Influence of Metal Rubber Ring on the Dynamics of the Active Magnetic Bearing System for Optimal Performance

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***Abstract*—A new trend in control of vibration levels of flexible, light weight rotor designs is developed. By adding Metal Rubber Ring (MRR), the flexibility of the bearing system beyond ordinary Active Magnetic Bearing (AMB) system is increased. As a result, effective stiffness and damping properties of the bearing are altered in a manner that can increase the modal damp and help restrain the vibrations. Rotor stability at critical frequencies places specific constraints on the equivalent stiffness and damping parameters of the bearing. An iterative design process comprising aspect of modeling, analysis and optimization is developed to avoid instability while selecting the optimum MRR parameters. The MRR parameters are specified so that the system experiences first two bending critical frequencies occurring below 400 Hz.**

Keywords-AMB;* MRR*; Modal Parameter; Sensitivity Analysis; Optimization

I. INTRODUCTION

The demand for high rotation speed and efficiency of modern rotor dynamic machineries has resulted to highly flexible light weight rotor designs. Compared with the traditional bearings, Active Magnetic Bearing (AMB) possesses numerous mouth-watering advantages such as contactless suspension, active control and so on, which make them a preferred choice as system support [1]. However, it is still difficult to apply the AMB into the actual flexible rotor system, because the supporting damp is so light that the vibration of the rotor is not easy to be restrained, especially when the system operates near or over the bending critical speeds.

Approaches based on unbalanced compensation and modern control theory had been adopted in attempt to providing solution to this problem, but the control strategy is too complicated for practical use.

To provide the additional damping yet maintaining the stability and simplicity of the system, Metal Rubber Ring (MRR) is sandwiched into conventional AMB by our research group to increase the bearing flexibility. The bearing thus formed is called Complexed Bearing (CB).

The flexibility of the bearing support beyond the fluid film had been shown to be capable of altering effective bearing stiffness and damping properties acting on a rotating shaft[2,3,4] and these properties play important roles in determining the dynamic responses to excitation forces. But how flexible will the MRR be to achieve a desired dynamic response is a question to be answered by the designer.

More so, the design of CB is an iterative procedure. It is the case that AMB parameters can, conveniently be adjusted through the controller, however the passive MRR cannot.

So, here we look into; i) how inclusion of passive MRR changes the rotor's dynamic response and; ii) use this information and the bearing variables to propose an optimum design.

II. COMPLEXED BEARING SYSTEM

The CB system is composed of AMB and MRR as shown in Figure 1.

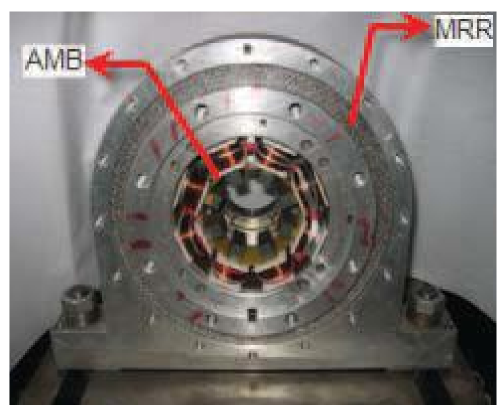

Figure 1. A Complexed Bearing

The main parameters of AMB are shown in Table 1.The MRR has a combine property of flexibility of a rubber and conductivity of metal. Here, we are more concerned on the former. The flexibility can be modeled by stiffness and damping coefficients. The model of the CB is illustrated in Figure 2.

(*National Natural Science Foundation of China –Grant No. 50675105*)

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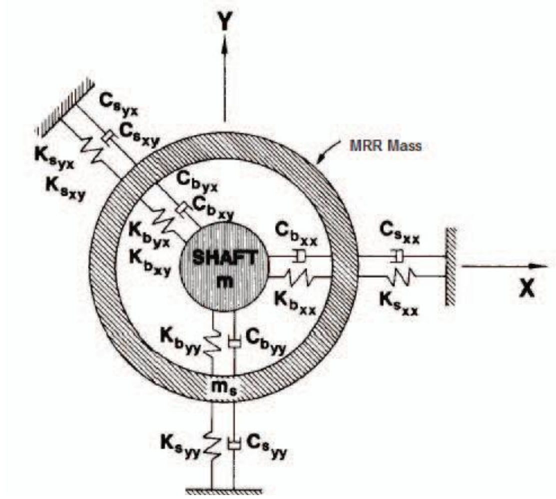

Figure 2. A Complexed Bearing

The total bearing flexibilities are sum of contributions from AMB and MRR assuming a rigid base. This is represented by (XX, YY) and cross-couple (XY-YX) stiffnesses $\boldsymbol{K}$, and damping coefficients $\boldsymbol{C}$. Figure 3 illustrates a more simplified model of the bearing system on the assumption of isotropic bearing properties and symmetrical rotor.

TABLE I. BEARING DESCRIPTION.

Description	Specification
Bearing load	80.0 *N*
Bore diameter	40.0 *mm*
Bearing width	20.0 *mm*
Number of turns	33.0 *turns*
Bias current	2.5 *A*
Clearance	2.5 *mm*
Displacement Stiffness-coefficient.	$-3.2\times10^5 N/m$
Current Stiffness-coefficient	32 *N/A*

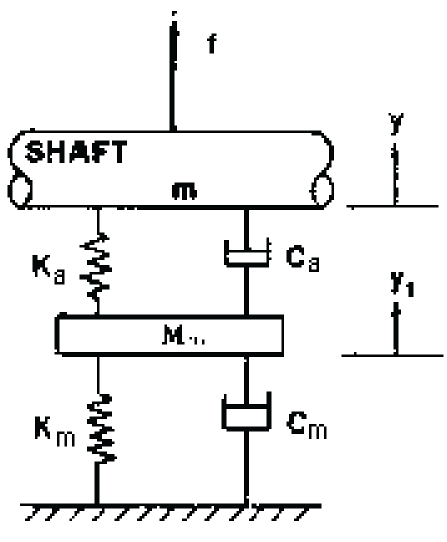

Figure 3. A simplified CB model

III. FINITE ELEMENT FORMULATION

The mathematical model is set up in MSC/MD Nastran-a finite element modeling and analysis software. The free vibration model equation of N-degree of freedom linear damped gyroscopic symmetrical system with homogeneous bearing properties can be written as:

$$\boldsymbol{M}\ddot{u}(t) + (\boldsymbol{C}_d + \boldsymbol{G})\dot{u}(t) + \boldsymbol{K}u(t) = F(t) \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{K}$, and $\boldsymbol{C}_d$ are $N \times N$ symmetrical mass, stiffness and damping matrices and contains bearing data $\boldsymbol{M}_m$, $\boldsymbol{K}_a$, $\boldsymbol{K}_m$ $\boldsymbol{C}_a$ and $\boldsymbol{C}_m$ inserted appropriately into rotor properties matrices, $\boldsymbol{M}_R$, $\boldsymbol{K}_R$, $\boldsymbol{C}_R$ on the main diagonal at position corresponding to the global coordinates of the main bearing , $\boldsymbol{G}$ is asymmetric and contains gyroscopic terms, $\ddot{u}(t)$ and $\dot{u}(t)$ are the first and second derivatives of time dependent vector $u(t)$ respectively. The full development of the equations can be seen in [5, 6, 8].

IV. COMPLEX EIGENVALUE ANALYSIS

Neglecting the excitation force, single value decomposition of (1) results to the right, u_i and left, v_i eigenvectors of $i-th$ eigenvalue λ_i given below;

$$(\lambda_i^2 \boldsymbol{M}^T + \lambda_i (\boldsymbol{C}_d + \boldsymbol{G})^T + \boldsymbol{K}^T)\{u_i\} = 0 \tag{2}$$

$$(\lambda_i^2 \boldsymbol{M}^T + \lambda_i (\boldsymbol{C}_d + \boldsymbol{G})^T + \boldsymbol{K}^T)\{v_i\} = 0 \tag{3}$$

$$\lambda_i = \alpha_i \pm \omega_i \tag{4}$$

The idea behind the optimization is that the modal parameters (damping coefficient, α_i and frequency ω_i) of a mode, i can conveniently be adjusted to assume a preassigned value. This is the job of optimization algorithm.

V. OPTIMAL DESIGN FORMULATION

The objective of this study is to maximize damping of the overall structure. In operation condition, such structure is subjected to excitation forces mainly from unbalance, as such; maximization of damping must be conducted with design constraints, such as total mass, operating frequency, maximum displacement, as well as physical constraints on design variables and objective functions.

The formulated problem is solved by the MSCAD Algorithm in MSC-MD Nastran [8] along with the developed finite element model, using sensitivity analysis.

The overall goal of the exercise is maximizing the damping coefficient of certain selected modes of interest within some frequency range. Here, the sum of weighted modal damping ratio and modal frequencies is chosen to be maximized subject to design constraints. The optimization problem is established thus;

Maximize:

$$f(p) = \sum_{i=1}^{N} A_i \xi_i + B_i \omega_i \tag{5}$$

$s.t.$

$$\frac{\omega_j}{\varpi_j} - 1 \le 0 \quad j = 1, ..., k \tag{6}$$

$$g_{k+1} : \frac{m}{m_{\max}} - 1 \le 0 \tag{7}$$

$$p_i^l \le p_i \le p_i^u, \quad i = 1, ..., n \tag{8}$$

where

A_i and B_i are weighting factors associated with each modal damping ξ_i and frequency ω_i respectively, ϖ_i is the desired modal frequencies for each mode. k is the total number of modes of interest, m and $m_{\max}$ are the overall mass and maximum allowable mass of the structure, respectively.

For the rotor structure under study, the natural choices for the design variables p_i in (8) are the bearing parameters; the stiffness and damping of the AMB (k_a, c_a), the stiffness and damping of MRR (k_m, c_m) and the mass of the AMB (m_a).

Individual analysis types each have their own techniques for performing sensitivity analysis.

The complex eigenvalue design sensitivity can be derived by differentiating (2) and (3) with respect to design variable. The approximation for the variation in the $k-th$ complex eigenvalue [9] is given by:

$$\Delta\lambda_i = \frac{\partial \lambda}{\partial p_i} = \frac{-\{v_i\}^T \left([\Delta M] \lambda_i^2 \left[\Delta \left(C_d + G \right) \right] \lambda_i + [\Delta K] \right) \{u_i\}}{\{v_i\}^T \left(2\lambda_i [M] + \left[\left(C_d + G \right) \right] \right) \{u_i\}} \tag{9}$$

where,

$p_i, i = 1, ...n$ are vectors of the design variables.

These analyses are carried out in MSC/MD Nastran Finite Element Software. MD Nastran is best suited for analyses involving complex systems compared to analytical methods.

VI. CASE STUDY

The rotor comprises of 828 mm long iron shaft with copper and iron cover sleeves at some selected cross-sections as shown in figure 4. The material properties of the composite shaft are; Modulus of Elasticity, $E = 2.0 \times 10^{11} Pa$,Poisson ratio, $\nu = .3$ and density, $\rho = 7850\ kg / m^3$.

A built-in electric motor is located half way between the two bearings at both ends supplies the rotational torque. The total weight of the composite rotor is 7.35 Kg. The rotor is suspended without mechanical contact by the bearings.

The shaft is discretized into108 Timoshenko beam element between 109 nodes. The sleeve is modeled as a shell element rigidly connected to the shaft at common nodes. Both supports are symmetrical and isotropic and modeled by bush element with stiffness and damping values approximated by the PID controller. Complex eigenvalue analysis solution sequence 107 was exercised using direct method and the eigenvalue and eigenvectors were extracted using complex LANCZOS method at rotation speed of 400 Hz (2513.27 rad/s).

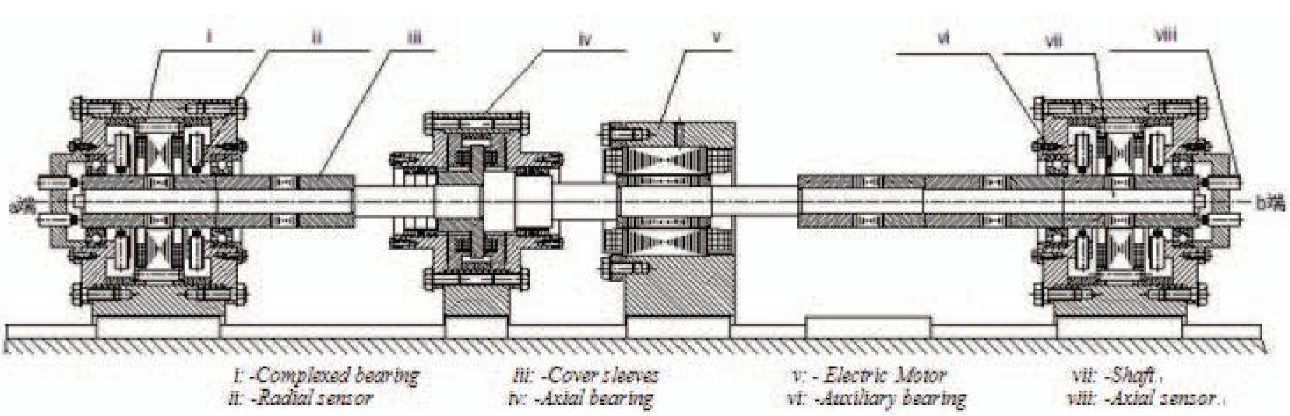

Figure 4. Schematic diagram of Complexed Bearing-rotor system.

Sequential linear programming (SLP) was used to estimate the value of the objective function between iterations. Other methods such as sequential quadratic programming, method of feasible direction and SUMT methods are also supported in Nastran. The modes of interest are the 1st and 2nd bending modes designated mode 1 and mode 2 respectively. The results are shown in Table 3 and Table 4 for modes 1 and 2 respectively. The pre-optimization modal natural frequency and damping ratio are 600.79 rad/s and 8.36% respectively for mode 1 and 1608.02 rad/s and 5.68% respectively for mode 2. Convergence was achieved in 19 iterative cycles of the re-design process and the optimum was found at 670.11rad/s, 43.18% for mode 1 and 1597.47 rad/s, 8.40% for mode 2. The bearing parameters for the final design are shown in Table 2.

TABLE II. SUMMARY OF OPTIMIZATION RESULT FOR MODE 1 AT ROTATIONAL FREQUENCY OF 400HZ

Cycle	Damping	Frequency(rad/s)	Frequency(Hz)	Damping ratio
1	-25.1273	-600.79	95.61876	0.083647
2	-28.4658	651.6313	103.7104	0.087368
3	-36.7618	664.4498	105.7505	0.110653
4	-47.8812	668.3218	106.3667	0.143288
5	-56.4744	-667.656	106.2608	0.169172
6	-73.2539	-671.48	106.8693	0.218186
7	-97.6147	672.0544	106.9608	0.290497
8	-105.619	-675.432	107.4983	0.312743
9	-114.76	670.702	106.7455	0.342209
10	-119.193	673.8929	107.2534	0.353743
11	-122.338	-671.94	106.9425	0.364135
12	-127.703	670.1882	106.6638	0.381094
13	-130.107	-672.204	106.9846	0.387105
14	-131.899	-669.923	106.6216	0.393773
15	-134.899	-670.113	106.6517	0.402615
16	-137.537	669.73	106.5908	0.410722
17	-138.499	-671.099	106.8087	0.412753
18	-141.762	-670.121	106.653	0.423093
19	-144.689	-670.106	106.6507	0.43184

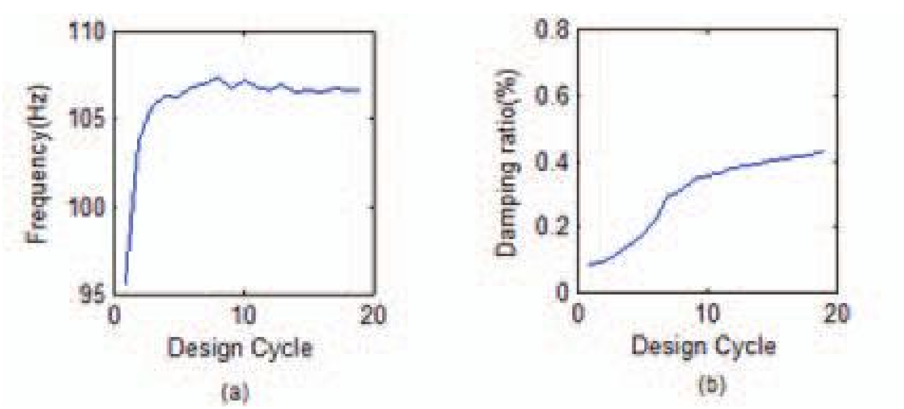

TABLE III. SUMMARY OF OPTIMIZATION RESULT FOR MODE 2 AT ROTATIONAL FREQUENCY OF 400 HZ.

Cycle	Damping	Frequency(rad/s)	Frequency(Hz)	Damping ratio
1	-45.6792	-1608.02	255.9245	0.056814
2	-65.1264	1609.487	256.1577	0.080928
3	-93.0148	-1598.52	254.4129	0.116376
4	-106.985	1556.855	247.7811	0.137438
5	-73.2967	1568.718	249.6691	0.093448
6	-74.693	1571.924	250.1794	0.095034
7	-75.2053	-1576.09	250.8423	0.095433
8	-67.8952	1588.065	252.7483	0.085507
9	-71.1291	-1588.04	252.7448	0.089581
10	-67.3129	1593.066	253.5443	0.084507
11	-66.2855	1594.372	253.7522	0.083149
12	-67.5772	-1594.43	253.7609	0.084767
13	-64.6881	1594.891	253.8347	0.081119
14	-66.1459	-1594.81	253.8224	0.082951
15	-66.4044	1595.427	253.9201	0.083243
16	-67.0703	1595.481	253.9286	0.084075
17	-65.9264	-1595.75	253.9722	0.082627
18	-67.0163	-1596.64	254.1132	0.083947
19	-67.0772	-1597.47	254.2456	0.083979

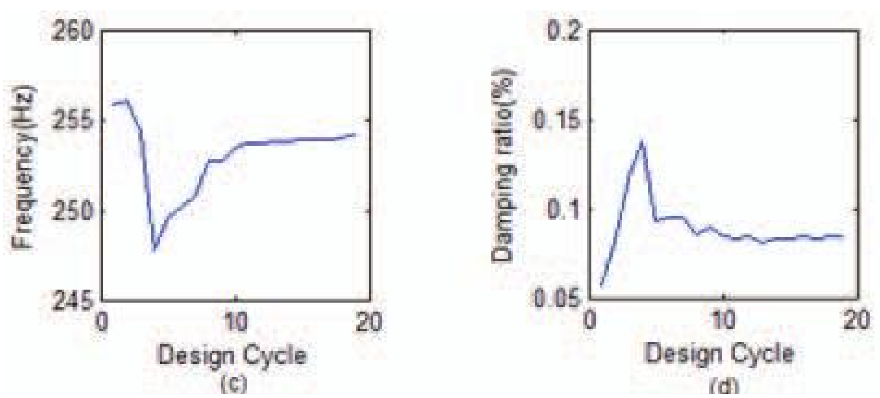

TABLE IV. OPTIMAL DESIGN FOR THE COMPLEXED BEARING SYSTEM.

Parameter	Analysis model (true)	Design model Initial (relative)	Design model final (relative)
AMB stiffness (*N/m*)	7.0×10^{5}	0.70	1.2700
AMB damping (*N.s/m*)	5.0×10^{2}	0.50	0.4591
MRR stiffness (*N/m*)	5.0×10^{5}	0.70	0.3502
MRR damping ((*N.s/m*)	4.0×10^{2}	0.30	1.2500
AMB mass (*Kg*)	2.095	0.80	0.2000

VII. CONCLUSION

Damping increase and maximization using MRR in AMB supported rotor system has been addressed in this paper. The optimization problem was formulated with bearing parameters; AMB- stiffness, damping and mass as well as MRR stiffness and damping, as design variables. The maximization of the modal damping ratio at first and second bending modes (mode 1 and mode 2 respectively) was sought.

Sequential linear programming was used, employing Nastrans MSCAD Algorithm for this purpose. Results of the optimization of the CB support shows that the MRR is instrumental in increasing the mode 1 damping ratio from 8.36% to 43.18% (for mode 1) and 5.68% to 8.4% (for mode 2).

ACKNOWLEDGMENT

To the glory of God and sponsorship from National Natural Science Foundation of China (Grant No. 50675105)

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