½, ½).

   and an optimal strategy for the row players.

   L = (1/6/m, 1/6/m, 1/6/m) = (1/3, 1/3, 1/3)

2. The value of the game V = 1/m – K

   Where K is the constant added to the entries.

Thus

V = 1/½ -2 = 2-2 = 0

The game with the payoff matrix P has the following solution.

(i) The optimal strategy for the row player is the mixed strategy (1/3, 1/3, 1/3).

(ii) The optimal strategy for the column player is the mixed strategy (0, ½, ½).

(iii) The value of the game is 0.

***Example** 3.6.*

P1 and P2 each extend either one, two or three fingers, and the difference in the amounts put forth is computed. If this difference is 1, the player putting forth the smaller amount wins 1, and if the difference is 2, the player putting forth the larger amount wins 2. Each player has 3 pure strategies.

Let Si denote P1's pure strategy of extending I fingers, $1 \leq i \leq 3$ and similarly define tj, $1 \leq j \leq 3$ for P2.

The payoff tableau is thus:

	t1	t2	t3
S1	0	1	-2
S2	-1	0	1
S3	2	-1	0

By symmetry it is reasonable to expect the value of this game to be zero.

To verify this and compute optimal strategies, we first add 2 to each entry of the above matrix, giving the following matrix which corresponds to a game with value at least 1 as all the entries in the last two rows are greater than or equal to 1.

$$\begin{bmatrix} 2 & 3. & 0 \\ 1 & 2 & 3 \\ 4 & 1 & 2 \end{bmatrix}$$

The associated Linear Programming Problem (LPP) corresponding to P2's determination of an optimal strategy and security level is to

Max $Y'_1 + y'_2 + y'_3$

Subject to

$2y'_1 + 3y'_2 \leq 1$

$y'_1 + 2y'_2 + 3y'_3 \leq 1$

$4y'_1 + y'_2 + 2y'_3 \leq 1$

$y'_1, y'_2, y'_3 \geq 0$

Adding three slack variables and solving leads to the tableaux of

Basic	Y'$_1$	Y'$_2$	Y'$_3$	Y'$_4$	Y'$_5$	Y'$_6$	Basic Solution
Y'$_4$	2	3	0	1	0	0	1
Y'$_5$	1	2	3	0	1	0	1
Y'$_6$	4	1	2	0	0	1	1
	-1	**-1**	**-1**	**0**	**0**	**0**	**0**
Y'$_4$	2	3	0	1	0	0	1
Y'$_3$	1/3	2/3	1	0	1/3	0	1/3
Y'$_6$	10/3	-1/3	0	0	-2/5	1	1/3
	-2/3	**-1/3**	**0**	**0**	**1/3**	**1**	**1/3**
Y'$_2$	2/3	1	0	1/3	0	0	1/3
Y'$_3$	-1/9	0	1	-2/9	1/3	0	1/9
Y'$_6$	32/9	0	0	1/9	-2/3	1	4/9
	-4/9	**0**	**0**	**1/9**	**1/3**	**0**	**4/9**
Y'$_2$	0	1	0	5/16	1/8	-3/16	¼
Y'$_3$	0	0	1	-7/32	5/16	1/32	1/8
Y'$_1$	1	0	0	1/32	-3/16	9/32	1/8
	0	**0**	**0**	**1/8**	¼	**1/8**	½

The value of the modified game is 2 and so the value of the original game is). (i.e. V=(1/m) – K).

Since the optimal value of the above problem is attained at (y'1, y'2, y'3) = (1/8, ¼, 1/80 and optimal strategy of P2, is 2(1/8, ¼, 1/80 = (1/4, ½, ¼).

Similarly, the solution to the dual problem is find in the bottom row is the slack variable columns is (y'4, y'5, y'6) = (1/8, ¼, 1/8) and so an optimal strategy for P1 is also 2(1/8, ¼,1/8) = (1/4, ½, ¼).

3.8 SOLVING GAMES USING APPROXIMATIONS METHOD

By modeling the learning process for two players we obtain a very basic (if slowly converging) approximation method. This can be illustrated with the following example

A farm has three implements for the same operation which differ in their suitability depending on soil conditions. The suitability (which may be expressed in operating days per week for the persons year) is shorn as a function of soil conditions in the folding table matrix A.

P_1\ P_2	DRY	NORMAL	WET
Implements			
I	1	0	2
II	3	0	0
III	0	2	1

Player P1 is the farm, Player P2 the weather and the game itself belongs in the category of how often we can expect to use each, if the three implements and what 'action' nature may take.

Approximation procedure consists of the following algorithms.

1. P1 arbitrary selects a row from matrix A, for example the first row, and writes it under the matrix.

2. P2 then chooses that column of matrix A which contains the lowest figure in that row (column 2) and writes it next to the matrix..

3. P1 selects the row that has the highest figure in it (the third row) and writes the sum of the last and then newly chosen row under the matrix.

4. P2 again chooses the column which contains the lowest figure in that row and writes the sum of the last and the new row next to the matrix.

	A			(2)	(4)	(6)	(8)	(10)	
	1	0	2	0	1	1	3	4	X1= 0/5
	3	0	0	0	[3]	3	3	[6]	X2= 2/5
	0	2	1	[2]	2	[4]	[5]	5	X3= 3/5
(1)	1	[0]	2						
(3)	[1]	2	3						
(5)	4	[2]	3						
(7)	4	4	[4]						
(9)	[4]	5	6						
	y1= 2/5	y2= 2/5	y3= 1/5						

The procedure continues, and if the smallest figure in a column or the highest figure in a row occurs more than once, an arbitrary choice is made. In addition, the figure selected each time are marked in □ when searching for the next row or column.

Approximation for the optimal strategy are obtained when this algorithm is followed. Count the □ figures in each of the three rows (next to A) formed by the columns and in each of the columns (under A) formed by the rows and divided by the number of steps. For example three figures are marked in the third row (2,4,5) and the number of steps is five, so that $x3 = 3/5$.

The value of the game, v is contained between the last marked figure in the columns (4 in the example) and the last marked figure in the rows (6 in the example) divided by the number of steps in each case.

The total result obtained after five steps for the optimal strategies x0 and y0 and for the value of the game, v is

$x^T = (0, 2/5, 3/5)$, $y^T = (2/5, 2/5\ 1/5)$

is $4/5 = v$, $6/5$

This becomes more practical after ten steps

$X^T = (1/10, 3/10, 6/10)$, $y^T = (3/10, 4/10, 3/10)$;

$9/10 \leq v \leq 11/10$

The last solution tells us that the implements will be used in the ratio of about 1;3:6 and the options of nature (dry – normal-wet) are about 3:4:3.

The conclusion is that implement number 3 should be ready for use at all times if possible. The value of the game is only symbolic.

CHAPTER FOUR

4.0 PRESENTATION AND ANALYSIS:

4.1 INTRODUCTION:

Computer facilitates the direct solution of many games problems, particularly the two-person zero-sum games. In this chapter consideration would be given to one of the languages of the computers in the application of two-person zero-sum games to data analysis.

Computer program for the solution of two person zero-sum game will be drawn to enhance faster operations and admiration of the computer would in the area of games.

4.2 SOFTWARE/PROGRAM IMPLEMENTATION AND DEVELOPMENT:

Software is the general term used to denote all forms of program that control the activities of a computer.

It refers to the set of computer programs procedures and associated documentation related to the effective operation of data processing system. Software enables us to exploit the capabilities of a computer. In this project a software that compute the value of a game using the 2 x 2 game solution method and simplex method is developed.

The implementation of the program will entails the process of coding, testing and documenting of the system. It involves development of quality assurance procedures, including data security backup and recovery and system controls. It also involved testing program with both artificial and live data and training users and operating personnel.

The development of the software involves the use of all the parameters in a system development cycle. Before the software is developed, the problem to be programmed is defined. For example the program as seen in appendix 1 is to compute the value of a game using the 2 x 2 solution method, while appendix 4 shows the computation of optimal strategy for row as well as column player using simplex method.

Feasibility study to determine whether a solution to the problem consider would be attained, chapter three of this project shows some problems and the solution, this is aimed at maximum user of the time/effort so that wastage is not done; equally exploration of alternative design options are made for problems whose solutions are not attainable.

Where there exists problems, a full study is undertaken to know the how/why the problem occurs and how to resolve the problem.

This analysis will lead to the use of both manual and computerized elements so that specification would be attained, with high quality of correctness, understandability, maintainability efficiency, portability, cost effectiveness, user-friendly and reliable.

4.3 FEATURES OF THE SOFTWARE:

The language used to develop the software in this project is BASIC an acronym for Beginners All-purpose Symbolic Instruction code, developed at Dartmouth College in 1963.

Basic is a high level language designed for people who have no prior programming experience and is one of the easiest of all programming languages to learn. It is widely used in programming scientific, mathematical and many business problems.

It encourages running the computer in an interactive mode. As soon as the user submits a program and some data to the computer, the computer executes the program; produces the result back to the user immediately.

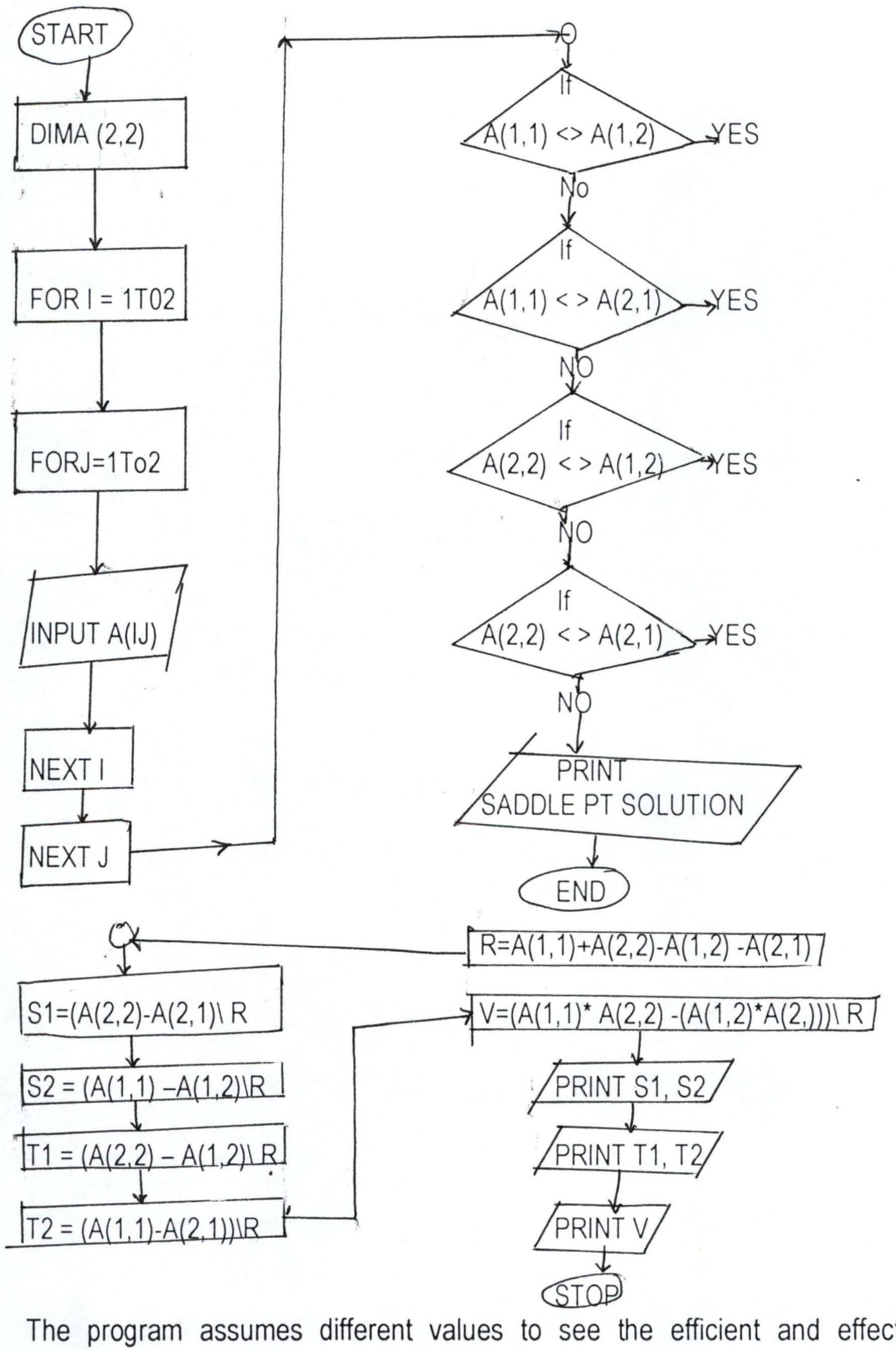
It is easy for the user to find out whether the program is working properly or there is a bug. It is English structured – like.

4.4 SYSTEM IMPLEMENTATION:

This stages of the program is concerned with making it fully operational. That is, it involves applying the program to solve the problem it is meant to solve.

The program in appendices 1 and 3 shown the details on the working as required to compute the value of a game using 2 x 2 game solution method and also using simplex method to find an optimal strategy for both Row and column player.

The flow chart below shows the pictorial representation of the procedure for solving the value of a game using the 2 x 2 game solution method.



The program assumes different values to see the efficient and effective working condition of the program using the method as in appendix 1 attached.

4.5 MAIN MENU

This stage describes the program in the proper form for users and to enhance maintainability. It describes the workings of the program and how expected problems could be solved.

The main menu in this program is a form of documentation internal in the program which exist I the form of comments. The program in appendix 1 and 3 contains internal documentation on what the program is intended to performed.

4.6 PROGRAM MAINTENANCE:

This includes whatever changes and enhancement needed to be made after the system is up and running. The program in this project allows and accepts values while the program is running.

CHAPTER FIVE

5.0 SUMMARY, DISCUSSIONS AND RECOMMENDATION:

5.1 PROGRAM DOCUMENTATION /LIMITATION:

The program in this project compute solution of game in particular, the two-person zero-sum games. The program uses the 2 x 2 game solution method to compute the optimal strategy for the row/column player and the value of the game.

The program uses the procedure of solving game problems as explained in chapter three of this project which was manually solve, then programmed using one of the computer programming language 'BASIC'

Values are tested after ensuring there is no any error in the programs, as seen in the output of the program the test explain in details the working of the program as required.

To expand the program, also the programs adjusted, So that it accept different values, so that Read /Data statement is replace with input statement and values are supplied as demanded.

The program also have embedded internally remarks (Documentation) for the understanding of the users. The user here knew whatsoever he is suppose to do at a particular point or what the program is about to output.

The program in appendix 4 uses the simplex method to generate solution for m x n matrix game.

The program is limited to two-person zero-sum game using 2 x 2 game solution method and simplex method. It doesn't work for problems or games outside this range. The output shows the optimal strategy for Row/Column player and the value of the game.

5.2 **SUMMARY**:

Game is a solution of conflict between two or more people, in which each contestant, player or participant has some, but not total control over the outcome of the conflict.

Game theory is the logical analysis of the situations of conflict and co-operations; specially, a game is refers to any situation in which there are at least two players, A player may be an individual, but it may also be more general entity like a company, a nation, or even a biological species. Each player has a number of possible strategies course of action which he or she may choose to follow; The strategy chosen by each player determine the outcome of the game is collection of numerical playoffs, one of each player. These payoff represent the value of the outcome to the different player.

This research project view the two-person zero-sum game a subset of the game theory. The basic players here are two only, that is it involves only two people who act on the understanding that the gains made by one player are equal to his losses. Gains and losses may be measured in sums of money and the total amount paid out in this case will be zero – This game is also known as matrix games, since they can be fully described by the matrix.

The graphical, simplex and approximation methods are used to compute two person zero sum games in this project also a computer program using 2 x 2 game solution method and the simplex method was drawn for the understanding of the game.

5.3 DISCUSSIONS:

The general problem of how to make decision in a competitive environment is very common and important one. The fundamental contribution of game theory is that it provides a basic conceptual framework for formulating and analysing such problem in simplex situations. However there is considerable gab between what the theory can handle and the complexity of most competitive situations arising in practice. Therefore it usually play just a supplementary role in dealing with these situations.

There are huge rewards systems at work and computer games can be constructed as satisfying or attempting to satisfy four central human needs. Firstly, there is the ever-present need for competitive – challenge needs to exert control over the unpredictable, the ever-present desire to transcend limitations, to enter into disguise, to indulge in fantasy and lastly and perhaps most importantly is the human desire after every day consciousness.

With all this achievements in the findings of situations in games, it is observed that games in particular the two-person zero-sum games if properly applied is the best tool for decision making

5.4 RECOMMENDATION:

In line with the research findings the researcher wishes to recommend that the federal government makes concerted effort to integrate game theory into the school mathematics/statistics and computer sciences curriculum. That is to say additional mathematics skills need to be incorporated in to the curriculum so as to provide students with basic skills they need in our modern society.

There is the need to train and retrain teachers in various strategies of computer- aided instruction in the use of game theory. To this end, the government should organize seminars, workshops and in- service courses to supply teachers with more information about games strengths and weaknesses Finally, further research should be carried out or is required in the subject matter for this academic and other discipline for this would facilitate the use of two- person zero- sum game in decision making process,.

REFERENCES

BURLEY,T. A & G.O. SULLIAV(1986): " Workout operational research" Macmillian Education Ltd. London

FEDERICK S. HILLER &GERALD J.LIEBERMAN(1986): 'Introduction to operations Research 4th ed'. Holden- Day Inc Oakland- California.

GULLET W. etal (1989):' The UNR concise encyclopedia of Mathematics 2nd ed'. New York.

GRATTAN- GUINESS (1994): 'Companies Ency clopped of the history & philosophy of Mathematical Sciences.' Mathematical Games, Recreational Mathematics vol ii pg 1555 New falter lane, London .

HAMDY A. TAHA (1987) Operations Research: An introduction 4th ed.' Macmillan Publisher New York

KAPLAN, E.L (1982); "Mathematical programming and game.' Wiley, New York.

OWEN, GUILLERMON(1982):' Game theory 2nd ed' Academic Press- New York.

PAUL R. THE (1988): 'An Introduction to linear Programming and Game theory 2nd ed Joh wiley & Sons Inc-Canada

PHILIP D. STRAFFIN (1993): "Game theory and Strategy Mathematical Association of America. USA

RICHARD BROWSON (1982)' Schaums outline of Theory & Process of operation Research. Mc Graw Hill Inc.

WAYNE L. WINSTON etal (1989)' Operations Research: Application & Algorithm 2nd ed.' Pws- Uent – Boston USA

WINFIELD. I.(1986)' Human Research and computing.' William Heinemann Ltd. London.

APPENDIX 1

```
10 CLS
20 REM: THIS PROGRAM COMPUTES THE VALUE OF A GAME
30 REM: USING THE 2*2 GAME SOLUTION METHOD.
40 DIM A(2, 2)
45 FOR i = 1 TO 2
50 FOR j = 1 TO 2
60 INPUT "ENTER VALUES FOR A (i,j)": A(i, j)
70 NEXT j: NEXT i
75 CLS
80 IF A(1, 1) <> A(1, 2) AND A(1, 1) <> A(2, 1) AND A(2, 2) <> A(1, 2) AND A(2, 2) <> A(2, 1) THEN 190
150 PRINT "SADDLE POINT SOLUTION"
160 END
170 REM *** OPTIMAL STRATEGY FOR
180 REM THE ROW PLAYER ***
190 LET R = A(1, 1) + A(2, 2) - A(1, 2) - A(2, 1)
200 S1 = (A(2, 2) - A(2, 1)) / R
210 S2 = (A(1, 1) - A(1, 2)) / R
220 REM *** OPTIMAL STRATEGY FOR THE
240 REM COLUMN PLAYER ***
250 T1 = (A(2, 2) - A(1, 2)) / R
260 T2 = (A(1, 1) - A(2, 1)) / R
280 V = ((A(1, 1) * A(2, 2)) - (A(1, 2) * A(2, 1))) / R
290 PRINT TAB(10); STRING$(65, "*"): PRINT
300 PRINT TAB(22); "output generated for the solution of a game"
310 PRINT TAB(30); "having a 2*2 payoff matrix"
320 PRINT TAB(10); STRING$(65, "*"): PRINT
330 PRINT : PRINT
340 PRINT TAB(15); "1. the optimal strategy for the row player is"
350 PRINT TAB(20); "S = ("; S1; ","; S2; ")": PRINT
360 PRINT TAB(15); "2. the optimal strategy for a column players is"
370 PRINT TAB(20); "T = ("; T1; ","; T2; ")": : PRINT
380 PRINT TAB(15); "3.The value of the game="; V: PRINT
390 PRINT TAB(10); STRING$(65, "*"): PRINT
400 FOR i = 1 TO 2: FOR j = 1 TO 2: PRINT A(i, j); : NEXT j: PRINT : NEXT i
410 END
420 REM ******************
430 REM SUBROUTINE FOR OUTPUT TO THE PRINTER
440 REM ******************
450 LPRINT : LPRINT : LPRINT
460 LPRINT TAB(10); STRING$(65, "*"): LPRINT
470 LPRINT TAB(22); "output generated for the solution of a game"
480 LPRINT TAB(30); "having a 2*2 payoff matrix"
490 LPRINT TAB(10); STRING$(65, "*"): LPRINT
500 LPRINT : LPRINT
510 LPRINT TAB(15); "1.The optimal solution for the row player is"
520 LPRINT TAB(20); "S" = "("; S1; ","; S2; ")": LPRINT
530 LPRINT TAB(15); "2. The optimal solution for the column player is"
540 LPRINT TAB(20); "T" = "("; T1; ","; T2; ")": LPRINT
550 LPRINT TAB(15); "3. The value of the game="; V: LPRINT
560 LPRINT TAB(10); STRING$(64, "*"): LPRINT
570 RETURN
```

PROGRAM OUT PUT 1

****

output generated for the solution of a game
having a 2*2 payoff matrix

****

1. the optimal strategy for the row player is
S = (-2 , 3)

2. the optimal strategy for a column players is
T = (2 ,-1)

3.The value of the game= 0

****

3 6
2 4

Press any key to continue

****

output generated for the solution of a game
having a 2*2 payoff matrix

****

1. the optimal strategy for the row player is
S = (.5833333 , .4166667)

2. the optimal strategy for a column players is
T = (.5833333 , .4166667)

3.The value of the game=-8.333334E-02

****

2 -3
-3 4

Press any key to continue

****

output generated for the solution of a game
having a 2*2 payoff matrix

****

1. the optimal strategy for the row player is
S = (-.25 , 1.25)

2. the optimal strategy for a column players is
T = (1.666667 ,-.6666667)

3.The value of the game= 0

****

y key to continue

E POINT SOLUTION

DATA → $\begin{bmatrix} 2 & 5 \\ 2 & 5 \end{bmatrix}$

ıy key to continue

APPENDIX 2.

```
10 CLS
20 K = 2
30 DIM A(8, 9)
40 FOR I = 1 TO 8
45 FOR J = 1 TO 9
50 READ A(I, J)
60 NEXT J: NEXT I
70 REM:COMPUTATION OF OPTIMAL STRATEGY FOR COLUMN PLAYER
80 A1 = A(1, 1) * (A(1, 5) * A(1, 9) - A(1, 8) * A(1, 6))
90 A2 = A(1, 2) * (A(1, 4) * A(1, 9) - A(1, 7) * A(1, 6))
100 A3 = A(1, 3) * (A(1, 4) * A(1, 8) - A(1, 7) * A(1, 5))
110 MAN = A1 - A2 - A3
120 X1 = A(2, 1) * (A(2, 5) * A(2, 9) - A(2, 8) * A(2, 6))
130 X2 = A(2, 2) * (A(2, 4) * A(2, 9) - A(2, 7) * A(2, 6))
140 X3 = A(2, 3) * (A(2, 4) * A(2, 8) - A(2, 7) * A(2, 5))
150 T1 = (X1 - X2 + X3) / MAN
160 X4 = A(3, 1) * (A(3, 5) * A(3, 9) - A(3, 8) * A(3, 6))
170 X5 = A(3, 2) * (A(3, 4) * A(3, 9) - A(3, 7) * A(3, 6))
180 X6 = A(3, 3) * (A(3, 4) * A(3, 8) - A(3, 7) * A(3, 5))
190 T2 = (X4 - X5 + X6) / MAN
200 X7 = A(4, 1) * (A(4, 5) * A(4, 9) - A(4, 8) * A(4, 6))
210 X8 = A(4, 2) * (A(4, 4) * A(4, 9) - A(4, 7) * A(4, 6))
220 X9 = A(4, 3) * (A(4, 4) * A(4, 8) - A(4, 7) * A(4, 5))
230 T3 = (X7 - X8 + X9) / MAN: W = T1 + T2 + T3
240 T11 = T1 / W: T22 = T2 / W: T33 = T3 / W
250 REM:COMPUTATION OF OPTICAL STRATEGY FOR ROW PLAYER
260 A4 = A(5, 1) * (A(5, 5) * A(5, 9) - A(5, 8) * A(5, 6))
270 A5 = A(5, 2) * (A(5, 4) * A(5, 9) - A(5, 7) * A(5, 6))
280 A6 = A(5, 3) * (A(5, 4) * A(5, 8) - A(5, 7) * A(5, 5))
290 MAS = A4 - A5 + A6
300 Y1 = A(6, 1) * (A(6, 5) * A(6, 9) - A(6, 8) * A(6, 6))
310 Y2 = A(6, 2) * (A(6, 4) * A(6, 9) - A(6, 7) * A(6, 6))
320 Y3 = A(6, 3) * (A(6, 4) * A(6, 8) - A(6, 7) * A(6, 5))
330 S1 = (Y1 - Y2 + Y3) / MAS
340 Y4 = A(7, 1) * (A(7, 5) * A(7, 9) - A(7, 8) * A(7, 6))
350 Y5 = A(7, 2) * (A(7, 4) * A(7, 9) - A(7, 7) * A(7, 6))
360 Y6 = A(7, 3) * (A(7, 4) * A(7, 8) - A(7, 7) * A(7, 5))
370 S2 = (Y4 - Y5 + Y6) / MAS
380 Y7 = A(8, 1) * (A(8, 5) * A(8, 9) - A(8, 8) * A(8, 6))
390 Y8 = A(8, 2) * (A(8, 4) * A(8, 9) - A(8, 7) * A(8, 6))
400 Y9 = A(8, 3) * (A(8, 4) * A(8, 8) - A(8, 7) * A(8, 5))
410 S3 = (Y7 - Y8 + Y9) / MAS
420 S11 = S1 / W: S22 = S2 / W: S33 = S3 / W: V = 1 / W - K
430 PRINT : PRINT : PRINT
440 PRINT TAB(10); STRING$(65, "*"): PRINT
450 PRINT TAB(22); "OUTPUT GENERATED FOR M*N GAME USING"
460 PRINT TAB(35); "SIMPLEX METHOD"
470 PRINT : PRINT
480 PRINT TAB(15); "1.THE OPTIMAL STRATEGY FOR THE ROW PLAYER IS": PRINT
490 PRINT TAB(20); "S="; "("; S11; ", "; S22; ", "; S33; ")": PRINT
500 PRINT TAB(15); "2.THE OPTIMAL STRATEGY FOR THE COLUMN PLAYER IS": PRINT
510 PRINT TAB(20); "T="; "("; T11; ","; T22; ","; T33; ")": PRINT
520 PRINT TAB(15); "3.THE VALUE OF GAME="; V: PRINT
530 PRINT TAB(10); STRING$(65, "*")
540 DATA 3,2,2,1,3,1,2,1,3
550 DATA 1,2,2,1,3,1,1,1,3
560 DATA 3,1,2,1,1,1,2,1,3
570 DATA 3,2,1,1,3,1,2,1,1
580 DATA 3,1,2,2,3,1,2,1,3
590 DATA 1,1,2,1,3,1,1,1,3
600 DATA 3,1,2,2,1,1,2,1,3
610 DATA 3,1,1,2,3,1,2,1,1
620 END
```

PROGRAM OUT PUT 2.

****

OUTPUT GENERATED FOR M*N GAME USING
SIMPLEX METHOD

IE OPTIMAL STRATEGY FOR THE ROW PLAYER IS

=(.8888889 , .8888889 , .8888889)

IE OPTIMAL STRATEGY FOR THE COLUMN PLAYER IS

=(0 , .5 , .5)

IE VALUE OF GAME= 3.333333

****

to continue

PROGRAM OUT PUT 2.

****

OUTPUT GENERATED FOR M*N GAME USING
SIMPLEX METHOD

IE OPTIMAL STRATEGY FOR THE ROW PLAYER IS

=(.8888889 , .8888889 , .8888889)

IE OPTIMAL STRATEGY FOR THE COLUMN PLAYER IS

=(0 , .5 , .5)

IE VALUE OF GAME= 3.333333

****

to continue

APPENDIX 3

```
.S
= 2
M A(8, 9)
)R I = 1 TO 8
)R J = 1 TO 9
RINT "ENTER VALUES FOR", "ROW"; I; "COL."; J: INPUT A(I, J)
EXT J: NEXT I
EM COMPUTATION OF OPTIMAL STRATEGY FOR COLUMN PLAYER
1 = A(1, 1) * (A(1, 5) * A(1, 9) - A(1, 8) * A(1, 6))
2 = A(1, 2) * (A(1, 4) * A(1, 9) - A(1, 7) * A(1, 6))
A3 = A(1, 3) * (A(1, 4) * A(1, 8) - A(1, 7) * A(1, 5))
MAN = A1 - A2 - A3
X1 = A(2, 1) * (A(2, 5) * A(2, 9) - A(2, 8) * A(2, 6))
X2 = A(2, 2) * (A(2, 4) * A(2, 9) - A(2, 7) * A(2, 6))
X3 = A(2, 3) * (A(2, 4) * A(2, 8) - A(2, 7) * A(2, 5))
T1 = (X1 - X2 + X3) / MAN
X4 = A(3, 1) * (A(3, 5) * A(3, 9) - A(3, 8) * A(3, 6))
X5 = A(3, 2) * (A(3, 4) * A(3, 9) - A(3, 7) * A(3, 6))
X6 = A(3, 3) * (A(3, 4) * A(3, 8) - A(3, 7) * A(3, 5))
T2 = (X4 - X5 + X6) / MAN
X7 = A(4, 1) * (A(4, 5) * A(4, 9) - A(4, 8) * A(4, 6))
X8 = A(4, 2) * (A(4, 4) * A(4, 9) - A(4, 7) * A(4, 6))
X9 = A(4, 3) * (A(4, 4) * A(4, 8) - A(4, 7) * A(4, 5))
T3 = (X7 - X8 + X9) / MAN: W = T1 + T2 + T3
T11 = T1 / W: T22 = T2 / W: T33 = T3 / W
REM:COMPUTATION OF OPTICAL STRATEGY FOR ROW PLAYER
A4 = A(5, 1) * (A(5, 5) * A(5, 9) - A(5, 8) * A(5, 6))
A5 = A(5, 2) * (A(5, 4) * A(5, 9) - A(5, 7) * A(5, 6))
A6 = A(5, 3) * (A(5, 4) * A(5, 8) - A(5, 7) * A(5, 5))
MAS = A4 - A5 + A6
Y1 = A(6, 1) * (A(6, 5) * A(6, 9) - A(6, 8) * A(6, 6))
Y2 = A(6, 2) * (A(6, 4) * A(6, 9) - A(6, 7) * A(6, 6))
Y3 = A(6, 3) * (A(6, 4) * A(6, 8) - A(6, 7) * A(6, 5))
S1 = (Y1 - Y2 + Y3) / MAS
Y4 = A(7, 1) * (A(7, 5) * A(7, 9) - A(7, 8) * A(7, 6))
Y5 = A(7, 2) * (A(7, 4) * A(7, 9) - A(7, 7) * A(7, 6))
Y6 = A(7, 3) * (A(7, 4) * A(7, 8) - A(7, 7) * A(7, 5))
S2 = (Y4 - Y5 + Y6) / MAS
Y7 = A(8, 1) * (A(8, 5) * A(8, 9) - A(8, 8) * A(8, 6))
Y8 = A(8, 2) * (A(8, 4) * A(8, 9) - A(8, 7) * A(8, 6))
Y9 = A(8, 3) * (A(8, 4) * A(8, 8) - A(8, 7) * A(8, 5))
S3 = (Y7 - Y8 + Y9) / MAS
S11 = S1 / W: S22 = S2 / W: S33 = S3 / W: V = 1 / W - K
PRINT : PRINT : PRINT
PRINT TAB(10); STRING$(65, "*")
PRINT TAB(22); "OUTPUT GENERATED FOR M*N GAME USING"
PRINT TAB(35); "SIMPLEX METHOD"
0 PRINT
0 PRINT TAB(15); "1.THE OPTIMAL STRATEGY FOR THE ROW PLAYER IS"
0 PRINT TAB(20); "S="; "("; S11; ", "; S22; ", "; S33; ")": PRINT
0 PRINT TAB(15); "2.THE OPTIMAL STRATEGY FOR THE COLUMN PLAYER IS"
0 PRINT TAB(20); "T="; "("; T11; ","; T22; ","; T33; ")": PRINT
20 PRINT TAB(15); "3.THE VALUE OF GAME="; V
30 PRINT TAB(10); STRING$(65, "*")
40 PRINT "THE DATA IS:"
50 FOR I = 1 TO 8: FOR J = 1 TO 9: PRINT A(I, J); : NEXT J: PRINT : NEXT I
60 END
```

PROGRAM OUTPUT 3

****

OUTPUT GENERATED FOR M*N GAME USING
SIMPLEX METHOD

1.THE OPTIMAL STRATEGY FOR THE ROW PLAYER IS
S=(8 , 4 , 8)

2.THE OPTIMAL STRATEGY FOR THE COLUMN PLAYER IS
T=(.5 , 0 , .5)

3.THE VALUE OF GAME= 6

****

IE DATA IS:

```
2 2 1 3 1 2 1 3
2 2 1 3 1 1 3 3
1 1 1 2 1 3 3 3
1 3 1 2 1 1 3 1
2 3 1 1 1 3 1 1
1 3 1 1 1 3 3 1
2 1 1 2 1 3 3 1
2 3 1 2 1 1 1 0
```

ss any key to continue