APPLICATION OF DYNAMIC PROGRAMMING

TO A

SINGLE CROPPING IRRIGATION PROBLEM

BY

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CERTIFICATION

This research project, The Application of Dynamic Programming to a Single Cropping Irrigation Problem was carried out by Patrick Chimezie OSUJI, Department of Maths/Computer Science, School of Science and Science Education, Federal University of Technology, (FUT) Minna, as meeting the requirement for the award of Master of technology (M-tech) degree in Applied Mathematics.

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This project is dedicated to our son and daughter, Godson and Peace.

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ABSTRACT

The management of agricultural and natural resource systems entails sequential decision processes, with decisions usually made under uncertainty. Solutions to many of these management problems can be obtained using the approach of dynamic programming.

A model was then developed and employed to relate irrigation, rainfall and water use parameters to i=th season crop and thus to yield. The model was validated using data from Hadija Jama'are Irrigation System, Kano, Nigeria.

The aim of this project is to illustrate how the principles of dynamic programming can be and has been applied to agricultural system.

Application which is discussed and described is a simple crop-irrigation problem. Emphasis is placed on numerical approach to problem formulation and solution. To aid with numerical solution, a computer program was used to solve the same problem.

The problem include selecting integer values of the height of water released from storage at the beginning of each season (u_i in meters) so that the present values of receipts from sale of the crop is maximized. The findings are supported with optimum decision path for the optimal sequence of irrigation.

Key Words:- Dynamic Programming, Irrigation sequence, Optimalality, Eigenvalues, Yield estimates, Decision stage, Additive objective function, Maximum Principle, Recursive functional equation, Decision variables, Stage return and Discount factor.

three dynamic programming arguments are used to derive optimality conditions for a particular resource.

A second reason for learning about dynamic programming is the important practical one that it provides a means of solving dynamic and stochastic resource problems numerically.

Three of the most frequently asked questions in the field of irrigation are "what are the yield benefits derived from irrigation ?", "how are these yield benefits related to certain water-use parameters ?" and "how are these benefits together with the parameters related to the season's crop ?". If data useful in answering these questions are collected, a model can be developed and employed, Throsby.

The techniques of dynamic programming are explored to develop a mathematical optimization model for crop irrigation system. This is illustrated by applying it to a simple crop-irrigation problem which was developed progressively in chapter four. A computer solution of the model for the same problem is included in chapter five and shows what level of water input will produce optimal yield for crop under irrigation, thus helping us to decide how to manage water most efficiently for irrigation purposes.

1.2 The Nature of Agricultural And Natural Resource Problems

Human survival and welfare have always depended on the successful management of agricultural and natural resources. However over time the management of these resources has become more complex as a consequence of growth of various sort. Increases in world population and material standard of living circumscribe old notions of natural resource being virtually limitless. As Boulding (1966) has argued in The Coming of the Spaceship Earth, until recently man could concentrate on maximizing resource flows through the economy, without much concern for the impact on resource stocks. It is increasingly important to consider the extent to which future consumption may be restricted as a result of current usage. In other words, the *user costs* of the current consumption of resources should be evaluated.

Along with population and economic growth there has been an increase in the scale of exploiting agricultural and natural resources. For example, widespread land clearing, fertilizing and fishing have placed strains on the maintenance of the resource base, leading to problem of pollution and damage to natural habitats.

Another type of growth has been in technical knowledge about the exploitation and conservation of resources. For example, research continues to reveal more about the effect of water management practices on grain yield and about the conversation of feed inputs to meat output. Because better decisions can usually be made with more information, the management of resource entails the processing of more information than there was previously.

The question of what is successful management is more difficult to answer now that it is widely recognized that the sum of individuals' interests may not be the same as the interest of the society. Problems at the individual level are more likely to take account of environmental constraints imposed by society, and problems at the aggregate level are more likely to be formulated with multi-dimensional goals.

Operations Research techniques can be used to tackle the increased complexity of resource management. Many resource problems entail decisions which are sequential, risky and irreversible. Dynamic programming is a versatile technique with considerable scope for helping to solve such problems.

The management of immobile and animate resources such as minerals involves exploration and extraction decisions which are clearly sequential and subject to risk. The management of living resources is even more complex because the interact with other natural systems which are uncontrollable and imperfectly understood. For example, crops respond to changes in temperature and water, which in turn are affected by and influence the weather.

Decisions are further complicated by the changing state of the biological unit as it passes through phases of birth, growth, reproduction and death.

The relationship between outputs and inputs not only changes through time, but is also uncertain. Uncertainty is increased if inputs such as water and solar energy are determined by the weather, if the biological units are free ranging or migratory, and if the biological units are part of a food chain which maintains a dynamic equilibrium with other living systems.

Another feature of the management of agricultural and natural resource system is that decisions can have irreversible effects on the natural resource base. In the case of inanimate resource there are examples of decisions which are for all intent and purposes irreversible, such as to exhaust a mine or exploit a wilderness area. The decisions are technically reversible but are economically irreversible because the cost of reversal are unacceptable high. Georgesco Roegen has interpreted the second law of thermodynamics as support for the proposition that the consumption of a resource in the economic process irreversibly reduces the usefulness or value for any further consumption. Decisions with respect to living systems are often reversible. Living systems are typically well-buffered and are capable of reacting against adverse stimuli. Biomass which is destroyed will often regenerate automatically given time. However, decisions which lead to the extinction of species are technically irreversible, at least at the present stage of genetic engineering.

In summary, current development in the usage of natural resources are increasingly raising complex and socially important issues. The next section briefly reviews some of the management techniques used to study resource problems, and considers the scope for the greater use of dynamic programming. Control variables employed in resource management are broadly classified prior to the formulation of a generalized resource problem. Optimality conditions for the generalized problem are derived using the reasoning of dynamic programming. The aim is to provide some intuitive insight into the conditions required for intertemporal optimality.

1.3 Management Techniques Applied To Farm Management

Given the variety of types of management problem encountered in agriculture and natural resource industries it is not surprising that all of the common operations research techniques, including dynamic programming, have been applied to problems in these industries. What is surprising is that dynamic programming is not used more frequently than it is. Some of the special problems that occur in agriculture and the techniques used to solved them, are considered.

There is a long tradition of applying operations research techniques to solving management problems, Throsby ; Martin. However it is probably fair to say that the most popular techniques are linear programming for whole-farm management and marginal analysis for single-enterprise management.

Linear programming was originally used to solve farm planning problems which were deterministic and static. The linear programming framework has been extended to solve more complex whole-farm problems which may be characterized as dynamic, stochastic, non-linear and discrete. Whilst linear programming is computational much more efficient than dynamic programming for solving deterministic problems with a linear objective function and linear constraints, dynamic programming may be more suitable for solving more intractable problems

Examples of the use of marginal analysis for solving static, deterministic single-enterprise problems are widespread in text dealing with production economics and farm management. The classical application is the decision on the optimal application of fertilizer to a growing crop. To the extent that the fertilizer problem can be typified as a single-period, deterministic, point-input, point-output process, straight forward calculus techniques can be applied.

Marginal analysis has been extended to stochastic, single-period, single-enterprise problems in order to provide further insight into the theory of the farm, Magnusson, and to aid the efficient management of farm enterprises, Dillion ; Anderson et al.,. If the statistical moments of financial outcome as a function of input levels are known and utility is a well-defined function of financial outcome, then stochastic utility maximization problems can be solved using calculus.

The extension of marginal analysis to the solution of multi-period, single-enterprise problems has been less straightforward. Dynamic systems are controlled by the timing as well as the level of inputs and outputs. Dillion, (p.97) notes that some early work in livestock production functions 'did not really comprehend the problem of profit maximization over time'. He gives the marginal conditions which must hold for the solution of point-input, point-output processes. The necessary marginal conditions for the more complex problems are best obtained by formulating the problem in an optimal control framework and applying the maximum principle. The approach is described in later section for the discrete time case. However, in practice the solution is often difficult.

There is a scope for the wider adoption of control theory formulations of crop and livestock problems. In general, time in such formulation is best treated as a discrete variable because decisions are made at intervals. There is a range of solution techniques besides dynamic programming for solving problems formulated in this way, such as iterative gradient methods. However dynamic programming is a technique particularly suited for obtaining numerical solutions to problems which involve functions which are non-linear and stochastic, and state and decision variables which are constrained to a finite range of values. Applications to agriculture is not considered in this project.

1.4 Control Variables In Resource Management

In this section some of the basic types of control required in the management of different resource systems are considered. The decision paves the way for the consideration of a generalized resource problem in the next section.

All resource systems are managed for the eventual harvesting of some product. However, the pattern of harvesting varies between resource systems. In the case of a renewable biological resource, harvesting may continue indefinitely, whereas in the case of an exhaustible resource harvesting finishes with depletion or with extraction costs high relative to resource price. Resource systems are classified by the pattern of harvesting in Table 1.1 which also show the basic type of control variables often used in the management of different resource systems. The management of an agricultural or naturally occurring resource may be typified as a series of decisions on the levels of inputs and outputs. As a generalization, the management of an agricultural resource requires a sequence of both input and output decisions, whereas the management of a natural resource requires a sequence of output decisions only. Types of input, output and replacement decisions are described in turn. The distinctions are important for determining the formulation of control problems.

Table 1.1

Enterprise	Input Decision		Output Decision		
	Acquisition	Continual maintenance	Continual produce	Repro- duction	Final biomass
(1) Once only harvested or culled					
Horticulture, grains	*	*			*
Poultry, pigs, beef	*	*		*	*
Forestry (no thinning)	*				*
(2) Continual harvesting until exhausted					
Mining	*		*		
(3) Continual harvesting until producing stock is culled					
Viticulture, fruits tropical sorghum	*	*	*		
Eggs, wool, diary	*	*	*	*	*
Forestry (with thinning)	*		*		*
(4) Continual harvesting indefinitely					
Aquaculture	*	*	*		
Ocean fishing	*		*		

1.4.1 Input decisions

Two types of input control are acquisition inputs and maintenance inputs. Acquisition inputs are the resources themselves or inputs required to gain access to the resource. In agriculture they may be seeds, land or young stock; they include hunting inputs in the case of the fishery and exploration in the case of minerals. Maintenance inputs are those inputs which must be supplied by man for the survival or growth of the resource. For the management of some natural resources no maintenance input may be required, as in the case of fishery or relatively few and mainly for protection, as in the case of forest stands and exhaustible resource. However in agriculture maintenance inputs are typically very important, being a major determinant of the timing, quality and quantity of the eventual produce. In crop production, maintenance inputs may be irrigation, fertilizers and pesticides; in livestock production they may be feed, water, shelter and veterinary supplies.

1.4.2 Output decisions

Three types of output can be distinguished; output continually harvested from the resource stock; young stock resulting from reproduction by the parent stock; and biomass of an adult stock after harvesting or slaughter.

Eggs, fruits, wool and milk are examples of continual agricultural outputs. Exhaustible resources mined and timber from thinning forest stands are examples of continual natural resource outputs. The two further categories of output relate only to living resources. The production of young stock can usually be controlled, although the case of the deep-sea fishery is an exception. Examples of valuable biomass of a once-living stock are vegetables, grains, meat and felled trees.

1.4.3 Timing and replacement decisions

The scope for deciding the timing of harvesting depends on the type of output. In the case of inanimate resource such as minerals the product is always available and from living resource either appear at a point in time (e.g. eggs and young stock) or mature ready for harvest over a relatively short time period (e.g. wool and milk). In these cases, the output decision is one of whether or not to harvest the product when it becomes available. Availability of product depends on time, weather and season, and on any inputs injected by man. The situation is difficult in the case of output which is final biomass because it accumulates continuously. There are many opportunities for harvesting the biomass, either continually (e.g. forest thinning) or finally (e.g. forest clear felling). In this case the timing of harvesting is an important decision variable.

For living-resource categories (1) and (3) in table 1.1 a decision to cull is accompanied by the decision on whether or not to replace the culled stock with new stock of the same or of different kind. The final harvesting decision is followed by the acquisition decision. If there is to be replacement with the same kind of stock, the replacement may be obtained either by retaining the output of young stock or from outside the system.

Whatever optimal decisions have to be made, whether they relate to the timing and levels of inputs or outputs, the same principle applies. The current consequences of a decision must be weighed against the future consequences of the decision. The way in which this should be done for continual maintenance and harvesting and decisions is demonstrated in the marginal conditions for intertemporal optimality which is derived in the next section using the logic of dynamic programming.

1.5 A Simple Derivation of the Conditions for Intertemporal Optimality

The consequence of an input into a resource system is an immediate loss exchanged for some future gain. For example, feed is fed to livestock in the expectation of marketable weight gain in the future. Conversely, the consequence of extracting an output from a resource system is an immediate gain exchanged for some future loss. Harvesting an additional fish now means that it cannot be harvested later; that the effort required to catch another fish later may be increased because there will be fewer fish per unit volume of sea; and that the growth in the biomass of the fish stock will be reduced if growth is an increasing function of biomass. If optimal decisions are to be made on the timing and level of a sequence of inputs or outputs, and the goal of the decision maker is a function of current and future returns, then both the immediate and future consequences of a current decision have to be taken into account. MacInerney (1976, 1978, 1981) gives a useful diagrammatic exposition of how they should be taken into account in a simple two-period model.

The rules which summarize the optimality conditions which hold for control problems in general are referred to as the maximum principle. The maximum principle can be defined for either continuous-time or discrete-time problems. One simple method for deriving the maximum principle for continuous-time problems is via the reasoning of dynamic programming, Sethi and Thompson. The maximum principle for discrete-time problems can be obtained using calculus an dynamic lagrange multiplier, Benavie. However, as shown below, the maximum principle for the discrete-time problems can also be derived easily and directly using the reasoning of dynamic programming. A discrete-time formulation is appropriate for most resource problems because in general outputs are obtained (inputs are applied) periodically rather than continuously. The derivation shows how dynamic programming can be used as an analytic device. For comparison, the same rules are obtained in Appendix 1.A by working with a dynamic Lagrangian expression.

1.5.1 The general resource problem without replacement

For the generalized problem let the level of resource stock at the start of the first decision period be x_i. Decisions on input or output levels u_i are made at the beginning of each n decision periods (subscripted by i) in the planning horizon. Any stock remaining at the end of the n-th decision period has a final value $F(x_{n+1}\}$. The period gain resulting from decision u_i is denoted by $a_i\{x_i, u_i\}$. The discount factor per decision period which applies aver all decision periods is α, and equals $1/(1+r)$ where r is the discount rate.

Table 1.2

Examples of Functions in Some Resource Control Problems

Natural resource	x_i	u_i	$g_i\{x_i\}$	$h_i\{x_i,u_i\}$	$a_i\{x_i,u_i\}$*
Mine	Resource mass	Level of extraction	-	Level of extraction	Net returns from extraction = $p_ih_i - C_i\{x_i,u_i\}$
Irrigated crop	Crop biomass	Level of irrigation	-	Irrigation induced growth ($h_i<0$)	Net returns from irrigation = $-p_nh_i(1+r)^{n-i} - c_nu_i$
Beef cattle	Liveweight	Level of feed	-	Weight gain ($h_i<0$)	Net returns from feeding = $-p_nh_i(1+r)^{n-i} - c_nu_i$
Timber thinning	Timber biomass	Level of thinning	Autonomous growth	Level of thinning	Net returns from thinning = $(p_i - c_i)u_i$
Fishery	Fish biomass	Level of fishing effort	Autonomous growth	Level of harvesting	Net returns from harvesting = $p_ih_i - C_i\{x_i,u_i\}$

* p_i = price of product; c_i = cost per unit of control; $C_i\{.\}$ = total control cost; r = rate of discount.

The present value of the initial resource stock, denoted by lower-case v_i, is

$$v_i\{x_i,u_i,\ldots,u_n\} = \sum_{i=1}^{n} \alpha^{i-1}a_i\{x_i,u_i\} + \alpha^n F\{X_{n+1}\} \qquad (1.1)$$

The problem is to maximize (1.1) with respect to $u_1,\ldots,u_n$ subject to the initial stock level x_1 and the stock dynamics equations

$$x_{i+1} = x_i + g_i\{x_i\} - h_i\{x_i,u_i\} \quad (i=1,\ldots,n) \qquad (1.2)$$

where $g_i\{.\}$ is the autonomous growth per period of the resource and $h_i\{.\}$ is the per period reduction in stock level consequent on x_i and u_i.

The nature of the functions $a_i\{x_i,u_i\}$, $g_i\{x_i\}$ and $h_i\{x_i,u_i\}$ depends upon the natural resource to be managed. Some examples are shown in Table 1.1.

The formulation applies most immediately to timber thinning and the fishery, for which u_i is the level of harvesting effort in period i. The functions $g_i\{x_i\}$ and $h_i\{x_i,u_i\}$ are usually distinguished in the case of the fishery to deal separately with the natural growth of the fish biomass and the harvest of biomass, respectively.

In the case of an irrigated crop and beef cattle, there is no need for both $g_i\{.\}$ and $h_i\{.\}$, so $g_i\{.\}$ is dropped. Here the u_i's are levels of inputs, and h_i's are negative representing *increases* in the resource stock. In the simple mining problem, u_i is the level of extraction equal to h_i. However, u_i could instead represent exploration effort, in which case h_i would be negative, representing discoveries of new resource stocks.

In many cases there are restrictions on u_i. There may be a limit to the amount of water available across periods for irrigating a crop, or the appetite of livestock (a function of liveweight) may limit the level of feed input. Any constraints on u_i are easily taken care of when solving problems numerically using dynamic programming. They are ignored for ease of exposition in the rest of the section.

Equation (1.1) states that the value of the initial resource stock equals the present value of all period gains arising from the management of the resource plus the final value of the resource. Alternatively, the value of the initial resource stock is the sum of the gain in period 1 and the value of the resource at the beginning of period 2 discounted one period. In symbols;

$$v_i\{x_1,u_1,\ldots,u_n\} = a_1\{x_1,u_1\} + \sum_{i=2}^{n} \alpha^{i-1}a_i\{x_1,u_1\} + \alpha^n F\{X_{n+1}\}$$

$$= a_1\{x_1,u_1\} + \alpha v_2\{x_2,u_2,\ldots,u_n\}.$$

Let u^{*i} denote the optimal level of u_i and the upper case $V_i\{x_i\}$ denote the value of the resource stock at the beginning of period i if control $u^*_i,\ldots,u^*_n$ are implemented.

Suppose the value of $V_2\{x_2\}$ has already been determined. Then the management problem at the beginning of the first period can be formulated as finding;

$$V_1\{x_1\} = v_1\{x_1,u^*_1,\ldots,u^*_n\}$$
$$= a_1\{x_1,u^*_1\} + \alpha v_2\{x_2,u^*_2,\ldots,u^*_n\}$$
$$= a_1\{x_1,u^*_1\} + \alpha v_2\{x_2\}$$
$$= \max_{u_i} \left[a_1\{x_1,u_1\} + \alpha v_2\{x_2\}\right] \quad \ldots (1.3)$$

Note that by the transformation of equation (1.2) x_2 is the following function of x_1 and u_1;

$$x_2 = x_1 + g_1\{x_1\} - h_1\{x_1,u_1\}$$

This is the approach of dynamic programming. A one-decision variable problem is abstracted from the original n-decision variable problem, although clearly the solution depends on $V_2\{x_2\}$ which is initially unknown. It may appear that the cart is being placed before the horse, but $V_2\{x_2\}$ can be determined through the process of backward induction described in Chapter four.

Assuming that $a_i\{x_i,u_i\}$, $V_1\{x_1\}$, and $V_2\{x_2\}$ can be differentiated, and for simplicity that the solution is an interior solution, from (1.3) and (1.2) a necessary condition for optimality is

$$\delta a_i/\delta u_i + \alpha(dV_2/dx_2)(\delta x_2/\delta u_1) = 0 \quad (1.4)$$

The term dV_2/dx_2 denotes the rate of change in the optimal value of the resource stock *in situ* at the beginning of period 2 with respect to x_2. It is the value of the marginal change in x_2 after allowing for optimal reaction in the control variables $u_2,\ldots,u_n$. It is conveniently written as λ_2 because it has the same interpretation as the Lagrange multiplier λ_2

used in the alternative derivation of the intertemporal optimality conditions in Appendix 1.A. Thus (1.4) can be written as

$$\delta a_1/\delta u_1 = -\alpha\lambda_2(\delta x_2/\delta u_1) = \alpha\lambda_2(\delta h_1/\delta u_1) \qquad (1.5)$$

Equation (1.5) shows that immediate gains (losses) must be balanced against the present value of future losses (gains) in determining u^*_1. It is required that u_1 be increased whilst the immediate marginal gains (present value of future losses) more than offset the present value of future losses (immediate gains) until (1.5) holds.

If u_i is the usage of a resource then the right-hand side of equation (1.5) represents the marginal *user cost* referred to in section 1.2. For a one-period problem (or if n = 1), the right hand side of equation (1.5) is zero.

Although x_1 is not a control variable, if the stock is optimally managed across all periods then the resource owner should be indifferent between leaving the marginal unit of the resource in place or extracting it. To see that this condition does indeed hold, differentiate both sides of equation (1.3) with respect to x_1 to obtain;

$$dV_1/dx_1 = \delta a_1/\delta x_1 + \alpha(dV_2/dx_2)(\delta x_2/\delta x_1)$$

or

$$\lambda_1 = \delta a_1/\delta x_1) + \alpha\lambda_2(1 + dg_i/dx_1 - \delta h_1/\delta x_1) \qquad (1.6)$$

Augmenting the current resource base by one unit has two impacts. The period gain is altered, and the resource stock at the beginning of period 2 is changed. Thus, the increase in the optimal value of the resource stock at the beginning of period 1 equals the additional period gain plus the increase in the optimal value of the resource stock at the beginning of period 2, discounted one period.

The generalized natural resource problem subsumes not only the first-period problem (1.3) but also the remaining sequence of single-period problems for i = 2,...,n. The two conditions for optimality, (1.5) and (1.6), generalize to

$$\delta a_i/\delta u_i = -\alpha\lambda_{i+1}(\delta x_{i+1}/\delta u_i) \qquad (i = 1,...,n) \qquad (1.7)$$

$$\lambda_i = \delta a_i/\delta x_i+\alpha\lambda_{i+1}(\delta x_{i+1}/\delta x_1) \qquad (i = 1,...,n) \qquad (1.8)$$

with boundary conditions

$$x_1 = \overline{x_1} \qquad (1.9)$$

$$\lambda_{n+1} = dF/dx_{n+1} \qquad (1.10)$$

The last boundary condition holds because the value of the resource after all decisions have been made, $V_{n+1}\{x_{n+1}\}$, is set equal to $F\{x_{n+1}\}$.

Equations (1.7) to (1.10) represent a special case of the discrete maximum principle, although (1.8) is usually rewritten to give the between period change in λ. Equation (A1.2) in Appendix 1.A shows the alternative version of (1.8). The results are related to the Hamiltonian used in control theory in Appendix 1.B.

It may be possible to use equations (1.7) to (1.10) to solve a resource problem analytically. However, the solution of a set of difference equations with two-point boundaries is not always straightforward. In Chapter four it is explained how numerical solutions to the problem formulated in (1.1) can be obtained directly using dynamic programming.

1.A Appendix: A Lagrangian Derivation of the Discrete Maximum Principle

A general control problem for the management of a natural resource was presented in Chapter one. The conditions which must hold for intertemporal optimality were derived using a dynamic programming approach. The derivation introduced the idea of recursive induction which is the heart of the dynamic programming approach. In this appendix the same conditions are derived by finding the conditions for which the relevant Lagrange expression is maximized. The alternative derivation is included for comparison of the two methods and for completeness. A Lagrangian expression for the problem presented in section 1.5 is

$$L = \sum_{i=1}^{n} \alpha^{i-1} a_i\{x_1,u_1\} + \lambda_1(\bar{x}_1 - x_1) + \alpha^n F\{X_{n+1}\}$$

$$+ \sum_{i=1}^{n} \alpha^i \lambda_{i+1}(x_i + g_i\{x_i\} - h_i\{x_i,u_i\} - x_{i+1})$$

$$= \sum_{i=1}^{n} \alpha^{i-1} [a_i\{x_1,u_1\} + \alpha^i \lambda_{i+1}(x_i + g_i\{x_i\} - h_i\{x_i,u_i\} - x_{i+1})$$

$$+ \lambda_1(\bar{x}_1 - x_1) + \alpha^n F\{X_{n+1}\}$$

where the Lagrange multiplier or costate variable, λ_i, is the contribution which an additional unit of the resource stock would make to the value of the resource stock at the beginning of period i. If a small change in u_i or x_i changes x_{i+1} by $\blacktriangle x_{i+1}$, the value at the beginning of period i of the change is $\alpha\lambda_{i+1}\blacktriangle x_{i+1}$. In some derivations of the discrete maximum principle, the Lagrange expression is formulated so that what is represented by $\alpha\lambda_{i+1}$ in this analysis is represented instead by λ_i.

Necessary conditions for an interior solution are

$$\delta L/\delta u_i + \alpha^{i-1}[\delta a_i/\delta u_i - \alpha^i\lambda_{i+1}(\delta h_i/\delta u_i)] = 0$$

which implies

$$\delta a_i/\delta u_1 = \alpha\lambda_{i+1}(\delta h_i/\delta u_i) \quad (i=1,\ldots,n) \qquad \text{(A1.1)}$$

$$\delta L/\delta x_i = \alpha^{i-1}[\delta a_i/\delta x_i + \alpha\lambda_{i+1}(1+dg_i/dx_i - \delta h_i/\delta x_i) - \lambda_i] = 0$$

which implies

$$\lambda_i = \delta a_i/\delta x_i + \alpha\lambda_{i+1}(1+dg_i/dx_i - \delta h_i/\delta x_i)$$

or, after rearranging and noting that $\alpha = 1/(1+r)$ where r is the rate of discount

$$\lambda_{i+1} - \lambda_i = r\lambda_i - \lambda_{i+1}(dg_i/dx_i - \delta h_i/\delta x_i) - (1+r)\delta a_i/\delta x_i \quad (i = 1,..,n)$$

$$\text{(A1.2)}$$

$$x_1 = \bar{x}_1 \qquad \text{(A1.3)}$$

$$\delta L/\delta x_{n+1} = \alpha^n[-\lambda_{i+1} + dF/dx_{n+1}] = 0$$

which implies

$$\lambda_{i+1} = dF/dx_{n+1} \qquad \text{(A1.4)}$$

No allowance has been made for any constraints on u_i. If there are constraints on u_i, they can be incorporated in the

Lagrangian expression. Another extension is to allow for the possibility of boundary solutions. Some of the conditions become inequalities, and the Kuhn-Tucker conditions apply.

Dorfman and Benavie present the derivation of the discrete maximum principle with useful economic interpretations. Clark presents derivations of both the continuous and discrete maximum principles, and applies them to a wide variety of problems encountered in the management of natural resources.

1.B Appendix: A Note on the Hamiltonian used in Control Theory

The conditions for the optimal management of a resource through time are often derived from optimal control theory. Although not immediately obvious, they can also be derived intuitively from the basic equation of dynamic programming. The derivation here is restricted to cases where all functions are continuously differentiable. The discrete maximum principle for the general resource problem formulated in Chapter one can be stated in terms of the expression

$$H_i\{x_i,u_i,\lambda_{i+1}\} = a_i\{x_i,u_i\}+\alpha\lambda_{i+1}(g_i\{x_i\}-h_i\{x_i,u_i\}) \quad (i=1,..,n) \qquad \text{(B1.1)}$$

The discrete maximum principle states that a necessary condition for optimality is that u_i maximizes $H\{.\}$ at each stage i. If the solution is an interior solution, the requirement is that

$$\delta H_i/\delta u_i = 0 \quad (i = 1,..,n) \qquad \text{(B1.2)}$$

The associated adjoint equation

$$\lambda_{i+1}-\lambda_i = r\lambda_i-(1+r)\delta H_i/\delta x_i \qquad (i=1,...,n) \qquad (B1.3)$$

relates the rate of change of the adjoint variable or dynamic Lagrange multiplier to $H_i\{.\}$ and the rate of interest r, where $\alpha = 1/(1+r)$.

The fundamental dynamic programming equation for optimal control is the general form of (1.3);

$$V_i\{x_i\} = \max_{u_i} [a_i\{x_i,u_i\}+\alpha V_{i+1}\{x_{i+1}\}]$$

$$= \max_{u_i} [a_i\{x_i,u_i\}+\alpha V_{i+1}\{x_i+g_1\{x_1\}-h_1\{x_1,u_1\}\}]$$

$$(i=1,...,n) \qquad (B1.4)$$

The purpose of this appendix is to show that (B1.4) implies (B1.2) and (B1.3). A necessary condition for an interior solution is

$$\delta a_i/\delta u_i - \alpha(dV_{i+1}/dx_{i+1})\delta h_i/\delta u_i = 0 \qquad (i=1,..,n) \qquad (B1.5)$$

which is the same as (B1.2) after substituting λ_{i+1} for dV_{i+1}/dx_{i+1} evaluated at x_{i+1}.

Denoting optimal u_i by u^*_i, (B1.4) can be rewritten as

$$V_i\{x_i\} = a_i\{x_i,u^*_i\}+\alpha V_{i+1}\{x_i+g_1\{x_1\}-h_1\{x_1,u^*_1\}\}$$

$$(i=1,...,n) \qquad (B1.6)$$

Because u^*_i is a function of x_i, (B1.6) can be totally differentiated with respect to x_i to give

$$dV_i/dx_i = \delta a_i/\delta x_i+\alpha dV_{i+1}(1+dg_i/dx_i-\delta h_i/\delta x_i)$$

$$(i=1,..,n) \qquad (B1.7)$$

After rearrangement and substitution for dV_i/dx_i, (B1.7) becomes

$$\lambda_i(1+r)-\lambda_{i+1} = (1+r)\delta a_i/\delta x_i+\lambda_{i+1}(dg_i/dx_i-\delta h_i/\delta x_i)$$

or

$$\lambda_{i+1}-\lambda_i = r\lambda_i-(1+r)\delta H_i/\delta x_i \qquad (1,..,n) \qquad \text{(B1.8)}$$

which is the same as (B1.3). The term $H_i\{.\}$ introduced in (B1.1) is the current value Hamiltonian of control theory, discounted one period.

Chapter two

WATER MANAGEMENT UNDER IRRIGATION

2.1 Irrigation Methods;

Irrigation is the application of water to soil to assist in the production of crops. Irrigation water is supplied to supplement the water available from rainfall and ground water. In many areas of the world, the frequency of rainfall is not adequate to meet the moisture requirements of crops.

Irrigation is an age-old art - as old as civilization. The pressure for survival and the need for additional food supplies are causing the rapid expansion of irrigation system throughout the world, Michael and Ojha.

The scope of irrigation science extends from the water shed to the farm and to the channel. Water being a limited resource, its efficient use is basic to the survival of the ever increasing population of the world. In the comprehensive strategy needed for the conservation and development of water resources, several factors are to be kept in view. These include the availability of water, its quality, location, distribution and variation in its occurrence, climatic conditions, nature of the soil, competing demands and socio-economic conditions. In dealing with each of these, every effort must be made to make the best use of water, so as to make possible a high level of continuous production. The objective of an efficient irrigation is to increase agricultural production per unit volume of water, per unit area of cropped land, per unit time, Irrigation Commission. Water for irrigation is obtained from natural streams or rivers, surface reservoirs and from underground reservoirs. Flood water from rivers is collected in surface reservoirs by constructing dams at suitable sites. Runoff water from small areas can also be collected by constructing ponds or tanks. Water from underground reservoirs is utilized by constructing wells and installing pumps or other water lifts. Water from surface reservoirs is taken through canals. Canals run from higher to lower elevations, and water flows to them by the force of gravity.

Based on whether the source of water is above or below the field surface, irrigation is classified into two main types: (a) Flow irrigation where the water reaches the field from the source by gravity flow;

(b) Lift irrigation requiring the water to be raised or lifted from its source to the field surface.

If stream, river or lake can be conveniently tapped to take water to canals, it is often the most economical source of water for irrigation. However, in most cases, this source of water is not adequate during dry season when water is needed most by the crop. Therefore, it is essential that the potential supplying power of the river or stream be considered before planning the irrigation system.

2.1.1 The Measurement, Conveyance and Control Of Irrigation Water On The Farm

The important factor in the efficient working of an irrigation system is the character of the structure used for transporting and distributing water. An additional factor is the measurement of irrigation water, which permits more intelligent use of this valuable natural resource.

Present day knowledge of soil-moisture plant relations permit irrigation system to be designed for applying water in correct quantities when needed. In order to use this knowledge efficiently a reasonable accurate measurement of water is necessary. Accurate measurement is necessary in field studies of soil-water plant relationship. Further, measurement enables the farmer to know the actual volume of water consumed.

Many researchers have measured the water use in irrigated crops using metal tanks installed in the middle of the field. Many experiments are conducted with three to six moisture regimes in tanks without bottoms to determine the most satisfactory regime in terms of optimal yield.

Various types of structures are used to control water in the farm. The purpose of the farm distribution system is to safely carry the required irrigation stream from the source to the individual furrow, basin or border. The system must provide a means of control so that the labour required for irrigation is held to a practical minimum, thereby ensuring easy and accurate application of irrigation water, and will help to make the best use of water supply Christiansen, J.E. 1953.

Water is a basic necessity for all life systems. It is essential not only for survival but also to the quality of life, In a recent publication, Food and Agriculture Organization (FAO) emphasized the importance of water as a key input in agriculture stating that "Notwithstanding the fact that land is indispensable for agricultural production, It is water rather than land which is the binding constraint. It is only when water constraint is removed that other technical constraints such as nutrients and pests become important".

The development of water resources especially for irrigation purposes in Nigeria dates back to the pre-colonial era. The traditional application of water to land for dry season farming in Northern Nigeria was one of the earliest attempts made towards increasing agricultural production. This notwithstanding, Nigeria has not developed irrigation to the same extent as other developing nations, particularly in Asia.

Even though irrigation has been practiced in Nigeria as far back as ninth century, it was not until the recurrence of drought and the attendant famine during the 1970's that concerted efforts were made to develop irrigation to give protection against the failure of crops and ameliorate the attendant environmental degradation of drought, World Bank.

The initial case for development of irrigation in Nigeria was based in part, therefore, on the need to sustain a growth in food supply that would broadly lead to national food security.

2.2 Effect of Irrigation on Yield Response

Management of limited water supplies in Irrigation system requires sound knowledge of the relationships between crop yield and water use. To optimize economic returns the way yield responds to varying conditions of water supply should influence the way in which water is distributed in an irrigation system, as well as the choice of crop variety and the use of inputs at the farm level. For best results, soil water should be kept close to field capacity, Aglibut et al,. Rayes. But the water environment in which crops are grown varies tremendously throughout Nigeria. Most tropical crops are grown under less than favourable conditions. Thus, since the new improved varieties were developed under favorable conditions, where water is not a limiting factor, International Rice Research Institute, the potential of new crop technology has not been widely reached. For adequately irrigated areas, relatively higher yields are associated with climatic factors that contribute to low evaporative demand conditions.

2.2.1 Effect of Irrigation on Sequential Cropping

For centuries, the crop and farmers of the humid tropics have adapted themselves to the rainy season. Crop varieties are mostly photoperiod-sensitive and the farmers usually plant whenever the monsoon rains begin, Abilay.

They harvest at a fixed date after the rains stop and the water recedes. As a result, farmers of the humid tropics, accustomed to growing one crop a year, seldom realize the importance of water management for better irrigation.

With the development of photoperiod-insensitive crop varieties in recent years, the Nigerian farmers are equipped to grow more crops in one year and to develop crop-oriented sequential cropping pattern, because they are not limited by temperature. The only environmental defect left to hamper cultivation of more crops in the humid tropics is rainfall availability. Development and measurement of water resources are the only way to remedy it. Experience from some locations in Nigeria is used to estimate rotational intervals under conditions of variable soils, sources of water, and seasons. It is hoped that through better water control and management double cropping crop culture can be established and further development of multi-cropping can be achieved rapidly throughout Nigeria, FMA&NR.

2.3 Diagonalization of Matrix In Irrigation Problem;

Data from an agricultural experiment with two-way classification form an r x c matrix table, where r and c denote the number of classes in the two classification criteria used. For example, if the grain yield in an irrigation trial are classified based on two criteria - water depth (1 to 3) and multi-cropping (1^{st} to 3^{rd} seasons) the resulting data represent a 3 x 3 matrix table.

In practice, if such matrix is reduced, the resulting diagonal elements or eigenvalues summarizes the matrix table and a simple and final statements regarding conditions for the feasibility of this reduction seems in order, Margenau and Murphy.

A matrix may be diagonalized if (a) if all the eigenvalues are distinct, for example, consider the case in which all eigenvalues A are different. Select one, say λ_i, and form the n linear equations;

$$Ax = \lambda_i x \qquad \text{... (2.1)}$$

They are homogeneous, and may be solved for the ratio of the components of the eigenvectors x_i. Remembering that each component contains an arbitrary constant that can be written as a column vector, Reju.

$$X_k = [x_{1k}, x_{2k}, \ldots., x_{nk}]$$

The remaining eigenvectors are determined in the same way, using each eigenvalues in turn. Finally a matrix x is formed whose columns are the eigenvectors of A which satisfies the equation

$$AX = [\lambda_i \delta_{ij}] \qquad \text{..(2.2)}$$

where δ_{ij} is the "kronecker" delta an represents a discontinuous factor which is taken to be unity when the two subscripts have the same value (i=j) but is zero when they are not equal. when equation (2.2) is multiplied by x^{-1} we obtain

$$X^{-1}AX = \Lambda = [\lambda_i \delta_{ij}] \qquad \text{..(2.3)}$$

It is thus shown that the matrix x which diagonalizes A, may be found by compounding the eigenvectors of A into a matrix. The reduction to diagonal form here described is unique except for the order in which the eigenvalues occur along the diagonal.

(b) If it is Hermitian or symmetric. Here the problem is that of diagonalizing the symmetric matrix A by means of a congruent transformation. Except for slight modification, the reduction of a Hermitian matrixes to diagonal form is similar to the procedure used for real symmetric matrices.

(c) If it is unitary for example, if a unitary matrix is indicated by U then from its definition $U = (U^{+})^{-1}$, hence

$$U^{+} = U^{-}; \quad UU^{+} = U^{+}U = E \qquad .... (2.4)$$

Suppose the element in a single column of U are given by U_j, then the next Hermitian scalar product of two columns

$$U_j^{+}U_k = \delta_{ij} \qquad .... (2.5)$$

A similar relation may be found between the rows. Hence the rows and columns of a unitary matrix of order n form a set of mutually perpendicular unit vectors in Hermitian space.

In cases (b) and (c) a unitary matrix can always be found to affect the transformation while in (a) a more general type of transforming matrix will be needed, Aiyesimi.

CHAPTER THREE

METHODS OF DYNAMIC PROGRAMMING

3.1 Introduction;

Since Bellman expounded and popularized dynamic programming in the 1950s, several papers on dynamic programming with management applications have appeared. However, the management applications have been mainly to business and industry. There does not have been a dynamic programming paper published with applications to the management of agricultural and natural resources. Books on operations research usually devote a chapter or two to dynamic programming. The treatment is inevitably limited, seldom progressing for example to the solution of infinite-stage problems.

In this chapter the solution of deterministic, finite-stage dynamic programming using backward recursion is explained. The solution procedure is valid for any problem for which Bellman's Principle of Optimality holds. The necessary structure and properties of a dynamic programming problem are discussed. The process of backward recursion is illustrated in the solution of a problem in Chapter four by compiling solution table. The computer solution to the same problem is also shown in Chapter five.

3.2 Backward Recursion Applied To General Resource Problem

In Chapter one an n-stage general resource problem was introduced. It was shown how the problem could be formulated as one of maximizing, with respect to the initial decision variable u_1, an objective function subsuming an (n-1)-stage problem. The same type of decision problem exist at all subsequent decision stages. The overall problem of finding $u^*_1,....,u^*_n$ can be solved by finding u^*_i which satisfies the following recursive functional equation;

$$V_i\{X_i\} = \max_{u_i} [a_i\{x_i,u_i\}+\alpha V_{i+1}\{x_i+g_i\{x_i\}-h_i\{x_i,u_i\}\}] \quad (i = n,..,1) \quad ... (3.1)$$

with

$$V_{n+1}\{X_{n+1}\} = F\{X_{n+1}\}$$

$$x_1 = \overline{x}_1$$

where

$V_i\{x_i\}$ = value derived from implementing $u^*_1,....,u^*_n$ given the level of the resource stock is x_i;

u_i = decision on level of input or output;

$a_i\{x_i,u_i\}$ = stage return;

$g_i\{x_i\}$ = autonomous addition to the resource stock between stage i and i+1;

$h_i\{x_i,u_i\}$ = controlled reduction in the resource stock between stage i and stage i+1;

$\overline{x}_1$ = stock of the resource available at stage 1;

$F\{X_{n+1}\}$ = final value of the resource stock remaining at stage n+1;

and

α = discount factor.

Equation (3.1) is a functional equation because functions appear on both sides. It is recursive because determining $V_{i+1}\{X_{i+1}\}$ enables $V_i\{X_i\}$ to be determined. The equation reflects Bellman's Principle of Optimality which is expounded more fully in section 3.2 in connection with numerical solution of problems.

Because equation (3.1) is recursive the general resource problem can be solved by a process of backward induction. The term $V_{i+1}\{.\}$ on the right-hand is usually unknown for all i except for i=n. In the case of i=n, $V_{n+1}\{X_{n+1}\} = F\{X_{n+1}\}$. Equation (3.1) can therefore be solved for i=n to give u^*_n and $V_n\{x_n\}$. The solution to the general resource problem is obtained by repeating the process for all i from i=n to i=1.

In practice the solution of a functional equation may nat be straight-forward. In Chapter one it was assumed that $V_i\{X_i\}$ and $a_i\{x_i,u_i\}$ were differentiable functions and the solutions were always interior solutions. It was also pointed out that many resource problems do not have these features, in which case it still may be possible to obtain solutions numerically.

A numerical formulation of the general resource problem restricts the values of x_i and u_i to finite sets for all i. A further restriction is that any feasible combination of x_i and u_i must imply access to a value of x_{i+1}. The dynamic programming problem can then be interpreted as one of finding the optimal path through a network of nodes. A perhaps obvious but exploitable characteristic of the optimal path is described in Bellman's Principle of Optimality.

3.3 The Principle Of Optimality

The Principle of Optimality is illustrated with reference to a simple mining problem. Suppose x_i represents the number of units of a mineral in the ground at stage i, and always takes one of the nine values between 0 and 80 shown in the grid in Fig. 3.1. The decision variable u_i represents the number of units mined and can take one of the values 0, 10 or 20, subject to $u_i \leq xi$. The amount of mine at each of four stages has to be decided. The net returns from mining, $a\{x_i,u_i\}$, are shown in Table 3.1.

$$\max_{u_1 \text{ to } u_4} \sum_{i=1}^{4} a\{x_i, u_i\}$$

Subject to

$$x_1 = 80$$

$$x_{i+1} = x_i - u_i$$

and to other constraints on u_i.

The feasible stock levels at each stage are bordered by the broken lines in Fig 3.1. At each decision stage, the three levels of mining 0, 10 and 20 are possible at each feasible state node.

Table 3.1
Mining Stage Returns a{x,u}

Stock (x)	Mined (u) 0	10	20
0	-10	-	-
10	-10	-60	-
20	-10	-40	-60
30	-10	-25	-40
40	-10	-15	-20
50	-20	0	-10
60	-20	10	10
70	-20	20	35
80	-20	30	70

As an example, the decisions are illustrated in Fig. 3.1 at stage 4 for stock level 20.

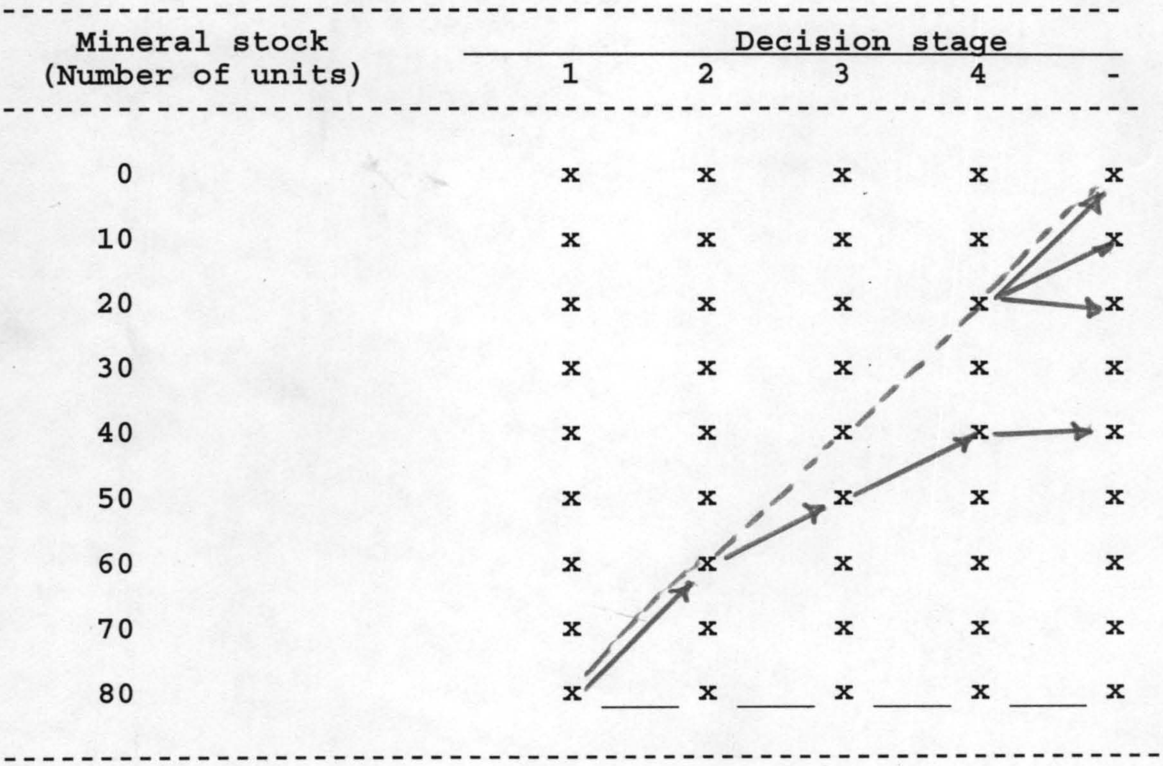

Fig 3.1 Grid of feasible stock levels.

The optimal path through the state nodes is 80, 60, 50, 40, 40 and is shown in Fig 3.1. The corresponding optimal mining policy is 20, 10, 10, 0 which gives a total return (70 + 10 + 0 - 10) = 70. The method of identifying the optimal path is not dealt on at this stage . The point to be made is that for the policy to be optimal, it must conform with Bellman's (p. 83) Principle of Optimality;

> An optimal policy has the property that, whatever the initial state and optimal first decision may be, the remaining decisions constitute an optimal policy with regard to the state resulting from the first decision.

In other words, if 80, 60, 50, 40, 40 really is an optimal path starting with x_1 = 80 then 60, 50, 40, 40 must be the optimal path starting with x_2 = 60; 50, 40, 40 must be the optimal path starting with x_3 = 50; 40, 40 must be the optimal path starting with x_4 = 40. It is simple to check the validity of these statements.

Note that the Principle of Optimality is not a definition of an optimal policy, for the principle states that an optimal policy consists of a nesting of shorter optimal policies. Rather it is a statement about the recursive nature of an optimal policy which suggests that sequential decision problems can be solved by a process of backward recursion. Referring to the example, it was argued by a process of contraction along the optimal path starting with x_1 = 80 that

40, 40 must be the optimal final path starting with $x_4 = 40$. Conversely, an optimal path over any number of decision stages can be found by a reverse process of expansion. Optimal final paths are determined from all possible values of x_4 (20 to 80) in the knowledge that one of them must be part of each of the optimal paths starting from x_3 (40 to 80). This enables the optimal paths from each possible value of x_3 to be determined in th knowledge that one of them must be part of each of the optimal paths starting from each possible value of x_2. In this way the optimal path starting with $x_1 = 80$ is found.

In the process of finding the overall optimal path, information is generated about many possible optimal sub-paths, only a few of which are actually embedded in the overall optimal path. Such information may be redundant. On the other hand, it may be useful for sensitivity analysis, or it may be useful for pursuing a feedback control strategy when transitions from one stock level to another are subject to uncertainty. In any event, the number of decision consequences explored using backward recursion is much less than the number using total enumeration for problems with many decision stages. In the example problem, $3^4 = 81$ decision consequences must be evaluated using total enumeration compared with (1x3) + (3x3) + (5x3) + (7x3) = 48 using backward recursion. In general, if the number of decision stages is n and the initial stock is large enough, the number of decision consequences which must be evaluated is 3^n by total enumeration and $3n^2$ by backward recursion. For n > 3, backward recursion requires the evaluation of fewer decision consequences.

3.4 The Structure Of Dynamic Programming Problems

In Chapter one it was stressed that one of the advantages of dynamic programming was that few constraints are placed on the function $a_i(x_i,u_i)$, $g_i(x_i)$ and $h_i(x_i,u_i)$. The functions could be non-linear, discontinuous and/or stochastic. However, for it to be possible to solve a problem by dynamic programming, it must have a particular structure. The structure must be the same or similar to that for the general resource problem already encountered.

To describe the required structure, the following terms are employed; decision, decision stage, state, transformation function, stage return function and objective function. The problem must consist of a sequence of decision, $u_1,..,_n$. A point in time at which a decision is made is a decision stage, often referred to merely as stage. Any decision u_i made at the i-th decision stage has two consequences. First, it results in a change in the state of the decision system from x_i at stage i to X_{i+1} at stage i+1. The change is expressed by the transformation function which for the general resource problem can be written as

$$X_{i+1} = x_i + g_i\{x_i\} - h_i\{x_i,u_i\}$$

but which can be more generally written as

$$X_{i+1} = t_i\{x_i,u_i\}$$

Secondly, the decision results in a return at each decision stage given by the stage return function, $a_i\{x_i,u_i\}$. The overall objective of the problem must be to select the decision sequence $u_1,\ldots,u_n$ so that a separable objective function of the n stage returns is optimized. The objective function most frequently encountered are the sum of stage returns, or the present value of stage returns

$$\sum_{i=1}^{n} \alpha^{i-1} a_i\{x_i,u_i\}$$

The final decision to be taken, u_n, determines the terminal state of the system, X_{i+1}. There may be some final value F{Xn+1} associated with the terminal state, in which case it is included in the objective function.

Although the additive objective function is the most commonly used one, there are other separable objective functions for which the principle of optimality holds. Nemhauser, following Mitten, has presented a sufficient condition on the objective function for the application of the principle of Optimality, besides separability. If the objective function is separable, then the objective function of the last two decision stages can be written

$$f_{n-1} = f_{n-1}\{a_{n-1}, f_n\{a_n, F\}\}$$

where a_n is the stage return at stage n and F is the terminal value. In general

$$f_i = f_i\{a_i, f_{i+1}\} \qquad (i = 1,\ldots,n-1)$$

The sufficient condition is that

$f_i\{a_i, f'_{i+1}\} \geq f_i\{a_i, f''_{i+1}\}$ if $f'_{i+1} > f''_{i+1}\}$ for all a_i $(i=1,..,n-1)$

In other words, f_i must be a monotonically non-decreasing function of f_{i+1} for all a_i. This condition is satisfied if the objective function is additive in stage returns, or the product of stage returns provided all stage returns are non-negative. The latter objective function has been used in crop-irrigation problems because it implies zero total return if any stage return is zero as a result of crop failure. Cooper and Cooper (1981) have pointed out that the monotonicity condition is not a necessary condition, but have also commented that so far, there has been no satisfactory statement of the necessary conditions on the objective function for the principle of Optimality to hold.[1]

It is important to note that the decision system must be fully described at any stage i by the state of the system, x_i, in the sense that X_{i+1} and a_i depend only on x_i and u_i and any other exogenous variable. Only to this extent may the behavior of the system be dependent on the history of the system prior to stage i. In other words, the decision system must process the Markov property.[2]

To sum up, a dynamic programming problem with an additive objective function has the following form:

$$\max_{u_1 \ldots u_n} \sum_{i=1}^{n} \alpha^{i-1} a_i(x_i, u_i) + \alpha^n F\{X_{n+1}\}$$

Subject to

$$x_1 = \bar{x}_1$$

$$x_{i+1} = t_i(x_i, u_i) \qquad (i= 1,..,n)$$

and other constraints on u_i.

The corresponding recursive equation for solving the problem is

$$V_i\{X_i\} = \max_{u_i} \; [a_i(x_i, u_i) + \alpha V_{i+1}\{t_i\{x_i, u_i\}\}] \qquad (i = n,...,1) \quad ..(3.2)$$

with

$$V_{n+1}\{X_{n+1}\} = F\{X_{n+1}\}$$

$$x_1 = \bar{x}_1$$

and other constraints on u_i.

[1] Hastings, pp 25-33, proves that two conditions, which he terms separability and optimality conditions, are necessary and sufficient for solving a multistage decision problem by dynamic programming.

[2] Problems with casualty lags greater than one period can be formulated to conform with the first-order Markov requirements, by suitably defining the state variable. For example, if x_{i+1} and a_i depend on x_{i-1} as well as x_i and u_i, the requirement is met by defining both x_i and x_{i-1} as state variables.

CHAPTER FOUR

The Irrigation Optimization Model

4.1 Introduction

Here, for exposition purposes, a highly simplified problem in the management of a growing resource is introduced. The problem is formulated as a dynamic programming problem and solved numerically.

4.2 Deriving The Yield Response Model

In deriving The Yield Response Model, Day, et al,. consider the yield of seasonal crops and the depth of water, w_i, received by each crop grown in the i-th season for a horticultural crop, like tomatoes, using the data from Hadejia-Jama'are Irrigation System, Kano, Nigeria, the yield response to moderately well drained sandy loam soil, an improved variety, (*Bataoto*), and fertilizer applied either at the split or one time application, at the rate of 90-120-90, ($N.P_2O_5.K_2O$), Simons, as shown in Table. 4.1 below.

Water Depth (cm)	Grain Yield (t/ha) 1st season crop	2nd season crop	3rd season crop
1	15	14	17
2	13	15	18
3	18	17	14

Table 4.1; Yield response of sequential cropping to the depth of water (w_i) received by each crop grown in the i-th season.

Arranging the data in matrix form, and reducing it to the diagonal form to obtain the eigenvalues, λ_i,

Thus, the matrix, X, was determined such that

$$X^{-1}AX = \Lambda = [\lambda_i \delta_{ij}] \qquad ..(4.1)$$

with,

$$\delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$$

making all matrix elements vanish except along the main diagonal elements which are now the eigenvalues, Fisher, et al,.

Table 4.1, in matrix form, gives the matrix:

$$A = \begin{vmatrix} 15 & 14 & 17 \\ 13 & 15 & 18 \\ 18 & 17 & 14 \end{vmatrix} \qquad .... (4.2)$$

Reducing matrix A to the diagonal form using a suitable numerical method on the computer, yields $\lambda_1 = 0.9$, $\lambda_2 = 1.6$ and $\lambda_3 = 2.1$, showing the eigenvalues to be distinct, Adeboye, thereby fulfilling the condition for diagonilization of matrix in irrigation problems of section 2.3 (a).

Result of Reducing Matrix A

$$A = \begin{vmatrix} 0.9 & 0.0 & 0.0 \\ 0.0 & 1.6 & 0.0 \\ 0.0 & 0.0 & 2.1 \end{vmatrix} \text{ or } \begin{vmatrix} \lambda_1 & 0.0 & 0.0 \\ 0.0 & \lambda_2 & 0.0 \\ 0.0 & 0.0 & \lambda_3 \end{vmatrix} \qquad ... (4.3)$$

where the eigenvalues occur along the diagonal and zero elsewhere, with respect to equation (4.1). If λ ...

the yield, Y, and λ the depth of water w_i received, then equation (4.1) can be rewritten as, Polis,

$$Y = w_i \delta_{ij} \quad .. \quad (4.4)$$

As a result of the diagonilization, table 4.1, now takes the form of equation (4.3),

Table 4.2 ; Crop yield of seasonal crop and the depth of water received.

Depth of water (w) cm	Crop yield (y) t/ha
1	0.9
2	1.6
3	2.1

It was observed that the functional relationship between the yield (y) and the depth of water w_i received is non-linear because the response is usually rapid at lower levels, slower at the intermediate levels and could become negative at higher levels of water depth.

Therefore, a quadratic function was considered and hence the yield response data (now represented by the eigenvalues) of table 4.2, are fitted to the quadratic equation model of the form;

$$Y = A + B X + C X^2$$

Using the technique that involves the linearization of the non-linear form of table 4.2, through the transformation of variables, Burt, to obtain:

$$Y = W - 0.1 W^2 \quad .... \quad (4.5)$$

Where A = 0.0; B = 1.0: and C = - 0.1

Hence in the prediction equation, the yield response of seasonal crop y to different depths of water w_i

received by the crop grown in the i-th season is given by

$$y_i = w_i - 0.1\, w_i^2 \qquad (i = 1 \text{ to } 3)$$

(See the graphical representation in Fig 4.1)

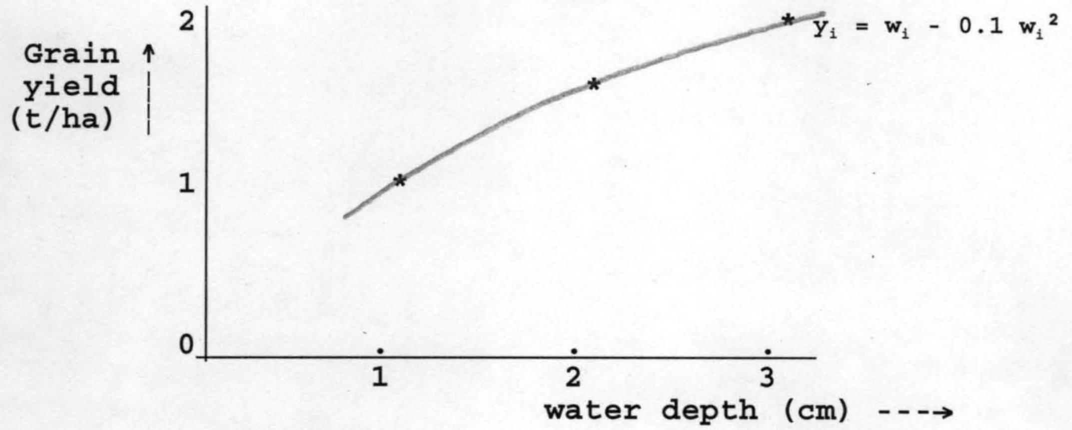

Fig 4.1; *Functional relationship of yield and water depth.*

4.2.1 Deriving The Water Balance Equation.

A water balance refer to an accounting of water movement into and out from farm according to the simplified equation,

Net Irrigation + rain fall = seepage & percolation + evapotran spiration + surface drainage

Irrigation, drainage, rainfall and evapotranspiration data are all expressed as depths of water in centimeters over the area of the farm. For a highly simplified example, the depth of water, w_i, received by the i-th season crop, in cm would consist of total supply of water, which is the sum of net irrigation, u, in meters plus rainfall, q, in centimeters which for varying depths and quantities we have the following equation;

$$w_i = u_i + q_i$$

which implies basic counting technique, Arrow, et al,.

4.2.2 Objective of the Farmer

Is to select integer values of u_i so that the present value of receipts from sale of the crop is maximized. The price (# per ton) received from the i-th season crop is b_i.

4.3 Dynamic Programming As A Method Of Conceptualizing Resource

Many agricultural economics have noted the need for an operational approach for solving dynamic problems. Johnson and Rausser p.164 comment that 'many agricultural economics for a number of years have been busily applying static neo-classical theory to intrinsically dynamic system'. Heady, p.38 has suggested 'perhaps conventionally optimizing theory was used more widely in recent decades because theory related to time and stochastic phenomena was not yet sufficiently operational' Hanf and Schiefer p. 16 notes that 'in most operational decision models the time dimension of marginal decisions is not considered adequate'.

One of the aim of this project has been to draw together an example of how dynamic programming, since its advocacy by partitioners such as Throsby and Burt in the early 1960's has already been applied to a dynamic resource problem. In so doing the image of dynamic programming as an arcane or esoteric technique may be rebutted and its simplicity and practicability demonstrated.

A useful insight into the nature of a dynamic resource problem is obtained just by formulating the basic recursive equation, regardless of whether the problem is solved using dynamic programming. The process of identifying state variables, decision variables, stage return function and state transformation functions help formalize the problem.

To summarize what is gained from the recursive equation, consider once more the equation for the general resource problem introduced in Chapter one;

$$V_i\{X_i\} = \max_{0 \le u_i \le u_i} [a_i\{x_i,u_i\}+\alpha v_{i+1}\{x_i+g_i\{x_i\}-h_i\{x_i,u_i\}\}]$$

$$= \max_{0 \le u_i \le u_i} [z_i\{x_i,u_i\}]$$

$$= \max [a_i\{x_i,u^*_i\}+\alpha v_{i+1}\{x_i+g_i\{x_i\}-h_i\{x_i,u^*_i\}\}]$$

$$(i = n,..,1) \quad ... (4.6)$$

Where 0 and u_i are lower and upper bounds on the control variable u_i, and u^*_i is the optimal value of the resource stock at stage i is given the usual economic imputation of being the capitalized value of the stream of optimal stage returns. Thus (4.5) is consistent with

$$V_i\{x_i\} = \sum_{j=1}^{n} \alpha^{j-i} a_j\{x_j,u^*_j\}.$$

It can be seen that from (4.6) that at any stage i, u^*_i depends simultaneously on the impact of u_i on the current stage return and on the present value of the resulting resource stock at stage i+1.

4.3.1 Forward Recursion and Stage Numbering

In general it is possible to solve numerical dynamic programming problems which are deterministic by forward as well as by backward recursion. Forward recursion has an advantage over backward recursion for problems with unc... planning horizons. The relevant planning horizon may be judged to be the one for which, if it were increased, there would be no change in the optimal first decision, or first few decisions. The relevant decision horizon must be found by experimenting with successive longer horizons until the requisite criterion is met. If backward recursion is used, a fresh problem would have to be solved for each planning horizon. If forward recursion were used, once calculations had been performed for a planning horizon of n stages, only calculations for one additional stage would need to be performed for a decision horizon of n+1 stages.

At this point,it is probable as well to point out that the ordering of stage subscripts in dynamic programming recursive equations is conventionally the reverse of that used here. That is, if backward recursion is used, stage subscript i denotes the number of decision stages remaining, instead of the number of the stage reached in the sequence in which decisions are actually taken. The convention of reverse ordering is perhaps useful when the process of backward recursion is actually being worked through. It is not followed here because temporal ordering of stage subscripts make the interpretation of the recursive equations more straightforward.

4.4 The Formulation of tho Problem;

A farmer grows three horticultural crop in successive seasons over one year on 100 ha. Each crop takes four months or one season to reach maturity from the time of planting.

The yield of each crop (in hundreds of tons per 100 ha), is given by;

$$y_i = w_i - 0.1 w_i^2 \qquad (i = 1 \text{ to } 3)$$

where, w_i, is the depth of water in cm. received by the crop grown in the i-th season. The depth of water received depends on the height of water released from storage at the beginning on each season (u_i, in meters) and rainfall received during each season (q_i, in cm). The area of the dam is 1 hectare, so

$$w_i = u_i - q_i$$

The dam is full at the beginning of the first season with water height of 3 m. The amount of water which can be released at the beginning of any season is limited to integer values of water and also by the amount in storage. Rainfall augments the water in storage. The catchment area is 100 ha, so 1 cm of rainfall raises the level of the dam x_i by 1 m. provided the dam is not full.

The problem can be formulated as;

$$\max_{u_1,u_2,u_3} \sum_{i=1}^{3} \alpha^{i-1} b_i (w_i - 0.1w^2{}_i).$$

Subject to

$$0 \le u_i \le x_i \le 3 \qquad u_i,\ x_i \text{ integer}$$

$$x_1 = 3$$

$$X_{i+1} = \min\ (x_i - u_i + q_i)$$

with data

b = [50, 100, 150]

q = [2, 1, 1]

$\alpha = 0.95$

4.5 The Solution Procedure

The backward recursive equation used to solve this problem is,

$$V_i\{X_i\} = \max_{0 \le b_i \le x_i} [b_i(w_i - 0.1w_i^2) + \alpha V_{i+1}\{x_i - u_i + q_i\}] \quad (i=3 \text{ to } 1)$$

with ..... (4.7)

$$w_i = u_i + q_i$$

$$V_4\{X_4\} = 0$$

The recursive equation is solved in appendix (4.A) and used in the compilation of Table 4.3. Consider the option which can be taken at stage two if the dam water level is 3 m. If no water is released, then only water received by the crop is 1 cm of rainfall. The value of the additional crop yield is #9,000.00. The dam water level will still be 3 m at stage 3 despite the rainfall because of overflow. The optimal return at stage 3 from a water level of 3 m has been calculated to be #36,000.00 in stage 3 section of Table 4.3. The return discounted one stage is #(0.95*36,000 = 34,200.00).
The value of releasing no water at stage 2 with 3 m of water is therefore #(9,000+34,200 = 43,200.00).

The value of releasing all of the available water is the sum of the value of the additional crop yield (#24,000.00) and the discounted value of return from the dam with 1 m of water depth remaining at stage 3 is #(0.95*24,000 = 22,800.00), which equals #46,800.00. The optimal amount of water to release can be seen to be 2 m, giving a total return of #50,900.00. If there is excessive water level, there will be overflow and this equals the amount of rainfall received during that season.

Optimal Irrigation Returns (#000)

Stage 3

Dam water level x_i (m)	Water released (ha m)				$V_3\{x_3\}$	$u^*_3\{x_3\}$
	$u_3 = 0$	$u_3 = 1$	$u_3 = 2$	$u_3 = 3$		
	$q_3 = 1; w_3 = 1$	$q_3 = 1; w_3 = 2$	$q_3 = 1; w_3 = 3$	$q_3 = 1; w_3 = 4$		
0	13.5+0.0 = 13.5				13.5	0
1	13.5+0.0 = 13.5	24.0+0.0 = 24.0			24.0	1
2	13.5+0.0 = 13.5	24.0+0.0 = 24.0	31.5+0.0 = 31.5		31.5	2
3	13.5+0.0 = 13.5	24.0+0.0 = 24.0	31.5+0.0 = 31.5	36.0+0.0 + 36.0	36.0	3

Stage 2

Dam water level x_i (m)	Water released (ha m)				$V_2\{x_2\}$	$u^*_2\{x_2\}$
	$u_2 = 0$	$u_2 = 1$	$u_2 = 2$	$u_2 = 3$		
	$q_2 = 1; w_2 = 1$	$q_2 = 1; w_2 = 2$	$q_2 = 1; w_2 = 3$	$q_2 = 1; w_2 = 4$		
0	9.0+22.8 = 31.8				31.8	0
1	9.0+29.9 = 38.9	16.0+22.8 = 38.8			38.8	0
2	9.0+34.2 = 43.2	16.0+29.9 = 45.9	21.0+22.8 = 43.8		45.9	1
3	9.0+34.2 = 43.2	16.0+34.2 = 50.2	21.0+29.9 = 50.9	24.0+22.8 = 46.8	50.9	2

Stage 1

Dam water level x_i (m)	Water released (ha m)				$V_1\{x_1\}$	$u^*_1\{x_1\}$
	$u_1 = 0$	$u_1 = 1$	$u_1 = 2$	$u_1 = 3$		
	$q_1 = 2; w1 = 2$	$q_1 = 2; w_1 = 3$	$q_1 = 2; w_1 = 4$	$q_1 = 2; w_1 = 5$		
0	8.0+43.6 = 51.6				51.6	
1	8.0+48.4 = 56.4	10.5+43.6 = 54.1			56.4	
2	8.0+48.4 = 56.4	10.5+48.4 = 58.9	12.0+43.6 = 55.6		58.9	
3	8.0+48.4 = 56.4	10.5+48.4 = 58.9	12.0+48.4 = 60.4	12.5+43.6 = 56.1	60.4	

4.6 Optimal Irrigation Decision Path

Table 4.3 shows that if the dam is full at the start of the growing season, the maximum return is #60,400.00. The optimal decision path shown in table 4.4 was derived by tracking forward through time from stage 3 to stage 1.

Table 4.4

Optimal Irrigation Sequence Starting With the Dam Full

Season	Dam water Level (m)	Water released (ha m)	Season return (#000)	Discounted total return (#000)
1	3	2	12.0	60.4
2	3	2	21.0	50.9
3	2	2	31.5	31.5

The three crop received a total of ten units of water. Six were supplied from the dam, and four were received as rainfall. The dam was emptied at the beginning of the third season, though subsequent rainfall left the dam with a water height of 1 m at the end of the third season.

Table 4.3 can be used to find the optimal sequence of irrigation starting with other water levels and seasons.

Appendix 4.A

The solution of equation (4.6) used in the compilation of table 4.3

$$V_i\{X_i\} = \max_{0 \le b_i \le x_i} [b_i(w_i - 0.1w_i^2) + \alpha V_{i+1}\{x_i - u_i + q_i\}] \quad (i=3 \text{ to } 1)$$

with ..... (4.7)

$$w_i = u_i + q_i$$

$$V_4\{X_4\} = 0$$

<u>Stage Three</u>

Applying the data to equation (4.7)

$u_3 = 0,\ q_3 = 1,\ w_3 = 1,\ x_3 = 0$

$V_3\{x_3\} = [150(1 - 0.1) + 0.95V_4\{x_{0-0+1}\}] = 13.5+0 = 13.5$

$u_3 = 0,\ q_3 = 1,\ w_3 = 1,\ x_3 = 1$

$V_3\{x_3\} = [150(1 - 0.1) + 0.95V_4\{x_{1-0+1}\}] = 13.5+0 = 13.5$

$u_3 = 1,\ q_3 = 1,\ w_3 = 2,\ x_3 = 1$

$V_3\{x_3\} = [150(2 - 0.4) + 0.95V_4\{x_{1-1+1}\}] = 24.0+0= 24.0$

$u_3 = 0,\ q_3 = 1,\ w_3 = 1,\ x_3 = 2$

$V_3\{x_3\} = [150(1 - 0.1) + 0.95V_4\{x_{2-0+1}\}] = 13.5+0 = 13.5$

$u_3 = 1,\ q_3 = 1,\ w_3 = 2,\ x_3 = 2$

$V_3\{x_3\} = [150(2 - 0.4) + 0.95V_4\{x_{2-1+1}\}] = 24.0+0 = 24.0$

$u_3 = 2,\ q_3 = 1,\ w_3 = 3,\ x_3 = 2$

$V_3\{x_3\} = [150(3 - 0.9) + 0.95V_4\{x_{2-2+1}\}] = 31.5+0 = 31.5$

$u_3 = 0,\ q_3 = 1,\ w_3 = 1,\ x_3 = 3$

$V_3\{x_3\} = [150(1 - 0.1) + 0.95V_4\{x_{3-0+1}\}] = 13.5+0 = 13.5$

$u_3 = 1,\ q_3 = 1,\ w_3 = 2,\ x_3 = 3$

$V_3\{x_3\} = [150(2 - 0.4) + 0.95V_4\{x_{3-1+1}\}] = 24.0+0 = 24.0$

$u_3 = 2,\ q_3 = 1,\ w_3 = 3,\ x_3 = 3$

$V_3\{x_3\} = [150(3 - 0.9) + 0.95V_4\{x_{3-2+1}\}] = 31.5+0 = 31.5$

$u_3 = 3,\ q_3 = 1,\ w_3 = 4,\ x_3 = 3$

$V_3\{x_3\} = [150(4 - 1.6) + 0.95V_4\{x_{3-3+1}\}] = 36.0+0 = 36.0$

<u>Stage Two</u>

$u_2 = 0,\ q_2 = 1,\ w_2 = 1,\ x_2 = 0$

$V_2\{x_2\} = [100(1-0.1) + 0.95V_3\{x_{0-0+1}\}] = 9.0+22.8 = 31.8$

$u_2 = 0,\ q_2 = 1,\ w_2 = 1,\ x_2 = 1$

$V_2\{x_2\} = [100(1-0.1) + 0.95V_3\{x_{1-0+1}\}] = 9.0+29.9 = 38.9$

$u_2 = 1,\ q_2 = 1,\ w_2 = 2,\ x_2 = 1$

$V_2\{x_2\} = [100(2-0.4) + 0.95V_3\{x_{1-1+1}\}] = 16.0+22.8 = 38.8$

$u_2 = 0,\ q_2 = 1,\ w_2 = 1,\ x_2 = 2$

$V_2\{x_2\} = [100(1-0.1) + 0.95V_3\{x_{2-0+1}\}] = 9.0+34.2 = 43.2$

$u_2 = 1,\ q_2 = 1,\ w_2 = 2,\ x_2 = 2$

$V_2\{x_2\} = [100(2-0.4) + 0.95V_3\{x_{2-1+1}\}] = 16.0+29.9 = 45.9$

$u_2 = 2,\ q_2 = 1,\ w_2 = 3,\ x_2 = 2$

$V_2\{x_2\} = [100(3-0.9) + 0.95V_3\{x_{2-2+1}\}] = 21.0+22.8 = 43.8$

$u_2 = 0,\ q_2 = 1,\ w_2 = 1,\ x_2 = 3$

$V_2\{x_2\} = [100(1-0.1) + 0.95V_3\{x_{3-0+1}\}] = 9.0+34.2 = 43.2$

$u_2 = 1,\ q_2 = 1,\ w_2 = 2,\ x_2 = 3$

$V_2\{x_2\} = [100(2-0.4) + 0.95V_3\{x_{3-1+1}\}] = 16.0+34.2 = 50.2$

$u_2 = 2,\ q_2 = 1,\ w_2 = 3,\ x_2 = 3$

$V_2\{x_2\} = [100(3-0.9) + 0.95V_3\{x_{3-2+1}\}] = 21.0+29.9 = 50.9$

$u_2 = 3,\ q_2 = 1,\ w_2 = 4,\ x_2 = 3$

$V_2\{x_2\} = [100(4-1.6) + 0.95V_3\{x_{3-3+1}\}] = 24.0+22.8 = 46.8$

<u>Stage one</u>

$u_1 = 0,\ q_1 = 2,\ w_1 = 2,\ x_1 = 0$

$V_1\{x_1\} = [50(2-0.4) + 0.95V_2\{x_{0-0+2}\}] = 8.0+43.6 = 51.6$

$u_1 = 0,\ q_1 = 2,\ w_1 = 2,\ x_1 = 1$

$V_1\{x_1\} = [50(2-0.4) + 0.95V_2\{x_{1-0+2}\}] = 8.0+48.4 = 56.4$

$u_1 = 1,\ q_1 = 2\ w_1 = 3,\ x_1 = 1$

$V_1\{x_1\} = [50(3-0.9) + 0.95V_2\{x_{1-1+2}\}] = 10.5+43.6 = 54.1$

$u_1 = 0,\ q_1 = 2,\ w_1 = 2,\ x_1 = 2$

$V_1\{x_1\} = [50(2-0.4) + 0.95V_2\{x_{2-0+2}\}] = 8.0+48.4 = 56.4$

$u_1 = 1,\ q_1 = 2,\ w_1 = 3,\ x_1 = 2$

$V_1\{x_1\} = [50(3-0.9) + 0.95V_2\{x_{2-1+2}\}] = 10.5+48.4 = 58.9$

$u_1 = 2,\ q_1 = 2,\ w_1 = 4,\ x_1 = 2$

$V_1\{x_1\} = [50(4-1.6) + 0.95V_2\{x_{2-2+2}\}] = 12.0+43.6 = 55.6$

$u_1 = 0,\ q_1 = 2,\ w_1 = 2,\ x_1 = 3$

$V_1\{x_1\} = [500(2-0.4) + 0.95V_2\{x_{3-0+2}\}] = 8.0+48.4 = 56.4$

$u_1 = 1,\ q_1 = 2,\ w_1 = 3,\ x_1 = 3$

$V_1\{x_1\} = [50(3-0.9) + 0.95V_2\{x_{3-1+2}\}] = 10.5+48.4 = 58.9$

$u_1 = 2,\ q_1 = 2,\ w_1 = 4,\ x_1 = 3$

$V_1\{x_1\} = [50(4-1.6) + 0.95V_2\{x_{3-2+2}\}] = 12.0+48.4 = 60.4$

$u_1 = 3,\ q_1 = 2,\ w_1 = 5,\ x_1 = 3$

$V_1\{x_1\} = [50(5-2.5) + 0.95V_2\{x_{3-3+2}\}] = 12.5+43.6 = 56.1$

CHAPTER FIVE

ANALYSIS OF RESULTS

5.1 Using Computer To Solve The Crop Irrigation Problem

The repetitive nature of the procedure for solving numerical problems is apparent from solving the crop irrigation problem. For problem of any realistic size computer programs are written to find solutions.

Hastings has developed a general purpose dynamic programming package which is available commercially, and known as DYNACODE. Morin refers to some other codes which are available. A sample set of the routines for solving general purpose dynamic programming problems (referred to as GPDP) is used here. The program is written in Basic to make them accessible to owners of micro computers.

The first step is to structure the data for writing to file using DPD. Table 5.1 shows the system of numbering and labelling states and decisions used.

Table 5.1

Identification of States and Decisions

Dam level (m) x	State number I	Label - -	Water released u (ha m)	Decision number D	Label - -
0	1	L_0	0	1	R_0
1	2	L_1	1	2	R_1
2	3	L_2	2	3	R_2
3	4	L_3	3	4	R_3

```
----Final  Stage     DP     Problem ------
Problem Name ?       CID
```

```
 Deterministic Dynamic Programming Solution      --CIDF---
Problem Parameters;
Number of Decisions              =  3
Rate of Discount  (%)            =  5.2632
```

Optimal State Sequence For All Initial States

Stages				Values
1	2	3	4	
1	3	3	2	51.6287
2	4	3	2	56.3787
3	4	3	2	58.8787
4	4	3	2	60.3787

Details of Optimal Path
Enter Initial State No. (or '0' to finish) ? 4

Stage No.	State No.	State Level	Decision No.	Decision Release	Discounted Stage Return	Value
1	4	L3	3	R2	12	60.3787
2	4	L3	3	R2	19.95	50.925
3	3	L2	3	R2	28.4287	31.5
4	2	L1	0		0	0

Details of Optimal Path
Enter Initial State No. (or '0' to finish) ? 0

Table 5.2 Solution To The Crop Irrigation Problem using FDP

The stage-3 data block CD and the stage-2 and -1 data for data block ED were drawn up as shown in Table 5.3. The layout is similar to that used in table 4.3. The data were transferred to copies of code sheets (not shown here) Because the crop irrigation problem is deterministic and has finite stages, CIDF was chosen as the problem name. Finally, Table 5.2 shows the solution obtained by running FDP for the problem CIDF. The first part of the printout from FDP gives the Optimal sequence of state numbers starting from each possible state at stage 1. The second part gives further details of the optimal sequence for any nominated initial state number, in this case the state No. 4 representing full dam.

Table 5.3

Stage Data For The Crop Irrigation Problem

Stage 3

I	D = 1		D = 2		D = 3		D = 4	
	J	R(I, J)	J	R(I, J)	J	R(I, J)	J	R(I, J)
1	2	13.5						
2	3	13.5	2	24.0				
3	4	13.5	3	24.0	2	31.5		
4	4	13.5	4	24.0	3	31.5	2	36.0

Stage 2

I	D = 1		D = 2		D = 3		D = 4	
	J	R(I, J)	J	R(I, J)	J	R(I, J)	J	R(I, J)
1	2	9.0						
2	3	9.0	2	16.0				
3	4	9.0	3	16.0	2	21.5		
4	4	9.0	4	16.0	3	21.5	2	24.0

Stage 1

I	D = 1		D = 2		D = 3		D = 4	
	J	R(I, J)	J	R(I, J)	J	R(I, J)	J	R(I, J)
1	3	8.0						
2	4	8.0	3	10.5				
3	4	8.0	4	10.5	3	12.0		
4	4	8.0	4	10.5	3	12.0	3	12.5

The results agree with those presented in Table 4.4 based on the calculations in Table 4.3, thereby helping farmers take decision on how best to apply water to a crop under irrigation.

This table show the result of the first part of the right hand side of equation (4.4) to show that the computer routine agrees with the manual solution.

5.3 Expected Development and Recommendations

Computer power is becoming increasingly available on farms with the spread of micro computers. Although computers are used primarily for book-keeping and taxation purposes, many software packages have been developed to aid management decision making in the areas of budgeting, control of farm inputs and predicting yield of crop and livestock products. An indication of the interest in the field is the publication of relevant newsletters and journals, such as Farm Computer News in Nigeria an the New International journal of computers and electronics in Agriculture.

It is unrealistic to expect many farm managers to have the time and inclination to invest in any detailed understanding of dynamic programming. However, the continued development of dynamic programming software should make only a general understanding of the scope of dynamic programming necessary in other to apply it.

Perhaps the greatest problem is the lack of relevant data. However, as Burt has pointed out, data are not collected unless there is a perceived need for them; and the need is not always perceived until models designed to answer practical questions are developed. There is a symbolic relationship between data available and model used. In the application of dynamic programming considered in this project there is area of data shortfall. Such application help to identify priorities in scientific research. Some requirements are for data specific to the farm. By using dynamic programming within the adaptive control framework it is possible to generate farm specific data. Thus if a farm specific parameter in a dynamic programming model is initially unknown, it may be set at some value which is an average across farms. Bayesian or other methods may be used to periodically revise estimates of the parameter. An example application to estimating the yield response of multiple cropping to depth of water received was presented in chapter Four.

There are many areas of application in agriculture and natural resources industries which have not been touched in this project. For example dynamic programming can be usefully applied to problems in Farm financial management and in other stochastic multistage decision process. Speculating on future application Denardo p. 394 singled out computer science, applied mathematics, economic growth models and agricultural resources as promising areas for further development.

5.4 Conclusion;

The question is not whether irrigation plays a critical role in the food production of this nation: it has already. The question is how it can continue to do this in the most sustainable manner, FAO. The answer to this will go a long way toward determining the outcome of tomorrow's agricultural research and extension services, practices and policies.

The paradox of all the foregoing scenario is that irrigated agriculture would have to produce much more food in future while using less water than it uses currently and the intensification of food production to attain these objective will lead to greater stress. Producing more food with le water at a lower financial and environmental cost will therefore be a major challenge for Nigeria in this millennium.

In this research project, for exposition purposes, a highly simplified management of a growing resource is introduced. the problem is formulated as a dynamic programming problem and solved numerically by hand. The result is the optimal decision path or the optimal sequence of irrigation starting with any water levels and seasons. These are only on broad lines which if further pursued could assist in achieving some useful results in our bid to producing more food with less water, thereby solving the major challenge for Nigeria in this millennium.

Organizations such as National Agricultural Research Project, (NARP); Strategic Grain Reserve Scheme, (SGRS); Agricultural Development Program, (ADP); River Basin Development Authorities, (RBDA); are interested in agriculture. This project may to an extent serve as basis for their research programs in irrigated agriculture, which will lead to double cropping or even multi-cropping practices and policies. This is what is new in the whole project.

So far dynamic programming has been seen as a technique for deducing optimality conditions or as an algorithm fo obtaining numerical solutions. My special contributions to knowledge here includes, the ability to derive a yi estimate (that relates the yield of each crop to the depth water received by the crop grown in the i-th season) f seasonal crop grown over three seasons with three moi regimes. Then the ability to formulate the crop irrigation problem using the logic of dynamic programming. The second order, non linear polynomial yield estimate, the price received for the i-th season crop, the rainfall received during each season, the water released from storage at the beginning of each season, the depth of water received by each crop grown in the i-th season, the dam water level and the rate of discount were implored into the backward recursive equation to obtain the optimal irrigation returns from where the optimal sequence of irrigation was derived. An important feature here is that optimal decisions are specified for the entire range of possible states which may be reached at each stage. An important point is that an optimal decision is known for whatever state is actually reached. The solutions are closed-loop or feedback policies. The computer routine should help those who want to solve relatively simple problems but are daunted by the prospect of a lot of tedious calculation, or of having to start programming a solution from the scratch.

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