ANALYSIS OF FLOW IN A CIRCULAR PIPE IN THE PRESENCE OF A MAGNETIC FIELD

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CERTIFICATION

This thesis titled: ANALYSIS OF FLOW IN A CIRCULAR PIPE IN THE PRESENCE OF A MAGNETIC FIELD by SHEHU BOLAJI meets the regulation governing the award of the degree of Masters of Technology in Mathematics, Federal University of Technology, Minna and is approved for its contribution to knowledge and literary presentation

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DEDICATION

This research work is dedicated to the Almighty God; the most compassionate the most merciful who reigns throughout eternity.

To my wife Tayo and my little boy Dayo; and my parents.

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I am using this opportunity to express my profound gratitude to all those who contributed one way or the other to the successful completion of an academic goal of this nature.

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ABSTRACT

In this research work, the equations arising from the flow in a circular pipe with a magnetic field in the fluid is solve analytically, and the numerical approximations of these equations obtained from varying the parameters η, λ, μ, B are derived by using a regular perturbation technique. The result shows that the higher the magnetic field in the fluid, the higher the velocity of the fluid.

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CHAPTER ONE

1.1 INTRODUCTION

We consider a steady state rectilinear flow along a non-conducting circular pipe of radius 'a' under the magnetic field that stems from a uniform imposed axial current in the fluid.

There are radial magnetic or pinch forces associated with the imposed current and its magnetic field but these are irrational and are balance by the pressure. No Hartmann layers occur because the field is parallel to the walls. Alfven propagation could occur around the field lines but we shall suppress it by neglecting the $\partial B/\partial t$ term.

The equations in polar coordinates are:

$$\frac{B}{a}\frac{\partial V_Z}{\partial \theta} + \lambda \nabla^2 B_Z = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{B}{\mu a}\frac{\partial B_Z}{\partial \theta} + \eta \nabla^2 V_Z = \ell \frac{\partial V_Z}{\partial t} + \frac{\partial \ell}{\partial Z} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Where the term are as defined in 1.2

The boundary conditions are that V_Z and B_Z vanish at the wall. There is no field induced outside the fluid.

The flow is in the Z-direction under a known, imposed, uniform transverse field B and an unknown field B_Z due to currents flowing in (x, y)-planes. The object is to find V_Z and B_Z as functions of r and θ. The Laplacians are two-dimensional.

In this work we discuss a numerical approximation of the equations using a method of regular perturbation and then solve the resulting order zero (0) and order one (1) equations analytically. The order zero (0) equation is solved by general method and the order one (1) equation is solved by separation of variable method.

1.2 DEFINITION OF TERMS

i. Field is a substance which deforms or yields continuously when shear stress is applied to it, no matter how small it is.

ii. Magnetohydrodynamics (MHD): This is the science of the motion of electrically conducting fluids under magnetic fields. The situation is essentially one of mutual interaction between the fluid velocity field and the electromagnetic field; the motion affects the magnetic field and the magnetic field affects the motion.

iii. Alfven wave is the magnetic field lines, under apparent tension and having the inerfia of the fluid 'frozen' to them. It represents the tendency of vorticity to propagate along field lines whenever the conductivity is high enough.

iv. Hartman effect: This occurs where a steady state of balance between vorticity suppression (or propagation) and diffusion is achieved.

v. Mass density (ℓ) is the mass per unit volume

vi. Magnetic flux density vector (B). This is the direction and strength of the field at any point. The closer the lines, the stronger the field.

vii. Electric field vector (E) this is the force on a charged particle due to changing magnetic field with time relative to a certain frame of reference.

viii. Viscosity (η) -this is the property of a fluid by which it offers Resistance to shear acting on it. According to Newton's law of viscosity the shear of acting between two layers of fluid is proportional to differences in their velocities and inversely proportional to the distance between them.

$$\text{i.e. } F = \mu A \frac{du}{dy}$$

$$\tau = F/A = \mu \frac{du}{dy}$$ which is the

shear stress where μ is the

constant of proportionality

$\frac{du}{dy}$ = rate of angular deformation.

ix. Permeability of free space $\mu = 4\pi \times 10^{-7}$

x. Magnetic diffusivity $\lambda = \frac{1}{\mu\sigma}$

xi. σ is the electrical conductivity

xii. The continuum Approximation: This is the postulate that the fluid may be treated as continuous and describable in terms of local properties such as pressure and velocity. These should strictly be defined as averages over elements large compared with the microscopic structure of matter but small enough in comparison with the scale of the macroscopic phenomena to permit the use of differential calculus to describe them.

1.3 THE ELECTRICAL AND MAGNETIC PROPERTIES OF THE FLUID

Magnetohydrodynamics differs from ordinary hydrodynamics in that the fluid is electrically conducting. It is not magnetic; it affects a magnetic field not by its mere presence but only by virtue of electric currents flowing in it.

The non-relativistic electromagnetic theory is necessary in MHD. This implies that all material velocities must be much smaller than C the velocity of light (3×10^8 m/sec approx.)

A charged particle such as an electron suffers forces of three kinds as follows:

A. It is repelled or attracted by other charged particles, the total force on the particle per unit of its charge being the electrostatic field E_S (Volt/m). That is the $E_S = -gradV$ where V is an electrostatic potential (Volts).

B. A charged particle moving with velocity vm/sec relative to a certain frame of reference suffers a magnetic for V x B (Newton's) per unit of its charge. The direction of B is that in which the particle must travel to feel no magnetic force. B is measured in Weber/m^2.

C. If the magnetic field B is changing with time relative to a certain frame of reference, then per unit of its charge a particle will suffer a further force E_i the induced electric field, defined by:

$$divE = 0 \text{ and Curl } E_i = -\partial B/\partial t \Rightarrow divB = 0 \quad \ldots\ldots(3)$$

1.4 FLUID-MECHANICAL ASPECTS

Implicit in hydrodynamic or continuum approximation is the assumption of local quasi-equilibrium which permits the state of the fluid at each point to be described by a few variables, related just as if the fluid were in equilibrium.

In this study we shall assume that conducting fluid is in local equilibrium and supports isotropic transport processes characterized by the scaler coefficients K(thermal conductivity).

$\eta(viscosity)$ and σ(electrical conductivity).

The stress relationships are applicable

Normal stresses

$$\pi_{xx} = -P + 2\eta\left(\partial V_Z / \partial x - \frac{1}{3} divV\right) \quad \ldots\ldots (4)$$

Shear Stresses

$$\pi_{xy} = \eta\left(\partial V_x/\partial y + \partial V_y/\partial x\right) \quad (5)$$

The thermodynamic pressure P has been made equal to the mean of any three perpendicular normal stresses.

1.5 CONTINUITY EQUATION

This is the mass conservation equation. It is given by:

$$divPV = -\partial p/\partial t \text{ or } pdivV = -Dp/Dt \quad (6)$$

where Dp/Dt denotes the substantive time derivative $\partial p/\partial t + (V.grad)p$ which represents differentiation through the history of a moving fluid particle.

1.5.1 INCOMPRESSIBLE FLUID

This is a fluid where each traveling fluid element changes its density negligibly, even though ℓ may be non – uniform. Then

$Dp/Dt = 0$ and therefore

$$divV = 0 \quad (7)$$

1.5.2 ACCELERATION

The acceleration of a fluid element is DV/Dt and is determined by the imbalance between gradients of pressure viscous stress and the J x B body force qE being negligible. This is expressed by the modified Navier-Stokes equation (all terms being per unit volume):

$$\ell\frac{DV}{Dt} = j \times B + \left(\frac{\partial\sigma_{xx}}{\partial x} + \frac{\partial\sigma_{xy}}{\partial y} + \frac{\partial\sigma_{xz}}{\partial Z}\right) \text{ etc.} \quad (8)$$

In case of an inviscid fluid, equation (4) degenerates to

$$\ell \frac{DV}{Dt} + gradP = j \times B, \quad \ldots\ldots (9)$$

In the case of an incompressible fluid, with η constant, to

$$\ell \frac{DV}{Dt} + gradP = j \times B + \eta \nabla^2 V \quad \ldots\ldots (10)$$

by virtue of equations (4 & 5) and (7)

Given enough boundary conditions these equations suffice to determine the unknown vector and scalar fields provided the dependencies of r, η and ℓ on P are known. The simplest case is that where r, η and ℓ may be taken as constants.

CHAPTER TWO

MATHEMATICAL REVIEW

2.0 PARTIAL DIFFERENTIAL EQUATION (P.D.E)

A partial differential equation with two or more independent variables.

e.g.

i. $$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0$$

ii. $$\frac{\partial^2 \phi}{\partial t}+\frac{\partial^2 \phi}{\partial x^2}=0$$

2.1 METHODS OF SOLVING A P.D.E.

There are several methods of solving a P.D.E. which can be classified either as analytical or numerical method.

1. **ANALYTICAL METHODS:** The methods produce exact solutions of the P.D.E. the following are among the most important

   a. **METHOD OF GENERAL SOLUTIONS:** In this method we first find the general solution and then that particular solution which satisfies the boundary conditions.

   b. **SEPARATION OF VARIABLES:** In this method it is assured that a solution can be expressed as a product of unknown functions each of which depends on only one of the independent variables. The success of the method hinges on being able to write the resulting equation so that one side depends on the remaining variables so that each side must be a constant. By repetition of this unknown functions can then be determined. Superposition of these solutions can then be used to find the actual solution. The method often makes use of Fourier series, Fourier integrals, Bassel series and Legendre series.

c. **LAPLACE TRANSFORM METHODS:** In this methods the Laplace transform of the P.D.E. and associated boundary conditions are first obtained with respect to one of the independent variables we then solve the resulting equation for the Laplace transform of the required solution, which is then found by taking the inverse Laplace transform. In cases where Laplace inversion is of some difficult the complex inversion formular can be used.

An exhaustive study of these methods is not the focus of this research work. As an illustration of the use of separation of heat in a slab of thickness L. The governing equation for temperature, denoted by U, is

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \qquad t > 0,\ 0 < x < L$$

if we let the boundaries be kept at the same temperature, say zero, then the boundary conditions are homogeneous

$$U(0,t) = U(L,t) = 0,\quad t > 0 \text{ and } U(x,0) = f(x)$$

Using separation of variables

$$U = X(x)T(t)$$

$$\frac{X''}{x} = \frac{T'}{KT} = -\lambda^2$$

Where λ is real. This leads to two ordinary differential equation

$$T^1 + \lambda^2 KT = 0 \text{ and}$$

$$X'' + \lambda^2 X = 0$$

The general solutions are:

$$T = e^{-k+\lambda^2 t}$$

And

$$X = A\sin\lambda x = 0 + B\cos\lambda x$$

The boundary condition for x are

$$x(0) = x(L) = 0$$

Consequently B = 0 and the Eigen value condition is

$$Sin\lambda L = 0$$

which gives

$$\lambda n = \frac{n\pi}{L} \qquad \text{n = 1, 2, 3...}$$

therefore eigen functions are

$$X_n = \sin\frac{n\pi}{L}x, \text{ n = 1, 2, 3...}$$

and the corresponding time factor is

$$T_n = \exp\left[-\left(\frac{n\pi}{L}\right)^2 kt\right] \text{ 1, 2, 3...}$$

Hence the final solution is of the form

$$U = \sum_{n=0}^{\infty} a_n \left[\exp\left(-\frac{n\pi}{L}\right)^2 kt\right] \sin\frac{n\pi x}{L}$$

Now the initial condition implies that

$$f(x) = \sum_{n=1}^{\infty} a_n \sin\frac{n\pi x}{L}$$

For a pair of sine, the following identities hold

$$\int_0^L \sin\frac{n\pi x}{L}\sin\frac{n\pi x}{L}\,dx = \begin{cases} 0 & m \neq n \\ L/2 & m = 0 \end{cases}$$

Multiplying both sides by $\sin\frac{n\pi x}{L}$ and

Integrating from O to L we get

$$a_0 = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx, \text{ n} = 1, 2, 3 \ldots$$

Thus the solution is completely determined therefore,

$$U(x,t) = \sum_{n=1}^{\infty}\left[\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx\right]\sin\frac{n\pi x}{L}\exp\left[-\left(\frac{n\pi x}{L}\right)^2 kt\right]$$

we now as an example of the Laplace transform in solving P.D.E. consider the concentration $C(x,t)$ of a difference surface of a given substance governed by

$$\frac{\partial c}{\partial t} = k\frac{\partial^2 c}{\partial x^2} \quad \text{O} < \text{x} < \text{L, t} > \text{O}$$

Subject to the boundary conditions

$$\frac{\partial c}{\partial x}(o,t) = 0, \quad c(L,t) = C_O$$

and the initial condition

$$c(x,o) = 0 \quad \text{O} < \text{x} < \text{L}$$

Taking the Laplace transform with respect to t and using the initial condition, we get from the LHS

$$\frac{\partial \overline{C}}{\partial t} = \overline{SC}$$

from the RHS we get

$$\frac{\partial^2 c}{\partial x^2} = \frac{\partial^2 \overline{c}}{\partial x^2}$$

therefore, the P.D.E. becomes

$$\frac{\partial^2 \bar{c}}{\partial x^2} = \frac{S^2 \bar{c}}{k} = 0 \quad O < x < L$$

the boundary conditions are

$$\frac{\partial^2 \bar{c}}{\partial x}(o,s) = 0$$ at the left end and

$$\bar{c}(L,s) = \int_0^\infty e^{-st} C(L,t)dt = \frac{c_o}{s}$$ at the right end

thus the P.D.E. has been reduced to o.d.e. the solution is

$$\frac{\bar{c}}{c_o} = \frac{\cosh \sqrt{s/kx}}{s \cosh \sqrt{s/kl}}$$

Inverse transform is

$$\frac{C}{C_0} = \frac{1}{2\pi.i} \int \frac{e^{st} \cosh \sqrt{S/K \times ds}}{S \cosh \sqrt{S/KL}}$$

The final result is

$$\frac{C}{C_0} = 1 - \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)n}{(2n+1)} \cos\left[\left(\frac{n+1}{2}\right)\pi x\right]^2$$

$$\exp\left\{kt\left[\left(n+\frac{1}{2}\right)\frac{\pi}{L}\right]^2\right\}$$

d. VARIATION OF PARAMETERS

This method applies even when the coefficients of the differential equation are functions of x, provided we know a fundamental solution set for the corresponding homogeneous equation

$$L[y](x) = y^{11} 4P(x)y^1 + q(x)y = R(x)$$

Where the coefficient of y^{11} is taken to be one if g, h is a fundamental solution set for the homogenous part of the differential equation

$$L[y](x) = 0$$

Then

$y_p = Ug + g + vh$ is the particular solution of the non-homogeneous equation

$$L[y](x) = y^{11}P(x)y^{1} + q(x)y = R(x),$$

Where

$$U = -\int \frac{R(x)g(x)dr}{W[g,h]}, \qquad V = \int \frac{R(x)h(x)dr}{W[g,h]},$$

Where $W[g,h]$ is the Wronskian of g and h

2.2 NUMERICAL METHODS

These are appropriate methods of solving a P.D.E. They include the finite difference scheme, and the finite element methods.

A special method of solving a P.D.E is the perturbation method, which is a semi-analytical scheme.

2.3 CHOICE OF METHOD OF SOLUTION

The analytical method of solving a P.D.E requires that the problem must be sufficiently idealized for techniques to be effective for more practical problems either the boundary geometry or the governing equations are less simple, and one must often be content with approximate solutions.

Among methods of approximation two are most important. If the problem is close to one solvable exactly, perturbation methods are powerful tools for getting analytical results. If however, the problem is far from anything that can be solved exactly, strictly numerical methods via discretization must be employed.

In general, analytical perturbation methods are much more effective in gaining a qualitative insight, while numerical methods are good in producing quantitative information. Sometimes the two can be mixed for studying small departures from a basic state that must itself be solved numerically.

2.4 PERTURBATION ANALYSIS

In this section we shall give a brief account of the analytical approach of perturbation method. As mentioned in the previous section, when a problem is closed to being solved analytically, perturbation methods are the most powerful tools for getting analytical results.

Perturbation analysis can be classified into two viz:-

(a) Regular perturbation

(b) Singular perturbation

2.4.1 REGULAR PERTURBATION ANALYSIS

We shall give an introductory outline of the typical ideas and procedure of regular perturbations.

(i) Identify a small parameter. This is very important first step, which must be taken by recognizing the physical scale relevant to the problem.

(ii) One then normalizes all variable with respect to this characteristic scale. In the normalized form, the governing equations will display certain dimensionless parameters of certain physical mechanisms. If one of the parameters say E, is much less than unity, then E can be chosen as the perturbation parameter.

(iii) Expand the solution as an ascending series of the small parameter. As an example a power series ε

$$U = U_0 + \varepsilon U_1 + \varepsilon^2 U_2 + \dots \qquad (2.1)$$

where U_n is called the nth order term collect terms of the same order in all governing equations and auxiliary conditions and get perturbation equations at each order.

(iv) Starting from the lowest order, solve the problem at each order successively up to a certain order say O(ε^m).

(v) Substitute the results for Un, n = 1, 2, … back into (2.1) to get the final results which is accurate up to some described order $O(\varepsilon^m)$Example: Let us examine the quadratic equation

$$U^2 + U - \varepsilon 1 = 0 \qquad (2.2)$$

Where ε is much less than unity, let us propose to find the solution as perturbation series

$$U = U_o + \varepsilon U_1 + \varepsilon^2 U_2 + \varepsilon^3 U_3 + \dots$$

And substitute this into equation (2.2)

$$\left(U_o + \varepsilon U_1 + \varepsilon_2 U_2 + \varepsilon_3 U_3 + \dots\right)^2 + \varepsilon\left(U_o + \varepsilon U_1 + \varepsilon^2 U_2 + \dots\right) - 1 = 0$$

Expanding and collecting terms of equal powers, we get

$$\left(U_0^2 - 1\right) + \varepsilon\left(2U_0 U_1 + U_0\right) + \varepsilon^2\left(2U_o U_2 + U_1^2 + U_1\right) + \dots = 0$$

With the coefficient of each power of ε to zero, a sequence of perturbation equation is obtained of various orders.

$O(\varepsilon^o)$: $\quad U_0^2 - 1 = 0$

$O(\varepsilon)$: $\quad 2U_0U_1 + U_0 = 0$

$O(\varepsilon^2)$: $\quad 2U_0U_2 + U_1^2 + U_1 = 0$

the lowest order solution is

$$U_o = \pm 1$$

With this result higher order problems are solved successively.

$U_1 = \frac{1}{2}$ and

$$U_2 = \frac{U_1^2 + U_1}{2U_o} = \pm \frac{1}{8}$$

In this case the efficacy of the approximate result can be judged by comparing with the exact solution

$$U = \frac{1}{2}\left(-\varepsilon \pm \sqrt[2]{1 + \frac{\varepsilon^2}{4}}\right)$$

$$= \pm 1 - \frac{\varepsilon}{2} \pm \frac{\varepsilon^2}{8} + 0(\varepsilon^4)$$

Clearly, this result confirms the perturbation series to the accuracy calculated.

2.4.2 SINGULAR PARTURBATION ANALYSIS

There are, however, many situations where the regular perturbation fails in some range of the independent variables. Then one must turn to the singular perturbation analysis which requires the following additional steps.

i. Diagnose the failure of the regular expansion check which of the original assumptions is violated when failure occurs. Examine the quantitative nature of the breakdown

ii. Choose new terms that should be important near breakdown and start a new perturbation analysis.

Example: Consider the following cubic equation

$U = 1 + \varepsilon U^3$ which can also be solved exactly. For a small ε let us try the straightforward expansion

$$U = U_0 + \varepsilon U_1 + \varepsilon^2 U_2 + \ldots$$

Substituting this series into the equation we have

$$1 + \varepsilon\left[U_0^3 + \varepsilon 3U_0^2 + U_1 + \varepsilon^2\left(3U_0^2 U_2 + \varepsilon 3U_0 U_1^2\right) + \ldots\right]$$

Equating equal powers of ε yields the perturbation equations

$O(\varepsilon^o)$ $\quad U_0 = 1$

$O(\varepsilon)$ $\quad U_1 = U_0^3$

$O(\varepsilon^2)$ $\quad U_2 = 3U_0^2 U_1$

The situations are obviously

$$U_0 = 1, U_, = 1, U_2 = 3$$

Hence the final solution is

$$U = 1 + \varepsilon + 3\varepsilon^2 + O(\varepsilon^3)$$

We notice that the other two solutions of the original cubic equation disappear. Hence we seek a better expansion, which leads to a singular perturbation.

We may assume

$U = X \in^{-1/2}$ so that

The original cubic equation becomes

$$X\varepsilon^{-1/2} = 1 + X^3 \varepsilon^{-1/2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots ^{**}$$

Substituting the new expansion

$X = x_0 + \varepsilon^{1/2} x_1, + \varepsilon x_2 + \varepsilon^{3/2} x_3 + \ldots$ into (**) and collecting powers of ε we get the perturbation equations

$O(\varepsilon^o)$ $\qquad x_0^2 - x_0 = 0$

$O(\varepsilon^1)$ $\qquad 3x_0^2 x_1 - x_1 + 1 = 0$

$O(\varepsilon^2)$ $\qquad 3x_0^2 x_2 - 3x_0 x_1^2 - x_2 = 0$

The solutions at successive order gives

$$x_0 = (0,1,-1)$$

$$x_1 = \frac{-1}{3x_0^2 - 1}$$

$$x_2 = \frac{-3x_0 x_1^2}{3x_0^2 - 1}$$

Specific value can be gotten depending on the value of x_0

e.g. When $x_0 = -1$

$$x = -1 - \frac{\varepsilon^{1/2}}{2} + \frac{3\varepsilon}{8} + \ldots$$

So that

$$U = -\varepsilon^{1/2} - \frac{1}{2} + \frac{3}{8}\varepsilon^{1/2} + \ldots$$

CHAPTER THREE

3.0 PROBLEM SOLUTION

Consider the magnetic field equation

$$\frac{B}{a}\frac{\partial V_z}{\partial \theta} + \lambda \nabla^2 B_z = 0 \qquad 3.0.0$$

$$\nabla^2 Bz = -\frac{B}{\lambda a}\frac{\partial V^2}{\partial \theta} \qquad 3.0.1$$

Let $\overline{B} = \frac{Bz}{B_0}$ and $V = \frac{Vz}{V_0}$

$Bz = \overline{B}.B_0$ and $Vz = V.V_0$

$$\Rightarrow \frac{\partial Bz}{\partial r} = Bo\frac{\partial \overline{B}}{\partial r}, \qquad \frac{\partial^2 Bz}{\partial r^2} = Bo\frac{\partial^2 \overline{B}}{\partial r^2},$$

$$\frac{\partial Bz}{\partial \theta} = Bo\frac{\partial \overline{B}}{\partial \theta}, \qquad \frac{\partial^2 Bz}{\partial r^2} = Bo\frac{\partial^2 \overline{B}}{\partial r^2},$$

$$\frac{\partial Vz}{\partial \theta} = Vo\frac{\partial V}{\partial \theta},$$

So, equation 3.0.1 becomes

$$\frac{\partial^2 Bz}{\partial r^2} + \frac{1}{r}\frac{\partial^2 Bz}{\partial r} + \frac{1}{r^2}\frac{\partial^2 Bz}{\partial \theta} = -\frac{B}{\lambda a}V_0\frac{\partial V}{\partial \theta}$$

$$\Rightarrow B_0\frac{\partial^2 \overline{B}}{\partial r^2} + \frac{B_0}{r}\frac{\partial \overline{B}}{\partial r} + \frac{B_0}{r^2}\frac{\partial^2 \overline{B}}{\partial^2 \theta} = -\frac{BV_0}{\lambda a}\frac{\partial V}{\partial \theta}$$

$$\Rightarrow \frac{\partial^2 \overline{B}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{B}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \overline{B}}{\partial \theta^2} = -\frac{BV_0}{\lambda a B_0}\frac{\partial V}{\partial \theta}$$

$$\Rightarrow \frac{\partial^2 \overline{B}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{B}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \overline{B}}{\partial \theta^2} = -\frac{BV_0}{\lambda a B_0}\cdot\frac{\partial V}{\partial \overline{B}}\cdot\frac{\partial \overline{B}}{\partial \theta} \qquad 3.0.2$$

Since $\frac{V_0}{B_0}\frac{\partial V}{\partial \overline{B}}$ is non-dimensional,

Let $\frac{V_0}{B_0}\frac{\partial V}{\partial \overline{B}} = \xi$

$$\Rightarrow \nabla^2 \overline{B} = -\frac{B}{\lambda a}\xi \frac{\partial \overline{B}}{\partial \theta} \qquad 3.0.3$$

By perturbation

$$\overline{B} = \xi^0 \overline{B}^{(0)} + \xi^1 \overline{B}^{(1)} + \xi^2 \overline{B}^{(2)} + \qquad 3.0.4$$

Putting 3.0.4 into 3.0.3 we have

$$\nabla^2\left[\xi^0 \overline{B}^{(0)} + \xi^1 \overline{B}^{(1)} + ...\right] = -\frac{B\xi}{\lambda a}\frac{\partial}{\partial \theta}\left[\xi^0 \overline{B}^{(0)} + \xi^1 \overline{B}^{(1)} + ...\right]$$

$$\xi^0 \nabla^2 \overline{B}^{(0)} + \xi^1 \overline{B}^{(1)} + \xi^2 .... = -\frac{B\xi^1}{\lambda a}\frac{\partial \overline{B}^{(0)}}{\partial \theta} - \frac{B\xi^2}{\lambda a}\frac{\partial \overline{B}^{(1)}}{\partial \theta} - \frac{B\xi^3}{\lambda a}\frac{\partial \overline{B}^{(2)}}{\partial \theta} .....$$

Collecting terms in order of ξ,

we have for order zero (ξ^0)

$$\nabla^2 \overline{B}^{(0)} = 0$$

For order one (ξ^1)

$$\nabla^2 \overline{B}^{(1)} = -\frac{B}{\lambda a}\frac{\partial \overline{B}^{(0)}}{\partial \theta}$$

For order of $k(\xi^k)$

$$\nabla^2 \overline{B}^{(k)} = -\frac{B}{\lambda a}\frac{\partial \overline{B}^{(k-1)}}{\partial \theta}$$

3.1 SOLUTION FOR ORDER ZERO

The order zero equation is

$$\nabla^2 \overline{B}^{(0)} = 0 \qquad 3.1.0$$

$$\Rightarrow \frac{\partial^2 \overline{B}^{(0)}}{\partial r^2} + \frac{1}{r}\frac{\partial \overline{B}^{(0)}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \overline{B}^{(0)}}{\partial \theta^2} = 0$$

Assume a solution of the form

$$\overline{B}^{(0)} = \overline{B}^{(0)}(r,\theta) = R(r)\Theta(\theta) \qquad 3.1.2$$

Where $R(r)$ is a function of r only and $\Theta(\theta)$ is a function of θ only.

So that

$$\frac{\partial \overline{B}^{(0)}}{\partial r} = \Theta\frac{dR}{dr}, \frac{\partial^2 \overline{B}^{(0)}}{\partial r^2}\Theta\frac{d^2R}{dr^2} \text{ and}$$

$$\frac{\partial^2 \overline{B}^{(0)}}{\partial \theta^2} = R\frac{d^2\Theta}{d\theta^2}$$

Therefore, equation (3) gives

$$\Theta\frac{d^2R}{dr^2} + \frac{1}{r}\Theta\frac{dR}{dr} + \frac{1}{r^2}\frac{Rd^2\Theta}{d\theta^2} = 0 \qquad 3.1.3$$

$$\Rightarrow R''\Theta + \frac{1}{r}R'\Theta + \frac{1}{r^2}R\Theta'' = 0$$

Dividing through by $R(r)\Theta(\theta)$, we have

$$\frac{1}{R}\left[r2\frac{d^2R}{dr^2} + r\frac{dR}{dr}\right] = -\frac{1}{\Theta}\frac{d^2\Theta}{d\theta^2} = n^2$$

$$\Rightarrow \frac{1}{\Theta}\Theta'' = -n^2$$

$$\frac{d^2\Theta}{d\theta^2} + n^2\Theta = 0 \qquad 3.1.4$$

The solution of which is

$$\Theta = A\cos n\theta + P\sin\theta \qquad 3.1.5$$

Also,

$$\frac{1}{R}\left[r^2R'' + rR'\right] = n^2$$

$$\Rightarrow r^2R'' + rR'' - n^2R = 0 \qquad 3.1.6$$

Since this is homogeneous, it takes the form

$$\frac{d^2R}{ds^2} - n^2R = 0$$

For $r = e^s$, we have

$$R = ce^{ns} + De^{-ns}$$

$$\Rightarrow R = Cr^n + Dr^{-n} \qquad 3.1.7$$

For n = 0, equation (6) gives

$$\frac{d^2\Theta}{d\theta^2} = 0$$

$$\therefore\ \Theta = Ao\theta + Po \qquad 3.1.8$$

and equation (8) gives

$$\frac{r^2d^2R}{dr^2} + r\frac{dR}{dr} = 0 \text{ or}$$

$$\frac{d^2R}{ds^2} = 0$$

$$\Rightarrow R = C_oS + Do$$

$$\therefore R = Co\ \log r + Do \qquad 3.1.9$$

From 3.1.8 and 3.1.9, we have

$$\overline{B}^{(0)} = (A\Theta + P)(C\log r + D)$$

From 3.1.5 and 3.1.7 we have

$$\overline{B}^{(0)} = (AnCosn\theta + PnSinn\theta)(C_n r^n + D_n r^{-n})$$

The general single-valued solution of (3) for all possible n is

$$\overline{B}^{(0)} = Ao\log r + \sum_{n=1}^{\infty}(AnCosn\theta + PnSinn\theta)(C_n r^n + D_n r^{-n}) + Co \qquad 3.1.10$$

Where Ao, An, Pn, Cn, Dn and Co are arbitrary constants.

APPLYING THE SOUNDARY CONDITIONS

i. $\overline{B}^{(0)} = 0$ when $\theta = 0$, for $0 < r \leq a$

ii. $\overline{B}^{(0)}$ is finite when $r \to 0$, and

iii. $\overline{B}^{(0)} = B$ when $r = a$, for $0 < \theta < \pi$

Since $\overline{B}^{(0)}$ is finite by condition (π), we have

$A_o = D_n = 0$

Therefore, equation 3.1.10 becomes

$$\overline{B}^{(o)} = \sum_{n=1}^{\infty} (d_n \cos_n \theta + b_n \sin n\theta)r^n + c_o \qquad 3.1.11$$

Where $d_n = A_n C_n$ and $b_n = P_n C_n$

By condition (I) Co = 0, hence 3.1.11 becomes

$$\overline{B}^{(o)} = \sum_{n=1}^{\infty} (d_n \cos_n \theta + b_n \sin n\theta)r^n \qquad 3.1.12$$

By condition (III), we have from 3.1.12

$$\overline{B}^{(o)} = \sum_{n=1}^{\infty} (d_n \cos n\theta + b_n \sin n\theta)a^n \qquad 3.1.13$$

$$\Rightarrow B = \sum_{n=1}^{\infty} (d_n \cos n\theta + b_n Sinn\theta) a^n$$

This gives

$$d_n = \frac{2}{\pi} \int_0^{\pi/2} \frac{B}{a_n} Cosn\theta d\theta = \frac{2B}{\pi a^n} \int_0^{\pi/2} Cosn\theta d\theta = 0$$

and

$$b_n = \frac{2}{\pi} \int_0^{\pi/2} \frac{B}{a_n} Sinn\theta d\theta = \frac{2B}{\pi a^n} \int_0^{\pi/2} Sinn\theta d\theta$$

$$= \frac{2B}{n\pi a^n}(1 - Cosn\pi)$$

Therefore, the required order zero solution is

$$\overline{B}^{(0)} = \sum_{n=1}^{\infty} \frac{2B}{n\pi a^n}(1 - Cosn\pi) r^n Sinn\theta \qquad 3.1.14$$

$$\overline{B} = \overline{B}^{(0)} + \xi \overline{B}^{(1)}$$

SOLUTION FOR ORDER ONE

The order one equation is

$$\Delta^2 \overline{B}^{(1)} = -\frac{B}{\lambda a} \frac{\partial \overline{B^{(0)}}}{\partial \theta} \qquad 3.2.0$$

But, the order Zero solution is

$$\overline{B}^{(0)} = \sum_{n=1}^{\infty} \frac{2B}{n\pi a^n}(1 - \cos n\pi) r^n Sinn\theta \qquad 3.2.1$$

Then

$$\frac{\partial \overline{B}^{(0)}}{\partial \theta} = \frac{2B}{n\pi} \sum_{n=1}^{\infty} \frac{1}{a^n}(1 - \cos n\pi) r^n \cos n\theta$$

Therefore, equation (1) becomes

$$\Delta^2 \overline{B}^{(1)} = -\frac{2B^2}{\lambda n\pi}\sum_{n=1}^{\infty} \frac{1}{a^{n+1}}(1-\cos n\pi)r^n \cos n\theta \qquad 3.2.2$$

$$\text{Let } -\frac{2B^2}{\lambda n\pi}\sum_{n=1}^{\infty} \frac{1}{a^{n+1}}(1-\cos n\pi) = K_1$$

Then

$$\Delta^2 \overline{B}^{(1)} = K_1 r^n \cos n\theta \qquad 3.2.3$$

$$\Rightarrow \frac{\partial^2 \overline{B}^{(1)}}{\partial r^2} + \frac{1}{r}\frac{2\overline{B}^{(1)}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \overline{B}^{(1)}}{\partial \theta^2} = K_1 r^n \cos\theta$$

Let us assume of solution of the form

$$\overline{B}^{(1)} = f(r)\cos n\theta \qquad \frac{\partial^2 \overline{B}^{(1)}}{\partial r^2} = f^{11}(r)\cos n\theta$$

$$\frac{\partial \overline{B}^{(1)}}{\partial r} = f^1(r)\cos n\theta,$$

$$\text{And } \frac{\partial \overline{B}^{(1)}}{\partial \theta} = nf(r)\sin n\theta, \qquad \frac{\partial^2 \overline{B}^{(1)}}{\partial \theta^2} = -n^2 f(r)\cos n\theta.$$

Substituting these into equation 3.2.3 we have

$$f''(r)\cos n\theta + \frac{1}{r}f'(r)\cos n\theta - \frac{n^2}{r^2}f(r)\cos n\theta = K_1 r^n \cos n\theta$$

$$\Rightarrow f''(r) + \frac{1}{r}f'(r) - \frac{n^2}{r^2}f(r) = K_1 r^n \qquad 3.2.4$$

$$\Rightarrow r^2 f''(r) + rf'(r) - n^2 f(r) = K_1 r^{n+2}$$

Which is a non-homogeneous ordinary differential equation (o.d.e).

For

$$r^2 f''(r) + rf'(r) - n^2 f(r) = 0 \qquad 3.2.5$$

Let

$r = e^z$

$\Rightarrow z = \log r$

$r = e^z$

$\Rightarrow z = \log r$

$$f''(r) = \frac{1}{r^2}\left\{\frac{d^2 f}{dz^2} - \frac{df}{dz}\right\}$$

$$f'(r) = \frac{1}{r}\frac{df}{dz}$$

Substituting these into 3.2.5, we have

$$r^2 \cdot \frac{1}{r^2}\left\{\frac{d^2 f}{dz^2} - \frac{df}{dz}\right\} + r \cdot \frac{1}{r}\frac{df}{dz} - n^2 f(z) = 0$$

$$\Rightarrow \frac{d^2 f}{dz^2} - n^2 f(z) = 0$$

$\Rightarrow m^2 - n^2 = 0,$

therefore,

$m = n$

$\Rightarrow f(z) = Ce^{nz} + De^{-nz}$

$\Rightarrow f(r) = Ce^{n\log r} + De^{-n\log r}$

$f(r) = Cr^n + Dr^{-n}$

$$U = -\int \frac{y_1 g(r)}{w} dr, V = \int \frac{y_2 g(r)}{w} dr$$

as defined in 2.1(d)

$$\begin{vmatrix} r^n & r^{-n} \\ nr^{n-1} & -nr^{-n-1} \end{vmatrix}$$

$w = -nr^n \cdot r^{-n-1} - nr^{n-1} \cdot r^{-n}$

$= -nr^{n-n-1} - nr^{n-1-n}$

$= -2nr^{-1}$

$$y_n = Cr^n + Dr^{-n}$$

$$\Rightarrow y_1 = r^n, y_2 = r^{-n}$$

$$U = -\int \frac{y_1 f(r) dr}{w}, V = \int \frac{y_2 f(r) dr}{w}$$

Therefore,

$$w = -\frac{2n}{r}$$

$$u = -\int \frac{r^n \cdot k_1 r^{n+2} dr}{-2\frac{n}{r}}$$

$$= \frac{k_1}{2n} \int r^{2n+3} dr$$

$$= \frac{k_1}{2n} \left[\frac{r^{n+4}}{2n+4} \right]$$

Therefore,

$$u = \frac{k_1 r^{2n+4}}{2n(2n+4)}$$

Also,

$$V = \int \frac{r^{-n} k_1 r^{n+2} dr}{-2\frac{n}{r}}$$

$$= -\frac{k_1}{2n} \int r^{-n+n+2+1} dr$$

$$= -\frac{k_1}{2n} \int r^3 dr$$

Therefore,

$$V = -\frac{k_1 r^4}{8n}$$

$$y_p = y_1 u + y_2 v$$

$$= \frac{r^n k_1 r^{2n+4}}{2n(2n+4)} + r^{-n}\left(\frac{-k_1 r^4}{8n}\right)$$

$$= \frac{k_1 r^{3n+4}}{2n(2n+4)} - \frac{k^1 r^{4-n}}{8n}$$

$$= \frac{k_1 r^{4-n}\left(2r^{4n} - n + 2\right)}{8n(n+2)}$$

$$\Rightarrow y = Cr^n + Dr^{-n} + \frac{k_1 r^{4-n}\left(2r^{4n} - n + 2\right)}{8n(n+2)} \qquad 3.2.7$$

Since

$$\overline{B}^{(1)} = f(r)\cos n\theta$$

$$\Rightarrow \overline{B}^{(1)} = \left[Cr^n + Dr^{-n} + \frac{k_1 r^{4-n}\left(2r^{4n} - n + 2\right)}{8n(n+2)}\right]\cos\theta \qquad 3.2.8$$

APPLYING THE BOUNDARY CONDITIONS

(i) $\overline{B}^{(1)} = 0$ when $\theta = 0$ for $0 < r \leq a$

(ii) $\overline{B}^{(1)}$ is finite when $r \to 0$, and

(iii) $\overline{B}^{(1)} = B$ when $r = a$ for $0 < \theta < \pi$

Therefore, equation 3.2.8 becomes

$$\overline{B}^{(1)} = \left[Cr^n + \frac{k_1 r^{4-n}\left(2r^{4n} - n + 2\right)}{8n(n+2)}\right]\cos n\theta \qquad 3.2.9$$

By condition (I), equation 3.2.9 becomes

$$\overline{B}^{(1)} = 0$$

Applying condition III on equation 3.2.7 we have

$$f(a) = Ca^n + \frac{k_1 a^{4-n}\left(2a^{4n} - n + 2\right)}{8n(n+2)} = B$$

$$\Rightarrow Ca^n = B - \frac{k_1 a^{4-n}\left(2a^{4n} - n + 2\right)}{8n(n+2)}$$

$$\Rightarrow C = Ba^{-n} - \frac{k_1 a^{4-2n}\left(2a^{4n} - n + 2\right)}{8n(n+2)}$$

$$\Rightarrow \overline{B}^{(1)} = \left[Ba^{-n} - \frac{k_1 a^{4-2n}\left(2a^{4n} - n + 2\right)}{8n(n+2)} + \frac{k_1 a^{4-n}\left(2a^{4n} - n + 2\right)}{8n(n+2)} \right] r^n \cos n\theta$$

$$\Rightarrow \overline{B}^{(1)} = Ba^{-n} - k_1\left[\frac{a^{4-2n}\left(2a^{4n} - n + 2\right) + r^{4-n}\left(2r^{4n-n+2}\right)}{8n(n+2)}\right]r^n \cos n\theta \qquad 3.2.10$$

But $k_1 = \dfrac{-2B^2}{\lambda n\pi}\displaystyle\sum_{n=1}^{\infty} \frac{1}{a^{n+1}}(1-\cos n\pi)$

$$\therefore \overline{B}^{(1)} = Ba^{-n} + \frac{2B^2}{\lambda n\pi}\sum_{n=1}^{\infty} \frac{1}{a^{n+1}}(1-\cos n\pi)\left[\frac{a^{4-2n}\left(2a^{4n} - n + 2\right) + r^{4-n}\left(2r^{4n-n+2}\right)}{8n(n+2)}\right]r^n \cos n\theta \qquad 3.2.11$$

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3.3 SOLUTION OF THE VELOCITY FIELD EQUATION

The equation is

$$\ell\frac{\partial Vz}{\partial t}+\frac{\partial \ell}{\partial z}=\frac{B}{\mu a}\frac{\partial Bz}{\partial \theta}+\eta\nabla^2 Vz \qquad 3.3.1$$

Since P is a function of time only

$$\frac{\partial P}{\partial t}=0 \text{ and for steady flow}$$

$$\frac{\partial Vz}{\partial t}=0$$

Therefore equation 3.3.1 becomes

$$\frac{B}{\mu a}\frac{\partial Bz}{\partial \theta}+\eta\nabla^2 Vz=0 \qquad 3.3.2$$

$$\Rightarrow \nabla^2 Vz=-\frac{B}{\eta\mu a}\frac{\partial Bz}{\partial \theta} \qquad 3.3.3$$

For $Vz=\frac{V}{Vo}$ and $Bz=\frac{B}{Bo}$

Equation 3.3.3 gives

$$Vo\frac{\partial^2 V}{\partial r^2}+\frac{V}{r}\frac{\partial V}{\partial r}+\frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2}=-\frac{BBo}{\eta\mu a}\frac{\partial \overline{B}}{\partial \theta}$$

$$\Rightarrow \frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}+\frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2}=-\frac{B}{\eta\mu a}\frac{Bo}{Vo}\frac{\partial \overline{B}}{\partial \theta}$$

$$\Rightarrow \nabla^2 V=-\frac{B}{\eta\mu a}\cdot\frac{Bo}{Vo}\cdot\frac{\partial \overline{B}}{\partial V}\cdot\frac{\partial V}{\partial \theta} \qquad 3.3.4$$

Since $\frac{Bo}{Vo}\cdot\frac{\partial \overline{B}}{\partial V}$ is dimensionless,

Let $\frac{Bo}{Vo}\cdot\frac{\partial \overline{B}}{\partial V}=B$

$$\Rightarrow \nabla^2 V = \frac{-B\beta}{\eta\mu a} \cdot \frac{\partial V}{\partial \theta} \qquad 3.3.5$$

By perturbation,

$$V = \beta^0 V^{(0)} + \beta^{(1)} V^{(1)} + \beta^2 V^{(2)} + \ldots\ldots\ldots\ldots\ldots\ldots \qquad 3.3.5$$

Putting 3.3.6 into 3.3.5, we have

$$\nabla^2\left[\beta^0 V^{(0)} + \beta^{(1)} V^{(1)} + \beta^2 V^{(2)} +\right] = -\frac{B\beta}{\eta\mu a}\frac{\partial}{\partial \theta}\left[\beta^0 V^{(0)} + \beta^{(1)} V^{(1)} + \beta^2 V^{(2)} + \ldots\right]$$

$$\Rightarrow \beta^0 \nabla^2 V^{(0)} + \beta^1 \nabla^2 V^{(1)} + \beta^2 \nabla^2 V^{(2)} \ldots = \frac{-B\beta^1}{\eta\mu a}\frac{\partial V^{(0)}}{\partial \theta} - \frac{B\beta^2}{\eta\mu a}\frac{\partial V^{(1)}}{\partial \theta} - \frac{B\beta^2}{\eta\mu a}\frac{\partial V^{(1)}}{\partial \theta}$$

Equation terms in order of β, we have for order zero (β^0)

$$\nabla^2 V^{(0)} = 0$$

For order one (β^1)

$$\nabla^2 V^{(1)} = -\frac{-B}{\eta\mu a}\frac{\partial V^{(1)}}{\partial \theta}$$

For order k (β^k)

$$\nabla^2 V^{(k)} = -\frac{B}{\eta\mu a}\frac{\partial V^{(k-1)}}{\partial \theta}$$

3.4 SOLUTION OF THE ORDER ZERO VELOCITY EQUATION

i.e

$$\nabla^2 V = 0 \qquad 3.4.0$$

$$\Rightarrow \frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} = 0 \qquad 3.4.1$$

Assuming

$$V^{(2)} = V(r,\theta) = R(r)\Theta(\theta) \qquad 3.4.2$$

Where R is a function of r only and Θ is a function of θ only.

$$\Rightarrow \frac{\partial V}{\partial r} = \Theta R', \ \frac{\partial^2 V}{\partial r^2} = \Theta R'', \text{ and } \frac{\partial^2 V}{\partial r^2} = R\Theta'',$$

Therefore, equation 3.4.1 becomes

$$\Theta R'' + \frac{1}{r}\Theta R' + \frac{1}{r^2} R\Theta'' = 0$$

Dividing through by $R(r)\Theta(\theta)$, we have

$$\frac{R''}{R} + \frac{1}{r}\frac{R'}{R} + \frac{1}{r^2}\frac{\Theta''}{\Theta} = 0$$

$$\frac{r^2 R''}{R} + \frac{rR'}{R} + \frac{\Theta''}{\Theta} = 0$$

$$\frac{r^2 R''}{R} + \frac{rR'}{R} + -\frac{\Theta''}{\Theta} = n^2$$

Where n is the separation constant.

$$\Rightarrow \frac{1}{\Theta}\frac{d^2\Theta}{d\theta^2} = -n^2$$

$$\Rightarrow \frac{d^2\Theta}{d\theta^2} + n^2\Theta = 0 \qquad 3.4.4$$

The solution of which is

$$\Theta = ACosn\Theta + P\sin n\Theta \qquad 3.4.5$$

Also,

$$\frac{1}{R}\left[r^2 R'' + rR'\right] = n^2$$

$$\Rightarrow r^2 R'' + rR' - n^2 R = 0$$

The solution of which is

$$R = Cr^n + Dr^{-n} \qquad 3.4.7$$

For n = 0, equation 3.4.4 becomes

$$\frac{d^2\Theta}{d\theta^2} = 0$$

$$\therefore \Theta = Ao\theta + Po \qquad 3.4.8$$

Also for n = 0, equation 3.4.6 becomes

$$r^2 R'' + rR' = 0 \text{, or}$$

$$\frac{d^2 R}{ds^2} = 0 \text{ for } r = e^s$$

$$\Rightarrow R = Cos + Do$$

$$\therefore R = Co\log r + Do \qquad 3.4.9$$

From 3.4.8 and 3.4.9, we have

$$V^{(0)} = (Ao\theta + Po)(C\log r + D) \qquad 3.4.10$$

Also from 3.4.5 and 3.4.7, we have

$$V(0) = (AnCosn\theta + PnSinn\theta)(C_n r^n + D_n r^{-n})$$

For all possible value of n, the general solution of the velocity equation is

$$V(0) = Ao\log r + Co + \sum_{n=1}^{\infty}(AnCosn\theta + PnSinn\theta)(C_n r^n + D_n r^{-n}) \qquad 3.4.11$$

Where An, Pn Cn Dn for n = 1,2,…. Are arbitrary constant

Applying the boundary conditions

i. $V = 0$ when $\theta = 0$ for $0 = \leq r < a$

ii. when $r \to a$, and

iii. V is finite when $r \to 0$, for $0 = < r < \pi$

By condition (iii)

$$V^{(0)} = C_o + \sum_{n=1}^{\infty}(d_n C_o sn\theta + b_n Sinn\theta)r^n \qquad 3.4.12$$

For $d_n = A_n C_n$ and $b_n = P_n C_n$

By conditions (i) and (ii), we have

$$C_o + \sum_{n=1}^{\infty}(d_n Cosn\theta + b_n Sinn\theta)a^n = 0$$

$$\Rightarrow C_o + \sum_{n=1}^{\infty} d_n a^n = 0$$

$$C_o = -d_n a^n = A_n$$

$$V^{(0)} = -\sum_{n=1}^{\infty} An + \sum_{n=1}^{\infty}(d_n Cosn\theta + b_n Sinn\theta) = 0$$

Let $\sum_{n=1}^{\infty} A_n = A$

$$A = \sum_{n=1}^{\infty}(d_n Cosn\theta + b_n Sinn\theta)a^n$$

$$d_n = \frac{2}{\pi}\int_0^{\pi/2}\frac{A}{a^n} Cosn\theta d\theta = \frac{2A}{\pi a^n}\int_0^{\pi/2} Cosn\theta d\theta = 0$$

$$b_n = \frac{2}{\pi}\int_0^{\pi/2}\frac{A}{a^n} Sinn\theta d\theta = \frac{2A}{\pi a^n}\int_0^{\pi/2} Sinn\theta d\theta = 0$$

$$= \frac{2A}{n\pi a^n}(r - Cosn\pi)$$

Therefore the order zero solution is

$$V^{(0)} = \sum_{n=1}^{\infty}\frac{2A}{n\pi a^n}(1 - Cosn\pi)r^n Sinn\theta = 0 \qquad 3.4.13$$

3.5 SOLUTION OF THE ORDER ONE EQUATION

The order one equation is

$$\nabla^2 V^{(1)} = -\frac{B}{\eta\mu a}\frac{\partial V^{(0)}}{\partial\theta} \qquad 3.5.1$$

From the solution of the order zero equation

$$V^{(0)} = \sum_{n=1}^{\infty}\frac{2A}{n\pi a^n}(1 - Cosn\pi)r^n Sin\theta$$

$$\frac{\partial V^{(0)}}{\partial\theta} = \frac{2A}{\pi}\sum_{n=1}^{\infty}\frac{1}{a^n}(1 - Cosn\pi)r^n Cosn\theta$$

$$\nabla^2 V^{(1)} = -\frac{-2AB}{\eta\mu a}\sum_{n=1}\frac{1}{an}(1 - Cosn\pi)r^n Cosn\theta \qquad 3.5.2$$

Let $k = \frac{-2AB}{\eta\mu\pi}\sum_{n=1}^{\infty}\frac{1}{a^n}(1 - Cosn\pi)$

$$\Rightarrow \nabla^2 V^{(1)} = Kr^n Cons\theta$$

$$\frac{\partial^2 V^{(1)}}{\partial r^2} + \frac{1}{r}\frac{\partial V^{(1)}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V^{(1)}}{\partial\theta} = Kr^n Cosn\theta \qquad 3.5.3$$

Assuming a solution of the form

$$V^{(1)} = f(r)Cosn\theta$$

$$r^2 f''(r) + rf'(r) - n^2 f(r) = Kr^{n+2} \qquad 3.5.4$$

For the homogeneous part

$$r^2 f''(r) + rf'(r) - n^2 f(r) = 0 \qquad 3.5.5$$

Let e^z

$\Rightarrow z = \log r$

$$f'(r) = \frac{1}{r}\frac{df}{dz}, \qquad f''(r) = \frac{1}{r^2}\left[\frac{d^2 f}{dz^2} - \frac{df}{dz}\right]$$

Substituting the equation into equation 3.5.5 we have

$$r^2 \cdot \frac{1}{r^2}\left[\frac{d^2 f}{dz^2} - \frac{df}{dz}\right] + r \cdot \frac{1}{r}\frac{df}{dz} - n^2 f(z) = 0$$

$$\Rightarrow \frac{d^2 f}{dz^2} - \frac{df}{dz} + \frac{df}{dz} - n^2(z) = 0$$

$$\Rightarrow \frac{d^2 f}{dz^2} - n^2 f(z) = 0$$

$$\Rightarrow m^2 - n^2 = 0$$

$$\Rightarrow m - n$$

$$\Rightarrow f(z) = Ce^{nz} + De^{-nz}$$

$$f(r) = Ce^{n\log r} + De^{-n\log r}$$

$$f(r) = Cr^n + D^{-n}$$

$$y_n = Cr^n + Dr^{-n}$$

$$y_1 = r^n, \qquad y_2 = r^{-n},$$

$$\Rightarrow U = -\int \frac{y_2 r dr}{w}, \qquad V = \int \frac{y_1 r dr}{w},$$

$$\begin{vmatrix} r^n & r^{-n} \\ nr^{n-1} & -nr^{-n-1} \end{vmatrix}$$

$$w = -nr^n \cdot r^{-n-1} - nr^{n-1} \cdot r^{-n}$$

$$-nr^{n-n-1} - nr^{n-1-n}$$

$$= -2nr^{-1}$$

Therefore,

$$w = -\frac{2n}{r}$$

$$\Rightarrow U = -\int \frac{r^{n} \cdot Kr^{n+2}}{-\frac{2n}{r}} dr$$

$$= \frac{K}{2n} \int r^{2n+3} dr$$

$$= \frac{K}{2n} \int \left[\frac{r^{2n+3}}{2n+4} \right]$$

Therefore,

$$U = \frac{Kr^{2n+4}}{2n(2n+4)}$$

Also,

$$V = \int \frac{r^{-n} \cdot Kr^{n+2}}{-\frac{2n}{r}} dr$$

$$= -\frac{K}{2n} \int r^{-n+n+2+1} dr$$

$$= -\frac{K}{2n} \int r^{-n+n+2+1} dr$$

Therefore,

$$V = \frac{-Kr^{4}}{8n}$$

$$y_p = y_1 U + y_2 V$$

$$= \frac{r^{n} Kr^{2n+4}}{2n(2n+4)} + r^{-n}(-\frac{Kr^{4}}{8n})$$

$$= \frac{Kr^{3n+4}}{2n(2n+4)} - \frac{Kr^{4-n}}{8n}$$

$$y = y_h + y_p$$

$$\Rightarrow y = Cr^n + Dr^{-n} + \frac{k\left[r^{3n+4} - (n+2)r - n^{4-n}\right]}{8n(n+2)}$$

$$f(r) = Cr^n + Dr^{-n} + \frac{k\left[r^{3n+4} - (n+2)r - n^{4-n}\right]}{8n(n+2)} \qquad 3.5.7$$

Since

$$V(r,\theta) = f(r)\cos\theta$$

$$\Rightarrow V^{(1)} = \left[Cr^n + Dr^{-n} + \frac{k\left[r^{3n+4} - (n+2)r - n^{4-n}\right]}{8n(n+2)}\cos n\theta\right]$$

$$V^{(1)} = \left[C + Dr^{-2n} + \frac{k\left[r^{2n+4} - (n+2)r - n^{4-2n}\right]}{8n(n+2)} r^n \cos n\theta\right] \qquad 3.5.8$$

APPLYING THE BOUNDARY CONDITIONS

(i) $V = 0$ when $\theta = 0$ for $0 \le r < a$

(ii) $V = 0$ when $r \to a$, and

(iii) V is finite when $r \to 0$ for $0 < \theta < \pi$

Applying condition (iii) implies that $D = 0$

$$\Rightarrow V^{(1)} = \left[C + \frac{k\left[r^{2n+4} - (n+2)r^{4-2n}\right]}{8n(n+2)} r^n \cos n\theta\right] \qquad 3.5.9$$

Applying conditions (i) and (ii), we have

$$V^{(1)} = \left[C + \frac{k\left[a^{2n+4} - (n+2)a^{4-2n}\right]}{8n(n+2)} a^n \cos n\theta = 0 \right]$$

$$\Rightarrow C = \frac{k\left[(n+2)a^{4-2n} - a^{2n-4}\right]}{8n(n+2)} \qquad 3.5.10$$

Equation 3.3.10 becomes

$$V^{(1)} = \frac{k\left[(n+2)a^{4-2n} - a^{2n+4}\right]}{8n(n+2)} + \frac{k\left[r^{2n+4} - (n+2)r^{4-2n}\right]}{8n(n+2)} r^n \cos n\theta$$

$$\Rightarrow V^{(1)} = \frac{k\left[(n+2)a^{4-2n} - a^{2n+4} + r^{2n+4} - (n+2)r^{4-2n}\right]}{8n(n+2)} r^n \cos n\theta$$

But $k = \dfrac{-2\mathrm{AB}}{\eta\mu\pi} \sum_{n=1}^{\infty} \dfrac{1}{a^n}(1 - \cos n\pi)$

$$\therefore V^{(1)} = \frac{-2\mathrm{AB}}{\eta\mu\pi} \sum_{n=1}^{\infty} \frac{1}{a^n}(1 - \cos n\pi) \left[\frac{a^{2n+4} - (n+2)a^{4-2n} - r^{2n+4} + (n+2)r^{4-2n}}{8n(n+2)} \right] r^n \cos n\theta \qquad 3.5.11$$

$$\therefore V = V^{(0)} + \beta V^{(1)}$$

$$V = \sum_{n=1}^{\infty} \frac{2A}{n\pi a^{n}} (1 - Cos n\pi) r^{n} Sin n\theta + \beta \left(\frac{-2\mathrm{AB}}{\eta\mu\pi} \sum_{n=1}^{\infty} \frac{1}{a^{n}} (1 - \cos n\pi) \left[\frac{a^{2n+4} - (n+2)a^{4-2n} - r^{2n+4} + (n+2)r^{4-2n}}{8n(n+2)} \right] r^{n} \cos n\theta \right) \qquad 3..5.12$$

CHAPTER FOUR

4.0 NUMERICAL RESULTS

The solution of the order zero (0) and the order one (1) equation for both $\overline{B}$ and V are hence given as below

$$\overline{B}^{(0)} = \sum_{n=1}^{\infty} \frac{2B}{n\pi a^n}(1-\cos n\pi)r^n \sin n\theta \qquad 4.0.1$$

$$\overline{B}^{(1)} = \mathrm{B}a^{-n} + \frac{2\mathrm{B}^2}{\lambda\pi}\sum_{n=1}^{\infty}\frac{i}{a^{n+1}}(1-\cos n\pi)\left[\frac{a^{4-2n} - \left(2a^{4n} - n + 2\right) + r^{4-n}\left(r^{4n} - n + 2\right)}{8n(n+2)} r^n \cos n\theta\right] \qquad 4.0.2$$

$$\overline{B} = \overline{B}^{(0)} + \xi\overline{B}^{(1)} \qquad 4.0.3$$

And

$$V^{(0)} = \sum_{n=1}^{\infty} \frac{2A}{n\pi a^n}(1 - Cos n\pi)r^n Sin n\theta \qquad 4.0.4$$

$$V^{(1)} = \frac{-2\mathrm{A}}{\eta\mu\pi}\sum_{n=1}^{\infty}\frac{1}{a^n}(1-\cos n\pi)\left[\frac{a^{2n+4} - (n+2)a^{4-2n} - r^{2n+4} + (n+2)r^{4-2n}}{8n(n+2)}\right]r^n \cos n\theta \qquad 4.0.5$$

$$V = V^{(0)} + \beta V^{(1)} \qquad 4.0.6$$

The tables and graphs in the subsequent pages are those of $\overline{\mathrm{B}}(r,\theta)$ and $V(r,\theta)$ obtained by varying the parameters, λ, μ, π and B

TABLE 4.1.2

$\mu := 0.3$ $\varepsilon := 0.001$ $B := 1$ $\eta := 1$

r =	$V_3\left(r, \frac{56}{deg}\right) =$	$V_3\left(r, \frac{80}{deg}\right) =$
0.05	0.0804984	0.0458758
0.1	0.0402492	0.0229379
0.15	0.0268325	0.0152918
0.2	0.0201235	0.0114683
0.25	0.0160963	0.0091732
0.3	0.013408	0.0076412
0.35	0.0114817	0.0065434
0.4	0.010027	0.0057143
0.45	0.0088806	0.005061
0.5	0.007942	0.0045261
0.55	0.0071444	0.0040716
0.6	0.0064398	0.00367
0.65	0.0057913	0.0033004
0.7	0.0051681	0.0029453
0.75	0.0045421	0.0025885
0.8	0.0038845	0.0022138
0.85	0.0031618	0.0018019
0.9	0.0023284	0.001327
0.95	0.0013142	0.000749

TABLE 4.1.3

$\mu := 0.5$ $\varepsilon := 0.001$ $B := 1$ $\eta := 1$

r =	$V_5\left(r, \frac{56}{deg}\right) =$	$V_5\left(r, \frac{80}{deg}\right) =$
0.05	0.048299	0.0275255
0.1	0.0241495	0.0137627
0.15	0.0160995	0.0091751
0.2	0.0120741	0.006881
0.25	0.0096578	0.0055039
0.3	0.0080448	0.0045847
0.35	0.006889	0.003926
0.4	0.0060162	0.0034286
0.45	0.0053284	0.0030366
0.5	0.0047652	0.0027157
0.55	0.0042866	0.0024429
0.6	0.0038639	0.002202
0.65	0.0034748	0.0019803
0.7	0.0031009	0.0017672
0.75	0.0027253	0.0015531
0.8	0.0023307	0.0013283
0.85	0.0018971	0.0010811
0.9	0.0013971	0.0007962
0.95	0.0007885	0.0004494

TABLE 4.1.4

$\mu := 0.7$ $\varepsilon := 0.001$ $B := 1$ $\eta := 1$

$r =$	$V_7\left(r, \frac{56}{deg}\right) =$	$V_7\left(r, \frac{80}{deg}\right) =$
0.05	0.0344993	0.019661
0.1	0.0172496	0.0098305
0.15	0.0114997	0.0065536
0.2	0.0086244	0.004915
0.25	0.0068984	0.0039314
0.3	0.0057463	0.0032748
0.35	0.0049207	0.0028043
0.4	0.0042973	0.002449
0.45	0.003806	0.002169
0.5	0.0034037	0.0019398
0.55	0.0030619	0.001745
0.6	0.0027599	0.0015729
0.65	0.002482	0.0014145
0.7	0.0022149	0.0012623
0.75	0.0019466	0.0011094
0.8	0.0016648	0.0009488
0.85	0.001355	0.0007722
0.9	0.0009979	0.0005687
0.95	0.0005632	0.000321

TABLE 4.1.5

$\mu := 0.9$ $\varepsilon := 0.001$ $B := 1$ $\eta := 1$

$r =$	$V_9\left(r, \frac{56}{deg}\right) =$	$V_9\left(r, \frac{80}{deg}\right) =$
0.05	0.0268328	0.0152919
0.1	0.0134164	0.007646
0.15	0.0089442	0.0050973
0.2	0.0067078	0.0038228
0.25	0.0053654	0.0030577
0.3	0.0044693	0.0025471
0.35	0.0038272	0.0021811
0.4	0.0033423	0.0019048
0.45	0.0029602	0.001687
0.5	0.0026473	0.0015087
0.55	0.0023815	0.0013572
0.6	0.0021466	0.0012233
0.65	0.0019304	0.0011001
0.7	0.0017227	0.0009818
0.75	0.001514	0.0008628
0.8	0.0012948	0.0007379
0.85	0.0010539	0.0006006
0.9	0.0007761	0.0004423
0.95	0.0004381	0.0002497

TABLE 4.1.6

$\mu := 1$ $\varepsilon := 0.001$ $B := 1$ $\eta := 1$

$r =$	$V_{10}\left(r, \frac{56}{deg}\right) =$	$V_{10}\left(r, \frac{80}{deg}\right) =$
0.05	0.0241495	0.0137627
0.1	0.0120748	0.0068814
0.15	0.0080498	0.0045875
0.2	0.006037	0.0034405
0.25	0.0048289	0.002752
0.3	0.0040224	0.0022924
0.35	0.0034445	0.001963
0.4	0.0030081	0.0017143
0.45	0.0026642	0.0015183
0.5	0.0023826	0.0013578
0.55	0.0021433	0.0012215
0.6	0.0019319	0.001101
0.65	0.0017374	0.0009901
0.7	0.0015504	0.0008836
0.75	0.0013626	0.0007766
0.8	0.0011653	0.0006641
0.85	0.0009485	0.0005406
0.9	0.0006985	0.0003981
0.95	0.0003943	0.0002247

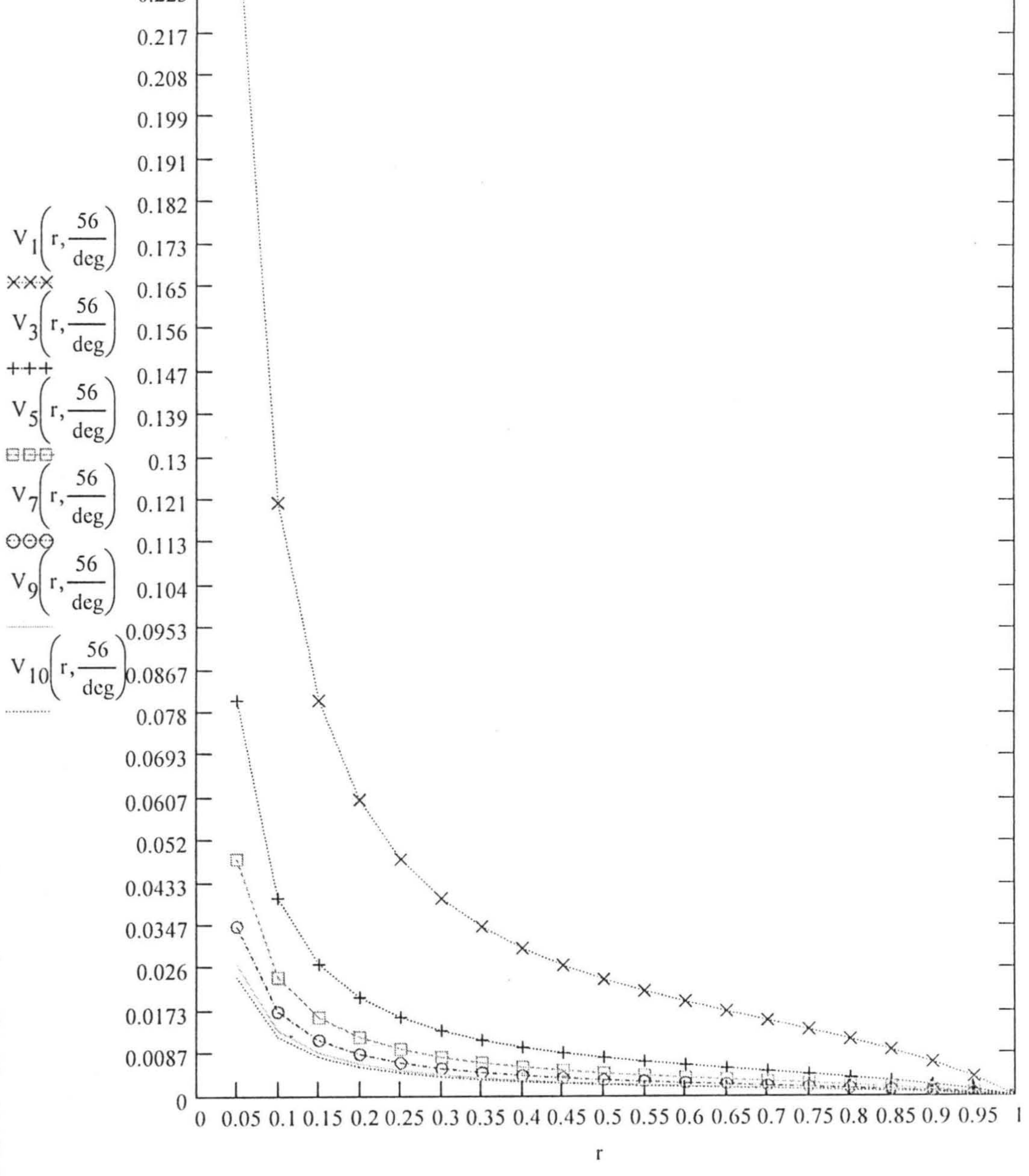

Figure 4.1.1: $\mu = 0.1, 0.3, 0.5, 0.7, 0.9, 1.0$ $\varepsilon = 0.001$, $\eta = 1$, $n = 5$, $B = 1$

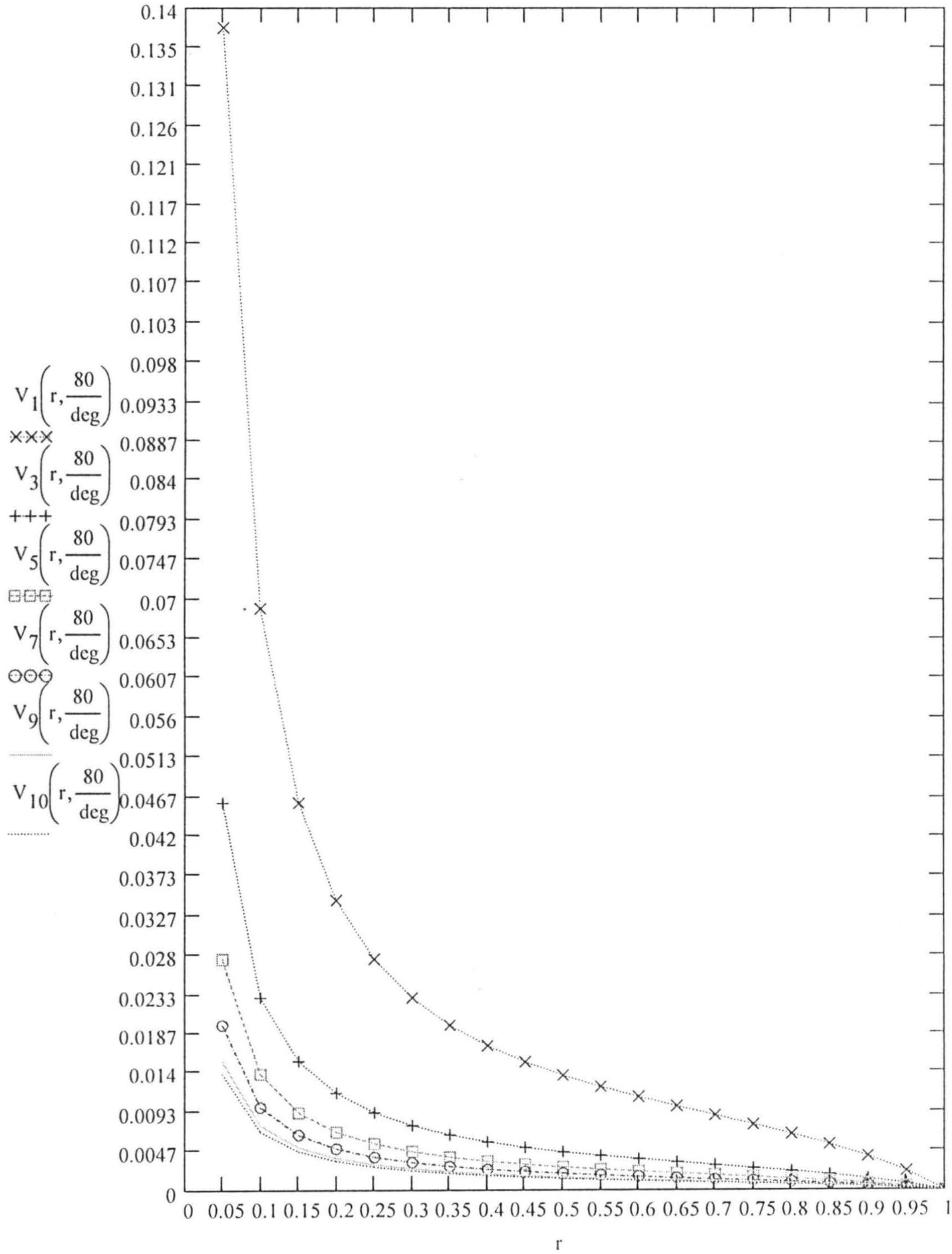

Figure 4.1.2: $\mu = 0.1, 0.3, 0.5, 0.7, 0.9, 1.0$ $\varepsilon = 0.001$, $\eta = 1$, $n = 5$, $B = 1$

TABLE 4.2.1

$\mu := 1$ $\varepsilon := 0.001$ $B := 1$ $\eta := 0.1$

$r =$	$V_{0.1}\left(r, \frac{56}{deg}\right) =$	$V_{0.1}\left(r, \frac{80}{deg}\right) =$
0.05	0.2414952	0.1376273
0.1	0.1207475	0.0688136
0.15	0.0804976	0.0458753
0.2	0.0603705	0.0344049
0.25	0.0482889	0.0275197
0.3	0.0402241	0.0229236
0.35	0.034445	0.0196301
0.4	0.0300809	0.017143
0.45	0.0266418	0.0151831
0.5	0.0238261	0.0135784
0.55	0.0214331	0.0122147
0.6	0.0193193	0.01101
0.65	0.0173739	0.0099013
0.7	0.0155044	0.0088359
0.75	0.0136263	0.0077656
0.8	0.0116535	0.0066413
0.85	0.0094853	0.0054056
0.9	0.0069853	0.0039809
0.95	0.0039426	0.0022469

TABLE 4.2.2

$\mu := 1$ $\varepsilon := 0.001$ $B := 1$ $\eta := 0.3$

$r =$	$V_{0.3}\left(r, \frac{56}{deg}\right) =$	$V_{0.3}\left(r, \frac{80}{deg}\right) =$
0.05	0.0804984	0.0458758
0.1	0.0402492	0.0229379
0.15	0.0268325	0.0152918
0.2	0.0201235	0.0114683
0.25	0.0160963	0.0091732
0.3	0.013408	0.0076412
0.35	0.0114817	0.0065434
0.4	0.010027	0.0057143
0.45	0.0088806	0.005061
0.5	0.007942	0.0045261
0.55	0.0071444	0.0040716
0.6	0.0064398	0.00367
0.65	0.0057913	0.0033004
0.7	0.0051681	0.0029453
0.75	0.0045421	0.0025885
0.8	0.0038845	0.0022138
0.85	0.0031618	0.0018019
0.9	0.0023284	0.001327
0.95	0.0013142	0.000749

TABLE 4.2.3

$\mu := 1$ $\varepsilon := 0.001$ $B := 1$ $\eta := 0.5$

$r =$	$V_{0.5}\left(r, \frac{56}{deg}\right) =$	$V_{0.5}\left(r, \frac{80}{deg}\right) =$
0.05	0.048299	0.0275255
0.1	0.0241495	0.0137627
0.15	0.0160995	0.0091751
0.2	0.0120741	0.006881
0.25	0.0096578	0.0055039
0.3	0.0080448	0.0045847
0.35	0.006889	0.003926
0.4	0.0060162	0.0034286
0.45	0.0053284	0.0030366
0.5	0.0047652	0.0027157
0.55	0.0042866	0.0024429
0.6	0.0038639	0.002202
0.65	0.0034748	0.0019803
0.7	0.0031009	0.0017672
0.75	0.0027253	0.0015531
0.8	0.0023307	0.0013283
0.85	0.0018971	0.0010811
0.9	0.0013971	0.0007962
0.95	0.0007885	0.0004494

TABLE 4.2.4

$\mu := 1$ $\varepsilon := 0.001$ $B := 1$ $\eta := 0.7$

$r =$	$V_{0.7}\left(r, \frac{56}{deg}\right) =$	$V_{0.7}\left(r, \frac{80}{deg}\right) =$
0.05	0.0344993	0.019661
0.1	0.0172496	0.0098305
0.15	0.0114997	0.0065536
0.2	0.0086244	0.004915
0.25	0.0068984	0.0039314
0.3	0.0057463	0.0032748
0.35	0.0049207	0.0028043
0.4	0.0042973	0.002449
0.45	0.003806	0.002169
0.5	0.0034037	0.0019398
0.55	0.0030619	0.001745
0.6	0.0027599	0.0015729
0.65	0.002482	0.0014145
0.7	0.0022149	0.0012623
0.75	0.0019466	0.0011094
0.8	0.0016648	0.0009488
0.85	0.001355	0.0007722
0.9	0.0009979	0.0005687
0.95	0.0005632	0.000321

TABLE 4.2.5

$\mu := 1$ $\varepsilon := 0.001$ $B := 1$ $\eta := 0.9$

r =	$V_{0.9}\left(r, \frac{56}{deg}\right) =$	$V_{0.9}\left(r, \frac{80}{deg}\right) =$
0.05	0.0268328	0.0152919
0.1	0.0134164	0.007646
0.15	0.0089442	0.0050973
0.2	0.0067078	0.0038228
0.25	0.0053654	0.0030577
0.3	0.0044693	0.0025471
0.35	0.0038272	0.0021811
0.4	0.0033423	0.0019048
0.45	0.0029602	0.001687
0.5	0.0026473	0.0015087
0.55	0.0023815	0.0013572
0.6	0.0021466	0.0012233
0.65	0.0019304	0.0011001
0.7	0.0017227	0.0009818
0.75	0.001514	0.0008628
0.8	0.0012948	0.0007379
0.85	0.0010539	0.0006006
0.9	0.0007761	0.0004423
0.95	0.0004381	0.0002497

TABLE 4.2.6

$\mu := 1$ $\varepsilon := 0.001$ $B := 1$ $\eta := 1$

$r =$	$V_{100}\left(r, \frac{56}{deg}\right) =$	$V_{100}\left(r, \frac{80}{deg}\right) =$
0.05	0.0241495	0.0137627
0.1	0.0120748	0.0068814
0.15	0.0080498	0.0045875
0.2	0.006037	0.0034405
0.25	0.0048289	0.002752
0.3	0.0040224	0.0022924
0.35	0.0034445	0.001963
0.4	0.0030081	0.0017143
0.45	0.0026642	0.0015183
0.5	0.0023826	0.0013578
0.55	0.0021433	0.0012215
0.6	0.0019319	0.001101
0.65	0.0017374	0.0009901
0.7	0.0015504	0.0008836
0.75	0.0013626	0.0007766
0.8	0.0011653	0.0006641
0.85	0.0009485	0.0005406
0.9	0.0006985	0.0003981
0.95	0.0003943	0.0002247

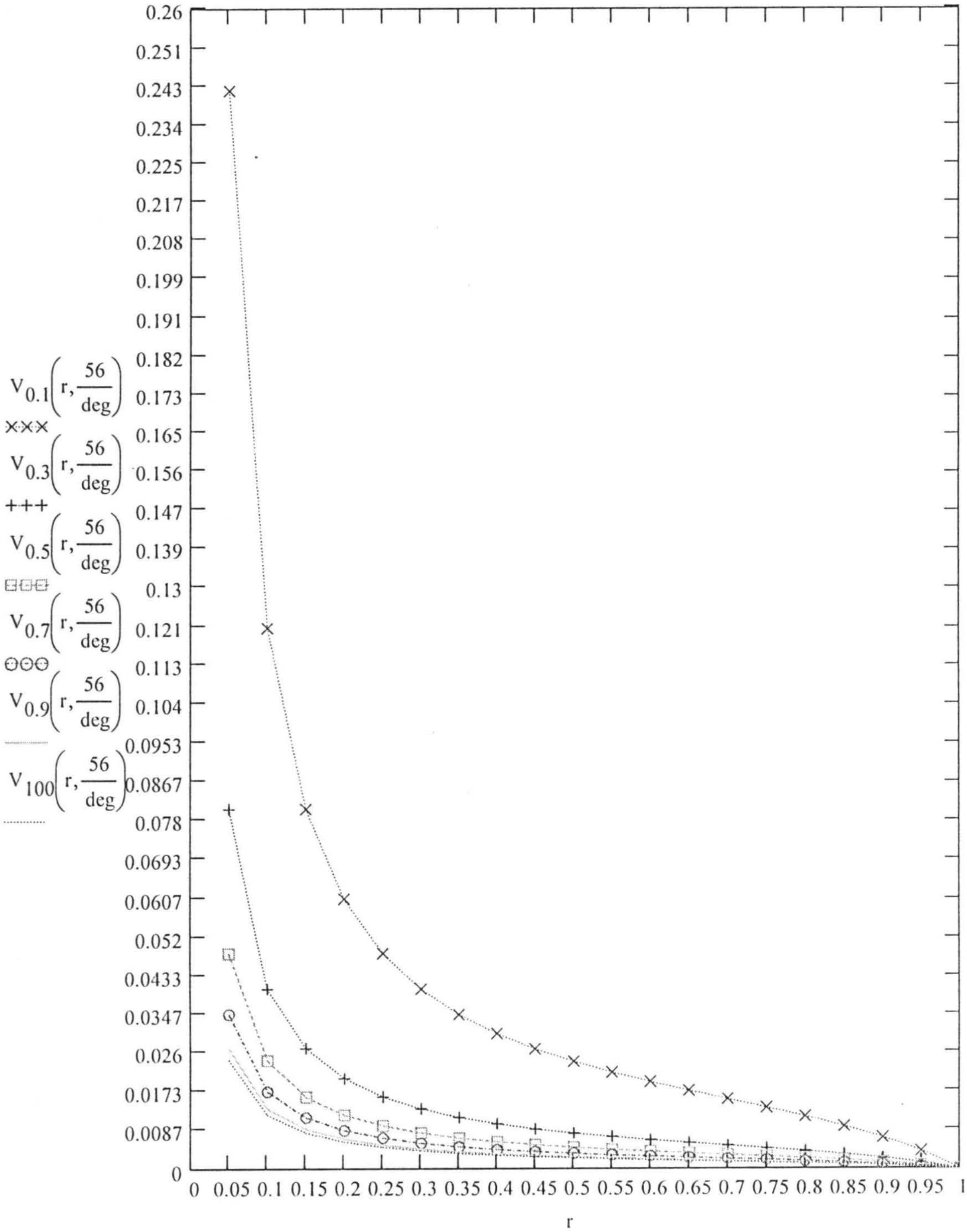

Figure 4.2.1: $\mu = 1$, $\varepsilon = 0.001$, $\eta = 0.1, 0.3, 0.5, 0.7, 0.9, 1.0$, $n = 5$, $B = 1$

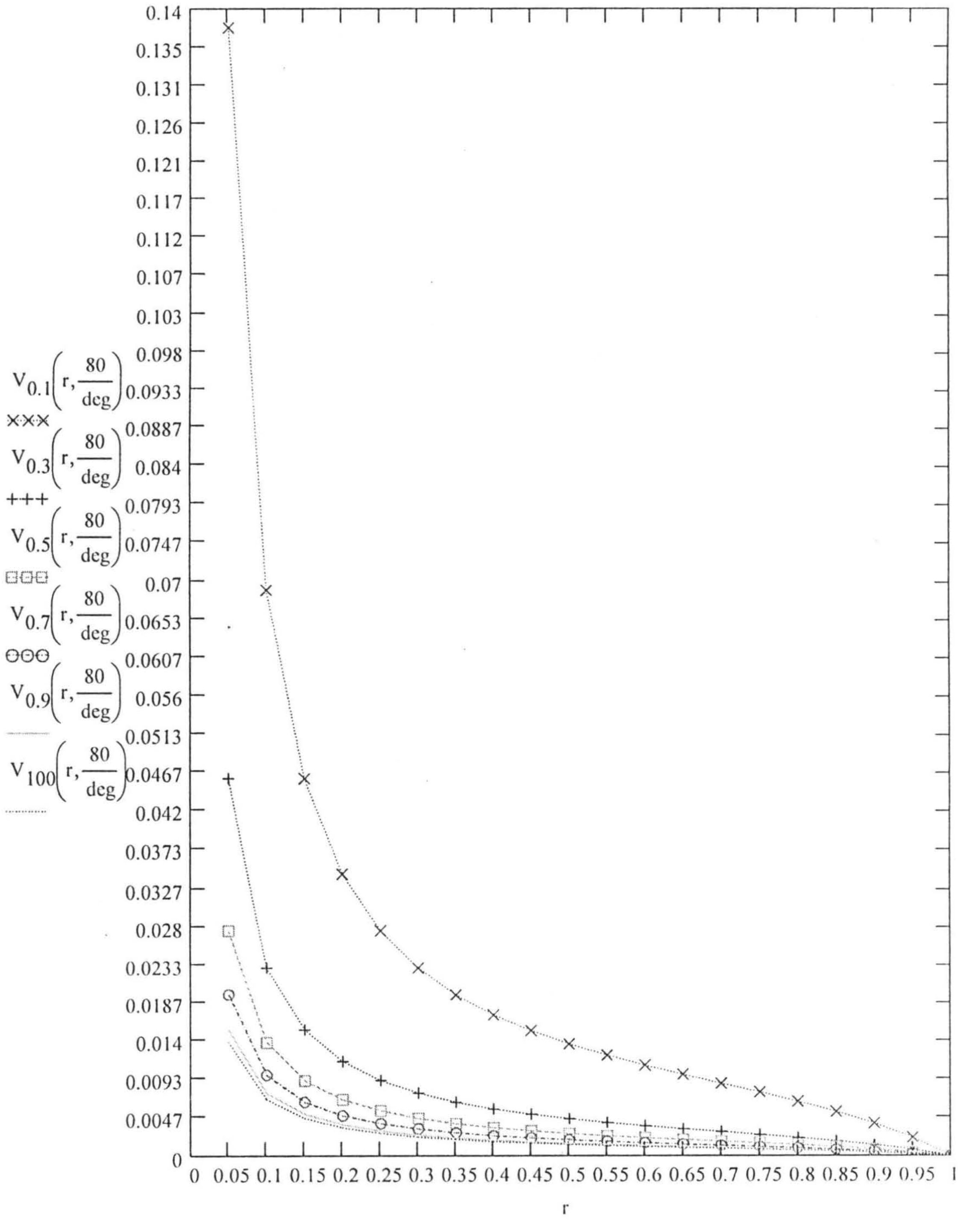

Figure 4.2.2: $\mu = 1$, $\varepsilon = 0.001$, $\eta = 0.1,0.3,0.5,0.7,0.9,1.0$, $n = 5$, $B = 1$

TABLE 4.3.1

$\mu := 1$ $\varepsilon := 0.001$ $B := 0.1$ $\eta := 1$

$r =$	$V_{01}\left(r, \frac{56}{deg}\right) =$	$V_{01}\left(r, \frac{80}{deg}\right) =$
0.05	0.0002415	0.0001376
0.1	0.0001207	0.0000688
0.15	0.0000805	0.0000459
0.2	0.0000604	0.0000344
0.25	0.0000483	0.0000275
0.3	0.0000402	0.0000229
0.35	0.0000344	0.0000196
0.4	0.0000301	0.0000171
0.45	0.0000266	0.0000152
0.5	0.0000238	0.0000136
0.55	0.0000214	0.0000122
0.6	0.0000193	0.000011
0.65	0.0000174	0.0000099
0.7	0.0000155	0.0000088
0.75	0.0000136	0.0000078
0.8	0.0000117	0.0000066
0.85	0.0000095	0.0000054
0.9	0.000007	0.000004
0.95	0.0000039	0.0000022

TABLE 4.3.2

$\mu := 1$ $\varepsilon := 0.001$ $B := 0.3$ $\eta := 1$

$r =$	$V_{03}\left(r, \frac{56}{deg}\right) =$	$V_{03}\left(r, \frac{80}{deg}\right) =$
0.05	0.0021735	0.0012386
0.1	0.0010867	0.0006193
0.15	0.0007245	0.0004129
0.2	0.0005433	0.0003096
0.25	0.0004346	0.0002477
0.3	0.000362	0.0002063
0.35	0.00031	0.0001767
0.4	0.0002707	0.0001543
0.45	0.0002398	0.0001366
0.5	0.0002144	0.0001222
0.55	0.0001929	0.0001099
0.6	0.0001739	0.0000991
0.65	0.0001564	0.0000891
0.7	0.0001395	0.0000795
0.75	0.0001226	0.0000699
0.8	0.0001049	0.0000598
0.85	0.0000854	0.0000487
0.9	0.0000629	0.0000358
0.95	0.0000355	0.0000202

TABLE 4.3.3

$\mu := 1$ $\varepsilon := 0.001$ $B := 0.5$ $\eta := 1$

r =	$V_{05}\left(r, \frac{56}{deg}\right) =$	$V_{05}\left(r, \frac{80}{deg}\right) =$
0.05	0.0060374	0.0034407
0.1	0.0030187	0.0017203
0.15	0.0020124	0.0011469
0.2	0.0015093	0.0008601
0.25	0.0012072	0.000688
0.3	0.0010056	0.0005731
0.35	0.0008611	0.0004908
0.4	0.000752	0.0004286
0.45	0.000666	0.0003796
0.5	0.0005957	0.0003395
0.55	0.0005358	0.0003054
0.6	0.000483	0.0002753
0.65	0.0004343	0.0002475
0.7	0.0003876	0.0002209
0.75	0.0003407	0.0001941
0.8	0.0002913	0.000166
0.85	0.0002371	0.0001351
0.9	0.0001746	0.0000995
0.95	0.0000986	0.0000562

TABLE 4.3.4

$\mu := 1$ $\varepsilon := 0.001$ $B := 0.7$ $\eta := 1$

$r =$	$V_{07}\left(r, \frac{56}{deg}\right) =$	$V_{07}\left(r, \frac{80}{deg}\right) =$
0.05	0.0118333	0.0067437
0.1	0.0059166	0.0033719
0.15	0.0039444	0.0022479
0.2	0.0029582	0.0016858
0.25	0.0023662	0.0013485
0.3	0.001971	0.0011233
0.35	0.0016878	0.0009619
0.4	0.001474	0.00084
0.45	0.0013054	0.000744
0.5	0.0011675	0.0006653
0.55	0.0010502	0.0005985
0.6	0.0009466	0.0005395
0.65	0.0008513	0.0004852
0.7	0.0007597	0.000433
0.75	0.0006677	0.0003805
0.8	0.000571	0.0003254
0.85	0.0004648	0.0002649
0.9	0.0003423	0.0001951
0.95	0.0001932	0.0001101

TABLE 4.3.5

$\mu := 1$ $\varepsilon := 0.001$ $B := 0.9$ $\eta := 1$

$r =$	$V_{09}\left(r, \frac{56}{deg}\right) =$	$V_{09}\left(r, \frac{80}{deg}\right) =$
0.05	0.0195611	0.0111478
0.1	0.0097805	0.0055739
0.15	0.0065203	0.0037159
0.2	0.00489	0.0027868
0.25	0.0039114	0.0022291
0.3	0.0032581	0.0018568
0.35	0.00279	0.00159
0.4	0.0024366	0.0013886
0.45	0.002158	0.0012298
0.5	0.0019299	0.0010998
0.55	0.0017361	0.0009894
0.6	0.0015649	0.0008918
0.65	0.0014073	0.000802
0.7	0.0012559	0.0007157
0.75	0.0011037	0.000629
0.8	0.0009439	0.0005379
0.85	0.0007683	0.0004379
0.9	0.0005658	0.0003225
0.95	0.0003194	0.000182

TABLE 4.3.6

$\mu := 1$ $\varepsilon := 0.001$ $B := 1$ $\eta := 1$

$r =$	$V_{010}\left(r, \frac{56}{deg}\right) =$	$V_{010}\left(r, \frac{80}{deg}\right) =$
0.05	0.0241495	0.0137627
0.1	0.0120748	0.0068814
0.15	0.0080498	0.0045875
0.2	0.006037	0.0034405
0.25	0.0048289	0.002752
0.3	0.0040224	0.0022924
0.35	0.0034445	0.001963
0.4	0.0030081	0.0017143
0.45	0.0026642	0.0015183
0.5	0.0023826	0.0013578
0.55	0.0021433	0.0012215
0.6	0.0019319	0.001101
0.65	0.0017374	0.0009901
0.7	0.0015504	0.0008836
0.75	0.0013626	0.0007766
0.8	0.0011653	0.0006641
0.85	0.0009485	0.0005406
0.9	0.0006985	0.0003981
0.95	0.0003943	0.0002247

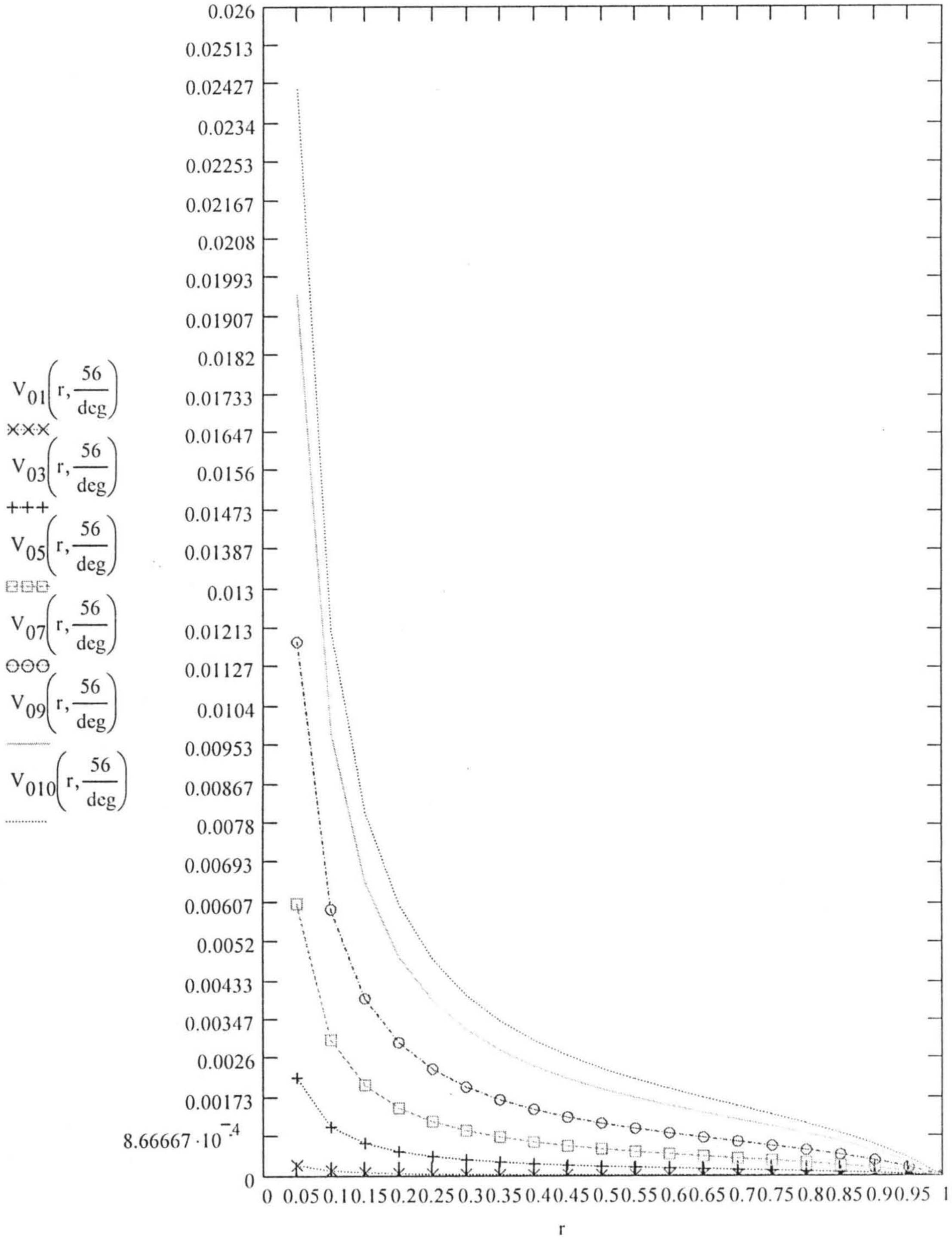

Figure 4.3.1: $\mu = 1$, $\varepsilon = 0.001$, $\eta = 1$, $n = 5$, $B = 0.1, 0.3, 0.5, 0.7, 0.9, 1.0$

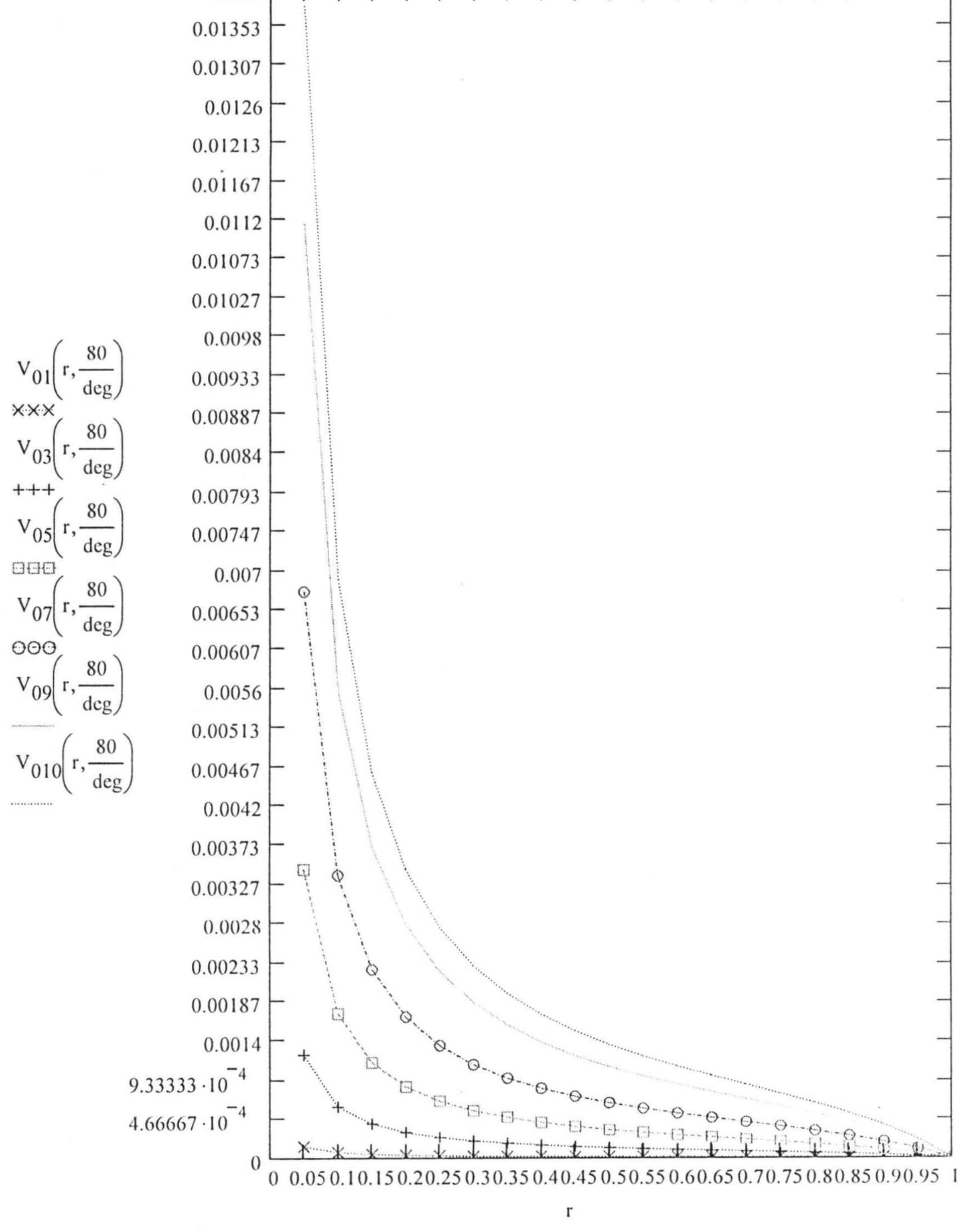

Figure 4.3.2: $\mu = 1$, $\varepsilon = 0.001$, $\eta = 1$, $n = 5$, $B = 0.1, 0.3, 0.5, 0.7, 0.9, 1.0$

TABLE 4.4.1

$\mu := 1$ $\varepsilon := 0.001$ $B := 1$ $\eta := 1$

$r =$	$B'_1\left(r, \frac{56}{deg}\right) =$	$B'_1\left(r, \frac{80}{deg}\right) =$
0.05	0.0144921	0.008259
0.1	0.2292751	0.1306631
0.15	1.1491657	0.6549057
0.2	3.600334	2.0518184
0.25	8.7238579	4.9716978
0.3	17.9737718	10.2431931
0.35	33.1169433	18.8732365
0.4	56.2300458	32.0453172
0.45	89.6843294	51.1108028
0.5	136.0982832	77.5619615
0.55	198.2129417	112.9609
0.6	278.5824929	158.7632415
0.65	378.8162209	215.8861116
0.7	497.7025052	283.639012
0.75	626.4795686	357.0286346
0.8	736.7047835	419.84562
0.85	748.8220944	426.7512355
0.9	450.6818574	256.8421002
0.95	-711.7799804	-405.6410572

TABLE 4.4.2

$\mu := 1$ $\qquad \varepsilon := 0.001$ $\qquad B := 1$ $\qquad \eta := 1$

$r =$	$B'_3\left(r, \frac{56}{deg}\right) =$	$B'_3\left(r, \frac{80}{deg}\right) =$
0.05	-0.0000248	-0.0000434
0.1	0.0000063	-0.0000448
0.15	0.0002585	0.0000931
0.2	0.0010037	0.0005282
0.25	0.0026177	0.0014774
0.3	0.0055802	0.0032161
0.35	0.0104741	0.0060782
0.4	0.0179843	0.0104551
0.45	0.0288931	0.0167927
0.5	0.0440639	0.0255825
0.55	0.0643996	0.0373375
0.6	0.0907396	0.0525331
0.65	0.1236067	0.0714635
0.7	0.1625818	0.0938844
0.75	0.2047275	0.1181151
0.8	0.2405444	0.1387337
0.85	0.2434934	0.1406057
0.9	0.1428355	0.0834053
0.95	-0.2461416	-0.1381478

TABLE 4.4.3

$\mu := 0.5$ $\varepsilon := 0.001$ $B := 1$ $\eta := 1$

$r =$	$B'_5\left(r, \frac{56}{deg}\right) =$	$B'_5\left(r, \frac{80}{deg}\right) =$
0.05	-0.0000267	-0.0000445
0.1	-0.0000243	-0.0000623
0.15	0.0001053	0.0000058
0.2	0.0005236	0.0002546
0.25	0.0014544	0.0008144
0.3	0.0031834	0.0018502
0.35	0.0060578	0.0035614
0.4	0.0104857	0.0061816
0.45	0.0169329	0.0099766
0.5	0.0259136	0.0152388
0.55	0.0379651	0.0222725
0.6	0.0535857	0.0313593
0.65	0.0730836	0.0426705
0.7	0.0962008	0.0560541
0.75	0.1211677	0.0704947
0.8	0.1422768	0.0827314
0.85	0.1435959	0.0836744
0.9	0.082672	0.0491184
0.95	-0.1513327	-0.0841166

TABLE 4.4.4

$\mu := 1$ $\varepsilon := 0.001$ $B := 1$ $\eta := 1$

$r =$	$B'_7\left(r, \frac{56}{deg}\right) =$	$B'_7\left(r, \frac{80}{deg}\right) =$
0.05	-0.0000275	-0.000045
0.1	-0.0000374	-0.0000697
0.15	0.0000396	-0.0000317
0.2	0.0003178	0.0001373
0.25	0.0009559	0.0005303
0.3	0.0021562	0.0012648
0.35	0.0041651	0.0024828
0.4	0.007272	0.0043502
0.45	0.0118071	0.0070555
0.5	0.0181349	0.0108057
0.55	0.026636	0.0158161
0.6	0.0376626	0.0222847
0.65	0.0514308	0.0303307
0.7	0.0677518	0.0398411
0.75	0.0853564	0.0500859
0.8	0.1001622	0.0587303
0.85	0.1007827	0.0592753
0.9	0.0568877	0.034424
0.95	-0.1107003	-0.0609603

TABLE 4.4.5

$\mu := 0.9$ $\varepsilon := 0.001$ $B := 1$ $\eta := 1$

$r =$	$B'_9\left(r, \frac{56}{deg}\right) =$	$B'_9\left(r, \frac{80}{deg}\right) =$
0.05	-0.000028	-0.0000453
0.1	-0.0000446	-0.0000739
0.15	0.0000031	-0.0000525
0.2	0.0002035	0.0000722
0.25	0.0006789	0.0003724
0.3	0.0015856	0.0009396
0.35	0.0031137	0.0018835
0.4	0.0054866	0.0033327
0.45	0.0089595	0.0054326
0.5	0.0138134	0.0083429
0.55	0.020342	0.0122292
0.6	0.0288165	0.0172433
0.65	0.0394015	0.0234752
0.7	0.0519468	0.0308339
0.75	0.0654612	0.0387477
0.8	0.0767651	0.0453964
0.85	0.0769976	0.0457203
0.9	0.042563	0.0262604
0.95	-0.0881268	-0.0480957

TABLE 4.4.6

$\mu := 1$ $\varepsilon := 0.001$ $B := 1$ $\eta := 1$

$r =$	$B'_{10}\left(r, \frac{56}{deg}\right) =$	$B'_{10}\left(r, \frac{80}{deg}\right) =$
0.05	-0.0000281	-0.0000453
0.1	-0.0000472	-0.0000753
0.15	-0.0000096	-0.0000597
0.2	0.0001635	0.0000494
0.25	0.000582	0.0003172
0.3	0.0013858	0.0008257
0.35	0.0027456	0.0016738
0.4	0.0048617	0.0029766
0.45	0.0079628	0.0048646
0.5	0.0123009	0.0074809
0.55	0.0181391	0.0109738
0.6	0.0257203	0.0154789
0.65	0.0351913	0.0210758
0.7	0.0464151	0.0276813
0.75	0.0584979	0.0347794
0.8	0.0685762	0.0407296
0.85	0.0686728	0.040976
0.9	0.0375494	0.0234031
0.95	-0.080226	-0.0435931

CHAPTER FIVE

5.1 DISCUSSION

The value of $B_Z(r,\theta)$ and $V_Z(r,\theta)$ are plotted against the values of r for different values of θ.It is discovered that the velocity of the fluid increases with lower angles and decreases with higher angles. The higher the viscosity (η) of the fluid the lower the velocity. Also the higher the imposed magnetic field (B) the higher the velocity.

5.2 SUMMARY AND RECOMMENDATIONS

We have succeeded in solving two dimensional linear problems concerning rectilinear flow in the Z-direction under a known, imposed uniform transverse field B and an unknown induced field B_Z due to currents flowing in (x, y) planes by analytical method through a regular perturbation analysis; and the corresponding velocity vector equation for V_Z

Interested researchers may decide to solve these problems numerically and determine even higher orders.

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