POPULATION PROJECTION USING COMPUTER SOFTWARE ON THE ANALYTICAL SOLUTION OF A NON-LINEAR AGE- STRUCTURED MATHEMATICAL MODEL

BY

BAKO, Deborah Ushafa
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A THESIS SUBMITTED TO THE POSTGRADUATE SCHOOL IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF MASTER OF TECHNOLOGY (M.TECH) IN MATHEMATICS IN THE DEPARTMENT OF MATHEMATICS AND STATISTICS, SCHOOL OF SCIENCE AND SCIENCE EDUCATION, FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA. NIGER STATE.

SEPTEMBER, 2011

DECLARATION

I hereby declare that this thesis has been written by me and that it is a record of my research work. All published and unpublished sources of information have been appropriately acknowledged.

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BAKO, Deborah Ushafa Date

CERTIFICATION

This thesis titled: Population Projection Using Computer Software on the Analytical Solution of a non -liear Age- Structured Mathematical Model by BAKO, Deborah Ushafa meets the regulations governin g the award of the degree of Master of Technology in Mathematics, Federal University of Technology, Minna and is approved for its contribution to knowledge and literary presentation.

Prof. N. I. Akinwande
Supervisor

.... 15/11/2011
Signature & Date

Prof. Y.M.Aiyesimi
Head of Department

.... 15/11/2011
Signature & Date

Prof. H. A. Suberu
Dean, School of Science
and Science Education

.... 16-11-11
Signature & Date

Prof. (Mrs). S.N. Zubairu
Dean, Postgraduate School

....
Signature & Date 7/3/12

DEDICATION

This research thesis is dedicated to God Almighty for His divine inspiration and wisdom throughout the course of this programme.

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Abstract

The study of population has been of great relevance to the growth of nations or communities over the ages because of the practical influence it has on human life. Population projection provides future estimates of population sizes needed in planning. In Nigeria a number of factors have contributed negatively to having an accurate and reliable data. Population projection can be made by examining the influence of fertility, mortality and migration as it affects population from time to time so as to determine future population. In this thesis, the analytical solution of the non –linear age- structure mathematical model of population dynamics was obtained. The influence of the female component on population dynamics was considered and a two compartmentalized kind of model of the Gurtin and MacCamy's one dimensional non- linear age structured mathematical model emphasizing the dominance of the female compartment on the reproduction aspect of the population was proposed. A computer software (MATLAB) was used for the plotting of graphical profiles of male and female component using local data collected from Bosso, Niger State, Nigeria. The analysis reveals that younger members (male and female) of population contribute higher than the older members with respect to new birth and the elderly have higher death rate than the younger members of the population for both male and female.

TABLE OF CONTENTS

Contents **Page**

CHAPTER TWO

CHAPTER THREE

CHAPTER FOUR

CHAPTER FIVE

CHAPTER ONE

1.0 INTRODUCTION

1.1 Background to the Study

The study of population has been of great relevance to the growth of nations or communities over the ages because of the practical influence it has on human life. Population plays a vital role in the economic success of a nation to the extent that she cannot survive without adequate understanding of her population dynamics. Population studies are important for both short and long term planning in fields such as education, health, employment, social security and environment preservation. Such studies also provide the information needed for the formulation of government policies so as to achieve economic and social objectives.

Population growth rate is influenced by three main factors: fertility, mortality and migration. (National Policy on Population for Sustainable Development) (NPPSD 2006). In several ecosystems, the mode of reproduction is heterosexual where the male and female couple give birth to new generations. In this work, we propose a two compartmentalized kind of model of the Gurtin and MacCamy's one – dimensional non-linear age-structure mathematical model emphasizing the dominance of the female compartment on the reproductive aspect of the population.

1.2 Aim

The main aim of the thesis is to project population using computer software on the analytical solution of a non-linear age structured mathematical model.

1.3 Objectives of the Study

The specific objectives of the study are to:

1. Propose a mathematical model of population dynamics with emphasis on the influence of female component on reproduction.
2. Obtain solutions of the model equations.
3. Use computer software to estimate population of male and female.

1.4 Significance of the Study

From the time of creation, population of human and animal communities or species at various times, steadily increase and in other several cases go into extinction due to several factors such as civilization, war, epidemics, migration, predation, harvesting and other ecological factors Akinwande (2009).

This study is centered on the female as a dominant factor in reproduction. Since the study of population is so vital, the contribution of female component to reproduction or population growth is significant in any bi-sexual population. Mathematical models have been of immense and reliable contribution in the study of various kinds of population dynamics. The model will be used to estimate populations of male and female over a period of time.

1.5 Scope and Limitation of the Study

We partitioned the total population $P(t)$ into two compartments of Male $P_1(t)$ and Female $P_2(t)$, with $P(t) = P_1(t) + P_2(t)$. Each compartment P_j, $j = 1,2$ has the density function $q_j(t,a)$ which are functions of time t and age a. In this wok, polygamy and monogamy forms of marriages are also considered. Accurate data are hard to come by due to lack of proper documentation from population commission and hospitals where children are given birth to or hospitals that conduct/receive delivery.

1.6 Mathematical Modeling

The term model is used in many different situations and in many different ways. Model may be defined as a simplified or idealized descriptions or conception of a particular system, situation or process. It may be categorized according to the medium in which they are expressed. Some types of models that have been identified in the philosophy of science literature are: Physical models including scale models, prototypes, computational models where the model is encoded as computer and Mathematical models where the language of mathematics is the primary medium for defining the content of the model.

Mathematical modeling, which is defined by Benyah, (2005) as "the process of creating a mathematical representation of some phenomenon in order to gain a better understanding of that phenomenon", has become an important scientific technique over the last two decades and is becoming more and more a powerful tool to solve problems arising from science, engineering, industries and the society in general.

It allows us to generate new hypothesis, suggest experiments and measure crucial parameters. Essentially, any real situation in the physical and biological world, whether natural or involving technology and human intervention is subject to analysis by modelling if it can be described in quantitative terms, Benyah, (2005).

Once a model has been developed and applied to the problem, the resulting solution must be analyzed and interpreted with respect to the problem. The model interpretations and conclusion are often modified to obtain a more accurate reflection of the observed reality of that phenomenon. Mathematical modeling is an evolving process. As new insight is gained, the process begins again as additional factors are considered. The success of a model depends on how easily it can be used and how accurate are its predictions. It is worthy of note however that the closer a mathematical model assumptions are to the reality of the dynamics, the more intractable and difficult the mathematical analysis, hence the need to simplify our assumptions reasonably without loosing track of the dynamics of the problem at hand.

1.7 Types of Mathematical Model

Most classifications distinguish model types on the basis of the techniques used to construct the model and the degree of understanding or knowledge of the system that the model represents. The following three types of mathematical models have been identified: -

a) Empirical Models: are mathematical models which are not derived from theory or physical laws. The relationships between variables are derived by studying the available data and selecting mathematical functions to represent relationship based on a compromise between accuracy of fit and simplicity of mathematics. This often lacks generalization.

b) Deterministic Models: are based on assumptions or knowledge concerning the relationship between variables in the modelled system, or on physical laws or principles and the outcome is a direct consequence of initial conditions and values. Typically, they offer greater generalization than empirical models.

c) Stochastic models: random events play a central role in this type of mathematical model. They are suitable where apparently random fluctuations in the system processes (and, hence, the system variables) make simple deterministic models inappropriate.

The Stages of Mathematical Modeling

Developing a mathematical model for a real-life situation requires a reasonable level of understanding of the underlying principles of the system to be modelled. During the process of building a model, the modeller will decide what factors are relevant to the problem and what factors can be de-emphasized. Different real-life problems may require very different methods of approach. A general approach to the formulation of a real-life problem in mathematical terms are outlined as follows:

(i) Identify the problem;

(ii) Identify the important variables and parameters;

(iii) Determine how the variables relate together stating the assumptions;

(iv) Develop the equation(s) that express the relationship between the variables and constants;

(v) Analyse and solve the resulting mathematical problem

2. (vi) Interpret the results.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Introduction

Population, size and composition, have far-reaching implications for change, development and the quality of life in the socio-economic life of a nation or community.

Population is a major asset and a resource for development. It constitutes the bulk of the producers of goods and services as well as the major consumers of the goods and services. Thus, the population of a country is a major determinant of the size of the nation.

In Nigeria, population has been a rather sensitive and controversial issue because of its implications for shaping regional (now geopolitical) state and ethnic relations and balance of power. It is the attitude towards the population question, in terms of its absolute size, as it affects the states and the sub-regions that constitute the background to the census controversies which the country has been associated with PAN (1990), Ottong (1983).

2.2 Population Size and Growth Rates

Nigeria is the most populous country in Africa and the tenth in the world. These include the major demographic features as obtained from two major sources, viz.: the 1991 census and the PRB's World population data sheet.

According to the final figures of the 1991 census, the population of Nigeria, at the time was 88.92 million. Projection of the population using a 3.0 per cent growth rate shows that, the population of

Nigeria could be about 106 million in 1999. The Population Reference Bureaus estimated total population of the country in 1999 to about 113.8 million.

Obviously, the population of Nigeria is large, which makes it a "giant" relative to the other African countries. The large population implies a large market for goods and services as well as a large pool of human resources for development. However, the impact of population on development depends not only on the absolute size but also on its quality. (Characterised by good education, health diet)

Population growth rate is influenced by the interplay of the three main demographic processes of fertility, mortality and migration. This yields annual growth rate (natural increase) of about 3.0 per cent the annual growth rate. Nigerian population is believed to have risen steadily from an estimated 2.8 per cent in the 1960s to around 3.3 per cent in the 1985 to 1990 period. Although a steady decline in the growth rate is believed to have been in progress in the 1990s, the rate is still relatively high for (economic) comfort.

For instance, a growth rate of 3.3 per cent per annum suggests a population doubling time of 22 years. The reality of this scenario might not necessarily be with the absolute size of the population but, more importantly, with the implications of the growth rate for the future size of the population, and the ability of the economy to grow commensurately with and, therefore, cope with the increase in population size.

The relatively low mortality of about 13 to 14 per 1000 (crude death rate) and a declining infant mortality rate, as well as the increasing life expectancy in a population, all suggest higher survival chances and therefore, a swell in the size of future population.

The major factor responsible for the rapid increase in the population of the country is the relatively high fertility level as portrayed by a total fertility rate of about 6.0 live-births per woman in the1990s.

The Nigerian fertility survey during 1981/82 put the average number of child birth per woman (i.e. total fertility rate). Although the data here suggest a slight decline, the level is still relatively high. It seems an appreciable fall in fertility level in the country would depend on achieving a significant change in the cultural, socio -psychological and economic attitude of Nigerians towards children.

A frontal approach was taken in pursuance of this goal in 1988, Nigeria adopted a National Population Policy which seeks to reduce population growth rate through voluntary fertility regulation, and to promote the health and welfare of mothers and children to improve the quality of life of all Nigerians. The main thrust of the policy is the recommendation to young couples not to have more than four children per family (or per woman) and to attain a reduction of the population of women bearing more than four children by 80 per cent by the year 2000.

2.3 Population Age and Sex Composition

The age and sex distribution of the population of Nigeria by the 1991 census show a high proportion of children in the population. Those under 15 years of age constituted about 45 per cent of the total population. The proportion of aged persons (60 years and above) in the population constituted only 3.3 per cent. The age structure of the population, according to the 1991 census, shows a very broad-based pyramid, reflecting the large proportion of children and young persons.

The large proportion of the population aged under 15 years portrays a large number of potential parents The data also demonstrate a high child (or youth) dependency ratio which, when combined with th aged dependency ratio, gives an overall dependency ratio of about 1 to 1. That is, for every supposedly active (i.e. productive) person in the population in the working age group of 25 to 64 years, one other person is dependent. This is a relatively large figure compared with the situation in the developed countries with e chief dependency ratio of about one child to three adults of the working age groups. Furthermore, the high level of unemployment in Nigeria means prolonged dependency of working-age adults on parents and on the economically active (working) population.

Obviously, a higher dependency ratio exposes considerable strain on the economy at both the family and national levels. The large amount of resources used to provide feeding and clothing as well as for the education and health care of young people has greatly reduced the level of savings, investment and capital formation in the country.

The high proportion of young people in the population has implications for future joblessness as the economy is not likely to expand (grow) rapidly enough to accommodate the population. It is observed that the high proportion of young people in the population is as a result of high fertility level and declining mortality level. The situation of a young and rapidly expanding population is likely to continue in the country for some time until fertility level falls and the proportion of children in the population starts lowering.

2.4 Net Reproduction Rate

According to Dharmalingham (2004) the **net reproduction rate** (**NRR**) is the average number of daughters that would be born to a female (or a group of females) if she passed through her lifetime conforming to the age-specific fertility and mortality rates of a given year. This rate is similar to the gross reproduction rate but takes into account that some females will die before completing their childbearing years. An NRR of **one** means that each generation of mothers is having exactly enough daughters to replace themselves in the population.

The NRR is particularly relevant where sex ratios at birth are significantly affected by the use of reproductive technologies or where life expectancy is low

2.5 Logistic Function

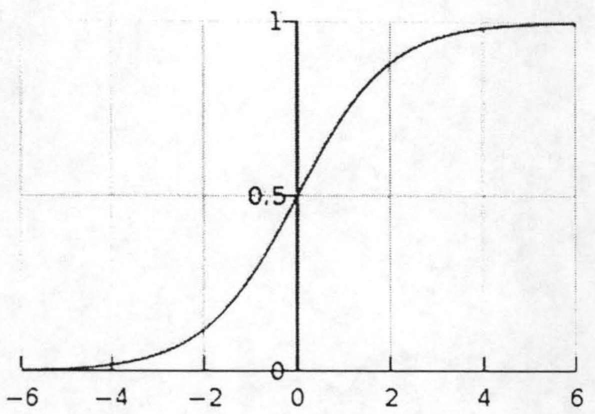

Figure 2.1 Standard logistic sigmoid function

A **logistic function** or **logistic curve** is a common sigmoid curve, given its name in 1844 or 1845 by Pierre François Verhulst who studied it in relation to population growth. It can model the "S-shaped" curve (abbreviated S-curve) of growth of some population *P*. The initial stage of growth is approximately exponential; then, as saturation begins, the growth slows, and at maturity, growth stops.

A simple logistic function may be defined by the formula

$$P(t) = \frac{1}{1+e^{-t}} \tag{2.1}$$

where the variable P might be considered to denote a *population* and the variable t might be thought of as *time*, Dharmalingham (2004). For values of t in the range of real numbers from $-\infty$ to $+\infty$, the S-curve shown is obtained. In practice, due to the nature of the exponential function e^{-t}, it is sufficient to compute t over a small range of real numbers such as [−6, +6].

The logistic function finds applications in a range of fields, including artificial neural networks, biology, biomathematics, demography, economics, chemistry, mathematical psychology, probability, sociology, political science, and statistics. It has an easily calculated derivative:

$$\frac{d}{dt}P(t) = P(t) \cdot (1 - P(t)). \qquad (2.2)$$

It also has the property that

$$1 - P(t) = P(-t). \qquad (2.3)$$

Logistic Differential Equation

The logistic function is the solution of the simple first-order non-linear differential equation

$$\frac{d}{dt}P(t) = P(t)(1 - P(t)) \qquad (2.4)$$

where P is a variable with respect to time t and with boundary condition $P(0) = 1/2$. This equation is the continuous version of the logistic map.

The qualitative behavior is easily understood in terms of the phase line: the derivative is 0 at P=0,1 and the derivative is positive for P between 0 and 1and negative for P above 1 or less than 0 (though negative populations do not generally accord with a physical model). This yields an unstable equilibrium at 0 and a stable equilibrium at 1, and thus for any value of P greater than 0 and less than 1, P grows to 1.

One may readily find the (symbolic) solution to be

$$P(t) = \frac{e^t}{e^t + e^c} \tag{2.5}$$

Choosing the constant of integration $e^c = 1$ gives the other well-known form of the definition of the logistic curve

$$P(t) = \frac{e^t}{e^t + 1} = \frac{1}{1 + e^{-t}} \tag{2.6}$$

More quantitatively as can be seen from the analytical solution, the logistic curve shows early exponential growth for negative t, which slows to linear growth of slope 1/4 near $t = 0$, then approaches $y = 1$ with an exponentially decaying gap.

2.6 Modeling Population Growth

Verhulst (1804–1849)

A typical application of the logistic equation is a common model of population growth, originally due to Pierre-François Verhulst in 1838, where the rate of reproduction is proportional to both the existing population and the amount of available resources, all else being equal. The Verhulst equation was published after Verhulst had read Malthus' *An Essay on the Principle of Population*. Verhulst derived his logistic equation to describe the self-limiting growth of a biological population. The equation is also sometimes called the *Verhulst-Pearl equation* following its rediscovery in 1920. Lotka derived the equation again in 1925, calling it the *law of population growth.*

Letting P represent population size (N is often used in ecology instead) and t represent time, this model is formalized by the differential equation:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right) \qquad (2.7)$$

where the constant r defines the growth rate and K is the carrying capacity.

In the equation, the early unimpeded growth rate is modeled by the first term $+rP$. The value of the rate r represents the proportional increase of the population P in one unit of time. As the population grows the second term, which multiplied out is $-rP^2/K$, becomes larger than the first as some members of the population P interfere with each other by competing for some critical resource, such as food or living space. This antagonistic effect is called the *bottleneck* and is modeled by the value of the parameter K. The competition diminishes the combined growth rate until the value of P ceases to grow (this is called *maturity* of the population).

Now setting $x = P / K$ gives the differential equation

$$\frac{dx}{dt} = rx(1 - x) \qquad (2.8)$$

For $r = 1$ we have the particular case with which we started.

$$\lim_{t\to\infty} P(t) = K. \qquad (2.9)$$

Which is to say that K is the limiting value of P: the highest value that the population can reach given infinite time (or come close to reaching in finite time). It is important to stress that the carrying capacity is asymptotically. In ecology, species are sometimes referred to as r-strategist or K-strategist depending upon the selective processes that have shaped their life history strategies. The solution to the equation (with P_0 being the initial population) is

$$P(t) = \frac{K P_0 e^{rt}}{K + P_0 \left(e^{rt} - 1\right)} \qquad (2.10)$$

where

reached independently of the initial value $P(0) > 0$, also in case that $P(0) > K$.A common choice for the activation or "squashing" functions, used to clip for large magnitudes to keep the response of the neural network bounded is Gershenfeld 1999

$$g(h) = \frac{1}{1 + e^{-2\beta h}} \qquad (2.11)$$

which we recognize to be of the form of the logistic function. These relationships result in simplified implementations of artificial neural networks with artificial neurons. Practitioners caution that sigmoidal functions which are symmetric about the origin (e.g. the hyperbolic tangent) lead to faster convergence when training networks with back propagation.

2.6 Population Growth Rate

Population growth rate (PGR) is the fractional rate at which the number of individuals in a population increases. Specifically, PGR ordinarily refers to the change in population over a unit time period often expressed as a percentage of the number *of* individuals in the population at the beginning of that period. This can be written as the formula:

$$\text{Growth rate} = \frac{\text{(Population at end of period} - \text{Population at beginning of period)}}{\text{Population at beginning of period}}$$

(In the limit of a sufficiently small time period.)

The above formula can be expanded to: **growth rate = crude birth rate — crude death rate + net immigration rate**, or **$\Delta P/P = (B/P) - (D/P) + (I/P) - (E/P)$**, where P is the total population, B is the number of births, D is the number of deaths, I is the number of immigrants, and E is the number of emigrants.

This formula allows for the identification of the source of population growth, whether due to natural increase or an increase in the net immigration rate. Natural increase is an increase in the native-born population stemming from a higher birth rate, a lower death rate, or a combination of the two. Net immigration rate is the difference between the number of immigrants and the number of emigrants.

The most common way to express population growth is as a ratio, not as a rate. The change in population over a unit time period is expressed as a percentage of the population at the beginning of the time period. That is:

$$\text{Growth ratio} = \text{Growth rate} \times 100\%.$$

A positive growth ratio (or rate) indicates that the population is increasing, while a negative growth ratio indicates the population is decreasing. A growth ratio of zero indicates that there were the same number of people at the two times net difference between births, deaths and migration is zero. However, a growth rate may be zero even when there are significant changes in the birth rates, death rates, immigration rates, and age distribution between the two times. Verhulst (1838). Equivalently, percent death rate = the average number of deaths in a year for every 100 people in the total population.

A related measure is the net reproduction rate. In the absence of migration, a net reproduction rate of more than one indicates that the population of women is increasing, while a net reproduction rate less than one (sub-replacement fertility) indicates that the population of women is decreasing.

Table 2.1

Approximately 4.03 billion people live in these ten countries, representing 58.7% of the world's population as of November 2010.

S/NO	COUNTRY	POPULATION	DATE	PERCENT AGE	SOURCE
1	People's Republic of China	1,341,170,000	December, 14,2010	19.5%	Chinese Official Population Clock
2	India	1,191,430,000	December, 14,2010	17.3%	Indian Official Population Clock
3	United States	310,906,000	December, 14,2010	4.51%	United States Official Population Clock
4	Indonesia	238,400,000	May, 2010	3.4%	Suluh Nusantara Indonesia Census report
5	Brazil	193,926,000	December, 14,2010	2.82%	Brazil Official Population Clock
6	Pakistan	171,306,000	December, 14,2010	2.49%	Official Pakistani Population Clock
7	Bangladesh	164,425,000	2010	2.39%	2008 UN estimate for year 2010

8	Nigeria	158,259,000	2010	2.3%	2008 UN estimate for year 2010
9	Russia	141,927,297	January 1, 2010	2.06%	Federal State Statistics Service of Russia
10	Japan	127,380,000	June 1, 2010	1.85%	Official Japan Statistics Bureau

Source: www.world population .com

2.7 Birth Rates

Birth rates are declining slightly on average, but vary greatly between developed countries (where birth rates are often at or below replacement levels), developing countries, and different ethnicities. Death rates can change unexpectedly due to disease, wars and catastrophes, or advances in medicine.

Before adding mortality rates, the 1990s saw the greatest absolute number of births worldwide, especially in the years after 1995, despite the fact that the birth rate was not as high as in the 1960s. In fact, because of the 163 million-per-year births after 1995, the time it took to reach the next billion reached its fastest pace (only twelve years), as world population reached six billion people in 1999; at the beginning of the decade, this figure was only expected to be met in 2000, at the earliest, by most demographers.

1985–1990 marked the period with the fastest yearly population change in world history. Even though the early 1960s had a greater growth *rate* than in the mid and late 1980s, the population change how ever around eighty-three million people in the five-year period, with an all-time record change of nearly eighty-eight million in 1990. The reason is that the world's population being around 5 billion in the mid- and late-1980s, compared to around 3 billion in the early 1960s, meant that the growth *rate* (which is a percentage) was not the major factor in the dramatic absolute increase in population.

The factors affecting global human population are very simple. They are fertility, mortality, initial population, and time. The current growth rate of ~1.3% per year is smaller than the peak which occurred a few decades ago (~2.1% per year in 1965-1970), but since this rate acts on a much larger population base, the absolute number of new people per year (~90 million) is at an all time high. The stabilization of population will require a reduction in fertility globally. In the most optimistic view, this will take some time.

2.9 Table 2.2

Nigeria Total Fertility Rate: Total fertility rate: 4.82 children born/woman (2010 est.)

Year	Total fertility rate	Rank	Percent Change	Date of Information
2003	5.4	30		2003 est.
2004	5.53	24	2.41 %	2004 est.
2005	5.53	23	0.00 %	2005 est.
2006	5.49	23	-0.72 %	2006 est.
2007	5.45	22	-0.73 %	2007 est.
2008	5.01	32	-8.07 %	2008 est.
2009	4.91	32	-2.00 %	2009 est.
2010	4.82	29	-1.83 %	2010 est.

Definition

This entry gives a figure for the average number of children that would be born per woman if all women lived to the end of their childbearing years and bore children according to a given fertility rate at each age. The total fertility rate (TFR) is a more direct measure of the level of fertility than the crude birth rate, since it refers to births per woman. This indicator shows the potential for population change in the country. A rate of two children per woman is considered the replacement rate for a population, resulting in relative stability in terms of total numbers. Rates above two children indicate populations growing in size and whose median age is declining. Higher rates may also indicate difficulties for families, in some situations, to feed and educate their children and for women to enter the labor force. Rates below two children indicate populations decreasing in size and growing older. Global fertility rates are in general decline and this trend is most pronounced in industrialized countries, especially Western Europe, where populations are projected to decline dramatically over the next 50 years.

Source: CIA World Factbook - November 3, 2010

In another study on population modeling with ordinary differential equation, it was revealed that, If $P(t)$ is the population of a community at time t, the Malthusian law which proposed the exponential growth model for populations is given by:

$$\frac{dP(t)}{dt} = \delta\, P(t) \qquad (2.12)$$

where δ is the growth modulus. We note the following on the proposed exponential model equation:

If we define

β = birth rate

μ = death rate

then $\delta = \beta - \mu$

and the population $P(t)$ will grow or decay exponentially when $\delta > 0$ or $\delta < 0$ respectively.

This model is not applicable where the population competes for space and or resources, in which case the growth modulus may depend on the population.

As a result, Verhulst proposed the logistic population model which assumes that the environment has a carrying capacity denoted K, this is the maximum population which the environment can sustain. The model equation is given by:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right); \qquad P(0) = P_0 \tag{2.13}$$

The analytical solution is given by:

$$P(t) = \frac{K P_0}{\left(K - P_0\right)e^{-rt} + P_0} \tag{2.14}$$

This model has been successfully applied to several population dynamics with a great measure of success.

The model equation has equilibrium at the points $P(0) = 0$ and $P(t) = K$; note that the equilibrium or steady state of the model is when the rate of change is zero, i.e. when

$$\frac{dP}{dt} = 0 \tag{2.15}$$

The equilibrium state $P(0) = 0$ is unstable, while the state $P(t) = K$ is stable.

The major deficiency of this model is that it gives no information regarding the age distribution of the population, hence assuming that the birth and death rates are independent of age. Lotka and Von Foerster addressed this crucial question in their age-dependent population model, giving rise to the use of partial differential equations as the population is now treated as depending on the two variables age a and time t. This is presented in the next sub-section.

(v) A graphical representation of the logistic model is given below

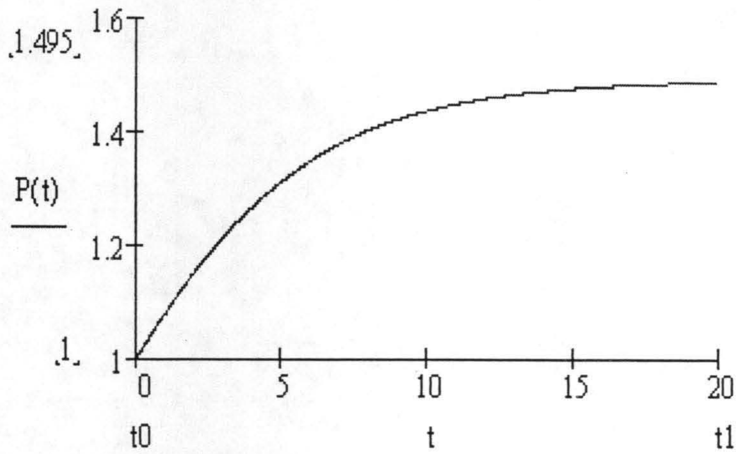

Figure 2.1 The logistic population model figure.

In an attempt to address the problem encountered in the Logistic population model, Lotka and Von Foerster in their age-dependent population model applied partial differential equation. Modeling of physical situations in which consideration is given to time and space leads to Partial Differential Equations rather than ordinary differential equation. In the case of population models, the space can be age grade. It can still be space, i.e. land mass, but in this work consideration is given only to age.

If $q(t, a)$ is the population of those in the age bracket or age grade $[a, a+da]$ at time t, then the total population $P(t)$ at time t is given by the integral value:

$$P(t) = \int_0^\infty q(t, a)\,da \qquad (2.16)$$

An interesting aspect of this model is that the age and time both increase with the same value over the time interval. If we consider members of the population with age a at time t, if the time t increased by h units, then the group will increase in age by the same quantity h units, hence the differential is given by:

$$\frac{\partial q(t, a)}{\partial t} + \frac{\partial q(t, a)}{\partial a} = \lim_{h \to 0} \frac{q(t+h, a+h) - q(t, a)}{h} \qquad (2.17)$$

From the principle of conservation, the rate of change add to the rate d of individuals (per unit age and time) of age a who die at time t must equal zero.

In the Lotka & co model, d is independent of the total population, it only depends on the time and age, and hence given by:

$$d = \lambda(a) q(t, a) \qquad (2.18)$$

while in the model by Gurtin & co, (1974) d depends on the total population and is given by:

$$d = \lambda(P(t), a) q(t, a) \qquad (2.19)$$

Similarly, the birth renewal equation below is independent on total population in Lokta's formulation while is dependent in Gurtin's.

The model equations are given by the set of equations below:

$$\frac{\partial q(t,a)}{\partial t}+\frac{\partial q(t,a)}{\partial a}+\lambda(P(t),a)q(t,a)=0 \tag{2.20}$$

$$P(t)=\int_0^{\infty} q(t,a)\,da \tag{2.21}$$

$$q(t,0)=\int_0^{\infty} \beta(P(t),a)q(t,a)\,da \tag{2.22}$$

$$q(0,a)=\phi(a) \tag{2.23}$$

Which is referred to as a boundary value problem in partial differential equations.

CHAPTER THREE

3.0 MATERIAL AND METHODS

3.1 Introduction

The total population $P(t)$ of a community in an ecosystem is partitioned into two compartments of Male $P_1(t)$ and Female $P_2(t)$, with $P(t) = P_1(t) + P_2(t)$. Each compartment P_j, $j = 1,2$ has the density function $q_j(t,a)$ which are functions of time t and age a; such that

$$P_j(t) = \int_0^{\Omega_j} q_j(t,a)da \qquad (3.1)$$

where Ω_j is the life expectancy of the j^{th} compartment.

3.2 The Model Equation

The equations for the proposed female – dominant age –structured mathematical model are given by (3.2) to (3.5) below.

$$\frac{\partial q_j(t,a)}{\partial t} + \frac{\partial q_j(t,a)}{\partial a} + \lambda_j(a)q_j(t,a) = 0 \qquad (3.2)$$

$$q_j(t,0) = B_j(t) = \int_0^{\Omega} \beta_j(a)q_2(t,a)da \qquad (3.3)$$

$$q_j(0,a) = \phi_j(a) \qquad (3.4)$$

$$\Omega = \max\{\Omega_j\} \qquad (3.5)$$

$\lambda_j(a)$= the death modulus for the j^{th} component.

$\beta_j(a)$= the birth modulus for the j^{th} component.

$B_j(t)$= the total birth at time t into the j^{th} component

$\phi_j(a)$= the population distribution by age at time $t = 0$.

Equation 3.3 is significant. The two equation incorporated here in the reproduction process. This is a peculiar feature in this study. One implication of this is that the more females the more the birth recorded in the population.

Next we present the one-dimensional age-structured non-linear population model of Gurtin and MacCamy (1974), and the corresponding analytic solution as obtained by the Akinwande (2009). This analytical solution will be used to postulate the analytic solution for the two-dimensional model equations proposed in this work.

In this sub-section we shall obtain the analytical solution of the one-dimensional non-linear age-structured population model given by equations (3.6) to (3.9); and utilize the theory of convolution Akinwande (2009) to solve the simultaneous integral equations arising which involve the total population P(t) and the total birth B(t) at time t.

$$\frac{\partial \rho(t,a)}{\partial t} + \frac{\partial \rho(t,a)}{\partial a} + \mu\left(P(t),a\right)\rho(t,a) = 0 \qquad (3.6)$$

$$P(t) = \int_0^\infty \rho(t,a)da \qquad (3.7)$$

$$\rho(t,0) = B(t) = \int_0^{\infty} \beta\big(P(t),a\big)\rho(t,a)da \tag{3.8}$$

$$\rho(0,a) = \phi(a) \tag{3.9}$$

where t is the time, a is the age; with $t \geq 0$, $a \geq 0$.

$\rho(t,a)$ is the population of those age a at time t. $P(t)$ gives the total population of the community at time t.

$\mu\big(P(t),a\big)$ is the death modulus which is assumed to depend on the total population and the age of individuals; while $\beta\big(P(t),a\big)$ is the birth modulus which is assumed also to depend on the total population and the age of individuals.

Using the method of characteristics and variation of parameters, we obtain the solution of the model equations as follow:

Noting that the partial derivative part of (3.6) is defined by:

$$\frac{\partial\, \rho(t,a)}{\partial\, t} + \frac{\partial\, \rho(t,a)}{\partial\, a} = \lim_{h \to 0} \frac{\rho(t+h,a+h) - \rho(t,a)}{h} \tag{3.10}$$

let

$$q(h) = \rho(t_0 + h, a_0 + h) \tag{3.11}$$

and

$$v(h) = \mu\big(P(t_0 + h), a_0 + h\big) \tag{3.12}$$

then the ordinary differential equation

$$\frac{dq(h)}{dh} + v(h)q(h) = 0 \tag{3.13}$$

$$\frac{dq(h)}{dh} = -v(h)q(h) \tag{3.14}$$

$$dq(h) = -v(h)q(h)dh \tag{3.15}$$

divide through by q(h)

$$\frac{dq(h)}{dh} = -v(h)dh \tag{3.16}$$

integrating from 0 to h, we have

$$\int_{o}^{h} \frac{dq(h)}{dh} = \int_{o}^{h} -v(s)ds \tag{3.17}$$

let the value of h=s

$$\log q(h) \quad \int_{o}^{h} -v(s)ds \tag{3.18}$$

$$\frac{q(h)}{q(0)} = \exp \int_{o}^{h} -v(s)ds \tag{3.19}$$

$$q(h) = q(0)\exp\left\{-\int_{0}^{h} v(s)ds\right\} \tag{3.20}$$

so, the ordinary differential equation has the unique solution

$$q(h) = q(0)\exp\left\{-\int_0^h v(s)ds\right\} \tag{3.21}$$

using (3.11) and (3.12) and substituting into equation (3.22) we have that

$$\rho(t_0 + h, a_0 + h) = \rho(t_0, a_0)\exp\left\{-\int_0^h \mu(P(t_0 + s), a_0 + s)ds\right\} \tag{3.22}$$

which gives the values of ρ at all points on the characteristic passing through the point (t_0, a_0). When $(t_0, a_0) = (0, a - t)$; $h = t$, we have that

$$\rho(t, a) = \phi(a - t)\exp\left\{-\int_0^h \mu(P(s), a - t + s)ds\right\}; \quad t \le a \tag{3.23}$$

$$\rho(t_0 + h, a_0 + h) = \rho(o + h, a - t + t) = \rho(t, a) \tag{3.24}$$

$$\rho(t_o + a_o) = \rho(o, a - t) = \phi(a - t) \tag{3.25}$$

then

$$(\rho(o + s), a - t + s) = (\rho(s), a - t + s) \tag{3.26}$$

also the boundary condition

also when

$(t_o, a_o) = (t-a, o)$; h=a, t>a

$t_o = t-a$, $a_o = o$

consider the left hand side of equation (3.22) to obtain equation (3.30)

$$\rho(t_0 + h, a_0 + h) = \rho(t - a + a, o + a) = \rho(t, a) \qquad (3.27)$$

$$\rho(t_o, a_o) = \rho(t - a, o) = \beta(t - a) \qquad (3.28)$$

$$\rho(t, a) = B(t - a)\exp\left\{-\int_0^a \mu\big(P(t - a + s), s\big)ds\right\};\ t > a \qquad (3.29)$$

From the theory of convolution, if f(t) and g(t) are piece wise continuous and of exponential order, where

F(s) = L{F(s) and G(s) = L {g(s)} (3.30)

F∗g = ∫ f(t-u) g(u)du (3.31)

Note that f∗g = g∗f

This is particularly useful in finding inverse Laplace transform in the solution of differential and integral equations (Adeboye, 2006).

The equations (3.24) and (3.31) give the solution of $\rho(t, a)$ in the positive quadrant $t \geq 0$; $a \geq 0$.

Suppose

$$\mu\big(P(t), a\big) = \mu(a);\ \ \beta\big(P(t), a\big) = \beta(a) \qquad (3.32)$$

i.e the death and birth modulli are only age dependent. (3.24) and (3.31) take the forms:

$$\rho(t, a) = \phi(a - t)\exp\left\{-\int_0^t \mu\big(a - t + s\big)ds\right\};\ t \leq a \qquad (3.33)$$

and

$$\rho(t, a) = B(t-a)\exp\left\{-\int_0^a \mu(s)ds\right\}; \quad t > a \qquad (3.34)$$

Using (3.34) and (3.35) in Equations (3.7) and (3.8) gives equation

$$P(t) = \int_0^t B(t-a)\exp\left\{-\int_0^a \mu(s)ds\right\}da + \int_t^\infty \phi(a-t)\exp\left\{-\int_0^t \mu(a-t+s)ds\right\}da \qquad (3.35)$$

and

$$B(t) = \int_0^t \beta(a)B(t-a)\exp\left\{-\int_0^a \mu(s)ds\right\}da + \int_t^\infty \beta(a)\phi(a-t)\exp\left\{-\int_0^t \mu(a-t+s)ds\right\}da \qquad (3.36)$$

Changing variables transform (3.36) gives:

$$P(t) = \int_0^t B(a)\exp\left\{-\int_0^{t-a} \mu(s)ds\right\}da + \int_0^\infty \phi(a)\exp\left\{-\int_0^t \mu(a+s)ds\right\}da \qquad (3.37)$$

similarly,

changing variables transform 3.37 to

$$B(t) = \int_0^t \beta(t-a)B(t-t+a)\exp\left\{-\int_0^{t-a} \mu(s)ds\right\}da + \int_0^\infty B(a)\phi(a)\exp\left\{-\int_0^t \mu(a+s)ds\right\}da \qquad (3.38)$$

or

$$B(t) = \int_0^t \beta(t-a)B(a)\exp\left\{-\int_0^{t-a} \mu(s)ds\right\}da + \int_0^\infty \beta(a-t)\phi(a)\exp\left\{-\int_0^t \mu(a+s)ds\right\}da \qquad (3.39)$$

Which is of the form

$$B(t) = \int_0^t K(a,t)B(a)da + g(t) = \int_0^\infty \beta(t-a)\phi(a)\exp\left\{-\int_0^t \mu(a+s)ds\right\}da \tag{3.40}$$

Equation (3.40) is an integral equation of the operator form

$$B(t) = g(t) + \int_0^t K(a,\, t)\, B(a)da \tag{3.41}$$

comparing (3.41) with the RHS of (3.40) only gives

$$\text{k(a,t)} = \beta\,\text{(t-a)exp}\left\{-\int_0^{t-a} \mu(s)ds\right\} \tag{3.42}$$

$$B(t) = g(t) + \int_0^t K(a,\, t)\, B(a)da \text{ - (bringing out B(t))} \tag{3.43}$$

$$B(t) = +\int_0^t K(a,t)B(a)da + g(t) = \int_0^\infty \beta(a-t)\phi(a)\exp\left\{-\int_0^t \mu(a+s)ds\right\}da \tag{3.44}$$

and

$$g(t) = \int_0^\infty \beta(a-t)\phi(a)\exp\left\{-\int_0^t \mu(a+s)ds\right\}da \tag{3.45}$$

and

$$B(t) = +\int_0^t K(a,t)B(a)da \equiv \int_0^t \beta(t-a)B(a)\exp\left\{-\int_0^{t-a} \mu(a+s)ds\right\}da \tag{3.46}$$

where K is the kernel of the integral equation given by

$$K(a, t) = \beta(t-a)\exp\left\{-\int_0^{t-a} \mu(s)ds\right\} \qquad (3.47)$$

from convolution kernel

K (s,t) = K (s-t)and from equation (3.30), we note that

$$K(a, t) = K(t-a) \qquad (3.48)$$

and so (3.46) takes the form:

$$K(u) = \beta(u)\exp\left\{-\int_0^{u} \mu(s)ds\right\} \qquad (3.49)$$

From equation (3.42) we have that

$$\int_0^t K(a,t)B(a)da = \int_0^t K(t-a)B(a)da = (K*B)(t) \qquad (3.50)$$

substituting the RHS of (3.48) into equation (3.42), we have

$$B(t) = g(t) + (K*B) \qquad (3.51)$$

where $(K*B)(t)$ is the convolution of K and B, Equation (3.42) then takes the form

$$B(t) = g(t) + (K*B)(t) \qquad (3.52)$$

Taking the Laplace transform of equation (3.50) gives

$$B(\eta) = g(\eta) + K(\eta)\,B(\eta) \qquad (3.53)$$

$$B(\eta) - K(\eta)B(\eta) = g(\eta)$$

where η is the transform parameter; giving

$$B(\eta) = [1 - K(\eta)]^{-1} g(\eta) \qquad (3.54)$$

The solution $B(t)$ of the integral equation (3.34) is then the inverse Laplace transform of the equation (3.38). Using the approximate Binomial expansion.

$$(1 - x)^{-1} \approx 1 - x \qquad (3.55)$$

then $[1 - K(\eta)]^{-1} \approx [1 - K(\eta)]$

$$B(\eta) = [1 - K(\eta)]g(\eta)$$

in the equation (3.53) gives:

$$B(\eta) = g(\eta) - K(\eta)\, g(\eta) \qquad (3.56)$$

and $B(t)$ is thus given by:

$$B(t) = g(t) - (K * g)(t) \qquad (3.57)$$

Next we substitute for $B(t)$ as given by (3.55) into (3.53) to obtain

$$P(t) = \int_0^t \left[g(a) - \int_0^a K(t-u)g(u)du \right] \exp\left\{ -\int_0^{t-a} \mu(s)ds \right\} da + \int_0^\infty \phi(a) \exp\left\{ -\int_0^t \mu(a+s)ds \right\} da \qquad (3.58)$$

which gives the population $P(t)$ profile of the community over time t.

From (3.44), (3.46) and (3.55), B(t) is given by:

B(t) = g(t) – (K*g) (a)

$$P(t)=\int_0^t [g(a)-(K*g)]\exp\left\{-\int_0^{t-a}\mu(s)ds\right\}da+\int_0^{\infty}\phi(a)\exp\left\{-\int_0^{t}\mu(a+s)ds\right\}da$$

$$=\left[\int_0^t g(a)-\int_0^t (K*g)(a)\right]da\ \exp\left\{-\int_0^{t-a}\mu(s)ds\right\}da+\int_0^{\infty}\phi(a)\exp\left\{-\int_0^{t}\mu(a+s)ds\right\}da \quad (3.59)$$

recall that

$$g(t)=\int_0^{\infty}\beta(a-t)\phi(a)\exp\left\{-\int_0^{t}\mu(a+s)ds\right\}da \quad (3.60)$$

$$K(a,t)=\beta(t-a)\exp\left\{-\int_0^{t-a}\mu(s)ds\right\} \quad (3.61)$$

$$B(t)=g(t)-(K*g)(t) \quad (3.62)$$

using equation (3.44) into (3.36) we have

$$B(t)=\int_0^{\infty}\beta(a-t)\phi(a)\exp\left\{-\int_0^{t}\mu(a+s)ds\right\}da-(K*g)(t) \quad (3.63)$$

$$B(t)=\int_0^{\infty}\beta(a-t)\phi(a)\exp\left\{-\int_0^{t}\mu(a+s)ds\right\}da-\int_0^{t}K(t-u)g(u)du \quad (3.64)$$

$$B(t)=\int_0^{\infty}\beta(a-t)\phi(a)\exp\left\{-\int_0^{t}\mu(a+s)ds\right\}da-\int_O^{T}K(t-u)\left[\int_o^{\infty}B(a-u)\phi(a)\exp\left\{-\int_o^{t}\mu(a+s)ds\right\}da\right] \quad (3.65)$$

using (3.27) for K(t-u)

$$B(t)= \int_0^{\infty} B(a-t)\phi(a)\exp\left\{-\int_0^t \mu(a+s)ds\right\}da \tag{3.66}$$

which gives:

$$B(t) = \int_0^{\infty} \beta(a-t)\phi(a)\exp\left\{-\int_0^t \mu(a+s)ds\right\}da \tag{3.67}$$

$$-\int_0^t\left[\beta(t-u)\exp\left\{-\int_0^{t-u}\mu(s)ds\right\}\int_0^{\infty}\beta(a-u)\phi(a)\exp\left\{-\int_0^u \mu(a+s)ds\right\}da\right]du \tag{3.68}$$

Recall equation (3.21) and (3.22)

$$P(t) = \int_0^t B(t-a)\exp\left\{-\int_0^a \mu(s)ds\right\}da+\int_t^{\infty}\phi(a-t)\exp\left\{-\int_0^t \mu(a-t+s)ds\right\}da \tag{3.69}$$

$$B(t) = \int_0^t \beta(a)B(t-a)\exp\left\{-\int_0^a \mu(s)ds\right\}da+\int_t^{\infty}\beta(a)\phi(a-t)\exp\left\{-\int_0^t \mu(a-t+s)ds\right\}da \tag{3.70}$$

we obtain the population distribution with time for the j^{th} component as

(j^{th} as a result of the partition) let $\beta = \beta_{j,}\mu = \lambda_{j,}\phi = \phi_{j,}\infty = \Omega, p = p_j$

substituting the above changes gives equation (3.40)

$$P_j(t) = \int_0^t B_j(t-a)\exp\left\{-\int_0^a \lambda_j(s)ds\right\}da + \int_t^{\Omega}\phi_j(a-t)\exp\left\{-\int_0^t \lambda_j(a-t+s)ds\right\}da \tag{3.71}$$

while the total birth at time λ^- for the j^{th} component will be obtained using equation (3.22).

Let λ_2 =female death modulus, B_2 = female birth modulus, ϕ_2= female population distribution.

B=B_2, $\beta = \beta_{J,} \mu = \lambda_{2,}$ $\phi = \phi_{2,} \infty = \Omega, P = Pj$

Substituting the above changes into equation (3.22) gives equation (3.41)

Also let t-a=a. while the total birth at time t for the j^{th} component will be given by:

$$B_j(t) = \int_0^t \beta_j(a) B_2(t-a) \exp\left\{-\int_0^a \lambda_2(s) ds\right\} da \tag{3.72}$$

$$+\int_t^\Omega \beta_j(a) \phi_2(a-t) \exp\left\{-\int_0^t \lambda_2(a-t+s) ds\right\} da \tag{3.73}$$

transforming variable of integration in (3.40) and (3.41) gives:

$$P_j(t) = \int_0^t B_j(a) \exp\left\{-\int_0^{t-a} \lambda_j(s) ds\right\} da + \int_0^{\Omega-t} \phi_j(a) \exp\left\{-\int_0^t \lambda_j(a+s) ds\right\} da \tag{3.74}$$

also let a =t-a, t= 0, $\beta_{j(a)} = \beta_j(a-t)$

$$B_j(t) = \int_0^t \beta_j(t-a) B_2(a) \exp\left\{-\int_0^{t-a} \lambda_2(s) ds\right\} da + \int_0^{\Omega-t} \beta_j(a-t) \phi_2(a) \exp\left\{-\int_0^t \lambda_2(a+s) ds\right\} da \tag{3.75}$$

Expressing (3.43) in operator form, gives

$$B_j(t) = g_j(t) + \int_0^t K_j(a, t) B_2(a) da \tag{3.76}$$

since $B(t) = g(t) + \int_0^t K(a, t)\, B(a)da$ (3.25) is the general form of the integral equation for both male and female.

Comparing (3.44) and (3.43) cross multiplying. B_2 is used in (44) because it is a female dominant model.

$$g_j(t) = \int_0^{\Omega-t} \beta_j(a-t)\phi_2(a)\exp\left\{-\int_0^t \lambda_2(a+s)ds\right\}da \tag{3.77}$$

$$K_j(a, t) = \beta_j(t-a)\exp\left\{-\int_0^{t-a} \lambda_2(s)ds\right\} \tag{3.78}$$

The expression for $B_2(t)$ from (3.44) in an operator form is given by let j =2, so that

$$B_2(t) = g_2(t) + \int_0^t K_2(a, t)B_2(a)da \tag{3.79}$$

Thus the explicit solution of $B_2(t)$, equation (3.31) to (3.36) gives (3.80)

$$B_2(t) = g_2(t) - (K_2 * g)(t) \tag{3.81}$$

from (42)

$$P_2(t) = \int_0^t B_2(a)\exp\left\{-\int_0^{t-a} \lambda_2(s)ds\right\}da + \int_0^{\Omega-t} \phi_2(a)\exp\left\{-\int_0^t \lambda_2(a+s)ds\right\}da \tag{3.82}$$

Substituting B_2 (t) as given by (3.48) into (3.48) gives (3.49) so the population distribution for the female component is given by

$$P_2(t) = \int_0^t \left[g_2(a) - \int_0^a K_2(t-u) g_2(u) du \right] \exp\left\{ - \int_0^{t-a} \lambda_2(s) ds \right\} da \tag{3.83}$$

$$+ \int_0^{\Omega - t} \phi_2(a) \exp\left\{ - \int_0^t \lambda_2(a+s) ds \right\} da \tag{3.84}$$

while the expression for the total birth for the male component from (3.44) is given by:

$$B_1(t) = g_1(t) + \int_0^t K_1(a, t) B_2(a) da \tag{3.85}$$

substituting B_2 (t) as given by (3.48) into (3.50) gives (3.51) which is explicitly given by

$$B_1(t) = g_1(t) + \int_0^t K_1(a, t) [g_2(a) - (K_2 * g)(a)] da \tag{3.86}$$

or

opening brackets in (3.51) to get (3.52)

$$B_1(t) = g_1(t) + \int_0^t K_1(a, t) g_2(a) da - \int_0^t K_1(a, t)(K_2 * g)(a) da \tag{3.87}$$

recall from (3.42) that

$$P_j(t) = \int_0^t B_j(a) \exp\left\{ - \int_0^{t-a} \lambda_j(s) ds \right\} da + \int_0^{\Omega - t} \phi_j(a) \exp\left\{ - \int_0^t \lambda_j(a+s) ds \right\} da \tag{3.88}$$

substituting (3.52) into the above to get and giving the population distribution with time for the male component as

$$P_1(t) = \int_0^{\Omega-t} \phi_1(a)\exp\left\{-\int_0^{t}\lambda_1(a+s)ds\right\}da + \int_0^{t} g_1(a)\exp\left\{-\int_0^{t-a}\lambda_1(s)ds\right\}da$$

$$+\int_0^{t}\left[\int_0^{a} K_1(u,\, a)g_2(u)du\right]\exp\left\{-\int_0^{t-a}\lambda_1(s)ds\right\}da$$

$$-\int_0^{t}\left[\int_0^{a} K_1(u,\, a)\left[\left(K_2 * g_2(u)\right)\right]du\right]\exp\left\{-\int_0^{t-a}\lambda_1(s)ds\right\}da \qquad (3.89)$$

or

$$P_1(t) = \int_0^{\Omega-t} \phi_1(a)\exp\left\{-\int_0^{t}\lambda_1(a+s)ds\right\}da + \int_0^{t} g_1(a)\exp\left\{-\int_0^{t-a}\lambda_1(s)ds\right\}da$$

$$+\int_0^{t}\left[\int_0^{a} K_1(u,\, a)g_2(u)du\right]\exp\left\{-\int_0^{t-a}\lambda_1(s)ds\right\}da$$

$$-\int_0^{t}\left[\int_0^{a} K_1(u,\, a)\left[\int_0^{u} K_2(t-v)g_2(v)dv\right]du\right]\exp\left\{-\int_0^{t-a}\lambda_1(s)ds\right\}da \qquad (3.90)$$

CHAPTER FOUR

4.0 RESULTS

4.1 Graphical Simulation of Male and Female Population

In order to obtain the graphical profiles of the male and female components of the model, we used the equations for male and female to generate the graphs i.e

$$P_1(t) = \int_0^{\Omega-t} \phi_1(a)\exp\left\{-\int_0^t \lambda_1(a+s)ds\right\}da + \int_0^t g_1(a)\exp\left\{-\int_0^{t-a} \lambda_1(s)ds\right\}da \qquad (4.1)$$

$$+\int_0^t\left[\int_0^a K_1(u,\ a)g_2(u)du\right]\exp\left\{-\int_0^{t-a}\lambda_1(s)ds\right\}da$$

$$-\int_0^t\left[\int_0^a K_1(u,\ a)\left[\int_0^u K_2(t-v)g_2(v)dv\right]du\right]\exp\left\{-\int_0^{t-a}\lambda_1(s)ds\right\}da$$

$$P_2(t) = \int_0^t\left[g_2(a) - \int_0^a K_2(t-u)g_2(u)du\right]\exp\left\{-\int_0^{t-a}\lambda_2(s)ds\right\}da \qquad (4.2)$$

$$+\int_0^{\Omega-t}\phi_2(a)\exp\left\{-\int_0^t\lambda_2(a+s)ds\right\}da$$

With these the population distributions with time are given below.

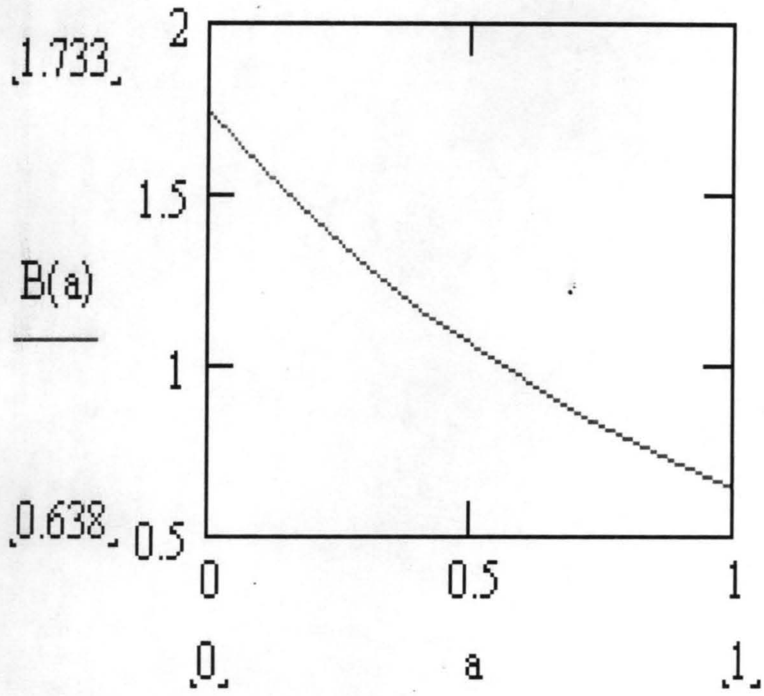

Figure 4.1 Showing the Birth Rate Profile

i.e younger members of population contribute higher than the older members w.r.t new birth

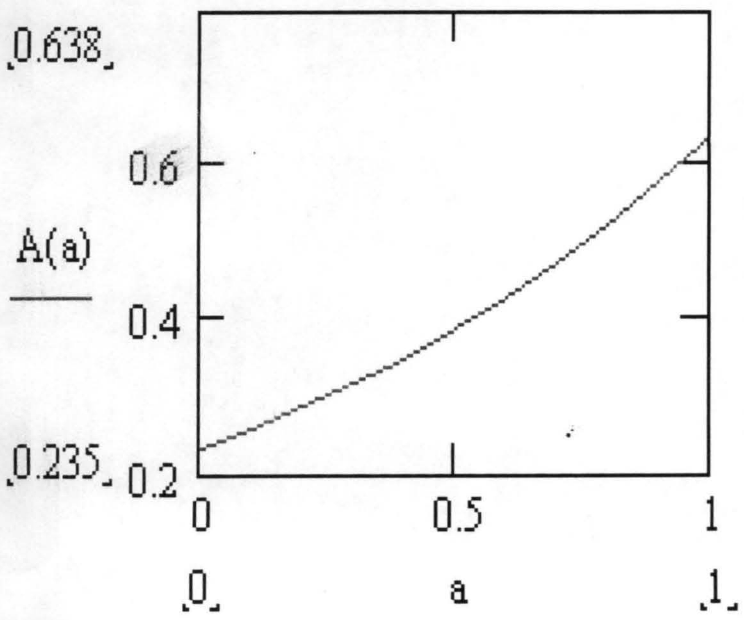

Figure 4.2 Showing Death Module Profile

i.e the elderly have higher death rate than the younger members of the population for male and female

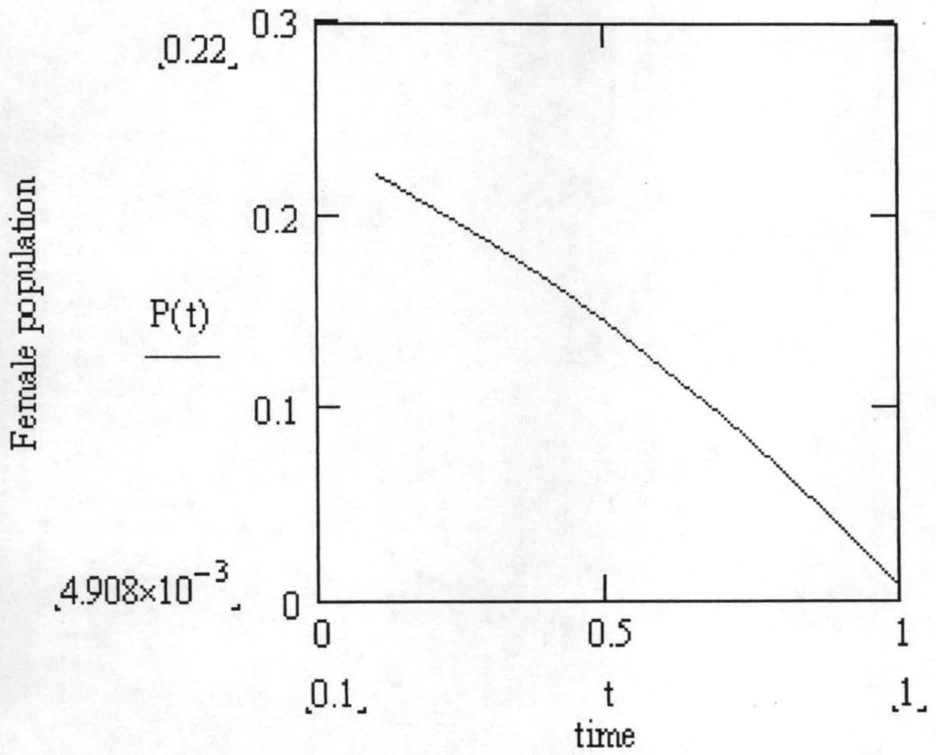

Figure 4.3 showing Female population over time

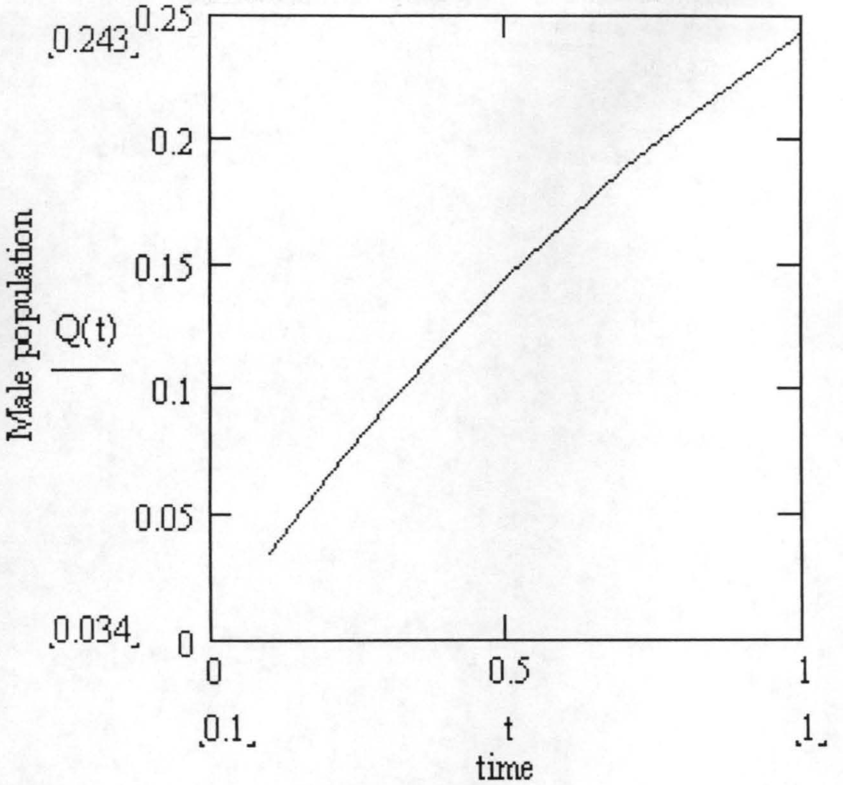

Figure 4.4 showing male population over time

CHAPTER FIVE

5.0 DISCUSSION OF RESULTS, SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Discussion of Results

In this section, we present our discussions/remarks based on the results obtained in the work. We developed a system of partial differential equations to study the dominance population model. We further present the one-dimensional age-structured non-linear population model of Gurtin and MacCamy to obtain the analytical solution of the model equations. The Matlab software was used to simulate the population profiles for the male and female compartments. however, from figure 4.1, it was observed that the birth modules is low for early and later age groups, while the middle age groups contribute significantly to the population growth via new births.

Also, from figure 4.2 which is the death module profile, it was observed that the elderly age group have higher death rate than the middle age group of the population for both male and female, putting other factors that could influence untimely death aside e.g war, accidents, polities etc, such events are not considered here, we assumed an ideal situation. Form this two graphs of birth and death modulus of the population, the male and female graphs were also considered resulting to the same facts in population projection for both male and female compartments.

5.2 Summary

Chapter one gives the introduction to the entire work where we discussed population growth rate, as its influenced by three main factors; fertility, mortality and migration. In this work we consider the influenced of the female component on population dynamics. Population projection provides future estimates of population sizes needed in planning. In Nigeria a number of factors have contributed negatively to having an accurate and reliable demographic data. Successive Nigeria Governments have embarked on population census in which huge amount of money was expected. Population projection can be made by examine the influence of demographic components(i.e birth, death and migration) of population as it affects population from time to time so as to determined future population. Chapter two gives a brief of literature reviews of some selected ideas from individuals on fertility, mortality and migration as it affects population growth while chapter three discussed the material and methods used. For this study the total population $P(t)$ of a community in an ecosystem is partitioned into two compartments of Male $P_1(t)$ and Female $P_2(t)$, with $P(t) = P_1(t) + P_2(t)$. Each compartment P_j, $j = 1,2$ has the density function $q_j(t,a)$ which are functions of time t and age a, such that

$$P_j(t) = \int_0^{\Omega_j} q_j(t,a)da \qquad (5.1)$$

where Ω_j is the life expectancy of the j^{th} compartment. Chapter four discusses the application of the model and results obtained while in chapter five, we give the discussion of results, summary, conclusion and recommendations.

5.3 Conclusion

In the solution of the population distributions for the male and female components of the population, we observe that the female population profile depends on the parameters related to the female only while the male population profile depends on the parameters of both the male and female. Apparently this is the essence of the term female-dominance. It will be of interest to identify the population ecologies where one of the reproductive classes is a domineering determinant factor to the existence of the other. This model may be applied to socio-economic relationship where particular sub-class dominates power/politics, economy or some vital sector. In essence beyond male-female relationship, other human or ecological interactions where dominance is evident can be relevant here.

In order to bring out implication of common trends in mortality; fertility and migration in years in the future, starting in the present population size and structure projections are like microscopes that magnify the differences in a given period and so help with analysis and the understanding of current rates. In Nigeria, population projections are important due to current economic and social transformatio-ns and changes in population dynamics. To this end there is need to embark on a comprehensive study of demography in Nigeria in particular reference to population projections and come up with an informal system that could help in real time information on Nigeria population. also more work is still neede on population projection by applying various methods of solution with respest to mortality, fertility and migration rates.

5.4 Recommendations

From our result, we recommend that further research should be done and Government should formulate public health policies which include:

1. More awareness on family planning methods: since a female is the dominant factor in Reproduction which result to over population, the methods should be with high emphasis especially in Govt. working sectors.
2. Age of marriage of a girl child: there should be a review on certain age limit that a girl child marries.

Early marriages should be discouraged to the minimal.

3. Providing strategic and comprehensive plan towards eradicating the problems associated with population growth rate.
4. Strengthening this research work towards the population growth and birth rate of women for both maternal and non-maternal cases.
5. The effect of migrating in and out of a population should be checked and moderated.

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