NUMERICAL INVESTIGATION OF FLOW PAST A SPHERE

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CERTIFICATION

This thesis titled "NUMERICAL INVESTIGATION OF FLOW PAST A SPHERE" by Abah, Sunday Ojima, meets the regulations governing the award of the degree of Masters of Technology in Mathematics. Federal University of Minna and is approved for its contribution to knowledge and literary presentation.

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DEDICATION

This research work is dedicated to the Almighty God; and my family.

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I would like to express my profound gratitude to all those who contributed to the successful completion of my academic goal of this nature, especially my parents Mr. And Mrs. Usman Abah and family members.

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ABSTRACT

In this research work, Numerical Investigation of Flow Past a Sphere we consider the flow of fluid past a sphere at low Reynolds number. Assuming the flow is steady and asymmetrical within the vicinity of the sphere. We neglect the Inertia terms and assume the absences of extraneous forces.

The method of regular perturbation analysis is being used in linearizing the governing equation; the resulting equation was solved analytically. The graphs of the variation of the flow pattern were plotted for specific values of the ratio of the distance from the sphere to the radius of the sphere.

TABLE OF CONTENTS

CHAPTER ONE

1.0 INTRODUCTION

In this research work we consider the flow of an inviscid fluid past a sphere at low Reynolds number. We will assume that flow is steady and axis symmetrical and flow is in the vicinity of the sphere is not valid at large distances from the sphere.

Under this assumption the equation is given by

$$\nabla^2 u - \nabla p = R(v.\nabla)v \qquad \nabla v = 0 \text{..............1.1}$$

Where v is the pressure, $R = {}^{ua}/_{v}$ is the Reynolds number. ρ is the density and v is the kinematic viscosity.

This problem was first solved by Stokes (1851) and Kaplun (1957) using a method of matched asymptotic expansion.

In this research work we neglect inertia terms and assume the absence of extraneous forces so that we have the problem in the form

$$\mu\nabla^2 u = \frac{\partial p}{\partial x} \text{........................1.2}$$

which is solvable using the method of regular perturbation analysis.

We now proceed to solve the order ε^o problem using the method of separation of variable which results into a Legendre polynomial which we solved using method of series solution.

1.1 Definition of Terms

(i) Fluid: A fluid is a substance which deforms or yields continuously when shear stress is applied to it, no matter how small it is.

(ii) Incompressible fluid: A fluid is said to be incompressible if the density is in variant with time and space.

$$\frac{d\rho}{dt} = \nabla \rho = 0 \quad \text{Where } \rho \text{ is density}$$

N.B Density (ρ) is the mass per unit volume.

(iii) Compressible fluid: A fluid is compressible if the density depends continuously on the time and space variable.

(iv) Specific volume is the volume per unit weight

(v) Viscosity: Viscosity is the property of a fluid by which it offers resistance to shear acting on it. According to Newton's law of Viscosity, the shear F acting between two layers of fluid is proportional to differences in their velocities and inversely proportional to the distance between them.

i.e. $$F = \mu A \frac{\Delta u}{\Delta y}$$

$$\Gamma = \frac{F}{A} = \mu \frac{\Delta u}{\Delta y}$$ Which is the shear stress.

Where μ is the constant of proportionality

$$\frac{du}{dy} = \text{rate of angular deformation}$$

Fluid are classified according to the relation between shear stress Γ and rate of angular deformation.

(a) Newtonian fluids: are fluid which obeys Newton's law of Viscosity

i.e. $\Gamma = \mu \frac{du}{dy}$ e.g. water and kerosene etc

(b) Non – Newtonian fluids: They do not obey Newton's law of Viscosity

i.e. $\Gamma = \mu \frac{(du)^n}{dy}$

e.g. blood, mud flow, suspensions and polymer solutions.

) Ideal fluids: These are fluids that have no Viscosity, Surface tension and are incompressible i.e. $\Gamma = 0$

(d) Idea Plastics or Bingham Plastics: These are fluids where

$$\Gamma = \text{constant} + \mu \frac{du}{dy}$$

(e) Thyxotropic fluids: are fluid where

$$\Gamma = \text{constant} + \mu \frac{(du)^n}{dy}$$

Fluid Mechanics: Fluids mechanics is the branch of engineering science which deals with the behavior of fluids under the conditions of rest and motion.

1.2. Kinematics of fluid flow

Kinematics of fluid flows deals with fluid motion in terms of displacements, velocities, acceleration, rotation of fluid without regard to the force or energy responsible for the motion.

If F is a flow or fluid property such as velocity, pressure, mass, density or temperature the following types of flow can be defined:

1 Steady flow: $\frac{\partial f}{\partial t} = 0$ at a point or section

2. Unsteady flow: $\frac{\partial f}{\partial t} \neq 0$ at a point or section

3. Uniform flow: $\frac{\partial f}{\partial s}_{t=t_0} = 0$

4. Non-Uniform flow: $\frac{\partial f}{\partial s_{t=t_0}} \neq 0$

5. One dimensional flow f = f(x,t) or f(s,t)

6. Two dimensional flow: f = f (x,y,t)

7. Three dimensional flow: f = f (x,y,z,t)

1.2.2 Streamlines Pathline and streaklines

An imaginary line in the flow field such that at every point along it the velocity vector is tangential to it is known as a streamline.

Equation of streamline is

$$\frac{u}{dx} = \frac{v}{dy} = \frac{w}{dz}$$

path line is the path followed by a fluid particle during its travel.

A steam line in the path followed by all the fluid particles passing through a given point n space. A stream tube consist of a group of streamlines.

1.2.3 Continuity Equation

The application of the principle conservation of mass to an elementary volume gives continuity equation in any co-ordinate system.

For compressible fluids,

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho_u}{\partial x} + \frac{\partial \rho_v}{\partial y} + \frac{\partial \rho_w}{\partial z} = 0$$

r in vector form, $$\frac{\partial \rho}{\partial t} + \nabla(\rho v) = 0$$

or incompressible (homogenous or non homogenous fluids)

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

2.4 Acceleration

cceleration is the rate of change of velocity with time and it is a vector quantity. Convective celeration is due to non – uniformity of flow where as local acceleration due to under adiness of flow. Tangential acceleration.

$$a_s = v_s \frac{\partial v_s}{\partial s}$$

It is along the streamline and it is due to change in magnitude of velocity. Local tangential acceleration is given by

$$\frac{\partial v_s}{\partial t}$$

Total tangential acceleration

$$\frac{\partial v_s}{\partial t} = \frac{\partial v_s}{\partial t} + v_s \frac{\partial v_s}{\partial s}$$

Convective normal acceleration due to change in direction of flow along a streamline is equal to

$$\frac{v^2 s}{R}$$

Where R is the radius of curvature of streamline.

Local normal acceleration = $\frac{\partial v_n}{\partial t}$

Where v_n normal component of velocity generate due to change in direction.

$$\frac{\partial v_n}{\partial t} = \frac{\partial v_n}{\partial t} + \frac{v^2 s}{R}$$

1.2.5 Rotation and circulation

Rotation ω about any and is defined as the average of angular velocities of two elements

The rotation velocity is given by

$$\omega = \omega_x i + \omega_y j + \omega_z k$$
$$= \frac{1}{2} \nabla x v$$

Circulaion Γ around a close curve C is defined as the line integral of v.ds alone the curve C, taken positive in anticlockwise direction.

$$\Gamma = \int_c v.ds$$
$$= \int_c (udx + vdy + wdz)$$

the quantity 2ω is known as Vorticity, which is also a vector quantity. If at every point in the flow $\omega_x = \omega_y = \omega_z$ is equal to zero, flow is called irrotational otherwise it is rotational flow.

.6 Velocity Potential and Stream Function.

Velocity Potential Function ϕ is a scalar function of any of x,y,z and t such that it negative derivative with respect to any of x,y,z gives the velocity component in that direction.

Thus:

$$\Phi = \phi(x, y, z, t) \text{ and}$$

$$-\frac{\partial \phi}{\partial x} = u$$

$$-\frac{\partial \phi}{\partial y} = v$$

$$-\frac{\partial \phi}{\partial z} = w.$$

for incompressible fluid , ϕ satisfies the Laplace equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

for 2- dimensional rotation or irrotational flow of incompressible fluids, a scalar function ψ (x,y,t) can be defined such that

$$-\frac{\partial \psi}{\partial y} = u + \frac{\partial \psi}{\partial y} = v$$

for irrotational flow ψ satisfies

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$$

i.e. $\nabla^2 \psi = 2\omega_z$

1.2.7 **Laminar Flow**

For Newtonian fluids, flow can be classified as laminar or turbulent,

depending on Reynolds number $R = \frac{\rho v l}{\mu}$

Where l is the characteristic length, ρ is the density of the fluid, v is the velocity of the flow and μ is the viscosity.

The characteristics of laminas flow are

1. No slip at the boundary that is because viscosity velocity of fluid at y = 0

   If boundary is stationary or if equal to the velocity of the boundary if it is in motion.

2. Because of viscosity there is shear between fluid layer which is given by

$\Gamma = \mu \frac{du}{dy}$ for flow in the u – direction

3. Flow is rotational
4. There is continuous dissipation of energy due to uncoil shear and energy hurt be supplied emotionally to maintain the flow
5. There is no mixing between different fluid layer except by molecular motion which is very small
6. Flow remains laminar as low as Reynolds number is less than the critical value.
7. Energy loss is proportional to first power of velocity and first power of viscosity.

laminar flower occurs in capillary tubes blood veins, in the case of flow pan tiny bodies is lubrication bearings, under ground flow etc. characteristics 1, 3, 4 are true for turbulent flow.

.2.8 Turbulent flows

As mentioned in laminar flow, when the Reynolds number exceed the critical value turbulent flow develops . Turbulent flows occur more often than in nature and in engineering applications than laminar flow. The characteristics of turbulent flow can be summarized as follows:-

1. No slip condition is satisfied at the boundary
2. Turbulent is generated due to instability of flow in region of high shear i.e. near the boundary or at the interface of two moving layers. The former is called wall turbulence while the later is called free turbulence
3. Local velocity component, pressure ,force or any other quality associated with flow

such as local concentration, of abetment show random fluctuation.

4. Vigorousness of turbulent at any given point is measured by turbulence intensity
5. Turbulent flow is characterize by the presence of circulation fluid mass known as eddies.
6. Presence of eddies is the flow maker it capable of efficient transport momentum, mass or energy across the flow.
7. Presence of turbulence fluctuations in velocities causes additional normal and tangential Stresses at any point.

1.2.9. Transition from laminar to turbulence Flow

Laminar flow takes place only at small values of Reynolds number. For pipe flow Reynolds number must be less than 2100; for open channel flow, Reynolds number must be less than 500. The Reynolds number at which flow cease to be laminar is known as critical Reynolds number.

CHAPTER TWO

MATHEMATICAL REVIEW

2.0 INTRODUCTION

In this chapter, we shall proceed to give a mathematical review of vectors, Divergence of a Vector and the curl of a Vector. We shall also give the Divergence and curl of a Vector in spherical coordinates.

A Vector is a quantity which posses both magnitude and direction and is normally denoted by bold lower case letters or a line segment such as $\vec{a}$ or $\overrightarrow{AB}$ where $\vec{a} = a_1 i + a_2 j + a_3 k$ 2.1.0

2.1 GRADIENT, DIVERGENCE AND CURL

Consider the Vector operator ∇ (del) defined by

$$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \quad \text{................2.1.1}$$

Then if Φ(x,y,z) and A(x,y,z) have continuous partial derivatives in a region, we can define the following

2.1.0 Gradient: The Gradient of Φ is defined by

$$\text{Grad } \Phi = \nabla\Phi = (i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z})\Phi$$

$$= i\frac{\partial\Phi}{\partial x} + j\frac{\partial\Phi}{\partial y} + k\frac{\partial\Phi}{\partial z} \quad ....2.1.2$$

An interesting interpretation is that if $\Phi(x, y, z) = c$

is the equation of a surface, then $\nabla\Phi$ is a normal to the surface.

2.1.1 **DIVERGENCE** The divergence of A is defined by

$$\text{divA} = \nabla . A = (i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}).(A_1 i + A_2 j + A_3 k)$$

$$= \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z} \quad ....2.1.3$$

2.1.2 **CURL** The curl of A is defined by

Curl A = ∇ x A

$$= (i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + \frac{\partial}{\partial z}) \text{x} (A_1 i + A_2 j + A_3 k)$$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$= i\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ A_2 & A_3 \end{vmatrix} - j\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ A_1 & A_3 \end{vmatrix} + k\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ A_1 & A_2 \end{vmatrix}$$

$$= \left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right)i - \left(\frac{\partial A_3}{\partial x} - \frac{\partial A_1}{\partial z}\right)j + \left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)k \quad \text{...............2.1.4}$$

2.1.3 Examples: If $\Phi = x^2yz^3$ and $A = xzi - y^2j + 2x^2yk$

find the following:

(a) $\nabla\Phi$

(b) $\nabla.A$

(c) $\nabla \times A$

(d) $\nabla.(\Phi A)$

(e) $\nabla \times (\Phi A)$

Solutions:

$$(a)\qquad \nabla \Phi = \left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z} \right)\Phi$$

$$= i\frac{\partial \Phi}{\partial x} + j\frac{\partial \Phi}{\partial y} + k\frac{\partial \Phi}{\partial z}$$

$$= i\frac{\partial}{\partial x}\left(x^2yz^3\right) + j\frac{\partial}{\partial z}\left(x^2yz^3\right) + k\frac{\partial}{\partial z}\left(x^2yz^3\right)$$

$$= 2xyz^3 i + x^2z^3 j + 3x^2yz^2 k$$

$$(b)\qquad \nabla . A = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k \right)\left(xzi - y^2 j + 2x^2yk\right)$$

$$= \frac{\partial}{\partial x}(xz) + \frac{\partial}{\partial y}\left(-y^2\right) + \frac{\partial}{\partial z}\left(2x^2y\right)$$

$$= z - 2y$$

(c) $\nabla X A = \left(\frac{\partial}{\partial x}i + \frac{\partial}{\partial y}j + \frac{\partial}{\partial z}k\right) X\left(xzi - y^2 j + 2x^2 yk\right)$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xz & -y^2 & 2x^2 y \end{vmatrix}$$

$$= i\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y^2 & 2x^2 y \end{vmatrix} - j\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ xy & 2x^2 y \end{vmatrix} + k\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ xz & -y^2 \end{vmatrix}$$

$$= i\left[\frac{\partial(2x^2 y)}{\partial y} - \frac{\partial(-y)}{\partial z}\right] - j\left[\frac{\partial(2x^2 y)}{\partial x} - \frac{\partial(xy)}{\partial z}\right] + k\left[\frac{\partial(-y^2)}{\partial x} - \frac{\partial(xz)}{\partial y}\right]$$

$$= 2x^2 i - (x - 4xy)j$$

(d) $\nabla.(\Phi A) = \nabla.\left[x^2 yz^3(xzi - y^2 j + 2x^2 yk)\right]$

$$= \nabla.\left(x^3 yzi - x^2 y^3 z^3 j + 2x^4 y^2 z^3 k\right)$$

$$= 3x^2 yz^4 - 3x^2 y^2 z^3 + 6x^4 y^2 z^2$$

(e) $\nabla \mathrm{x}(\phi A) = \nabla \mathrm{x}(x^3yz^4 i - x^2yz^3 j + 2x^4y^2z^3 k)$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3y^3z^3 & -x^2y^3z^3 & 2x^4y^2z^3 \end{vmatrix}$$

$$= i\begin{vmatrix} \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -x^2y^3z^3 & 2x^4y^2x^3 \end{vmatrix} - j\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial z} \\ x^3yz^4 & 2x^4y^2z^3 \end{vmatrix}$$

$$+ k\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ x^3yz^4 & -x^2y^3z^3 \end{vmatrix}$$

$$= i\left[\frac{\partial}{\partial y}(2x^4y^2z^3) - \frac{\partial}{\partial z}(-x^2y^3z^3)\right] - j\left[\frac{\partial}{\partial x}(2x^4y^2z^3) - \frac{\partial}{\partial z}(x^3yz^4)\right]$$

$$+ k\left[\frac{\partial}{\partial x}(-x^2y^3z^3) - \frac{\partial}{\partial y}(x^3yz^4)\right]$$

$$= (4x^4yz^3 + 3x^2y^3z^2)i + (4x^3yz^3 - 8x^3y^2z^3)j - (-2xy^3z^3 - x^3z^4)$$

$$+ (-2xy^3z^3 - x^3z^4)k$$

2.2 ORTHOGONAL CURVILINEAR COORDINATES

The transformation equation

$$x = f(u_1, u_2, u_3),$$
$$y = g(u_1, u_2, u_3) \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.2.1$$
$$z = h(u_1, u_2, u_3)$$

establish a one-to-one correspondence between points u_1, u_2, u_3 in x,y,z and rectangular coordinate system.

In vector notation, the transformation (2.2.1) can be written as

$$r = xi + yj + zk$$
$$= f(u_1,u_2,u_3)i + g(u_1,u_2,u_3)j + h(u_1,u_2,u_3)k \ldots\ldots 2.2.2$$

A point P in space can then be defined not only by rectangular coordinates (x,y,z) but by the coordinates (u_1,u_2,u_3) as well. We call (u_1,u_2,u_3) the curvilinear coordinates of the point.

From (2.2.2)

$$dr = \frac{\partial r}{\partial u_1}du_1 + \frac{\partial r}{\partial u_2}du_2 + \frac{\partial r}{\partial u_3}du_3 \ldots\ldots 2.2.3$$

The vector $\frac{\partial r}{\partial u_1}$ is tangent to the u_1 coordinate curve at P. If e_1 is a unit vector

at p in this direction, we can write $h_1 = \left|\frac{\partial r}{\partial u_1}\right|$

where $h_1 = \left|\frac{\partial r}{\partial u_1}\right|$.

Similarly we can write

$$\frac{\partial r}{\partial u_2} = h_2e_2 \text{ and } \frac{\partial r}{\partial u_3} = h_3e_3$$

where $h_2 = \left|\frac{\partial r}{\partial u_2}\right|$ and $h_3 = \left|\frac{\partial r}{\partial u_3}\right|$ respectively.

Therefore (2.2.3) becomes

$$dr = h_1 du_1 e_1 + h_2 du_2 + h_3 u_3 e_3 \ldots\ldots\ldots\ldots\ldots\ldots 2.2.4$$

where h_1, h_2, h_3 are scale factors.

If e_1, e_2, e_3 are mutually perpendicular at P, then the curvilinear coordinates are called orthogonal.

Therefore the element of length ds is given by

$$\begin{aligned} ds^2 &= dr.dr \\ &= h_1^2 du_1^2 + h_2^2 du_2^2 + h_3^2 du_3^2 \ldots\ldots\ldots\ldots\ldots 2.2.5 \end{aligned}$$

and corresponds to the square of the length of the diagonal in a parallelopiped.

Also the volume of the parallelopiped is given by

$$\begin{aligned} dv &= \left|(h_1 du_1 e_1).(h_2 du_2 e_2).(h_3 du_3 e_3)\right| \\ &= h_1 h_2 h_3 du_1 du_2 du_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.2.6 \\ &= \left|\frac{\partial r}{\partial u_1} . \frac{\partial r}{\partial u_2} X \frac{\partial r}{\partial u_3}\right| du_1 du_2 du_3 \\ &= \left|\frac{\partial (x,y,z)}{\partial (u_1, u_2, u_3)}\right| du_1 du_2 du_3 \ldots\ldots\ldots\ldots\ldots\ldots 2.2.7 \end{aligned}$$

where $\frac{\partial(x,y,z)}{\partial(u_1,u_2,u_3)}$ is the Jacobian of the transformation.

2.2.1 GRADIENT, DIVERGENCE, CURL AND LAPLACIAN IN ORTHOGONAL CURVILINEAR COORDINATES

If Φ is a scalar function and $A = A_1e_1 + A_2e_2 + A_3e_3$ a vector function of orthogonal curvilinear coordinates u_1, u_2, u_3 we have the following results

(i) $$\nabla \Phi = \frac{1}{h_1}\frac{\partial \Phi}{\partial u_1}e_1 + \frac{1}{h_2}\frac{\partial \Phi}{\partial u_2}e_2 + \frac{1}{h_3}\frac{\partial \Phi}{\partial u_3}e_3$$

(ii) $$\nabla . A = \frac{1}{h_1h_2h_3}\left\{\frac{\partial}{\partial u_1}(h_2h_3A_1) + \frac{\partial}{\partial u_2}(h_3h_1A_2) + \frac{\partial}{\partial u_3}(h_1h_2A_3)\right\}$$

(iii) $$\nabla X A = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1e_1 & h_2e_2 & h_3e_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1A_1 & h_2A_2 & h_3A_3 \end{vmatrix}$$

(iv) $$\nabla^2\Phi = \frac{1}{h_1h_2h_3}\left\{\frac{\partial}{\partial u_1}(\frac{h_2h_3}{h_1}\frac{\partial \Phi}{\partial u_1}) + \frac{\partial}{\partial u_2}(\frac{h_3h_1}{h_2}\frac{\partial \Phi}{\partial u_2}) + \frac{\partial}{\partial u_3}(\frac{h_1h_2}{h_3}\frac{\partial \Phi}{\partial u_3})\right\}$$

These reduce to the usual expressions in rectangular coordinates if we replace (u_1, u_2, u_3) by (x, y, z) in which case e_1, e_2, e_3 are replaced by i, j *and* k and $h_1 = h_2 = h_3 = 1$

2.2.2 SPHERICAL COORDINATES

Given the spherical coordinates (r, θ, ϕ) the transformation equation is given by

$$x = rSin\theta Cos\phi$$
$$y = rSin\theta Sin\phi$$
$$z = rCos\theta$$

where $r \geq 0,\ 0 \leq \theta \leq \pi,\ 0 \leq \phi \leq 2\pi$

Scale factor $h_1 = 1, \quad h_2 = r, \quad h_3 = rSin\theta$

Element of arc length ds^2 is given by

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 Sin^2\theta d\phi^2$$

Jacobian $\dfrac{\partial(x, y, z)}{\partial(r, 0, \phi)} = r^2 Sin0$

element of volume $dr = r^2 Sin\theta dr d\theta d\phi$

Laplacian:

$$\nabla^2 U = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial U}{\partial r}) + \frac{1}{r^2 Sin\theta}\frac{\partial}{\partial \theta}(Sin\theta\frac{\partial U}{\partial \theta}) + \frac{1}{r^2 Sin^2\theta}\frac{\partial^2 U}{\partial \phi^2}$$

2.3 LEGENDRE FUNCTION

2.3.1 LEGENDRE DIFFERENTIAL EQUATION

The Legendre's functions arises as solutions of the differential equations

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0 \ldots\ldots\ldots\ldots\ldots\ldots 2.3.1$$

which is called the Legendre differential equation.

The general solution of (2.3.1) in the case when $n=0, 1, 2, 3, \ldots\ldots$ is given by;

$$y = c_1 P_n(x) + c_2 Q_2(x)$$

where $P_n(x)$ are polynomials called Legendre polynomials and $Q_n(x)$ are called Legendre functions of the second kind which are unbounded at $x = \pm 1$

2.3.2 LEGENDRE POLYNOMIALS

The Legendre polynomials are defined by

$$P_n = \frac{(2n-1)(2n-3)\ldots\ldots 1}{n!}\left\{x^n - \frac{n(n-1)x^{n-2}}{2(n-1)} + \frac{n(n-1)(n-2)(n-3)x^{n-4}}{2.4.(2n-1)(2n-3)\ldots\ldots} - \ldots\ldots\right\}\ldots\ldots\ldots\ldots 2.3.2$$

where P_n is a polynomial of degree n

The Legendre polynomial can also be expressed as

$$P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2 - 1)^n \qquad 2.3.3$$

2.3.3 ASSOCIATED LEGENDRE FUNCTIONS

The differential equation

$$(1 - x^2)y'' - 2xy' + \left\{ n(n+1) - \frac{m^2}{1 - x^2} \right\} y = 0 \qquad 2.3.4$$

Is called Legendre's associated differential equation. If $m=o$, this reduce to Legendre's equation. Solution to (2.3.4) are called associated Legendre's functions.

2.4 Problem Formulation

In this section we shall proceed to given a brief formulation of the problem of the flow of fluid past a sphere.

Let **a** be the radius of the sphere and **U** be the speed of the uniform streaming motion at in infinity assumed to be parallel to the positive x-axis of a system of coordinates based on the center of the sphere. The velocity field **UV** and the space coordinates can be non-dimensionalized with the aid of **U** and **a** respectively and the equations of the motion will then be

$$\nabla^2 U - \nabla p = R(V.\nabla)V, \quad \nabla V = 0 \text{..........} 2.4.1$$

Where $\rho v U p / a$ is the pressure, $R = Ua/v$ is the Reynolds number, ρ is the density and v is the kinematic viscosity.

Neglecting the inertia terms in the absence of extraneous forces, the equation is in the form

$$\mu \nabla^2 U = \frac{\partial p}{\partial x} \text{........................} 2.4.2$$

CHAPTER THREE
SOLUTION OF PROBLEM

3.0 METHOD OF SOLUTION

Considering the equation (2.4.2)

$$\mu\nabla^2 U = \frac{\partial P}{\partial x} \quad \text{.................................3.0.1}$$

Which can be rewritten as

$$\mu\nabla^2 U = \frac{\partial p}{\partial u}.\frac{\partial u}{\partial x} \quad \text{..........................3.0.2}$$

Taking $\varepsilon = \dfrac{\partial p}{\partial u}$, we have

$$\mu\nabla^2 U = \varepsilon\frac{\partial u}{\partial x} \quad \text{.......................3.0.3}$$

Where ε is the perturbation parameter.

we shall now proceed to expand U in terms of ε, we have

$$U = \varepsilon^0 U_0 + \varepsilon^1 U_1 + \varepsilon^2 U_2 + \varepsilon^3 U_3 + \ldots \qquad 3.0.4$$

Putting (3.0.4) into (3.0.3) we have

$$\varepsilon^0\mu\nabla^2U_0 + \varepsilon^1\mu\nabla^2U_1 + \varepsilon^2\mu\nabla^2U_2 + \ldots$$

$$= \varepsilon^1\frac{\partial u_0}{\partial x} + \varepsilon^2\frac{\partial u_1}{\partial x} + \varepsilon^3\frac{\partial u_2}{\partial x} + \ldots \qquad 3.0.5$$

Rearranging (3.0.5) in terms of order ε, we have

$$\varepsilon^0;\mu\nabla^2U_0 = 0$$

$$\varepsilon^1;\mu\nabla^2U_1 = \frac{\partial U_0}{\partial x}$$

$$\varepsilon^2;\mu\nabla^2U_2 = \frac{\partial U_1}{\partial x}$$

and so on for higher order of ε

3.1. SOLUTION OF ORDER ε^o EQUATION

We shall proceed to solve the order ε^o equation given by

$$\mu\nabla^2U_0 = 0 \Rightarrow \nabla^2U_0 = 0 \ldots\ldots\ldots\ldots\ldots\ldots 3.1.0$$

Writing this order ε^o equation in spherical coordinates, we have;

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U_0}{\partial r}\right) + \frac{1}{r^2 Sin\theta}\frac{\partial}{\partial\theta}\left(Sin\theta\frac{\partial U_0}{\partial\theta}\right) + \frac{1}{r^2 Sin^2\theta}\frac{\partial^2 U_0}{\partial\phi^2}$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.1.1$$

Our method of procedure will be as follows,

We try a solution of the form

$$U(r,\theta,\phi) = R(\mathrm{r})\Theta(\theta)\Phi(\phi)\text{.....................}3.1.2$$

Substituting this into (3.1.1) and dividing through by $U = R\Theta\,\Phi$ and multiplying by r^2, we have

$$\frac{1}{R}\frac{\partial}{\partial r}(r^2\frac{\partial R}{\partial r}) + \frac{1}{\Theta Sin\theta}\frac{\partial}{\partial\theta}(Sin\theta\frac{\partial\Theta}{\partial\theta}) + \frac{1}{\Phi Sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2}\text{...............}3.1.3$$

The first term depends only on r and the second (3.1.1) and third term (taken together) only on θ and ϕ (3.1.3) can be written as

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) = \lambda\text{.........................}3.1.4$$

and

$$\frac{1}{\Theta Sin\theta}\frac{d}{d\theta}(Sin\theta\frac{d\Theta}{d\theta})+\frac{1}{\Phi Sin^2\theta}\frac{d^2\Phi}{d\phi^2}=-\lambda \qquad 3.1.5$$

from (3.1.4)

$$\Rightarrow \quad \frac{1}{R}\left[2r\frac{dR}{dr}+r^2\frac{d^2R}{dr^2}\right]=\lambda$$

Therefore,

$$2r\frac{dR}{dr}+r^2\frac{d^2R}{dr^2}=\lambda R$$

$$\Rightarrow r^2\frac{d^2R}{dr^2}+2r\frac{dR}{dr}-\lambda R=0 \qquad 3.1.6$$

(3.1.6) can be reduced by substitution

$$r=\exp t(r=e^t)$$

and writing $R(r)=S(t)$ to

$$\frac{d^2S}{dt^2}+\frac{dS}{dt}-\lambda S=0$$

This has the solution

$$S(t)=Ar^{\lambda_1 t}+Br^{\lambda_2 t}$$

Therefore the solution to the radial equation is

$$R(r)=Ar^{\lambda_1}+Br^{\lambda_2}$$

Where $\lambda_1 + \lambda_2 = -1$, and $\lambda_1\lambda_2 = -\lambda$ and we can take λ_1 and λ_2 as given by l and $-(l+1)$

λ then has the form $l(l+1)$

Hence we have the separated variable solution, which will have the form

$$U(r,\theta,\phi) = (Ar^l + Br^{-(l+1)})\Theta(\theta)\Phi(\phi) \qquad \text{.....................3.1.7}$$

Where Θ and Φ must satisfy (3.1.5) with $\lambda = l(l+1)$

Multiplying (3.1.5) through by $Sin^2\theta$ and substituting for λ,
It takes a separated form.

$$\left[\frac{Sin\theta}{\Theta}\frac{d}{d\theta}\left(Sin\theta\frac{d\Theta}{d\theta}\right) + l(l+1)Sin^2\theta\right] + \frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} = 0 \qquad \text{...........3.1.8}$$

Taking the separation constant as m^2, the equation in the azimuthal angle ϕ has the same solution as cylindrical polars.

$$\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} = -m$$

$$\frac{d^2\Phi}{d\phi^2} = -m^2\Phi$$

$$\Phi(\phi) = CCosm\phi + DSinm\phi$$

For $m = 0$, we have $\Phi(\phi) = C$

Having settled the form of $\Phi(\phi)$, we are left only with the equation satisfied by $\Theta(\theta)$, which is

$$\frac{Sin\theta}{\Theta}\frac{d}{d\theta}\left[Sin\theta\frac{d\Theta}{d\theta}\right]+l(l+1)Sin^2\theta=m^2 \text{........}3.1.9$$

A change of independent variable from θ to z = Cosθ, will reduce this to a form, from which solutions are known.

Putting $z = Cos\theta$, $\frac{dz}{d\theta} = -Sin\theta$, $\frac{d}{d\theta} = -(1-z^2)^{1/2}\frac{d}{dz}$

The equation for $M(z) \equiv \Theta(\theta)$ reads;

$$\frac{d}{dz}\left[\left(1-z^2\right)\frac{dM}{dz}\right]+\left[l(l+1)-\frac{m^2}{1-z^2}\right]M = 0 \text{.............}3.1.10$$

This equation is the associated Legendre equation.

From (3.1.10),

$$\Rightarrow (1-z^2)\frac{d^2M}{dz^2}-2z\frac{dM}{dz}+\left[l(l+1)-\frac{m^2}{1-z^2}\right]M = 0$$

For $m = 0$, we have

$$(1-z^2)\frac{d^2M}{dz^2}-2z\frac{dM}{dz}+l(l+1)M = 0 \text{......................}3.1.11$$

(3.1.11) is the Legendre's equation which we shall proceed to solve using series solution method

method.

Assuming a solution of the form

$$M(z) = \sum_{n=0}^{\infty} a_n z^n$$

then

$$\frac{dM}{dz} = \sum_{n=0}^{\infty} n a_n z^{n-1}$$

$$\frac{d^2 M}{dz^2} = \sum_{n=0}^{\infty} n(n-1) a_n z^{n-2}$$

Substituting into (3.1.11), we have

$$\sum_{n=0}^{\infty} \left[n(n-1) a_n Z^{n-2} - n(n-1) a_n Z^n - 2n a_n Z^n + l(l+1) a_n Z^n \right] = 0$$

which on collecting terms gives

$$\sum_{n=0}^{\infty} \left[(n+2)(n+1) a_{n+2} - \left[n(n+1) - l(l+1) \right] a_n \right] Z^n = 0$$

The recurrence relation is therefore

$$a_{n+2} = \frac{\left[n(n+1) - l(l+1)\right]a_n}{(n+1)(n+2)}$$

For n=0,1,2,3…

When n = 0, we have

$$a_2 = \frac{-l(l+1)}{2}a_0$$

When n = 1, we have

$$a_3 = \frac{\left[2 - l(l+1)\right]}{2(3)}a_0$$

Choosing $a_0 = 1$ and $a_1 = 0$, we have

$$M_1(z) = 1 - \frac{l(l+1)}{2!}z^2 + \frac{(l-2)l(l+1)(l+3)}{4!}z^4 \ldots\ldots\ldots\ldots\ldots 3.1.12$$

choosing $a_0 = 0$ and $a_1 = 1$, we have

$$M_2(z) = z - \frac{(l-1)(l+2)}{3!}z^3 + \frac{(l-3)(l-1)(l+2)(l+4)}{5!}z^5 \ldots\ldots\ldots\ldots\ldots 3.1.13$$

Since (3.1.12) contains only even powers of z and (3.1.13) contains only odd powers, these two solutions cannot be proportional to one another and are therefore linearly independent.

Hence

$$M(z) = EM_1(z) + FM_2(z)$$

is the general solution to (3.1.10).

But for general M, $M_1(z)$ and $M_2(z)$ are the associated Legendre function, which can be written as $P_l^m(z)$ and $Q_l^m(z)$.

Therefore, $M_l^m(z) = EP_l^m(z) + FQ_l^m(z)$

Now that the solution of each of the three ordinary differential equation R, θ and Φ have been obtained, we may assemble a complete separated variable solution in spherical polars. It is

$$U(r,\theta,\phi) = \left(Ar^l + Br^{-(l+1)}\right)(CCosm\phi + DSinm\phi)\left[EP_l^m(Cos\theta) + FQ_l^m(Cos\theta)\right].$$

........................(3.1.14)

Since the flow is symmetric about the sphere, we have m = 0, therefore (3.1.14) becomes

$$U(r,\theta,\phi) = Ar^l + Br^{-(l+1)}(C)\left[EP_l(Cos\theta) + FQ_l(Cos\theta)\right] \text{......(3.1.15)}$$

Since Q_l denotes an infinite series, which can exist mathematically but is invariably unreasonable on physical ground, because the solution is expected to be finite, it cannot contain Q_l

Therefore 3.1.15 becomes

$$U(r,\theta) = \sum_{l=0} \left[A_l r^l + B_l r^{-(l+1)} \left[P_l(Cos\theta) \right] \right] \text{..........3.1.16}$$

(3.1.16) is the general form of the solution.

Using the boundary condition at $r = \infty$, $U = 0$

$$A_l = 0$$

We have

$$\Rightarrow \quad U(r,\theta) = \sum_{l=0}^{\infty} B_l . r^{-(l+1)} . \left[P_l .(Cos\theta) \right] \text{...............3.1.17}$$

Using the boundary condition $r = a$, $U = U_0$ we have

$$U(a,\theta) = \sum_{l=0}^{\infty} B_l . a^{-(l+1)} . \left[P_l .(Cos\theta) \right] \text{........................3.1.18}$$

$$U(a,\theta) = U_0$$

We shall now obtain the value of B_l. Using the mutual orthogonality of Legendre Polynomials

We have

$$B_l a^{-(l+1)} = \frac{2l+1}{2} U_0 \int_0^1 P_l(z) dz \quad \text{where} \quad z = Cos\theta$$

Therefore

$$B_l \Rightarrow a^{l+1} = \frac{2l+1}{2} U_0 \int_0^1 P_l(z) dz \quad \text{from 3.1.17}$$

$\Rightarrow$ the solution required will become

$$U(r,\theta) = \sum_{l=0}^{\infty} a^{l+1} \frac{2l+1}{2} U_0 \int_0^1 P_1(z)dz.r^{-(l+1)}\left[P_1(Cos\theta)\right]$$

$$U(r,\theta) = \frac{U_0 a}{2r}\left[1 + \frac{3a}{2r} P_1(Cos\theta) - \frac{7a^3}{8r^3} P_3(Cos\theta) + \dots\right] \dots\dots\dots(3.1.19)$$

CHAPTER FOUR
NUMERICAL RESULTS

Considering the solution of the order ε^0 equation then

$$U(r,\theta)=\sum_{l=0}^{\infty} a^{(l+1)}\frac{2l+1}{2}U_0\int_0^1 P_l(z)dz.r^{-(l+1)}\left[P_l(Cos\theta)\right]$$

$$U(r,\theta)=\frac{U_0 a}{2r}\left[1+\frac{3a}{2r}P_1(Cos\theta)-\frac{7a^3}{8r^3}P_3(Cos\theta)+\ldots.\right].4.0.1$$

We proceed to analyse the above solution using **MATHCAD**. The tables and graphs in the subsequent pages shows the different values of $U(r,\theta)$ against θ for $a/r=\mu$ taken as a constant.

The equation 4.0.1 is written in MATHCAD as follows:

b:=0 c:=10 d:=360

b:=b.deg c:=c.deg D:=d.deg

U_0:= 0.1 μ_1:= 0.1,0.11..0.2 θ:= b,c..d B:= 0,10..360

$$U_1(\theta,\mu_1):=\frac{U_0}{2}.\mu_1.\left[\begin{array}{l}1+\frac{3}{2}.\mu_1-\frac{3}{2}.\mu_1.(Sin(\frac{1}{2}.\theta))^2-\frac{7}{8}.\mu_1{}^3\ldots\\ +84.\mu_1{}^3.(Sin(\frac{1}{2}.\theta))^2-\frac{210}{8}.\mu_1{}^3.(Sin(\frac{1}{2}.\theta))^4\end{array}\right]$$

Table 4.1 Flow distribution for values of a/r =0.1 to 0.14

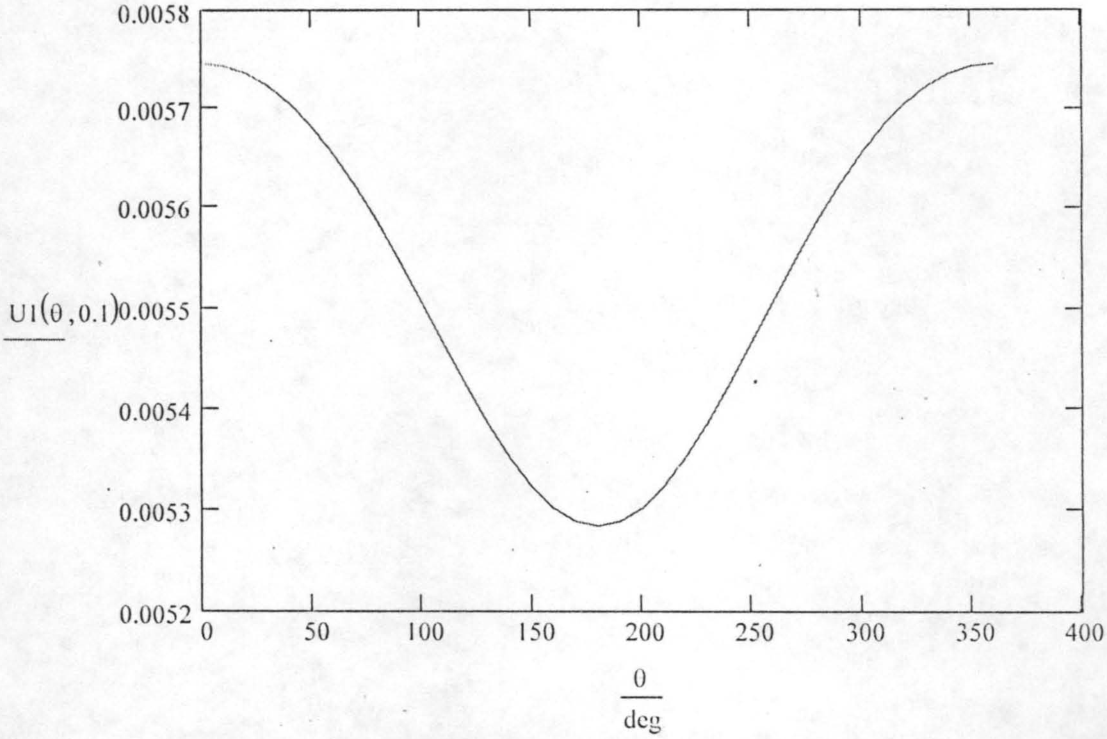
θ = (deg)	U1(θ,0.1) =	U1(θ,0.11) =	U1(θ,0.12) =	U1(θ,0.13) =	U1(θ,0.14) =
0	$5.746\cdot10^{-3}$	$6.401\cdot10^{-3}$	$7.071\cdot10^{-3}$	$7.755\cdot10^{-3}$	$8.453\cdot10^{-3}$
10	$5.743\cdot10^{-3}$	$6.399\cdot10^{-3}$	$7.069\cdot10^{-3}$	$7.754\cdot10^{-3}$	$8.454\cdot10^{-3}$
20	$5.736\cdot10^{-3}$	$6.392\cdot10^{-3}$	$7.064\cdot10^{-3}$	$7.753\cdot10^{-3}$	$8.457\cdot10^{-3}$
30	$5.723\cdot10^{-3}$	$6.381\cdot10^{-3}$	$7.056\cdot10^{-3}$	$7.749\cdot10^{-3}$	$8.461\cdot10^{-3}$
40	$5.705\cdot10^{-3}$	$6.364\cdot10^{-3}$	$7.043\cdot10^{-3}$	$7.742\cdot10^{-3}$	$8.463\cdot10^{-3}$
50	$5.682\cdot10^{-3}$	$6.343\cdot10^{-3}$	$7.025\cdot10^{-3}$	$7.731\cdot10^{-3}$	$8.463\cdot10^{-3}$
60	$5.655\cdot10^{-3}$	$6.316\cdot10^{-3}$	$7.002\cdot10^{-3}$	$7.715\cdot10^{-3}$	$8.458\cdot10^{-3}$
70	$5.623\cdot10^{-3}$	$6.284\cdot10^{-3}$	$6.973\cdot10^{-3}$	$7.692\cdot10^{-3}$	$8.446\cdot10^{-3}$
80	$5.587\cdot10^{-3}$	$6.247\cdot10^{-3}$	$6.938\cdot10^{-3}$	$7.663\cdot10^{-3}$	$8.426\cdot10^{-3}$
90	$5.548\cdot10^{-3}$	$6.207\cdot10^{-3}$	$6.898\cdot10^{-3}$	$7.627\cdot10^{-3}$	$8.399\cdot10^{-3}$
100	$5.507\cdot10^{-3}$	$6.163\cdot10^{-3}$	$6.855\cdot10^{-3}$	$7.586\cdot10^{-3}$	$8.364\cdot10^{-3}$
110	$5.465\cdot10^{-3}$	$6.118\cdot10^{-3}$	$6.808\cdot10^{-3}$	$7.541\cdot10^{-3}$	$8.322\cdot10^{-3}$
120	$5.424\cdot10^{-3}$	$6.074\cdot10^{-3}$	$6.761\cdot10^{-3}$	$7.493\cdot10^{-3}$	$8.277\cdot10^{-3}$
130	$5.386\cdot10^{-3}$	$6.031\cdot10^{-3}$	$6.716\cdot10^{-3}$	$7.446\cdot10^{-3}$	$8.231\cdot10^{-3}$
140	$5.352\cdot10^{-3}$	$5.993\cdot10^{-3}$	$6.674\cdot10^{-3}$	$7.403\cdot10^{-3}$	$8.187\cdot10^{-3}$
150	$5.323\cdot10^{-3}$	$5.961\cdot10^{-3}$	$6.639\cdot10^{-3}$	$7.365\cdot10^{-3}$	$8.148\cdot10^{-3}$
160	$5.302\cdot10^{-3}$	$5.937\cdot10^{-3}$	$6.612\cdot10^{-3}$	$7.337\cdot10^{-3}$	$8.118\cdot10^{-3}$
170	$5.289\cdot10^{-3}$	$5.921\cdot10^{-3}$	$6.595\cdot10^{-3}$	$7.318\cdot10^{-3}$	$8.099\cdot10^{-3}$
180	$5.284\cdot10^{-3}$	$5.916\cdot10^{-3}$	$6.59\cdot10^{-3}$	$7.312\cdot10^{-3}$	$8.092\cdot10^{-3}$
190	$5.289\cdot10^{-3}$	$5.921\cdot10^{-3}$	$6.595\cdot10^{-3}$	$7.318\cdot10^{-3}$	$8.099\cdot10^{-3}$
200	$5.302\cdot10^{-3}$	$5.937\cdot10^{-3}$	$6.612\cdot10^{-3}$	$7.337\cdot10^{-3}$	$8.118\cdot10^{-3}$
210	$5.323\cdot10^{-3}$	$5.961\cdot10^{-3}$	$6.639\cdot10^{-3}$	$7.365\cdot10^{-3}$	$8.148\cdot10^{-3}$
220	$5.352\cdot10^{-3}$	$5.993\cdot10^{-3}$	$6.674\cdot10^{-3}$	$7.403\cdot10^{-3}$	$8.187\cdot10^{-3}$
230	$5.386\cdot10^{-3}$	$6.031\cdot10^{-3}$	$6.716\cdot10^{-3}$	$7.446\cdot10^{-3}$	$8.231\cdot10^{-3}$
240	$5.424\cdot10^{-3}$	$6.074\cdot10^{-3}$	$6.761\cdot10^{-3}$	$7.493\cdot10^{-3}$	$8.277\cdot10^{-3}$
250	$5.465\cdot10^{-3}$	$6.118\cdot10^{-3}$	$6.808\cdot10^{-3}$	$7.541\cdot10^{-3}$	$8.322\cdot10^{-3}$
260	$5.507\cdot10^{-3}$	$6.163\cdot10^{-3}$	$6.855\cdot10^{-3}$	$7.586\cdot10^{-3}$	$8.364\cdot10^{-3}$
270	$5.548\cdot10^{-3}$	$6.207\cdot10^{-3}$	$6.898\cdot10^{-3}$	$7.627\cdot10^{-3}$	$8.399\cdot10^{-3}$
280	$5.587\cdot10^{-3}$	$6.247\cdot10^{-3}$	$6.938\cdot10^{-3}$	$7.663\cdot10^{-3}$	$8.426\cdot10^{-3}$
290	$5.623\cdot10^{-3}$	$6.284\cdot10^{-3}$	$6.973\cdot10^{-3}$	$7.692\cdot10^{-3}$	$8.446\cdot10^{-3}$
300	$5.655\cdot10^{-3}$	$6.316\cdot10^{-3}$	$7.002\cdot10^{-3}$	$7.715\cdot10^{-3}$	$8.458\cdot10^{-3}$
310	$5.682\cdot10^{-3}$	$6.343\cdot10^{-3}$	$7.025\cdot10^{-3}$	$7.731\cdot10^{-3}$	$8.463\cdot10^{-3}$
320	$5.705\cdot10^{-3}$	$6.364\cdot10^{-3}$	$7.043\cdot10^{-3}$	$7.742\cdot10^{-3}$	$8.463\cdot10^{-3}$
330	$5.723\cdot10^{-3}$	$6.381\cdot10^{-3}$	$7.056\cdot10^{-3}$	$7.749\cdot10^{-3}$	$8.461\cdot10^{-3}$
340	$5.736\cdot10^{-3}$	$6.392\cdot10^{-3}$	$7.064\cdot10^{-3}$	$7.753\cdot10^{-3}$	$8.457\cdot10^{-3}$
350	$5.743\cdot10^{-3}$	$6.399\cdot10^{-3}$	$7.069\cdot10^{-3}$	$7.754\cdot10^{-3}$	$8.454\cdot10^{-3}$
360	$5.746\cdot10^{-3}$	$6.401\cdot10^{-3}$	$7.071\cdot10^{-3}$	$7.755\cdot10^{-3}$	$8.453\cdot10^{-3}$

Table 4.2 **Flow distribution for values of a/r =0.15 to 0.2**

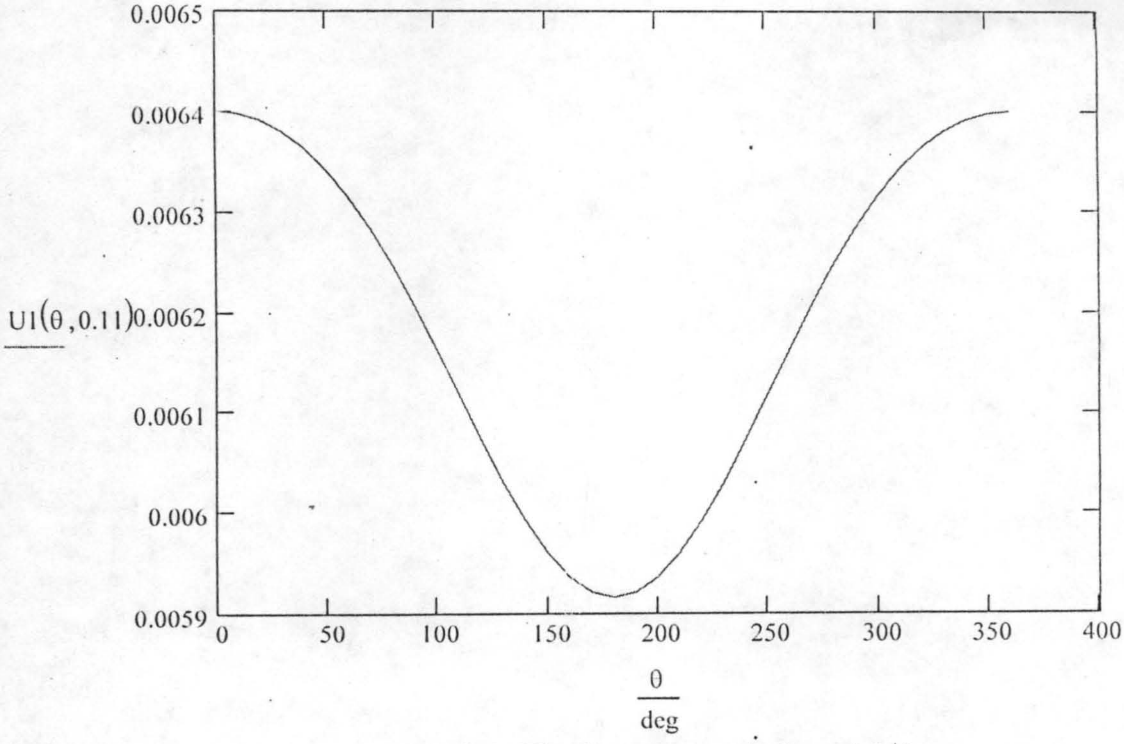
θ =		U1(θ,0.15) =	U1(θ,0.16) =	U1(θ,0.17) =	U1(θ,0.18)	U1(θ,0.1	U1(θ,0.2)
0	deg	$9.165 \cdot 10^{-3}$	$9.891 \cdot 10^{-3}$	0.011	0.011	0.012	0.013
10		$9.169 \cdot 10^{-3}$	$9.898 \cdot 10^{-3}$	0.011	0.011	0.012	0.013
20		$9.178 \cdot 10^{-3}$	$9.916 \cdot 10^{-3}$	0.011	0.011	0.012	0.013
30		$9.192 \cdot 10^{-3}$	$9.943 \cdot 10^{-3}$	0.011	0.012	0.012	0.013
40		$9.208 \cdot 10^{-3}$	$9.977 \cdot 10^{-3}$	0.011	0.012	0.012	0.013
50		$9.223 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
60		$9.234 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
70		$9.238 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
80		$9.233 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
90		$9.219 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
100		$9.194 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
110		$9.161 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
120		$9.121 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.015
130		$9.077 \cdot 10^{-3}$	$9.995 \cdot 10^{-3}$	0.011	0.012	0.013	0.015
140		$9.035 \cdot 10^{-3}$	$9.956 \cdot 10^{-3}$	0.011	0.012	0.013	0.015
150		$8.996 \cdot 10^{-3}$	$9.919 \cdot 10^{-3}$	0.011	0.012	0.013	0.015
160		$8.966 \cdot 10^{-3}$	$9.89 \cdot 10^{-3}$	0.011	0.012	0.013	0.015
170		$8.946 \cdot 10^{-3}$	$9.87 \cdot 10^{-3}$	0.011	0.012	0.013	0.015
180		$8.94 \cdot 10^{-3}$	$9.864 \cdot 10^{-3}$	0.011	0.012	0.013	0.015
190		$8.946 \cdot 10^{-3}$	$9.87 \cdot 10^{-3}$	0.011	0.012	0.013	0.015
200		$8.966 \cdot 10^{-3}$	$9.89 \cdot 10^{-3}$	0.011	0.012	0.013	0.015
210		$8.996 \cdot 10^{-3}$	$9.919 \cdot 10^{-3}$	0.011	0.012	0.013	0.015
220		$9.035 \cdot 10^{-3}$	$9.956 \cdot 10^{-3}$	0.011	0.012	0.013	0.015
230		$9.077 \cdot 10^{-3}$	$9.995 \cdot 10^{-3}$	0.011	0.012	0.013	0.015
240		$9.121 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.015
250		$9.161 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
260		$9.194 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
270		$9.219 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
280		$9.233 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
290		$9.238 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
300		$9.234 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
310		$9.223 \cdot 10^{-3}$	0.01	0.011	0.012	0.013	0.014
320		$9.208 \cdot 10^{-3}$	$9.977 \cdot 10^{-3}$	0.011	0.012	0.012	0.013
330		$9.192 \cdot 10^{-3}$	$9.943 \cdot 10^{-3}$	0.011	0.012	0.012	0.013
340		$9.178 \cdot 10^{-3}$	$9.916 \cdot 10^{-3}$	0.011	0.011	0.012	0.013
350		$9.169 \cdot 10^{-3}$	$9.898 \cdot 10^{-3}$	0.011	0.011	0.012	0.013
360		$9.165 \cdot 10^{-3}$	$9.891 \cdot 10^{-3}$	0.011	0.011	0.012	0.013

Table 4.3 Flow distribution for values of a/r =0.3 to 1.0

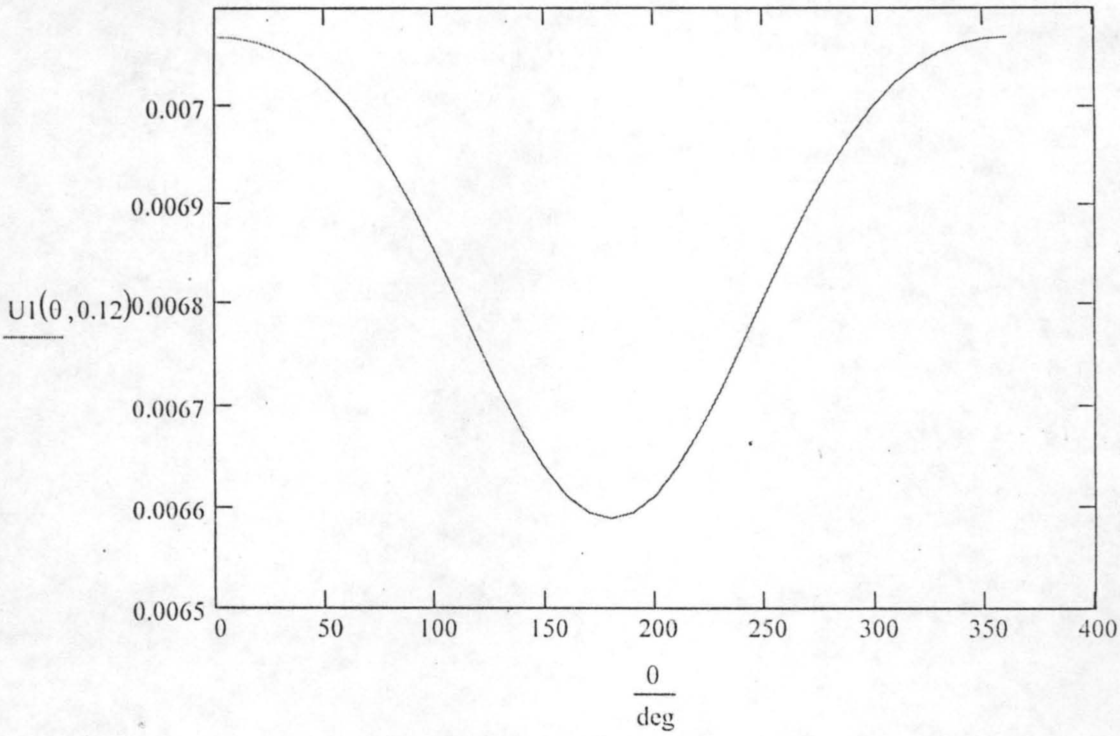
θ = (deg)	U1(θ,0.3) =	U1(θ,0.5) =	U1(θ,0.8) =	U1(θ,1) =	U1(θ,0.15	U1(θ,0.2)
0	0.021	0.041	0.07	0.081	0.012	0.013
10	0.022	0.043	0.083	0.113	0.012	0.013
20	0.022	0.048	0.12	0.204	0.012	0.013
30	0.023	0.057	0.18	0.352	0.012	0.013
40	0.024	0.068	0.258	0.546	0.012	0.013
50	0.026	0.082	0.352	0.776	0.013	0.014
60	0.028	0.097	0.455	1.03	0.013	0.014
70	0.029	0.112	0.562	1.296	0.013	0.014
80	0.031	0.128	0.669	1.562	0.013	0.014
90	0.032	0.142	0.772	1.816	0.013	0.014
100	0.034	0.156	0.866	2.05	0.013	0.014
110	0.035	0.168	0.95	2.258	0.013	0.014
120	0.036	0.178	1.022	2.437	0.013	0.015
130	0.037	0.186	1.081	2.584	0.013	0.015
140	0.037	0.192	1.128	2.7	0.013	0.015
150	0.038	0.197	1.162	2.787	0.013	0.015
160	0.038	0.2	1.186	2.847	0.013	0.015
170	0.038	0.202	1.2	2.882	0.013	0.015
180	0.038	0.203	1.205	2.894	0.013	0.015
190	0.038	0.202	1.2	2.882	0.013	0.015
200	0.038	0.2	1.186	2.847	0.013	0.015
210	0.038	0.197	1.162	2.787	0.013	0.015
220	0.037	0.192	1.128	2.7	0.013	0.015
230	0.037	0.186	1.081	2.584	0.013	0.015
240	0.036	0.178	1.022	2.437	0.013	0.015
250	0.035	0.168	0.95	2.258	0.013	0.014
260	0.034	0.156	0.866	2.05	0.013	0.014
270	0.032	0.142	0.772	1.816	0.013	0.014
280	0.031	0.128	0.669	1.562	0.013	0.014
290	0.029	0.112	0.562	1.296	0.013	0.014
300	0.028	0.097	0.455	1.03	0.013	0.014
310	0.026	0.082	0.352	0.776	0.013	0.014
320	0.024	0.068	0.258	0.546	0.012	0.013
330	0.023	0.057	0.18	0.352	0.012	0.013
340	0.022	0.048	0.12	0.204	0.012	0.013
350	0.022	0.043	0.083	0.113	0.012	0.013
360	0.021	0.041	0.07	0.081	0.012	0.013



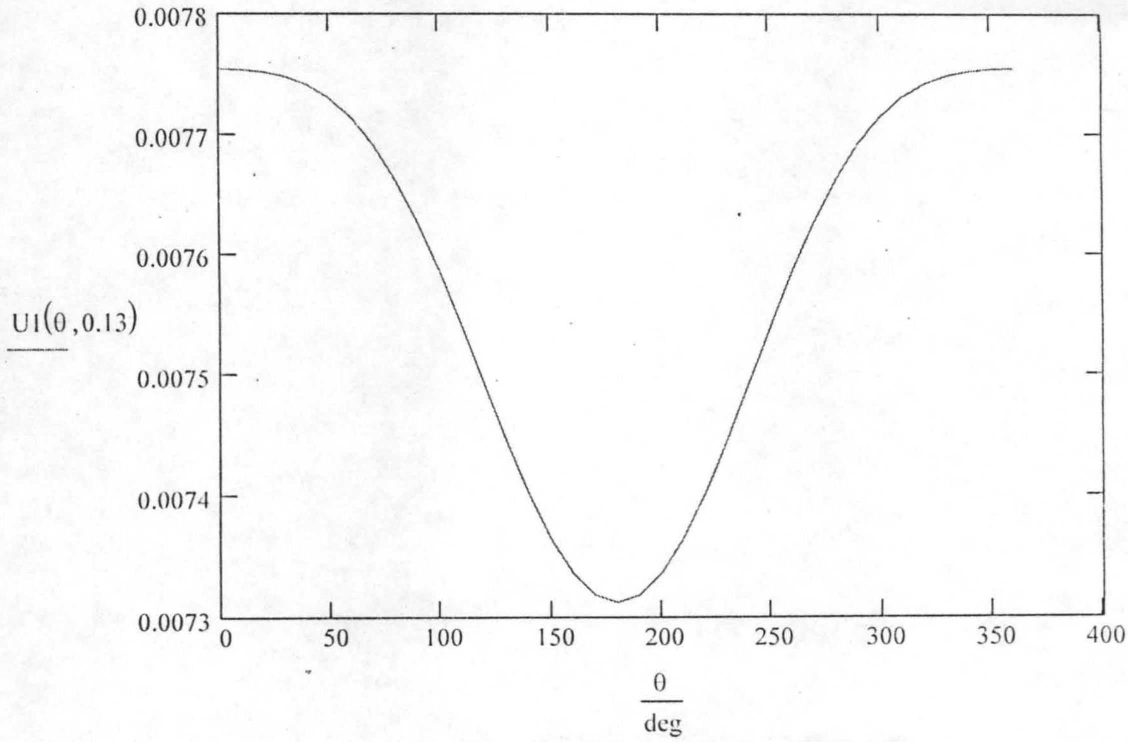
Graph 1: The flow pattern (mhu = 0.1)



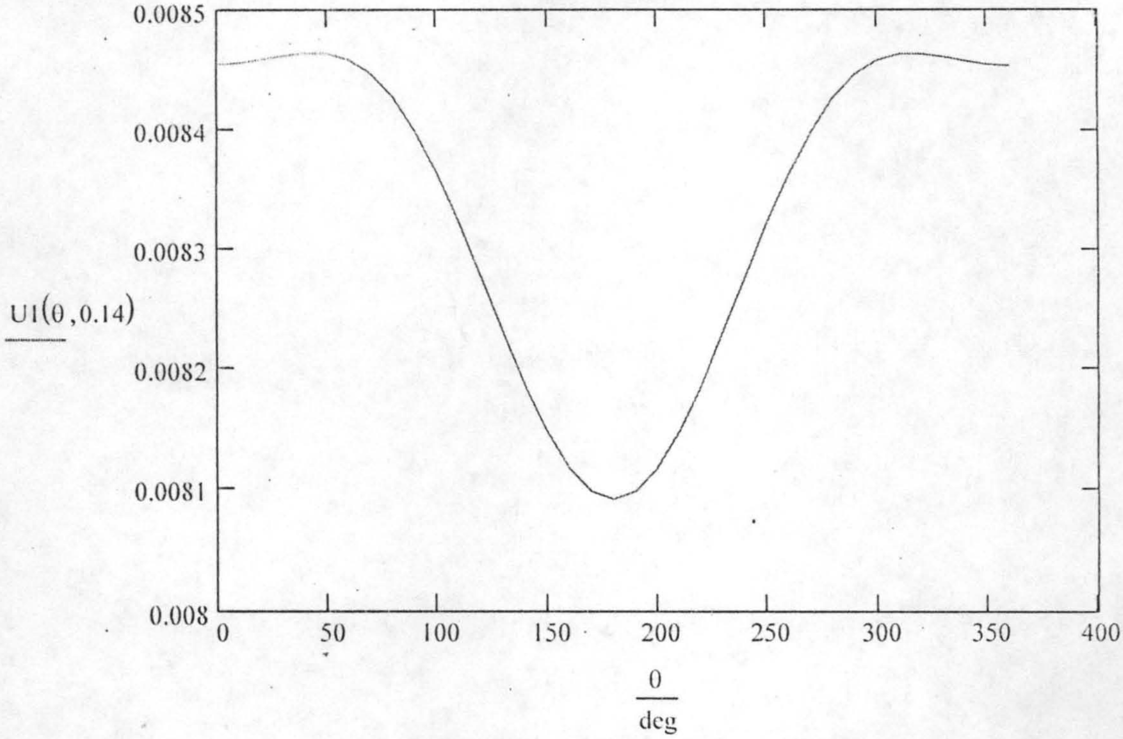
Graph 2: The flow pattern (mhu = 0.11)



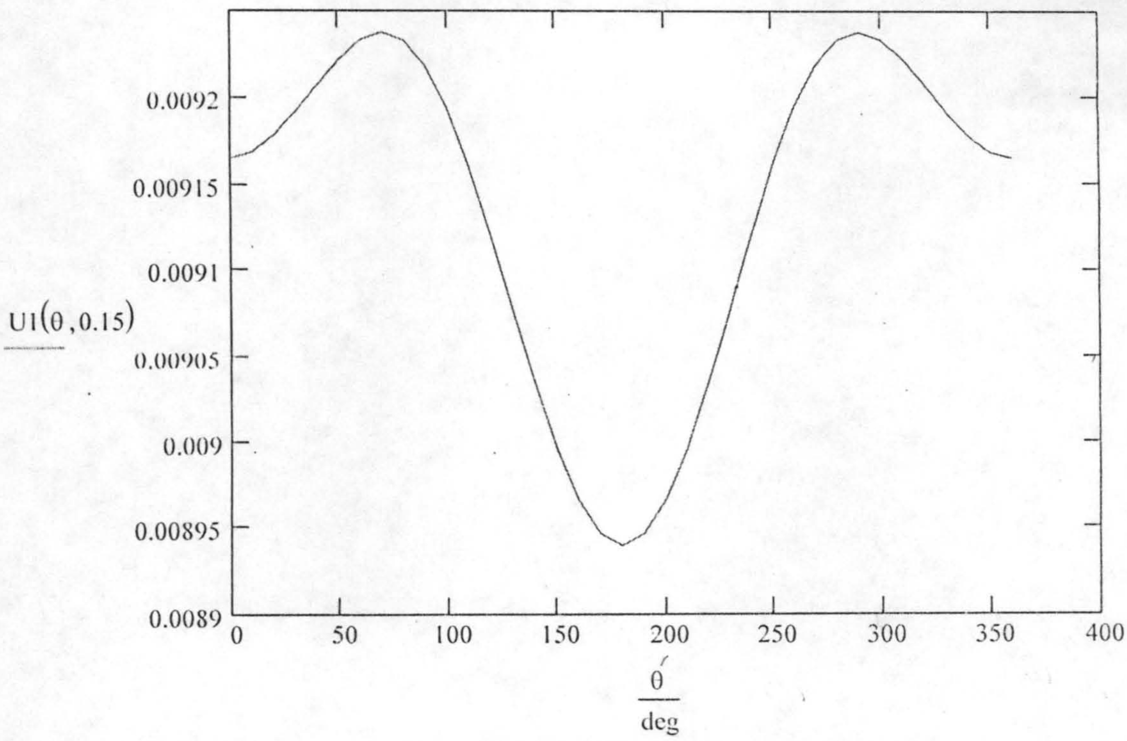
Graph 3: The flow pattern (mhu = 0.12)



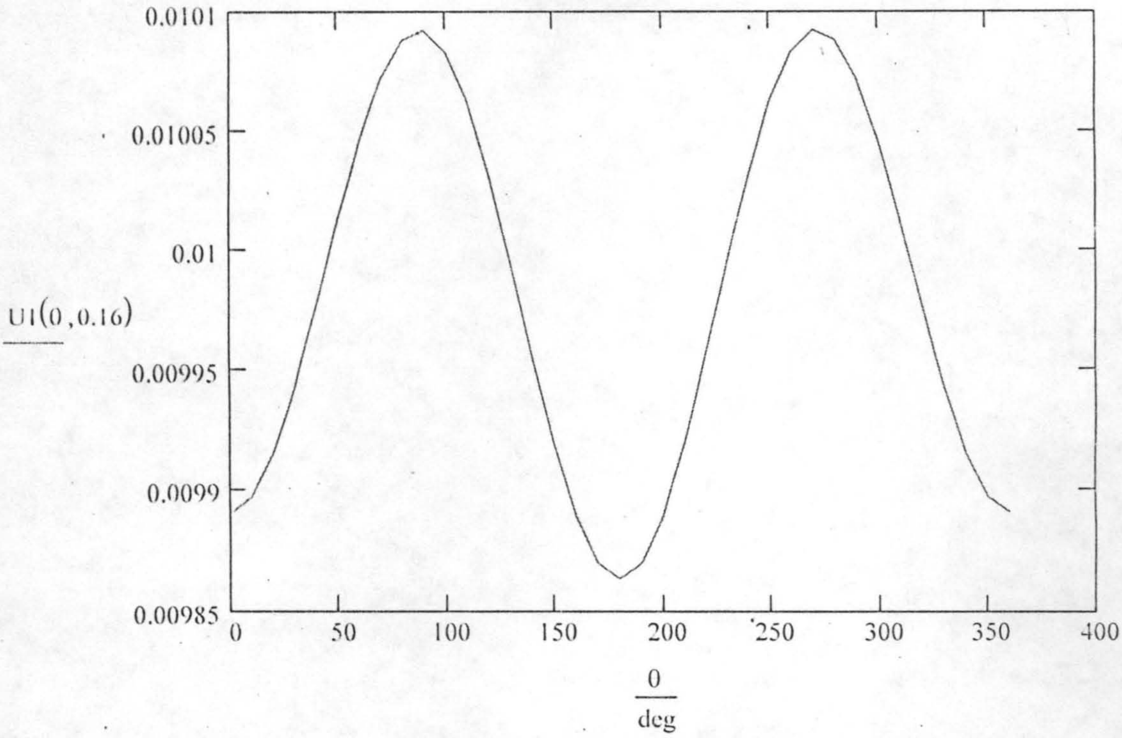
Graph 4: The flow pattern (mhu = 0.13)



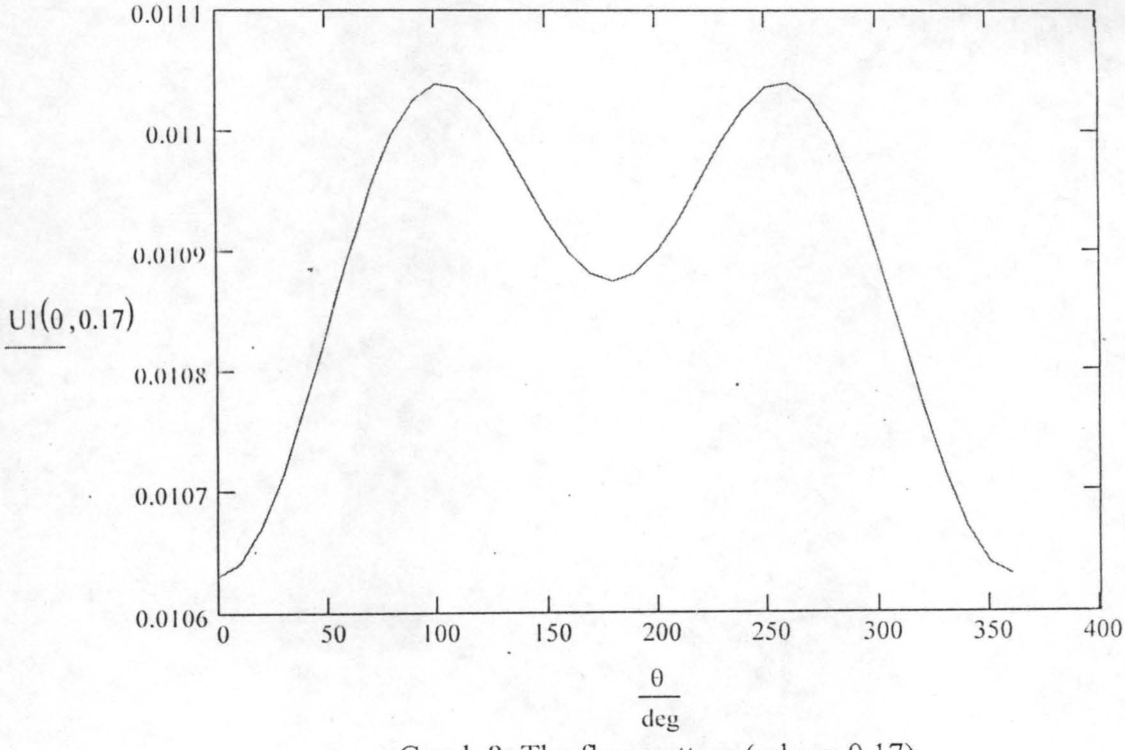
Graph 5: The flow pattern (mhu = 0.14)



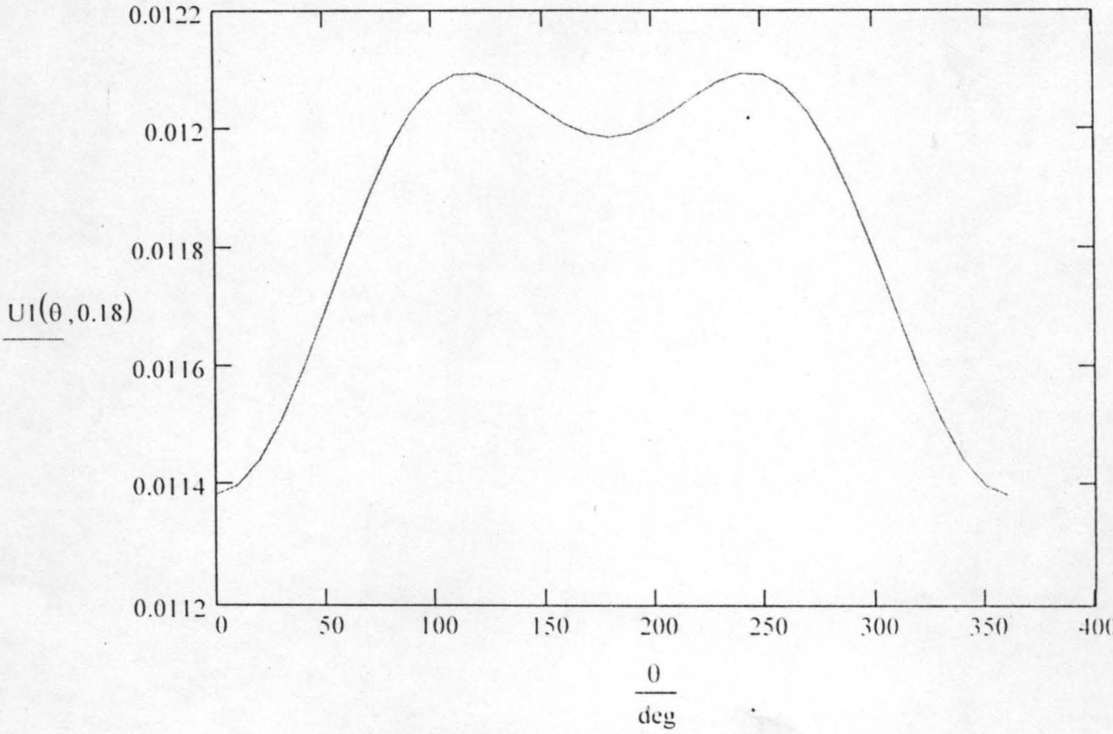
Graph 6: The flow pattern (mhu = 0.15)



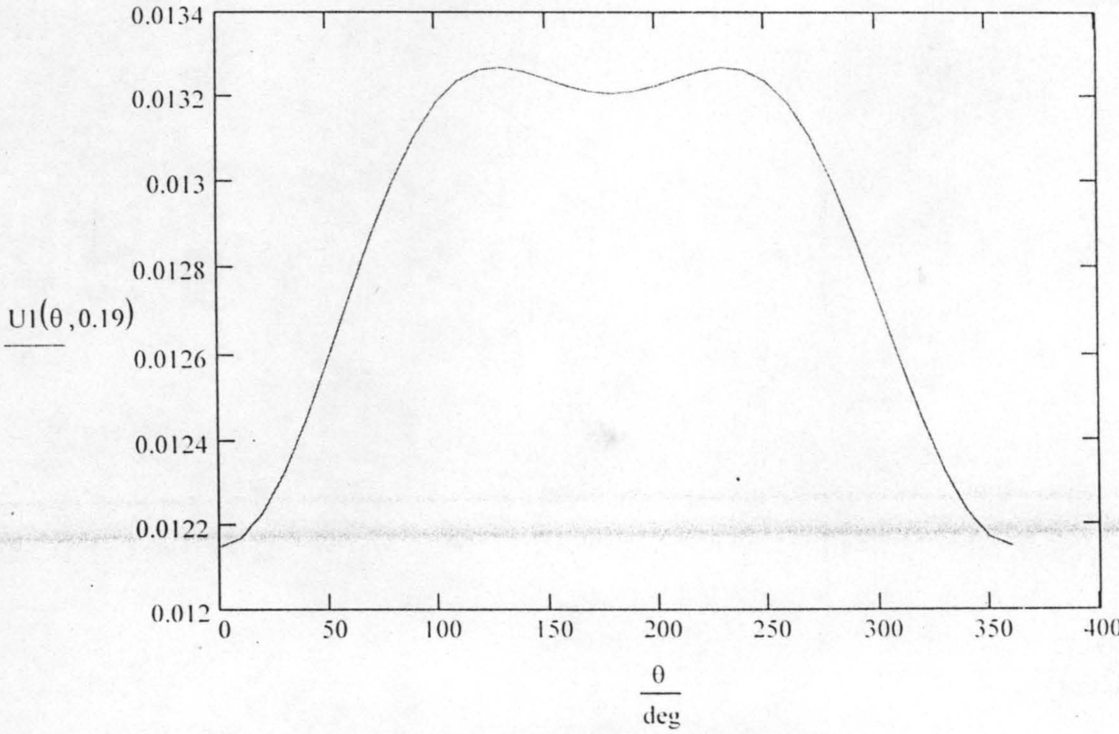
Graph 7: The flow pattern (mhu = 0.16)



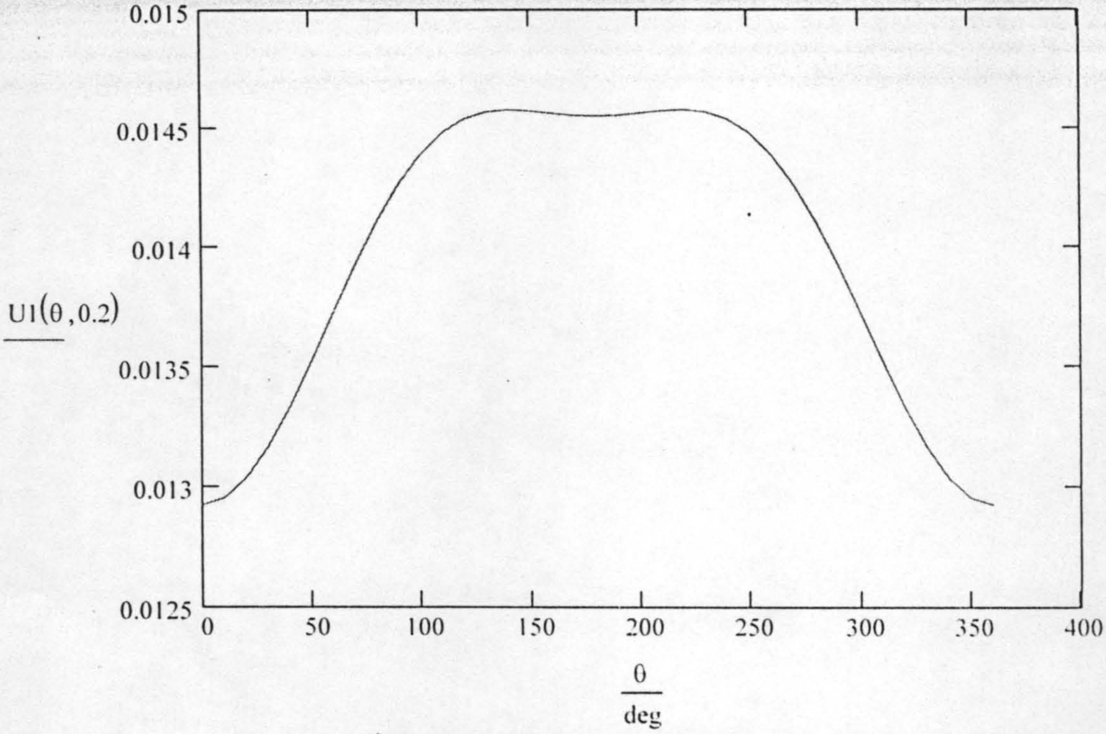
Graph 8: The flow pattern (mhu = 0.17)



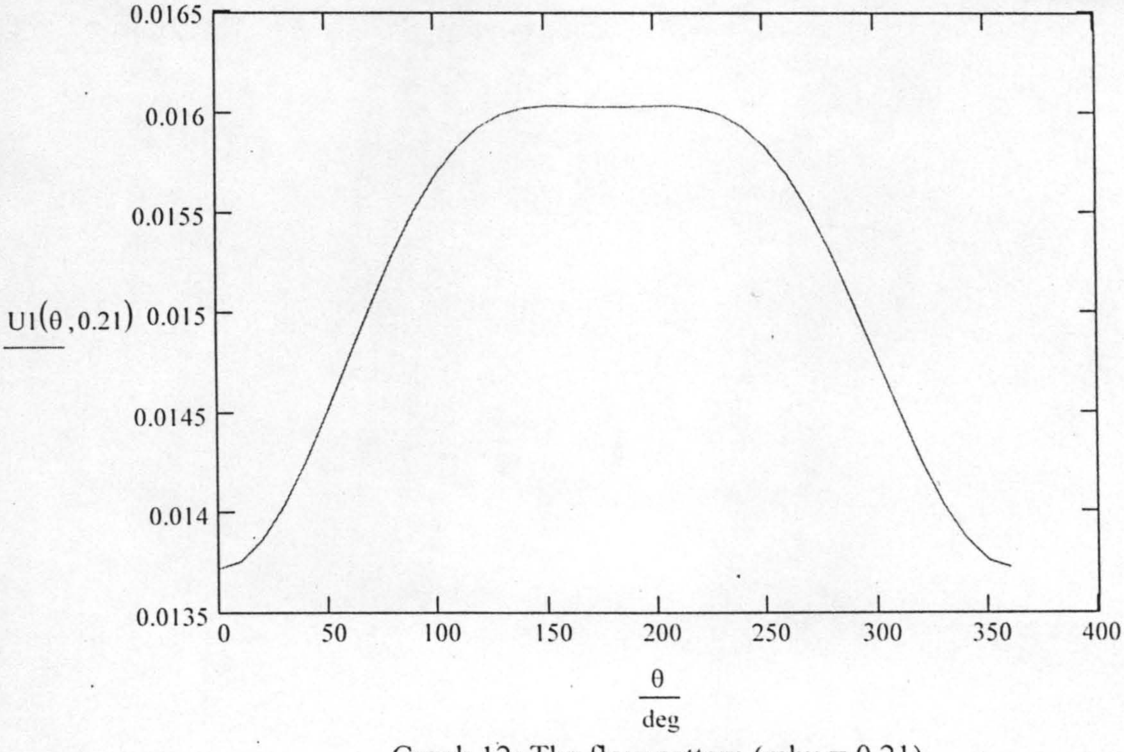
Graph 9: The flow pattern (mhu = 0.18)



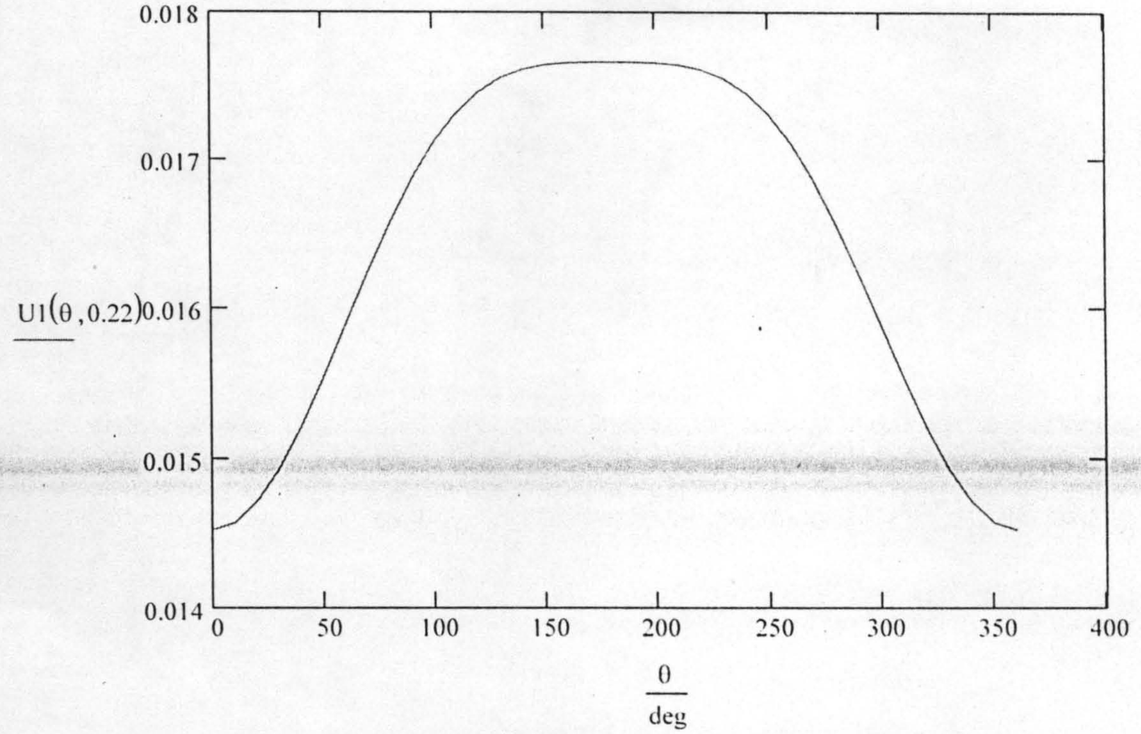
Graph 10: The flow pattern (mhu = 0.19)



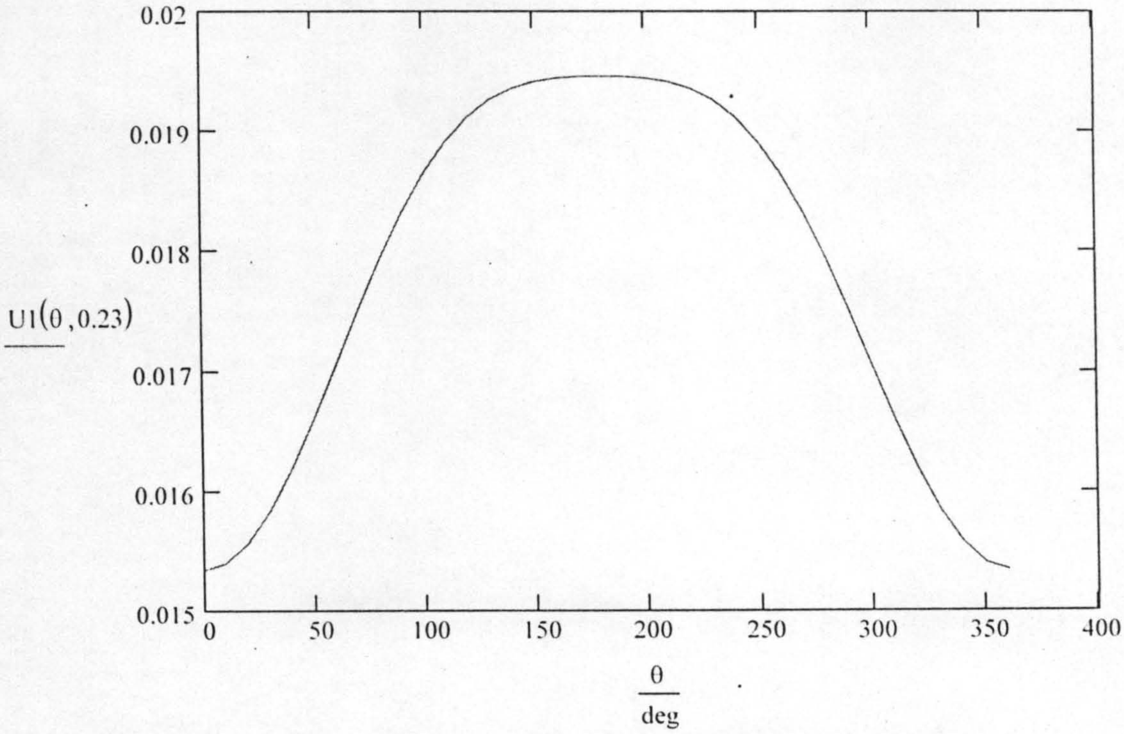
Graph 11: The flow pattern (mhu = 0.2)



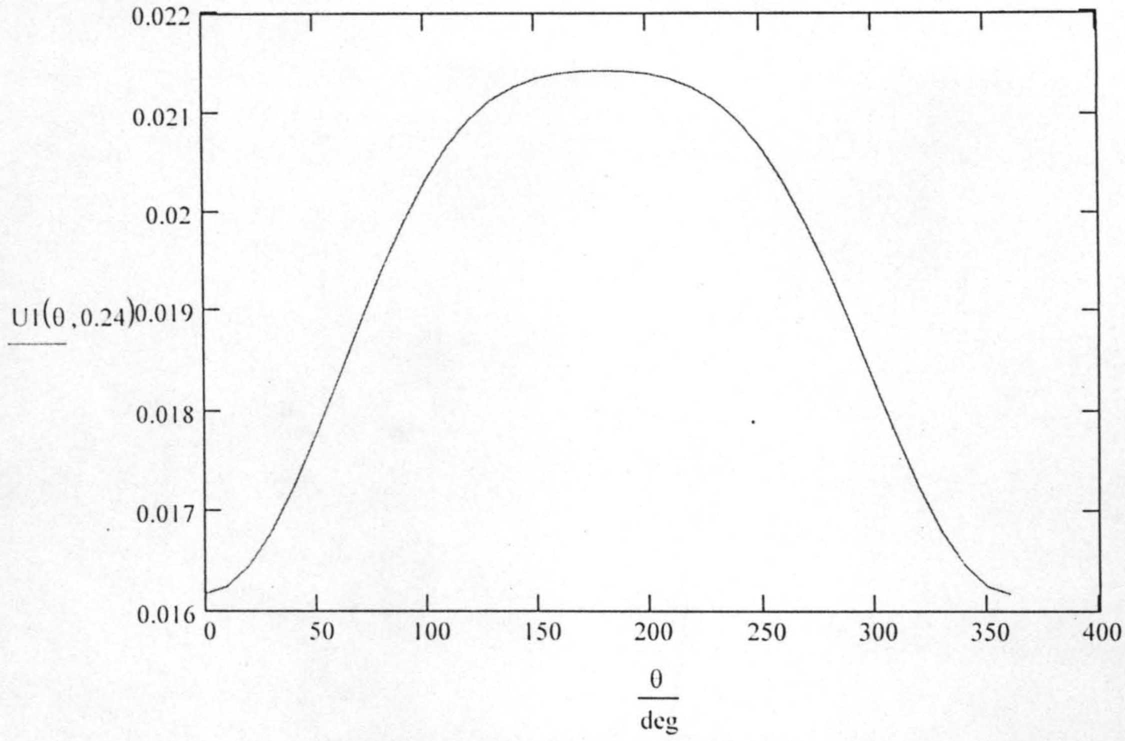
Graph 12: The flow pattern (mhu = 0.21)



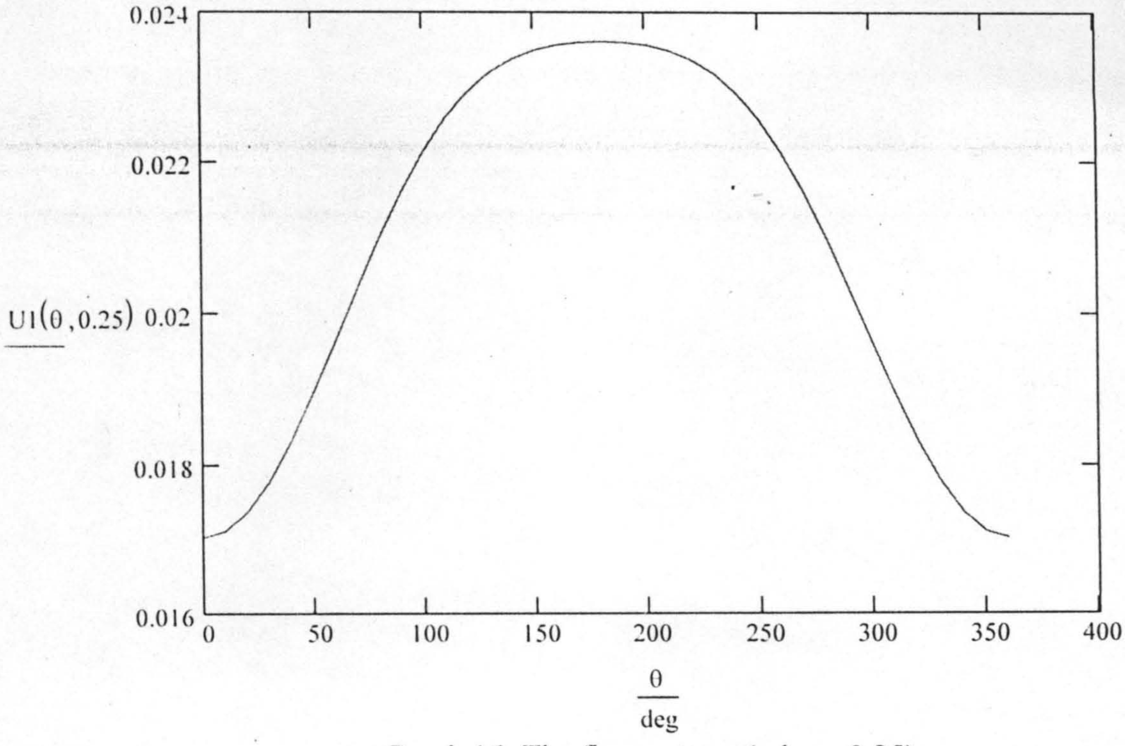
Graph 13: The flow pattern (mhu = 0.22)



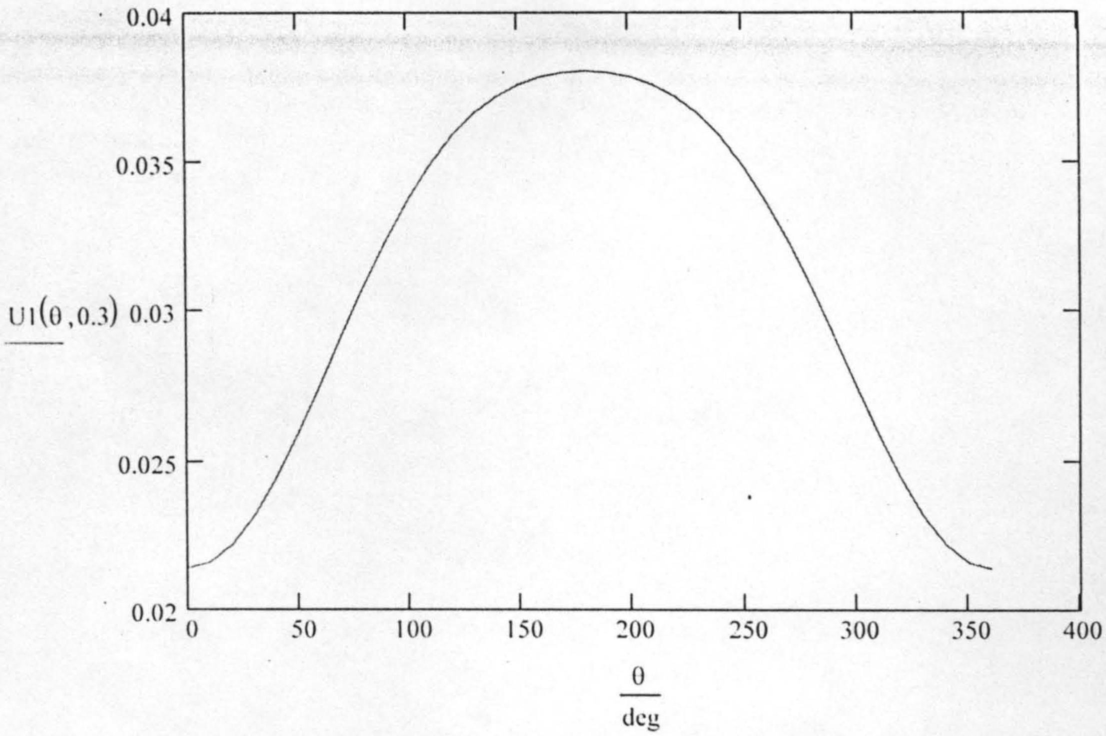
Graph 14: The flow pattern (mhu = 0.23)



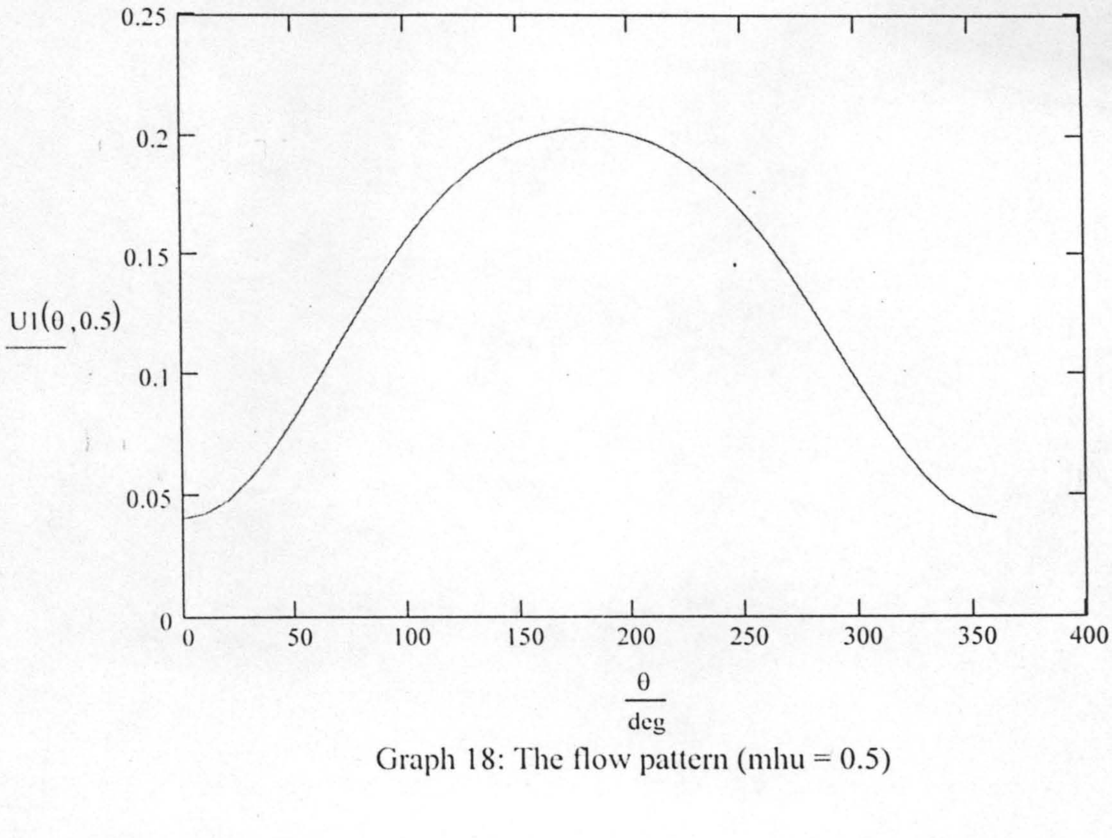
Graph 15: The flow pattern (mhu = 0.24)



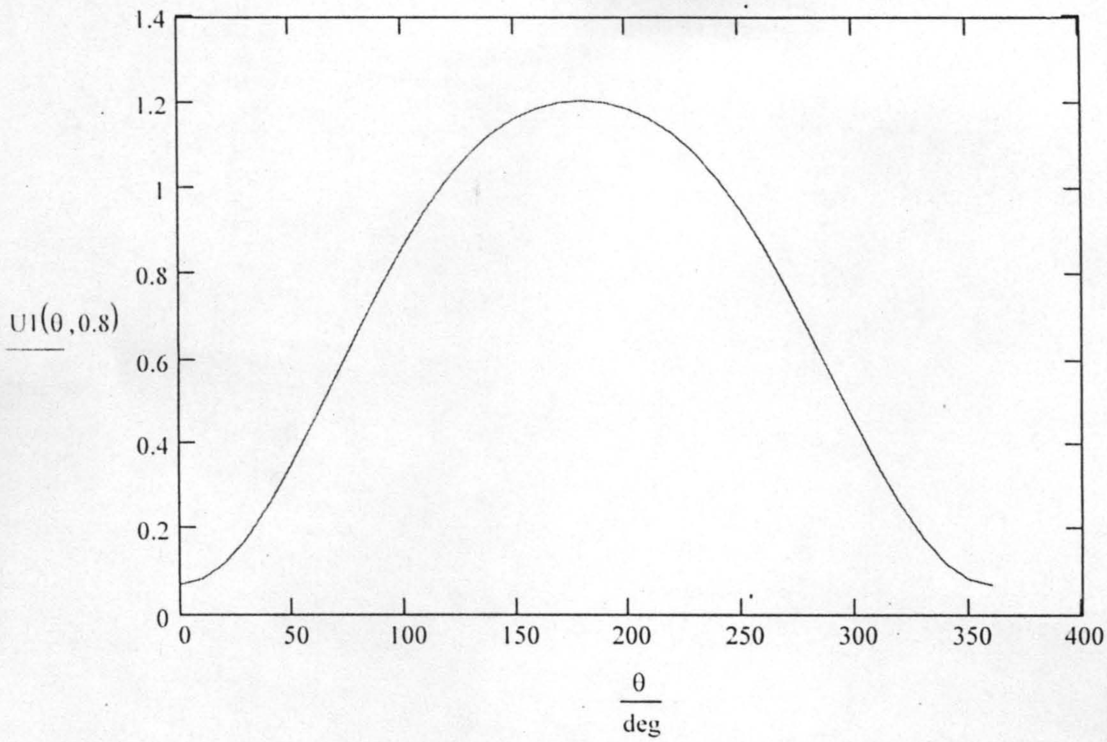
Graph 16: The flow pattern (mhu = 0.25)



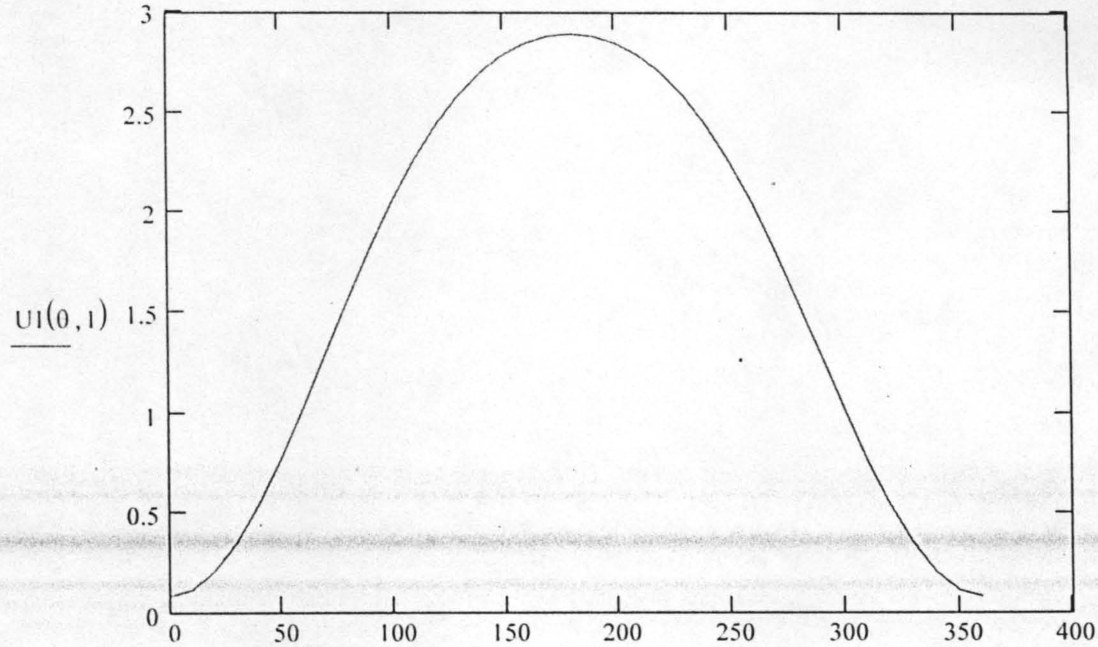
Graph 17: The flow pattern (mhu = 0.3)



Graph 18: The flow pattern (mhu = 0.5)



Graph 19: The flow pattern (mhu = 0.8)



Graph 20: The flow pattern (mhu = 1)

CHAPTER FIVE

5.1.0 Discussion and Summary

The flow past a sphere has been solved to the order ε^0 equation and graphs of the flow were plotted for various values of a/r.

From the graphs, the flow pattern is discovered to be gaussian which is symmetrical about 180°

The flow also increases steadily to a peak and then decreases, which shows that the flow is Lamina.

5.2.0 Conclusion and Recommendation

In this research, we have succeeded in analysing the flow past a sphere using the Legendre Polynomial functions for the order ε^0 pertubation parameter and graphs were plotted showing the flow pattern.

Interested researcher may solve this problem for higher orders.

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