# The EFFECT OF DROPLETS PARTICLE TRAJECTORIES IN SPRAY DRYERS 

## BY

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## A PROJECT SUBMITTED TO THE DEPARTMENT OF CHEMICAL ENGINEERING, FEDERAL UNIVERSITY OF TECHNOLOGY, minNA

IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE AWARD OF MASTER DEGREE (M.ENG) IN CHEMICAL ENGINEERING

## DECLARATION

, ALIYU MOHAMMED (M.ENG/SEET/2001/547), declare that this thesis, 'THE EFFECT OF DROPLETS PARTICLE TRAJECTORIES IN SPRAY DRYERS", presented for the award of Master Degree of Engineering in the Department of Chemical Engineering has not been presented elsewhere for any other degree.

## CERTIFICATION

This thesis titled "The Effect of Droplets Particle Trajectories in Spray Dryers" by Aliyu Mohamed meets the regulations governing the degree of M.Eng. of the Federal University of Technology, Minna and is approved for its contribution to scientific knowledge and literary presentation.


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## EXTERNAL EXAMINER



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## DEDICATION

This project work is dedicated to Almighty Allah, my parents, and beloved wife

## ACKNOWLEDGEMENT

I am particularly indebted to my project supervisor, Dr. K. R. Onifade for his encouragement and his many helpful suggestion and guidance during the period of this work. I am also grateful to Dr. Duncan A. for his assistant whenever he is approached.

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#### Abstract

In many unit operations involving particulate systems, it often becomes desirable to determine the paths described by the components parts either for purposes of design or improved operation, in particular, knowledge of spray trajectories in spray dryer where the slurry sprayed and drying agent (e.g. air ) are countercurrent, can improve the design of the column and choosing the optimum droplet and drying parameters. This work calculates the particle trajectories of different sized droplets sprayed from a rotary atomizer with different initial velocities. The droplets diameters employed include (a) 0.000027 m (b) 0.00003465 m (c) 0.0000446 m (d) 0.0000574 m (e) 0.00007445 m (f) $0.00009855 \mathrm{~m}(\mathrm{~g}) 0.00013655 \mathrm{~m}$ (h) 0.00021095 m and (i) 0.0004128 m sprayed with the different velocities of (a) $3.656 \mathrm{~m} / \mathrm{s}$ (b) $4.478 \mathrm{~m} / \mathrm{s}$ and (c) $5.171 \mathrm{~m} / \mathrm{s}$. The results showed that the greater the droplet diameter and initial velocity, the greater the horizontal and vertical components of the distance traversed by the droplets and time required before free fall. This also translates to requirement for bigger chamber dimension of the spray dryer.


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## NOMENCLATURE

$\mathrm{A}=$ particle area projected normal to direction of motion, $\mathrm{m}^{2}$.
$\mathrm{A}_{1}=$ cross sectional area available for liquid flow at pressure nozzle, $\mathrm{m}^{2}$
$\mathrm{A}_{\mathrm{a}}=$ cross sectional area available for air flow in pneumatic nozzle, $\mathrm{m}^{2}$
Cor $\mathrm{CD}=$ drag coefficient $=2 \mathrm{~F} / \mathrm{p} \mathrm{u}^{2} \mathrm{~A}$
$\mathrm{C}_{\mathrm{f}}=$ heat capacity of feed, $\mathrm{J} / \mathrm{Kg} .{ }^{\circ} \mathrm{k}$
$\mathrm{C}_{\mathrm{ps}}=$ heat capacity of solid, $\mathrm{J} / \mathrm{kg}^{\circ}{ }_{\mathrm{k}}$
$\mathrm{C}_{\mathrm{s} i}, \mathrm{C}_{\mathrm{s} 2}=$ heat capacity of inlet and outlet humid air, $\mathrm{J} / \mathrm{kg} .{ }^{\circ} \mathrm{k}$
$\mathrm{D}_{0}=$ Diameter of nozzle
$\mathrm{D}_{\mathrm{r}}=$ diffusivity, $\mathrm{m}^{2 / \mathrm{s}}$
$\mathrm{d}=$ mass median drop diameter, $\mu \mathrm{m}$
di $=$ diameter of each class of droplets, $\mu_{\mathrm{m}}$
$\mathrm{dm}=$ diameter of largest droplet, $\mu \mathrm{m}$
$\mathrm{dv}_{\mathrm{s}}=$ mean sauter diameter, $\sum n i \mathrm{di}^{3} / \sum n \mathrm{idi}^{2}, \mu \mathrm{~m}$
$\mathrm{E}_{\mathrm{x}}=$ amount of evaporation up to section $\mathrm{x} \mathrm{kg} / \mathrm{s}$
$\mathrm{g}=$ acceleration due to gravity
$\mathrm{G}=$ man velocity, $\mathrm{kg} / \mathrm{s}$ ( m of wetted periphery)
$\mathrm{H}=$ average absolute humidity at any section, kg of water vapor $/ \mathrm{kg}$ of dry air
$\mathrm{H}_{\mathrm{w}}=$ saturation humidity, kg of water vapor $/ \mathrm{kg}$ of dry air
$\mathrm{k}_{\mathrm{a}}=$ thermal conductivity of air, J/m.s. ${ }^{\circ} \mathrm{k}$
$\mathrm{L}_{\mathrm{w}}=$ wetted periphery of centrifugal- disk atomizer, m
$\mathrm{M}_{\mathrm{e}}=$ mass flow rate of entrainment, $\mathrm{kg} / \mathrm{s}$
$\mathrm{n}_{\mathrm{i}}=$ number of each dam of droplets
$\mathrm{N}_{\mathrm{u}}=$ nusselt Number, hdi / Ka
$r=$ radial position of droplet, $m$
$\mathrm{Rd}=$ radius of contractual - disk atomizer, m
$\mathrm{S}_{\mathrm{h}}=$ Sherwood number, kdiM/Dvpa
$t_{a}=$ temperature of drying air, ${ }^{\circ} \mathrm{k}$
$\mathrm{t}_{\mathrm{p}}=$ product temperature, ${ }^{\circ} \mathrm{k}$
$\mathrm{t}_{\mathrm{s}}=$ droplet temperature, ${ }^{\circ} \mathrm{k}$
$\mathrm{t}_{\mathrm{w}}=$ wet bulb temperature of drying air, ${ }^{\circ} \mathrm{k}$
$\mathrm{v}=$ volume of feed sample, $\mu \mathrm{m}^{3}$
$\mathrm{V}_{\mathrm{at}}, \mathrm{v}_{\mathrm{ar}}, \mathrm{V}_{\mathrm{av}},=$ absolute value of tangential, radial and axial velocities of air respectively, $\mathrm{m} / \mathrm{s}$
$\mathrm{V}_{\mathrm{c}}=$ velocity of air on the center-line of the spray, $\mathrm{m} / \mathrm{s}$
$\mathrm{V}_{\mathrm{f}}^{\prime}=$ cumulative volume fraction less than given size
$\mathrm{V}_{\mathrm{o}}=$ average velocity of liquid leaving atomizer and in the case of pneumatic nozzle air velocity at nozzle exit, m/s
$\mathrm{V}_{\text {rel }}=$ relative velocity between the air stream and the liquid stream at the nozzle m/s
$\mathrm{V}_{\mathrm{t}}, \mathrm{V}_{\mathrm{r}}, \mathrm{V}_{\mathrm{v}}=$ absolute values of tangential, radial and axial velocities of droplet respectively, $\mathrm{m} / \mathrm{s}$
$\mathrm{W}_{1}=$ flow rate of feed $\mathrm{kg} / \mathrm{s}$
$\mathrm{W}_{\mathrm{A}}=$ atomizing air flow rate, $\mathrm{kg} / \mathrm{s}$
$\mathrm{W}_{\mathrm{g}}=$ flow rate of drying air, (free air, measured at $294^{\circ} \mathrm{k}$ and 1 atm ) kg of drying air /s
$\mathrm{W}_{\mathrm{w}}=$ chamber capacity, in kg of water evaporated/s
$\mathrm{x}=$ Vertical distance from nozzle
$\mathrm{x}=$ dimensionless distance

## GREEK

$\Delta d=$ the spread of the droplet size distribution
$\delta=$ parameter to express the uniformity of spray
$\Gamma=$ gamma function
$\rho_{\alpha}=$ density of the surrounding of the jet, $\mathrm{Kg} / \mathrm{m}^{3}$
$\rho=$ density at nozzle exit
$\mathrm{w}=$ angular velocity of droplet, $\mathrm{rad} / \mathrm{s}$

## SUBSCRIPTS

$1=$ at the inlet conditions of spray dryer
$2=$ at the outlet condition of spray dryer
$a=$ of drilling air
$\mathrm{O}=$ at nozzle exit
$i=$ pertaining to $i$ ith class of droplets
in $=$ at initial conditions
$\mathrm{p}=$ of the product
$x=$ at a vertical distance $x$ below the nozzle
$\mathrm{w}=$ web bulb temperature of the air
$\mathrm{d}=$ at datum temperature

## CHAPTER ONE

### 1.0 INTRODUCTION

Spray drying is a process of converting a feed that is in the form of slurry, solution or paste into a dry powder, granules or agglomerates. This is achieved by atomizing a feed into a hot drying medium usually air which rapidly evaporates the moisture in the feed (See Figure 1.1). Spray drying is a widely used dehydration method. It is ideal for drying materials that are heat sensitive because the process is rapid even though the drying medium is at comparatively low temperatures.

Much research work has been done on processes in spray drying, with a view to gaining deeper knowledge of the phenomena in the dryer helping a more efficient use of existing equipment performing a better design of a spray dryer equipment (Toper, 1975). The growing importance in spray drying is abundantly evident from the ever-increasing number of industrial applications in the production of pharmaccuticals, detergents, food products, pigment, ceramics and a large number of organic and inorganic chemical compounds. Inspite oi these impressive developments and of the large number of fragmented experimental studies, which have appeared in the technical literature during the past three decades, the design of spray dryers has remained largely empirical. It is still mainly based on the extensive experience and the rest body of operating data, which the manufacturers have acquired over the years (Beard and Pruppacher, 1969).


Figure 1.1 Schematic diagram of a typical spay dryer (Gauvin and Kafta, 1975).

Prediction of droplet trajectories and residence times is essential for the sound design and efficient operation of spray dryers. Such knowledge can also be applied in the design of a number of other equipment involving the contacting of a dispersed phase of droplets or particles with a conveying gas such as spray coolers, gas scrubbers and absorbers, cyclone evaporators, pneumatic transport sectors and combustion devices involving fuel sprays (Katta and Gauvin, 1975)

In many unit operations involving particulate systems, it often becomes desirable to determine the paths described by the component parts either for purposes of design or improved operation. The fundamentals of particle motion are generally expressed in a drag coefficient (C) vs. Reynolds number (Re) plot, first suggested by Rayleigh. (Lapple and Shepherd, 1940.) Over the range of Reynolds number generally encountered in unit operations, the flow around suspended bodies may be divided into three regions - streamline, intermediate and turbulent. At low Reynolds numbers, streamline or viscous flows exist around the particle, while at high Reynolds numbers the flow around the particles is completely turbulent and the drag coefficient is approximately constant. Between the region of streamline and turbulent motion there exist an intermediate or transition region, in which the flow partakes of both forms to some extent

A particle moving in a fluid with an initial velocity will be decelerated by the action of frictional forces and will be either accelerated or decelerated by the action of external forces such as gravity. The net force acting on a particle
in a given direction will be the vectorial sum of the frictional force and the external forces. The frictional drag in each case will depend inherently- on the instantaneous velocity of the particle

The general procedure for the determination of the equation of motion involves the determination of a velocity- time relation for the particle motion in component directions. Once this has been obtained, the distance traveled in a component direction may be obtained as a function of the time by a simple graphical integration under a curve in which the respective velocity component is plotted against time, according to the relation
$\mathrm{S}=\int u d t$
Where S and U are vectors in the same direction

The calculation of particle trajectories in the design of dryer is based on the calculation of the path a droplet assumed when it is being atomized. This study is carried out due to the fact that the design was based on the criterion that the dimensions of the chamber shall be such that the largest droplet in the initial spray shall be dry (or, more exactly, shall reach the specified residual moisture content) before it reaches the chamber walls (Katta and Gauvin, 1976).

### 1.1 AIM

The aim of this work is to be able to calculate the particle trajectories of a cement droplets sprayed from an atomizer for a given droplet diameters and initial velocities.

## CHAPTER TWO

### 2.0 LITERATURL REYIEW

A spray dryer designer often needs to predict behavior or performance of dryers before they are built. For that models are necessary. The design of a spray dryer is usually accomplished by modifications of previous design and/or by application of empirical or simplified expressions for droplet trajectories and drying rates.

A typical dryer operates at air temperatures from $100^{\circ} \mathrm{C}-700^{\circ} \mathrm{C}$ with droplets less than $1000 \mu \mathrm{~m}$ and droplets Reynolds number less than 1000. The feed rate of water is of the order of $5 \%$ of the air flow rate (Crowe and Arnason, 1978).

### 2.1 PROCESS STAGES OF SPRAY DRYING

The final product of spray drying tower is powder. There are lots of different factors that influence the powder quality. Two most important of them are the mean size of particles and the size distribution. Both determine the bulk density of the dry powder. Drying process can be generally divided into the following stages:

1. Atomization of the solution or slurry to form droplets
2. Spray-Air contact
3. Evaporation of liquid from droplet
4. Separation of dried product from the gas medium

### 2.1.1 ATOMIZATION

The term atomization is used in its usual sense of break up of bulk liquid into droplets. Although details of the drop forming process vary widely among the different types of apparatus, certain events normally occur in all of these processes.

1. The applied force, whether due to pressure or rotation, separate protuberances; filaments, or sheets of liquid from the bulk liquid.
2. These protuberances, filaments, or sheets are broken up (owing to their inherent instability and to interaction with a mass of gas or liquid) into small particles.
3. These particles assume spherical shape if they are small enough. Or, if large enough to be unstable, they break into smaller masses, which then becomes spherical.

Atomization is the most important process in the spray drying operation, since it governs the size of the product, and the initial drop size distribution of the spray generated by the atomizer forms the basis of the chamber design.

The three principal types of atomizers used today are: centrifugal disk atomizers; pressure nozzles; and pneumatic nozzles. Centrifugal disk atomizers are generally employed for high capacities owing to their flexibility and ease of maintenance. Pressure nozzles are preferred for highly viscous feeds and where multiple atomizers are desired. Pneumatic nozzles are used only for small capacities due to the high cost of compressed air and their low efficiency.

### 2.1.1.1 CENTRIFUGAL DISK ATOMIZERS

Centrifugal disk atomizers are by far the most commonly used atomizers today. Masters (1972) presented a large number of correlations available in the literature to predict the Droplet Size Distribution (DSD) from various types of such atomizers. A very useful correlation is that obtained by Friedman et - al (1952). They correlated the Sauter mean diameter to the disk radius in terms of dimensionless groups to obtain

$$
\frac{d v s}{R d}=0.4\left(\frac{G}{\rho_{f} N R_{d^{2}}}\right)^{0.6}\left(\frac{\mu}{G}\right)^{0.2}\left(\frac{\sigma \rho_{f} L_{w}}{G^{2}}\right)^{0.1} .
$$

They found the following relationship between the largest droplet size and the Sauter mean diameter:

$$
d m=3 d v s
$$

## ROTARY ATOMIZERS

The rotary atomizers utilizes centrifugal force to accelerate liquid to such a velocity that the cohesive surface tension and viscous forces are overcome and disruption takes place since the accelerating force can be independently controlled. This type of atomizer is extremely versatile. It can handle successfully wide range of feed rate with liquids having a wide range of property, from thin solutions to highly viscous gels or slurries.

The wide variety of existing designs can be classified into three groups:

1. Flat disk (Figure 2.1(a))
2. Bowls or saucers (Figure 2.1(b))
3. Vaned disks or slotted wheels (Figure 2.1(c))


Figure 2.1 Characteristic rotary atomizers: (a) Sharp-edge flat disc; (b)Bowl; (c) Vaned disc; and (d) Air-blast bowl atomizer. (Fraser, et al, 1957).

In each classification the diameter may vary from 0.25 m to 0.45 m with rotary speeds up to $60,000 \mathrm{rpm}$. Disk speeds for diameter of 1 ft and upwards, are of the order $12,000 \mathrm{rpm}$, and such large disk have capacities for feed liquor of more than 5 tons an hour.

When highly viscous liquids are being atomized, it is advantageous to use spinning bowls in conjunction with an additional gas blast around the periphery for further atomization (see Figure 1(d)).

Under these conditions, the task of the disk is to film the liquid so that the energy transfer between gas and liquid can occur with high efficiency.

## Liquid Flow In Rotary Atomizers

When liquid is centrifuged from the center of a rotating surfaces, it's velocity at any point on the surface may be resolved into a radial and tangential component. For a frictionless liquid, the radial velocity at any point is equal to the tangential velocity at that point (i.e. $\pi N D$ ). The resultant velocity is therefore equal to $\sqrt{2} \pi \mathrm{ND}$, and the maximum energy imparted to the liquid is given by $m \pi^{2} N^{2} D^{2}$.

$$
\begin{aligned}
& \text { Where } m=\text { mass flow rate; }(\mathrm{kg} / \mathrm{s}) \\
& \mathrm{N}=\text { rotary speed; (radian per second) and } \\
& \mathrm{D}=\text { rotor diameter. }(\mathrm{m})
\end{aligned}
$$

The theoretical power required to produce thin energy can thus be written as

$$
\text { Power }(\mathrm{P})=42.5 \times 10^{-12} \mathrm{~m} \mathrm{~N}^{2} \mathrm{D}^{2} \text { horse power. }
$$

This relation is plotted in Figure 2.2 and shows the power per unit mass flow rate as a function of disk diameter and speed. At low speeds and feedrates,


Figure 2.2 Theoretical power requirements above no-load conditions for various diameter discs and rotary speeds (Fraser, et al, 1957).
when there is no slip, the tangential velocity of the liquid leaving a rotating surface is equal to its peripheral velocity. At rotary speeds and feed rates greater than approximately 1000 rpm and $4.2 \mathrm{lb} / \mathrm{hr}$ respectively, polished flat disks do not impart maximum tangential velocity because of slip between the disk surface and the liquid, although this loss in velocity is reduced with increase of disk diameter. They are therefore unstable for efficiently atomizing liquids at practical flow rates. Slip is considerably reduced with saucer-or-bowl-shaped disks where the liquid is being continually forced against the rotor surface, but above a critical flow rate per unit wetted perimeter, slip tends to increase. The condition for severe slip occur when the value of the relation $m / \eta \pi D$ is greater than 1440 Fraser et al (1957)

$$
\text { Where } \quad \begin{aligned}
\mathrm{m} & =\text { mass of flow rate, } \mathrm{kg} / \mathrm{s} ; \\
\eta & =\text { Viscosity }, \text { Nsec } / \mathrm{m}^{2} ; \text { and } \\
\mathrm{D} & =\text { Disk diameter, } \mathrm{m} .
\end{aligned}
$$

Slip can completely be eradicated by constructing the disk with radial vertical vanes so that the liquid is forced against the leading edge of each vane. Although this design is more efficient, it is more complicated and more difficult to balance dynamically. Care must also be taken to ensure that impeller action is at a minimum; otherwise considerable power may be wasted in pumping air. This type of design is unsuited for erosive liquids, since severe erosion is caused as the liquid impinges on the rear edge of the vanes. With spinning bowls, erosion can be minimized by placing relatively cheap expendable liners over the main bowl.

## Processes Of Drop Formation From Rotors

The manner by which drops are produced from rotary atomizers depend on the disk diameter, the rotary speed, the efficiency of energy transmission from the rotary member to the liquid, the liquid flow rate and it's surface tension, viscosity and density, and finally on the degree of wetting of the surface. Atomization occurs when surface tension and viscous forces are overcome by centrifugal forces, and at high peripheral velocities, this process is aided by air friction. Three stages of drop formation can be distinguished.

In the first stage, liquid fed at a small rate near the axis of the rotor spreads out towards the periphery where it forms a stationary toroidal ring. As liquid continues to flow into the ring, it's inertia increases and overcome the restraining surface tension. Disturbances appear on the outer edge and grow in size until liquid is centrifuged off as discrete drops. See figure 2.3.The second stage of drop formation seen in Figure 2.4(a) occurs when the feed rate is increased. The retaining thread grows in thickness and, resulting from the longer time of breakdown, forms long jets. Whereas the first stage of drop formation is essentially an intermittent process, atomization is now fairly continuous with the jets uniformity of diameter of the threads depend upon the peripheral velocity and becomes more uniform as the velocity is increased.

At high peripheral velocities (Figure 2.4(d)), the sheet is disintegrated by air action. Holes and tears appear on the sheet, and the resulting fragments are rapidly atomized as they start to contract into threads. With increase in


Figure 2.3 Process of formation of drops of uniform size from a spinning disc atomizer (a-r), ( Fraser, et al, 1957).
rotary speed, the region of disintegration recedes back towards the disk and thus an approximately constant sheet thickness is maintained at break-up.

## Drop Sizes Produced From Rotary Atomizer

The mean drop size, defined as surface mean diameter, is the diameter of a drop with the same surface-volume ratio as the whole spray cloud. It increases with increased liquid flow rate, viscosity and surface tension and diminishes with increased rotary speed, disk diameter, wetted periphery and liquid density. The relations depend on the operating conditions.

Disk operating at low feedrates of $30 \mathrm{lb} / \mathrm{hr}$, and forming a wide drop spectrum at the periphery can be made to produce homogeneous sprays if the drops are separated in sizes by gravity. At high and more practical feedrates, and especially with high peripheral speeds, turbulent air streams entrained around the disk inhibit a clean separation.

Because of difficulties involved in sampling dense clouds of heterogeneous liquid drops, formulae relating their sizes to the operating variables are only approximate and vary from one worker to another Fraser (1957).

One formula, which correlates data over a wide range of variables, can be written as:
$\operatorname{SMD}=7.4 \times 10^{3}(1 / \mathrm{N})^{0.6}(1 / \rho)^{0.5}(\eta \mathrm{~m} / \mathrm{D})^{0.2}(\gamma / \mathrm{x})^{0.1}$
Where
SMD = Surface mean diameter, m;


Figure2.4 Drop formations from a spinning disc atomizer (a-d)
(Fraser, et al, 1957).
$\mathrm{N}=$ rotary speed, radian per second;
$\mathrm{n}=$ number of vanes,
$b=$ height of vanes, $m$;
$\mathrm{m}=$ mass flowrate, $\mathrm{kg} / \mathrm{s}$;
$\mathrm{D}=$ disk diameter, m ;
$\rho=$ liquid density, $\mathrm{kg} / \mathrm{m}^{3}$
$\gamma=$ surface tension, $\mathrm{mN} / \mathrm{m}$; and
$\mathrm{x}=$ wetted periphery, m .
$=\pi \mathrm{D}$ for flat disks and bowls
$=\mathrm{nb}$ for vaned disks
This equation is valid for the following operating range:
$\mathrm{D}=0.05-0.20(\mathrm{~m}) ;$
$\mathrm{N}=90-1885$ (radian per second);
$\mathrm{M}=0.004-0.510(\mathrm{~kg} / \mathrm{s})$
$\gamma=74-100(\mathrm{mN} / \mathrm{m})$
$\rho=999.55-1409.62\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$ and
$\eta=.0011-9.9\left(\mathrm{Nsec} / \mathrm{m}^{3}\right) \mathrm{cp}$
The maximum drop size for a very wide range of flow rates is approximately three times the mean drop size as given by equation 2.4. The equation shows that for a given liquid, the most important factors influencing the drop size are the rotor speed and diameter. The length of wetted perimeter has a relatively small effect.

## Drop Penetration In The Atmosphere

With pressure nozzles and blast atomizers, the spatial distribution of the resulting spray cloud can, within certain limits, be reasonably controlled. In rotary atomizers, except where peripheral air blast is employed (Figure 2.1(d)), there is no control of the drop cloud after it has left the periphery. A rotary atomizer with a vertical axis ejects drops in a horizontal direction. The drops soon decelerate by air friction and begin to fall under gravity. Small drops will decelerate earlier than larger ones and thus fall away nearer the atomizer. Under still ambient conditions it is thus possible to deposit a spectrum of drops continuously ranging from small sizes near the rotor axis to large sizes away from it.

The size and distance of projection of a drop depend on its initial velocity of ejection. As the rotary speed is increased, the drop gain further momentum and is projected farther away. However, since the drop diameter is an inverse function of the rotary speed, drag forces increase until at a critical diameter, a maximum distance is reached. Where after, the distance is progressively reduced even though drops have greater initial velocities. The distance of projection is also a function of the feed rate. An increased liquid flow increases the drop diameter and thus it's path in the air (Fraser1957).

### 2.1.1.2 PRESSURE NOZZLES

The drop size distribution from pressure nozzles has been found (Marshall, 1954) to follow the Rosin-Rammler relationship
$1-V_{f} 1=\exp \left\lfloor-0.693\left(d_{i} / d_{p}\right) \delta\right\rfloor$
Where $\mathrm{dp}=$ partial diameter above which $36.79 \%$ of the volume or mass of the spray has drops of large diameter,
$\delta=$ Constant which depends on the uniformity of the spray sauter mean diameter

$$
\mathrm{dv}_{\mathrm{s}}=\mathrm{dp} / \Gamma(1-1 / \delta) .
$$

Several correlations are available in the literature to predict the mean droplet size from various types of centrifugal pressure nozzles (masters, 1972).

Most of atomizers employed in this group employ nozzles design, which force feed through an orifice. The pressure energy within this liquid bulk is converted into kinetic energy to form thin moving liquid sheets. The mechanism of the liquid sheets break-up near the orifice due to increase in pressure follow the findings of Ohnesorge for simple jets (Master, 1972). They break up to form a spray under the influence of the physical properties of the liquid and the frictional effects of the drying medium. The sheet length is directly proportional to surface tension, pressure, sheet velocity and frictional effects, which increase turbulence.

Two types of sheets are formed, the conical and fan sheets. Conical sheets give rise to conical spray patterns while fan sheets produce fan spray patterns.

## Conical Spray Pattern

Centrifugal pressure nozzles mostly produce conical spray patterns. The conical sheet, which results in the spray, is produced where the liquid is
flowing in radial lines when caused to flow through a narrow divergent annular orifice. In commercial atomizers, the conical sheet lengths are very short and can hardly be seen. Also the process of liquid disintegration proceeds very fast. So it is difficult to distinguish individual phases of liquid breakup experimentally.

The hollow cone spray and the solid cone spray are the two types of conical spray pattern obtained.

## Fan Spray Pattern

Deflector atomizers, impinging jet nozzles and fan spray nozzles, produce fan spray patterns. The deflector atomizer has a circular orifice. Liquid discharge from this orifice is deflected from the nozzle axis by a curved deflector plate. The impinging jet nozzle incorporate a design in which two or more jets is caused to impinge outside the nozzle. In fan spray nozzle with a single orifice design, two streams of liquid are made to impinge behind the orifice to produce sheet in a plane perpendicular to the plane of the streams.

### 2.1.1.3 PNEUMATIC NOZZLES

Pneumatic atomization is a process of producing sprays by the disruptive action of a high-velocity gas upon a liquid jet. Because two fluid streams are involved it is sometimes called two-fluid atomization. Pneumatic nozzles are particularly well suited to produce fine spray, that is, less than $50 \mu$ in mass median diameter. Other methods of atomization such as pressure
nozzles and spinning disks are generally capable of producing sprays of this drop-size range except under extreme operating conditions. However, the finesize production from pneumatic atomizers is accomplished only with relatively modest capacities, several gallons per hour at most. Nevertheless, this type of atomization has an advantage in that liquid and air streams can be controlled independently. Pneumatic nozzles better than pressure nozzles can atomize some viscous fluids and thick suspensions.

Knowledge of the drop-size distribution in sprays from pneumatic nozzles permits predictions of the performance of equipment using such sprays.

The drop size distribution from pneumatic nozzles has been shown to follow (Masters, 1975)

$$
\mathrm{n}_{\mathrm{i}} / \Delta \mathrm{d}=\mathrm{ad}_{\mathrm{i}}{ }^{2} \exp \left(-b d_{\mathrm{i}}{ }^{9}\right) .
$$

Where q was shown to be constant for a given nozzle and should be determined experimentally.

From a mathematical analysis, Nukiyama and Tanasawa,(1940) determined the relations between the constants $a$ and $b$ and $d v_{s}$ for different values of q . The following relations were presented when q had a value of 2 (Masters, 1975)

$$
\mathrm{b}=2.25 / \mathrm{dv}_{\mathrm{s}}^{2} \text { and } \mathrm{a}=1.91 \mathrm{~b}^{3} \mathrm{v} .
$$

Based on the novel experimental technique of spray cooling molten wax or melts of wax polyethylene mixtures, Kim and Marshal(1971) proposed the following volume frequency function for droplets produced by pneumatic atomizer.
$V_{f}^{1}=[16.7 \exp (-2.18 \mathrm{~d} / \mathrm{d})] /[1+6.67 \exp (-2.18 \mathrm{~d} / \mathrm{d})]^{2}$
Where $\mathrm{d}=160.86^{0.41} \mu^{0.32} /\left(\mathrm{V}_{\text {rel }}{ }^{2} \rho_{\mathrm{a}}\right)^{0.53} \mathrm{~A}^{0.36} \mathrm{\rho f}^{0.16}+1573\left(\mu^{2} / \mathrm{ff} \sigma\right)^{0.17}\left(\mathrm{~W}_{\mathrm{A}} / \mathrm{W}_{1}\right)^{\mathrm{n}} /$

$$
\mathrm{V}_{\mathrm{rel}}{ }^{0.54} .
$$

Here $\mathrm{n}=-1$, if $\left(\mathrm{W}_{\mathrm{A}} / \mathrm{W}_{1}\right)<3$ and $\mathrm{n}=-0.5$, if $\left(\mathrm{W}_{\mathrm{A}} / \mathrm{W}_{1}\right)>3$
Although pneumatic atomizers are seldom used in large scale in industrial applications these equation are nonetheless very useful because of the frequent use of this type of atomizing device for preliminary testing of the drying characteristics of a feed. Material it is to be noted (Gauvin and Katta, 1957) that equation (2.10) include certain parameters which are characteristics of the nozzle design and in this way differs from Nukiyama and Tanasawa's well known equation. The same authors also presented a slightly modified form of equation (2.10) to predict the value of $\mathbb{d}$ in the case of concentric double-air nozzle atomizer.

The average drop size from a pneumatic nozzle decreases with increase of air - to - liquid ratio (Burton, 1958; Lewis, 1948; nukiyama, 1940; Radcliff, 1953). This fact appears to be a consistent observation of all investigators,(Kim,1971) even though the degree or extent of it's effect is varied. An additional finding has been that the increase in relative velocity decreases drops size (Nuikyama, 1940; Wigg, 1964).

The effects of nozzle dimensions on drop size reported by various workers, however, appear to be contradictory (Kim, 1971). Thus Gretzinger, (1961) reported a drop size decrease; Nukiyama and Tanasawa (1938), no
effect; Wetzel (1951) and Wig (1964), a drop -size increase with increase with increase of liquid nozzle diameter.

Most investigations (Kim, 1971) to date suggest that the drop sizes from pneumatic nozzles are primarily a function of liquid rate, atomizing gas rate, and viscosity. The atomizing gas pressure is not especially fundamental in evaluating the performance of an atomizer, since the gas velocity density can better express atomization pressure.

### 2.1.2 SPRAY-AIR CONTACT

Knowledge of the manner in which air and spray are contacted and about the flow pattern of both air and the droplets is essential for the design and performance of spray dryer. An accurate prediction of droplet trajectories will also enhance a good estimation of spray drier capacity and efficiency.

Spray - air contact is determined by the position of the atomizer relative to the drying air inlet. There are three principal arrangements (Belcher, 1963 and Masters, 1968), the co-current flow; the counter current flow; and the mixed flow.

In co-current flow, both air and feed enter the spray tower from the same end. In counter-current flow the air and feed enters from opposite ends, and the mixed flow is a mixture of the first two. The product to be dried determines how best to contact the spray with the drying medium. Of the three arrangements counter-current flow has been claimed to give the best performance in most spray drying process because
trajectories for changes in atomization condition for drops produced by low speed disk atomizer.

### 2.1.2.2 FLOW PATTERN OF AIR

## Nozzle zone.

The zone traversed by the droplets as they decelerate from their high initial velocity to the point where they begin to be entrained by the swirling drying gas is designated as the nozzle zone, since in that region the droplets are mostly affected by the atomizing nozzle. It is also characterized by the large amount of drying gas, which is entrained into it from the surroundings. The gas flow patterns which the spray were shown to be similar to those in a gas jet by Rasbash and Stark (1962), Gluckert (1962), Briffa and Dombrowski (1966) and Benatt and Eisenklam(1969) who studied the properties of sprays in detail.

In the case of a pressure nozzle, the decay of the center line velocity of the gas within the spray can be correlated by the following equations based on the work carried out by Gluckert (1962) and Briffa an Dombrowski(1966):
$\mathrm{V}_{\mathrm{c}} / \mathrm{V}_{\mathrm{o}}=3.2\left(\mathrm{D}_{\mathrm{o}} / \mathrm{x}\right)$
$\mathrm{V}_{\mathrm{o}}=$
$\mathrm{W}_{1} /\left(\rho_{\mathrm{f}} \mathrm{A}_{1}\right)$
Where $V_{o}$ is the average velocity of the liquid leaving the pressure nozzle and $\mathrm{A}_{1}$ is the cross-sectional area available for liquid flow at the nozzle. The radius of the air core can be estimated to be half the radius of the nozzle on the basis of the result correlated by Nelson and Stevens (1961) showing the effect of the spray angle on the air core.

The radial velocity of the induced air jet was as Zero on the basis of the work carried out by Benatt and Eisenklan (1969).

Free Entrainment Zone: The zone where the droplet motion is unaffected by the atomizing nozzle and is governed by the entraining drying gas only is designated as the free entrainment zone.

The tangential velocity obtained obtained experimentally by Bank (1975) in the spray drying chamber were represented by
$\mathrm{V}_{\mathrm{at}}=\mathrm{C}_{1}\left(\mathrm{r} / \mathrm{R}_{\mathrm{x}}\right)^{0.5}$.
Where $C_{1}$ is a constant, which depends on the axial distance from the roof. The radius of the chamber at any axis distance x from the chamber in the conical portion can be obtained from
$R_{x}=1.469 R_{c}-0.469 x$
Bank (1957) observed that the flow patterns were not affected by the entrance volumetric flow rate.

The entrainment rates in the nozzle zone of a pressure atomizer can be obtained from the following correlation presented by Bennatt and Eisenkalam (1969) based on both experimental and theoretical studies
$\mathrm{E}=\mathrm{Me} /\left[\left(\mathrm{x}-\mathrm{x}^{1}\right) \sqrt{ } \mathrm{W}_{1} \mathrm{~V}_{0} \rho_{0}\right]$.
$\mathrm{X}^{1}=0.5+1.66\left(\mathrm{~W}_{1} / \rho_{1} \sqrt{ } \mathrm{P}_{1}\right)^{0.5} \operatorname{Cos} \theta$.

And E, the entrainment parameter, is given by

$$
\mathrm{E}=0.96 \tan \theta / \sqrt{ } \cos \theta \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots . . \ldots \ldots . . \ldots \ldots
$$



Figure 2.5 Psychrometric chart (Gauvin and Katta, 1976).
indicated that hollow particles of large diameter, with wall thickness of 5 to $10 \mu$, could be formed.

As a preliminary design consideration, the assumption of surface saturated condition and neglect of a drying falling rate period, with resulting constant droplet surface temperature is probably adequate.

Finally, the instantaneous diameter of any droplet was obtained from the following equation based on the assumption that the droplets are dense:

$$
\mathrm{d}_{\mathrm{i}}=\mathrm{d}_{\text {in }}\left(\rho_{\text {in }} \mathrm{C}_{\text {in }} / \mathrm{C}_{\mathrm{i}} \rho_{\mathrm{i}}\right)^{0.33}
$$

Where the subscripts $i$ and in represent the instantaneous values and the initial values, respectively, of the same droplet class.

### 2.1.3.1 PROPERTIES OF DRYING GAS

The amount of drying air $\mathrm{W}_{\mathrm{g}}$ needed can be estimated from the following overall energy balance on the spray-drying chamber over the datum $\mathrm{t}_{\mathrm{w}}$ :
$\mathrm{W}_{\mathrm{g}} \mathrm{C}_{\mathrm{s}}\left(\mathrm{t}_{1}-\mathrm{t}_{2}\right) \mathrm{W}_{\mathrm{w}} \lambda+\mathrm{w}_{\mathrm{w}} \mathrm{C}_{\mathrm{f}}\left(\mathrm{t}_{2}-\mathrm{t}_{\mathrm{w}}\right)+\mathrm{W}_{1} \mathrm{C}_{\mathrm{f}}\left(\mathrm{t}_{\mathrm{w}}-\mathrm{t}_{\mathrm{f}}\right)+\mathrm{W}_{\mathrm{p}} \mathrm{C}_{\mathrm{ps}}\left(\mathrm{t}_{\mathrm{p}}-\mathrm{t}_{\mathrm{w}}\right)+\mathrm{q}_{1}$
The temperature of the drying gas in the nozzle zone is obtained from an energy balance in this zone:

$$
\left(t_{1}-t_{\mathrm{x}}\right)=\left[\mathrm{E}_{\mathrm{x}} \lambda+\mathrm{W}_{\mathrm{p}} \mathrm{C}_{\mathrm{ps}}\left(\mathrm{t}_{\mathrm{p}}-\mathrm{t}_{\mathrm{w}}\right)+\mathrm{q}_{1 \mathrm{x}}\right] / \mathrm{M}_{\mathrm{e}} \mathrm{C}_{\mathrm{st1}} .
$$

The average humidity at any section is determined from the rate of evaporation and the rate of entrainment:

$$
\mathrm{H}_{\mathrm{x}}=\left(\mathrm{E}_{\mathrm{x}} / \mathrm{M}_{\mathrm{e}}\right)+\mathrm{H}_{\mathrm{l}}
$$

The temperature of the drying gas in the free entrainment zone can be calculated from the following energy balance between the inlet and the section under consideration:
$\left(t_{1}-t_{x}\right)=\left[E_{x} \lambda+\left(E_{x}-W_{p}\right)\left(t_{2}-t_{w}\right) C_{f}+q_{1 x}+W_{p} C_{p s}\left(t_{p}-t_{w}\right)\right] /\left(W_{g} C_{s}\right) \ldots . .2 .36$
The average humidity in this zone is obtained from the flow rate of the drying gas and the rate of evaporation:

$$
\mathrm{H}_{\mathrm{x}}=\left(\mathrm{E}_{\mathrm{x}} / \mathrm{W}_{\mathrm{g}}\right)+\mathrm{H}_{1}
$$

It was assumed that the radial temperature and humidity gradients in the drying gas are negligible. The heat losses were first calculated for the whole chamber by using an overall heat transfer coefficient. For any given section of the chamber, it was then assumed that the heat losses were proportional to the extent of evaporation. It can be seen that the above equations include the amount of heat supplied to the solids, a relatively small item.

### 2.1.3.2 SPRAY OF DROPS

A spray is considered as a system of fluid droplet in a fluid continuous phase. By specifying "droplet" microscopic cases are eliminated wherein large individual drops slugs, or columns predominate, and microscopic cases wherein dispersion is of molecular order, or at least where droplets are sufficiently small to prevent detection of phase boundaries. By specifying a "system" we imply that the droplets have a common origin, usually a body of liquid.(Mugele and Enstein, 1951)

Examples of natural sprays are rains, fogs, and waterfall mist, ocean spray and sneeze spray. Among artificial sprays are fountain sprays, atomizer sprays entrained liquid in a fractionating column, and the disperse phase in a solvent extraction column.

## Spray of pure liquid drops

For spray of pure liquid droplets, many attempts have been made to analyze the spray evaporation. These attempts could be grouped into (a) Theoretical (b) Practical (c) Mathematical and (d) Computational analysis.

Theoretical approach was used by Probert (1946) who considered the droplets to have no relative velocity and the temperature driving forces to be negligible. He observed that the rate at which evaporation proceeds to completion was a function of the spray homogeneity.

A practical method in which the size distribution is divided into small size groups was used by Marshall (1953). Evaporation for each group was then considered individually for selected time interval until evaporation was complete.

Mathematical approach was used by Shapiro and Erikson (1957) for one dimensional spray motion. This approach was similarly used by Bose and Pei (1964) who studied evaporation water sprays in a co - current flow nozzle dryer. They concluded that a substantial part of the evaporation took place during spray deceleration and that the relative velocity between spray and air contributed greatly to heat and mass transfer rates. In addition, they claimed that the relative velocity could not replaced by terminal velocity in an equation involving evaporation analysis.

Computational methods were used by Dickinson and Marshall (1960) to determine the evaporation history of water sprays. They obtained equation for spray evaporation of non - uniform distribution in terms of mean diameter, size distribution, droplets population, drying air and droplet temperature and air velocity (Onifade, 1983).

## Spray of droplets containing solids

The analysis of evaporation of spray of droplets containing solids is very complex. Consideration has to be given to the following:
a. Crust is formed simultaneously by all the droplets within the size distribution.
b. Vapor pressure lowering depends upon droplet size.

The evaporation history of sodium nitrate was studied by Baltas and Gauvin (1967) using computer programming for stepwise techniques. Even though simple system of spray movement at terminal velocity in the free fall zone of a single nozzle dryer was chosen, the computations were complex. The results obtained could still not give accurate prediction of spray evaporation due to the difficulty in obtaining representative data for spray at drying parameters.

## Evaporation from drops containing Dissolved solids

The aim of the investigation reported here was to develop a method of predicting the drying rates of drops containing dissolved solids for application to design calculations for spray dryers and to obtain fundamental information on the phenomena occurring during the drying process (Charlesworth and Marshal, 1960). Considerable attention was given to the problem of predicting
the formation of solid phase in drying a droplet of an inorganic salt solution. The need for such information has become more pressing as spray-drying techniques have become widely applied to the production of dried foodstuffs, pharmaceuticals and tonnage chemicals.

As in most drying operations, the spray - drying process can be subdivided into periods. Evaporation from a free liquid surface, the second characterize the first period by evaporation from or through a solid structure, which forms at an intermediate point in the process.

Theoretical Considerations: The complexity of the process of evaporation from a droplet having a velocity relative to the surrounding air is greatly increased by the presence of non-volatile dissolved solids in the droplet. Whereas, in the consideration of heat and mass transfer from a droplet of pure liquid, conditions within the droplet may justifiably be assumed uniform, they cannot be if there is non-volatile material present.

In general, the concentration of solute is initially uniform in the droplets; however, as soon as some evaporation has taken place from the droplet surface, concentration gradient are set up and there is diffusion of solvent toward the surface and solute towards the center. In addition, since the drop is necessarily decreasing in size, the solute in the surface layer is swept inward by the retreating interface, which is impermeable to the solute.

If the evaporation rate were constant over the entire surface, as would be the case for zero relative velocity between the droplet and the surroundings, the concentration gradients within the droplet should have spherical symmetry.

However, if the relative velocity is not zero, boundary-layer theory predicts, and Frossling (1938) has demonstrated experimentally, that the evaporation rate varies from a maximum at the point of impingement at the front of the drop, through a minimum at some latitude beyond the equator, to a second but lower maximum at the rear. As a result, diffusion has a tangential component toward the rear of the drops in addition to the radial one. The concentration profiles are consequently not simple.

It is postulated (Charlesworth and Marshal, 1960) that the surface solute concentration does not remain uniform but rather rises smoothly from its initial value. The rate of accumulation at the surface increases since the rate of evaporation is almost constant (depending on the vapor - pressure concentration relation and any changes in the Reynolds number of the flow) and the surface concentration is increasing. When the surface concentration reaches the saturation value, it can no longer continue to increase and solute must be deposited as a solid phase. Consequently, if the surface concentration of solute could be expressed as a function of time and drying conditions, the time of formation of the solid phase could be predicted. If rigorous, such a function would be complicated by the lack of spherical symmetry and by internal circulation.

Using this concept of the process as a basis and assuming internal transfer by diffusion only, one can express the surface concentration of solute as a function of time and drying conditions and estimate the time of formation of the solid phase.

Experimental Procedures: For study of the drying of droplets of aqueous solutions, a technique was developed wherein an individual droplet was suspended in a controlled air stream, and it's weight or temperature was measured as the drying progressed. Visual observations of the appearance and size of the particle were recorded.

Experimental Results: The generalized description of the appearance of a suspended droplet of an aqueous solution of an inorganic salt drying will be useful in understanding the experimental data obtained. (Figure 2.6).

Up to a point, the sequence of events, which were observed, were similar for all droplets no matter the solute or the drying conditions. During the first portion of the drying the droplet decreased in size with no appearance of a solid phase. The presence of a solid phase was first evidenced by the formation of crystals at the bottom of the drop. As drying progressed, more crystals appeared, forming a surface crust, which grew steadily up the sides of the droplet. If the droplet did not rotate, the advancing front of the crust remained symmetrical about the vertical axis and continued to rise until the droplet was completely encased in a layer of solid material. The rate of advance of the solid front was initially relatively fast. It slowed markedly as it approached the droplet equator, then once past it, it speeded up, completing the upper half of the crust in appreciably less time than was required for the formation of the lower portion. If the drop (or it's surface) rotated, swinging the solid up the side of the drop, there was a strong tendency for the crust to re-form in its previous position. That portion of the crust, which bodily advanced past the latitude of the old solid front rapidly redissolved.

Also the crust quickly grew over any liquid surface, which was exposed below the normal front latitude. The redissolution process was generally more rapid than the reformation process. The result of further drying after the completion of the crust about the droplet different depending on the nature of the solute and the surrounding air temperature.

If the air temperature was below the boiling point of the solution, several different phenomena were possible as seen in Figure. 2.6 above. Somewhat different results were observed after the closure of the crust about the droplet, if the surrounding air temperature was appreciably above the boiling point of the solution.

In virtually all cases, the final particle consisted of a hollow, thin, nearly spherical crust. The outer surface was usually quite smooth, whereas the inner surface was rough and uneven. Occasionally the inner core contained an open network of large crystals.


Rigure 2.6 Appearance changes in drying dople: (Charlesivorth and
Marshall, 1960)

### 2.1.4 SEPARATION OF DRIED PRODUCTS

This is the final stage of spray drying operation. It involves the separation of dried product from air followed by the removal of dried product from the dryer. Two types of design are usually employed:
a. The one point discharge; and
b. Two point discharge

In one point discharge, the dried product and air are both conveyed to the collecting equipment for separation. This type is employed mostly in the co-current spray dryers. In the case of two-point discharge, there is a primary product discharge, usually from the base of the dryer followed by recovery of fines from the collecting equipment. Counter-current and mixed flow spray dryers favour this type.

Equipments for separation range from conventional types. Such as cyclones, bag fillers, scrubbers, electrostatic precipitators (Celenzer, 1970) to recent ones such as intensive filter, fabric filter, differential bag shelving equipment and the ultra jet air pulse fabric filter.

The following factors are considered before final selection is made:
a. Operational procedure: The equipment must operate either continuously or batch wise according to the demands of the spray dryer operation.
b. Collection efficiency: This factor is very important especially where consideration has to be given to environmental pollution.
c. Cost: ensure a profitable running of the whole plant, the investment and operational cost of the equipment must fit in well with the total operational cost of the spray dryer-plant.
d. Product handling suitability: The equipment chosen must be able to withstand the operating conditions of the drying process such as temperature and air flow rates. It must also be able to cope with characteristics of the product such as the powder properties.
e. Space area requirement of the equipment: This has to fit well with the overall plant layout of the spray dryer.

### 2.2 MATHEMATICAL MODELING

Derived from its Latin root "Modus", the word " model" is generally understood to stand for an object that represent a physical entity with a change of scale.

A mathematical model is a set of mathematical equations representing a process or a system. It is a mathematical idealization, of a real world phenomenon. In the process of idealization, some simplifications would have been made in obtaining the mathematical model. Therefore, the mathematical model is less real than the system it is supposed to represent. Nevertheless, it is an essential step in the construction of a theory.

In recent years, mathematical modeling has become a powerful tool to solve complex, interconnected and interacting phenomena arising from the rapid development, taking place in science and technology. The explosive
growth of mathematical modeling activity has been a driving force behind the development of high-speed digital computers, which in turn aided model solutions in a symbiotic relationship.

In the process of mathematical modeling, the objective of the modeler is, in general to construct a model of an observed phenomenon and use the model to predict its future course, In some cases, modeling is used to explain the known facts and lay a foundation for the theory behind the phenomenon. Mathematical modeling procedure in some cases begins with the desire to construct a model from a set of observations made on the behavior of a system. Once the problem is defined, a mathematical model of the real-world phenomenon is to be constructed. The real-world phenomena are exceedingly complex. It is nearly impossible to construct a model that replicates the phenomenon in its totality. The modeler must sort out the essential and significant features that need to be incorporated in the model. Here, experience and intuition come into play. The modeler translates the feature into mathematical entities and relates them under certain simplifying but realistic assumptions and constraints. A myriad of mathematical solution techniques are available to extract the system behavior from the model. By using one or more of these techniques, the solution of the model is obtained and interpreted from the viewpoint of accuracy and stability. Determination of how closely the solution approaches the original, real-world phenomenon is the next step in the modeling process. If the solution meets the imposed limits of acceptability, the model is considered valid and then put into practice to predict a feature event,
or to make a decision. Otherwise the model is invalidated and steps B-D (Figure 2.7) are repeated by revising one or more ingredient(s) in the process. Thus, the process of mathematical modeling is iterative in nature.

In engineering, the purpose of modeling has been primarily for research. In research, even though the immediate use of the model is not clear, one pursues it for gain in comprehension, for interpreting knowledge and for formulating clues for further investigation. In addition to research, mathematical models are used for design and control. Here, the components of the total engineering system have to be expressed in a model compatible with design criteria which may include some or all of stability limits, error criterion, economic yield and safety criterion. Also, modeling for design is not just for creating a new system, but also for the adaptation of an existing system for a higher (or different) performance compatible with design criteria which may include some or all of stability limits, error criterion economic yield and safety criterion. Also, mottling for design is not just for creating a new system, but also for the adaptation of an existing system for a higher (or different) performance.


Figure 2.7 Schematic of mathematical modeling procedure.


Figure 2.8 Basic steps in the modeling process.

## CHAPTER THREE

### 3.0 MATHEMATICAL MODEL FOR THE SPRAY DRYER SYSTEM

### 3.1.0 CALCULATION OF PARTICLE TRAJECTORIES

### 3.1.1 INTRODUCTION

Among the many drying methods available, spray drying has gained a unique and important position in industrial applications where low particle temperatures and short residence times are especially advantageous, such as in the drying of foods, drugs and temperature sensitive materials in general.

As earlier mentioned the prediction of the droplets trajectories is essential because the design of spray dryers is based on the fact that the largest droplet in the initial spray are maintained in the drying air long enough to dry (or, more exactly shall reach the specified residual moisture content) before striking the walls of the chamber.

The general procedure for the calculation of the particle trajectories as stated earlier on, involves the determination of a velocity - time relation for the particle motion in component directions. Once this is done the distance traveled in those component directions are calculated using graphical integration of velocity - time curves.

### 3.1.2 DEVELOPMENT OF EQUATIONS

The following model has been derived for a special case of droplet from a centrifugal pressure nozzle for the droplet moving in the presence of a gravitational field with an initial velocity.

Consider that the spray issued from the nozzle at a cone angle and representative velocity U (Figure 3.1 ). If M is the mass of the droplet, it follows from a balance of forces that

$$
\left.\begin{array}{l}
\mathrm{M}\left(\frac{\mathrm{~d} \mathrm{U}_{\mathrm{h}}}{\mathrm{dt}}\right)=-\mathrm{Fsin}(\theta / 2) \\
\text { and } \mathrm{M}\left(\frac{\mathrm{dUv}}{\mathrm{dt}}\right)=\mathrm{Mg}\left(\frac{\rho_{\mathrm{s}}-\rho_{a}}{\rho_{s}}\right)-\mathrm{F} \cos (\theta / \ldots . . . . . . . . . . . \\
2
\end{array}\right) .
$$

Where $\mathrm{U}_{\mathrm{h}}$ and $\mathrm{U}_{\mathrm{v}}$ are velocity components in the horizontal and vertical directions respectively. But by definition

$$
\begin{align*}
& \mathrm{F}=1 / 2 \rho_{a} \mathrm{U}^{2} \mathrm{C} . \mathrm{A} . \\
& \text { and } \mathrm{C}=\mathrm{F}(\mathrm{Re})=\mathrm{F}\left(\frac{\mathrm{DU} \rho_{\mathrm{a}}}{\mu}\right) \text {. } \\
& \text { Also from the relation between vectors, } \\
& \operatorname{Sin}(\theta / 2)=\frac{U_{h}}{U} \text {. } \\
& \operatorname{Cos}(\theta / 2)=\frac{U v}{U} .
\end{align*}
$$



Figure 3.1 Foree diagram for droplet trajectory

From 3.5 and 3.6 respectively
$\mathrm{U}_{\mathrm{h}}=\mathrm{U} \sin (\theta / 2)$
$\mathrm{U}_{\mathrm{v}}=\mathrm{U} \cos (\theta / 2)$.
$\mathrm{U}_{\mathrm{h}}^{2}+\mathrm{U}_{\mathrm{v}}^{2}=\mathrm{U}^{2}\left(\sin ^{2}(\theta / 2)+\cos ^{2}(\theta / 2)\right)$
$\mathrm{U}_{\mathrm{h}}^{2}+\mathrm{U}_{\mathrm{v}}^{2}=\mathrm{U}^{2}$
Substituting 3.3, 3.5 and 3.6 into 3.1 and 3.2 give
$\frac{\mathrm{dU}_{\mathrm{h}}}{\mathrm{dt}}=-\frac{\rho_{a} \mathrm{C} \cdot \mathrm{A} . . \mathrm{U} \cdot \mathrm{U}_{\mathrm{h}}}{2 \mathrm{~m}}$
and
$\frac{\mathrm{dU}_{\mathrm{v}}}{\mathrm{dt}}=\mathrm{g}\left(\frac{\rho_{s}-\rho_{a}}{\rho_{s}}\right)-\frac{\rho_{a} \text { C.A..U.U. } \mathrm{U}_{\mathrm{v}}}{2 \mathrm{~m}}$
Since the droplets are spherical, therefore,
Volume $=1 / 6 \pi \mathrm{D}^{3}$ and Area $=\pi \mathrm{D}^{2} / 4$
$\therefore \mathrm{m}=1 / 6 \pi D^{3} \rho_{s}$
Equations 3.10 and 3.11 become
$\frac{\mathrm{dU}_{\mathrm{h}}}{\mathrm{dt}}=-\frac{3 \rho_{a} \mathrm{C} \cdot \mathrm{U} \cdot \mathrm{U}_{\mathrm{h}}}{4 \mathrm{D} \rho_{\mathrm{s}}}$.
$\frac{\mathrm{d} \mathrm{U}_{\mathrm{v}}}{\mathrm{dt}}=g\left(\frac{\rho_{\mathrm{s}}-\rho_{a}}{\rho_{\mathrm{s}}}\right)-\frac{3 \rho_{a} \mathrm{C} \cdot \mathrm{U} \cdot \mathrm{U}_{\mathrm{v}}}{4 \mathrm{D} \rho_{\mathrm{s}}}$
The solution of Equations 3.12 and 3.13 will give $U_{h}$ and $U_{v}$ at different times $t$ while the vectorial sum of $U_{v}$ and $U_{h}$ will give $U$ at a particular time. The integration of the values of $U_{h}$ and $U_{v}$ against $t$ will also give distances $S_{h}$ and $\mathrm{S}_{\mathrm{v}}$ in the X and Y directions and therefore map out the trajectory of the particle.

### 3.1.3 SOLUTIONS OF EQUATIONS OF MOTION

Equations 3.12 and 3.13 can be rewritten in incremental form as follows
$\Delta \mathrm{U}_{\mathrm{v}}=\left(\Delta \mathrm{U}_{\mathrm{v} 3}+\Delta \mathrm{U}_{\mathrm{ho}}\right) / 2$. the first and second average are compared. If they agree to within a reasonable tolerance, $\mathrm{U}_{\mathrm{hA}}$ and $\mathrm{U}_{\mathrm{vA}}$ are taken as velocities at the end of $\Delta t$ and $U$ for the next movement is calculated using Equation 3.16. If however, they do not, $\mathrm{U}_{\mathrm{h} 2}$ and $\mathrm{U}_{\mathrm{v} 2}$ are calculated from the addition of second average to $\mathrm{U}_{\mathrm{ho}}$ and $\Delta \mathrm{U}_{\mathrm{vo}}$ (i.e. $\Delta \mathrm{U}_{\mathrm{h}}+\mathrm{U}_{\mathrm{ho}}$ and $\left(\Delta \mathrm{U}_{\mathrm{v}}+\mathrm{U}_{\mathrm{vo}}\right)$ respectively. $\mathrm{U}_{\mathrm{h} 2}$ and $\mathrm{U}_{\mathrm{v} 2}$ are now used to compute $\Delta \mathrm{U}_{\mathrm{h} 4}$ and $\Delta \mathrm{U}_{\mathrm{v} 4}$ from 3.14 and 3.15 respecticely. The $3^{\text {rd }}$ average $\Delta \mathrm{U}_{\mathrm{h}}$ and $\Delta \mathrm{U}_{\mathrm{v}}$ are calculated from $\Delta \mathrm{U}_{\mathrm{h} 4}$ and $\Delta \mathrm{U}_{\mathrm{ho}}$ [i.e. $\Delta \mathrm{U}_{\mathrm{h}}=$ $\left(\Delta \mathrm{U}_{\mathrm{h} 4}+\Delta \mathrm{U}_{\mathrm{ho}}\right) / 2$ ] and $\Delta \mathrm{U}_{\mathrm{v} 4}$ and $\Delta \mathrm{U}_{\mathrm{vo}}$ [i.e. $\Delta \mathrm{U}_{\mathrm{v}}=\left(\Delta \mathrm{U}_{\mathrm{v} 4}+\Delta \mathrm{U}_{\mathrm{vo}}\right) / 2$ ] respectively. This third average and any subsequent average is compared with one before it until the tolerance is acceptable. $\mathrm{U}_{\mathrm{h}}$ and $\mathrm{U}_{\mathrm{v}}$ used to generate the last set of $\Delta \mathrm{U}_{\mathrm{h}}$ and $\Delta \mathrm{U}_{\mathrm{v}}$ are taken as velocities at the end of the time increment. The distance ' X ' and ' Y ' at the end of an interval are then calculated.

The whole process is repeated for the next time increment $\Delta t$ until ' $X$ ' and ' Y ' or either of them exceeds the value of the width and axial length of the spray column respectively.

A Fortran program was developed to carry out this calculation. The program appears in Appendix B. The flow chart for the program is shown in Figure 3.2.


Fig. 3.2 A Flowchart of the program for Solving the equation of motion

## CHAPTER FOUR

### 4.0 RESULTS AND DISCUSSION

### 4.1 RESULTS

The values used in the program drawn up to solve the equations in the model for particle trajectory (Chapter 3) are shown below.

Slurry-Cement
Density- $1040 \mathrm{~kg} / \mathrm{m}^{3}$
Surface tension- $0.0715 \mathrm{~N} / \mathrm{m}$
Air density $-1.2 \mathrm{~kg} / \mathrm{m}^{3}$
Air viscosity $-1.76 \times 10^{-5} \mathrm{kgm} / \mathrm{s}$
Atomizer- Rotary atomizer
Spray angle- $60^{\circ}$
Spray nozzle - SDX (SE- 703-056)
Diameters: 1. 0.00002700 m
2. 0.00003465 m
3. 0.00004460 m
4. 0.00005740 m
5. 0.00007445 m
6. 0.00009855 m
7. 0.00013655 m
8. 0.00021095 m
9. 0.00041280 m

## Initial velocities: 1. $3.656 \mathrm{~m} / \mathrm{s}$

2. $4.478 \mathrm{~m} / \mathrm{s}$
3. $5.171 \mathrm{~m} / \mathrm{s}$

The results obtained from the program are shown in the tables, which are listed in the Appendix A. These tables are used in producing Figures 4.10 to 4.18 as shown in Section 4.2.A sample of the result is shown below.

## SLURRY FLOWRATE $=7.72 \mathrm{e}-3 \mathrm{Kg} / \mathrm{s}$

## INITIAL VELOCITY $=3.656 \mathrm{~m} / \mathrm{s}$

## DIAMETER $=\mathbf{0 . 0 0 0 0 2 7} \mathbf{m}$

## PARTICLE TRAJECTORY

| Tim | Velocity ( $\mathrm{m} / \mathrm{s}$ ) |  |  | Distance (m) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | izont | Verti | R |  |  |
| . 00 | 1.828 | 3.166 | 3.656 | . 0000 | . 0000 |
| . 01 | . 016 | . 050 | . 053 | . 0032 | . 0058 |
| . 02 | . 000 | . 024 | . 024 | . 0033 | . 0060 |
| . 03 | . 000 | . 023 | . 023 | . 0033 | . 0063 |
| . 04 | . 000 | . 023 | . 023 | . 0033 | . 0065 |
| . 05 | . 000 | . 023 | . 023 | . 0033 | . 0068 |
| . 06 | . 000 | . 023 | . 023 | . 0033 | . 0070 |

## DIAMETER $=0.00003465 \mathrm{~m}$ PARTICLE TRAJECTORY

| Time | Velocity $(\mathrm{m} / \mathrm{s})$ |  |  | Distance (m) |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| (S) |  | Horizontal Vertical | Resultant | Horizontal Vertical |  |
| .00 | 1.828 | 3.166 | 3.656 | .0000 | .0000 |
| .01 | .077 | .167 | .184 | .0048 | .0085 |
| .02 | .006 | .049 | .049 | .0051 | .0093 |
| .03 | .000 | .039 | .039 | .0051 | .0098 |
| .04 | .000 | .039 | .039 | .0051 | .0102 |
| .05 | .000 | .039 | .039 | .0051 | .0105 |
| .06 | .000 | .039 | .039 | .0051 | .0109 |
| .07 | .000 | .039 | .039 | .0051 | .0117 |

### 4.2. DISCUSSION OF RESULT

Figure 4.1 is for the following diameters; $0.000027 \mathrm{~m}, 0.00003465 \mathrm{~m}$, and 0.00004460 m . Each plot represents the vertical and horizontal distance of the droplets plotted against time. Figure 4.2 is for diameters; 0.0000574 m , 0.00007445 m and 0.00009855 m while Figure 4.3 is a plot for diameters; $0.00013655 \mathrm{~m}, 0.00021095 \mathrm{~m}$ and 0.0004128 m . Figures 4.1 to 4.3 are plotted for initial droplet velocity of $3.656 \mathrm{~m} / \mathrm{s}$.

Figures 4.4 to 4.6 are plotted for initial droplet velocity of $4.478 \mathrm{~m} / \mathrm{s}$ as in Figures 4.1 to 4.3 while Figures 4.7 to 4.9 are for initial droplet velocity of $5.171 \mathrm{~m} / \mathrm{s}$ also as in Figures 4.1 to 4.3 respectively.

Figures 4.10 to 4.18 are plotted for the following diameters; 0.000027 m , $0.00003455 \mathrm{~m}, \quad 0.0000446 \mathrm{~m}, \quad 0.0000574 \mathrm{~m}, ~ 0.00007445 \mathrm{~m}, ~ 0.00009855 \mathrm{~m}$, $0.00013655 \mathrm{~m}, 0.00021095 \mathrm{~m}$, and 0.0004128 m respectively each with initial velocities of $3.656 \mathrm{~m} / \mathrm{s}, 4.478 \mathrm{~m} / \mathrm{s}$ and $5.171 \mathrm{~m} / \mathrm{s}$ plotted against time.

For droplets of diameter 0.000027 m , in Figure 4.1, the horizontal component of the distance traveled increased rapidly from $\mathrm{t}=0$ with time until it attained a distance of 0.0033 m at time $\mathrm{t}=0.01 \mathrm{~s}$. Beyond this time, the distance traveled by droplets became constant with time. Similar trend was obtained in the vertical component of the distance. However, the distance increased more rapidly in vertical than the horizontal component. After the droplets had traveled a distance of 0.0058 m , attained at time $\mathrm{t}=0.01 \mathrm{~s}$, the rapidness in the distance covered with time reduced and the increase in distance eventually became constant after the droplets had traveled a vertical distance of 0.0063 m
and horizontal distance of 0.0033 m at time $\mathrm{t}=0.03 \mathrm{~s}$. This signifies that the moisture content of the droplets has become constant and those are the values of those parameters required to dry the droplets of that size and subject to that air velocity. For a diameter of 0.00003465 m , the horizontal component of the distance reached up to 0.0051 m before the droplets attained a free fall velocity i.e. the distance traveled in that direction ceased to increase. The vertical component of the distance reached was 0.0093 m . For a diameter of 0.0000446 m , the droplets traveled a distance of 0.0082 m horizontally and 0.0147 m in the vertical direction before a free fall velocity was attained. At this point $\mathrm{t}=0.02 \mathrm{~s}$.

It can therefore be seen in Figure 4.1 that the greater the particle diameter, the longer time it takes toattain a free fall as such the greater the distances traveled in both the horizontal and vertical directions. These results are in agreement with the results of Lapple and Shepherd (1940) who observed that "at a start the horizontal component of the distance increased rapidly at a low distance downwards from the spray point. As the vertical distance of the droplets increases with time the horizontal component decreases until a point where a free fall velocity is attained where the horizontal distance tends to be constant as the vertical component of the distance increases".


Figure 4.1 Graph of initial velocity $3.656 \mathrm{~m} / \mathrm{s}$ with diameters $\mathrm{d} 1, \mathrm{~d} 2 \& \mathrm{~d} 3$

In Figure 4.2 for droplets with diameter 0.0000574 m the horizontal component of the distance traveled was 0.0132 m and vertical component was 0.0216 m at time $\mathrm{t}=0.05 \mathrm{~s}$ before the distance traveled became constant with time. A free fall velocity was attained at a vertical distance of 0.02266 m . At this point, the rate of change of distance with time became constant. For droplet with a diameter of 0.00007445 m , a free fall velocity was observed at a distance of 0.0210 m in the horizontal direction and 0.0491 m in the vertical direction at time $t=011 \mathrm{~s}$. In the case of droplets with diameter 0.00009855 m , a free fall velocity was attained when the droplets traveled 0.0338 m in the horizontal direction and 0.0994 m in the vertical direction at time $\mathrm{t}=0.19 \mathrm{~s}$. Again, it can be observed that like in Figure 4.1, as diameter increased, the distance traveled with time also increased.


Figure 4.2 Graph of initial velocity $3.656 \mathrm{~m} / \mathrm{s}$ with diameters $\mathrm{d} 4, \mathrm{~d} 5 \& \mathrm{~d} 6$

In Figure 4.3 the droplets with diameter 0.00013655 m attained a free fall velocity when the vertical and horizontal components of the distance were 0.2140 m and 0.0576 m respectively at a time $\mathrm{t}=0.32 \mathrm{~s}$. For a diameter of 0.00021095 m the free fall velocity was attained at $\mathrm{t}=0.39 \mathrm{~s}$ when the distances in both the horizontal and vertical directions were 0.1119 m and 0.4400 m respectively. The largest droplets with diameter 0.0004128 m attained a free fall velocity at time $t=0.65 \mathrm{~s}$ when the horizontal and vertical components of the distance were 0.2721 m and 1.2820 m respectively. The trends of results were also in consonance with those observed in Figure 4.2


Figure 4.3: Graph of initial velocity $3.656 \mathrm{~m} / \mathrm{s}$ with droplet diameters d 7 , d8 \& d9

From Figures 4.1 to 4.3 , it can be observed that the greatest distance traveled was in Figure 4.3 with the droplets diameter of 0.0004128 m . These droplets also took longer time before attaining a free fall velocity. This implies that the greater the particle diameter the greater the distance (both horizontally and vertically) traveled and hence the greater the chamber dimensions needed to accomplish the drying.

The observations and conclusions drawn from Figure 4.1 to 4.3 equally hold in Figures 4.4 to 4.6 and in Figures 4.7 to 4.9 for initial velocities of $4.478 \mathrm{~m} / \mathrm{s}$ and $5.171 \mathrm{~m} / \mathrm{s}$ respectively.


Figure 4.4 Graph of initial vel. $4.478 \mathrm{~m} / \mathrm{s}$ with droplet diameters d 1 , d 2 , \&d3


Figure 4.5 Graph of initial vel. $4.478 \mathrm{~m} / \mathrm{s}$ with droplet diameters $\mathrm{d} 4, \mathrm{~d} 5$ \& d 6


Figure 4.6 Graph of initial velocity $4.478 \mathrm{~m} / \mathrm{s}$ with droplet diameters $\mathrm{d} 7 \mathrm{~d} 8 \& \mathrm{~d} 9$



Figure 4.8 Graph of initial velocity $5.171 \mathrm{~m} / \mathrm{s}$ with droplet diameters d 4 , d 5
\& d6


Figure 4.9 Graph of initial velocity $5.171 \mathrm{~m} / \mathrm{s}$ with droplet diameters $\mathrm{d} 7, \mathrm{~d} 8$ \& d9

Figures 4.10- 4.18 plotted the distances traveled horizontally and vertically by a particular droplets diameter with three different initial velocities of $3.656 \mathrm{~m} / \mathrm{s}, 4.478 \mathrm{~m} / \mathrm{s}$ and $5.171 \mathrm{~m} / \mathrm{s}$. Figure 4.10 showed a plot for a droplet diameter 0.00027 m . For velocity of $3.656 \mathrm{~m} / \mathrm{s}$, the free fall was attained at distances 0.0060 m and 0.0033 m in the vertical and horizontal directions respectively. For a velocity of $4.478 \mathrm{~m} / \mathrm{s}$ free fall was attained at distances 0.0067 m and 0.0036 m in vertical and horizontal directions respectively, while for the velocity $5.171 \mathrm{~m} / \mathrm{s}$ the distances were 0.0074 m and 0.004 m respectively. The freefall velocities were all attained at time $t=0.02 \mathrm{~s}$. These results show that the bigger the initial velocity, the larger the horizontal and vertical distances traversed by the droplets before a free fall was attained. It can also be inferred from the above that the greater the initial velocity of a spray the longer it takes to get dried and the larger the drying tower dimension required. The same trend was also observed in Figures 4.11 to 4.18 for other droplet diameters using the same initial velocities as in Figure 4.10. The observation and conclusion also agree with the results of Lapple and Shepherd (1940) who stated that the greater the particle diameter, the more time the droplets take in suspended air before drying or reaching the required moisture content.


Figure 4.10 Graph of droplet diameter $2.7 \mathrm{e}-5 \mathrm{~m}$ with initial velocities V1, V2 \& V3


Figure 4.11 Graph of droplet diameter $3.465 \mathrm{e}-5 \mathrm{~m}$ with initial velocities $\mathrm{V} 1, \mathrm{~V} 2 \& \mathrm{~V} 3$


Figure 4.12 Graph of droplet diameter $4.46 \mathrm{e}-5 \mathrm{~m}$ with initial velocities $\mathrm{V} 1, \mathrm{~V}$ 2 \& V 3


Figure 4.13 Graph of droplet diameter $5.74 \mathrm{e}-5 \mathrm{~m}$ with initial velocities V1,V2\& V3


Figure 4.14 Graph of drplet diameter $7.445 \mathrm{e}-5 \mathrm{~m}$ with initial velocities V1, V2 \& V3


Figure 4.15 Graph of droplet diameter $9.855 \mathrm{e}-5 \mathrm{~m}$ with initial velocities V1, V2, \& V3.


Figure 4.16 Graph of droplet diameter $1.3655 \mathrm{e}-4 \mathrm{~m}$ with initial velocities V1,V2 \& 3 .



Figure 4.18 Graph of droplet diameter $4.128 \mathrm{e}-4 \mathrm{~m}$ with initial velocities V1,V2 \& V3

## CHAPTER FIVE

### 5.0 CONCLUSIONS

From the results of the models obtained for the particle trajectories of the droplets the following conclusions may be drawn;

1. The distance covered by a droplet and the time it takes to get dried are in direct proportion to the droplet diameter.
2. For a particular droplet diameter, the initial velocity is in direct proportion to the distances covered in both the horizontal and vertical directions. This implies that the greater the initial velocity of a spray and the droplet diameter the greater the distance the droplet will cover thus leading to longer time to dry before hitting the chamber walls. This will therefore result in bigger dryer dimensions.

From the above, it can be seen that knowledge about the droplet trajectories is very important in the design or improved performance of a spray dryer. The information ganed from studies of the droplet trajectories can be used along with other design equations to determine the dimensions of the dryer chamber.

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## APPENDICES

## APPENDIX A

TABLES OF RESULTS FOR DROPLETS TRAJECTORIES.

## SLURRY FLOWRATE $=7.72 \mathrm{e}-3 \mathrm{Kg} / \mathrm{s}$

INITIAL VELOCITY $=\mathbf{3 . 6 5 6} \mathbf{m} / \mathrm{s}$

## DIAMETER $\mathbf{= 0 . 0 0 0 0 2 7} \mathbf{m}$

## PARTICLE TRAJECTORY

| Time | Velocity $(\mathrm{m} / \mathrm{s})$ |  |  | Distance $(\mathrm{m})$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| (S) | Horizontal | Vertical | Resultant | Horizontal Vertical |  |
| .00 | 1.828 | 3.166 | 3.656 | .0000 | .0000 |
| .01 | .016 | .050 | .053 | .0032 | .0058 |
| .02 | .000 | .024 | .024 | .0033 | .0060 |
| .03 | .000 | .023 | .023 | .0033 | .0063 |
| .04 | .000 | .023 | .023 | .0033 | .0065 |
| .05 | .000 | .023 | .023 | .0033 | .0068 |
| .06 | .000 | .023 | .023 | .0033 | .0070 |

## DIAMETER $=\mathbf{0 . 0 0 0 0 3 4 6 5 m}$ <br> PARTICLE TRAJECTORY

| Time | Velocity (m/s) |  |  | Distance (m) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (S) | Horizontal | Vertical | Resultant | Horizontal Vertical |  |
| .00 | 1.828 | 3.166 | 3.656 | .0000 | .0000 |
| .01 | .077 | .167 | .184 | .0048 | .0085 |
| .02 | .006 | .049 | .049 | .0051 | .0093 |
| .03 | .000 | .039 | .039 | .0051 | .0098 |
| .04 | .000 | .039 | .039 | .0051 | .0102 |
| .05 | .000 | .039 | .039 | .0051 | .0105 |
| .06 | .000 | .039 | .039 | .0051 | .0109 |
| .07 | .000 | .039 | .039 | .0051 | .0113 |
| .08 | .000 | .039 | .039 | .0051 | .0117 |

## DIAMETER $=\mathbf{0 . 0 0 0 0 4 4 6 0} \mathbf{~ m}$ PARTICLE TRAJECTORY

| Time | Velocity $(\mathrm{m} / \mathrm{s})$ |  |  | Distance $(\mathrm{m})$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (S) |  | Horizontal | Vertical | Resultant | Horizontal Vertical |
| .00 | 1.828 | 3.166 | 3.656 | .0000 | .0000 |
| .01 | .216 | .418 | .470 | .0070 | .0123 |
| .02 | .042 | .130 | .136 | .0080 | .0147 |
| .03 | .009 | .078 | .078 | .0082 | .0156 |
| .04 | .002 | .067 | .067 | .0083 | .0163 |
| .05 | .000 | .065 | .065 | .0083 | .0170 |
| .06 | .000 | .064 | .064 | .0083 | .0176 |
| .07 | .000 | .064 | .064 | .0083 | .0183 |
| .08 | .000 | .064 | .064 | .0083 | .0189 |
| .09 | .000 | .064 | .064 | .0083 | .0196 |
| .10 | .000 | .064 | .064 | .0083 | .0202 |
| .11 | .000 | .064 | .064 | .0083 | .0208 |
| .12 | .000 | .064 | .064 | .0083 | .0215 |

## DIAMETER $=\mathbf{0 . 0 0 0 0 5 7 4 0} \mathrm{m}$ PARTICLE TRAJECTORY

| Time | Velocity $(\mathrm{m} / \mathrm{s})$ |  |  | Distance $(\mathrm{m})$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| (S) Horizontal |  | Vertical | Resultant | Horizontal Vertical |  |
| .00 | 1.828 | 3.166 | 3.656 | .0000 | .0000 |
| .01 | .430 | .798 | .906 | .0094 | .0166 |
| .02 | .138 | .317 | .346 | .0119 | .0216 |
| .03 | .050 | .176 | .183 | .0128 | .0240 |
| .04 | .019 | .130 | .131 | .0131 | .0254 |
| .05 | .007 | .113 | .114 | .0132 | .0266 |
| .06 | .003 | .108 | .108 | .0133 | .0277 |
| .07 | .001 | .105 | .105 | .0133 | .0288 |
| .08 | .000 | .105 | .105 | .0133 | .0299 |
| .09 | .000 | .104 | .104 | .0133 | .0309 |
| .10 | .000 | .104 | .104 | .0133 | .0320 |
| .11 | .000 | .104 | .104 | .0133 | .0330 |
| .12 | .000 | .104 | .104 | .0133 | .0340 |
| .13 | .000 | .104 | .104 | .0133 | .0351 |
| .14 | .000 | .104 | .104 | .0133 | .0361 |
| .15 | .000 | .104 | .104 | .0133 | .0372 |
| .16 | .000 | .104 | .104 | .0133 | .0382 |
| .17 | .000 | .104 | .104 | .0133 | .0392 |
| .18 | .000 | .104 | .104 | .0133 | .0403 |
| .19 | .000 | .104 | .104 | .0133 | .0413 |

## DIAMETER $=\mathbf{0 . 0 0 0 0 7 4 4 5} \mathrm{m}$

PARTICLE TRAJECTORY

| Time | Velocity (m/s) |  |  |  | Distance (m) |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| (S) Horizontal | Vertical | Resultant | Horizontal Vertical |  |  |  |
| .00 | 1.828 | 3.166 | 3.656 | .0000 | .0000 |  |
| .01 | .691 | 1.261 | 1.438 | .0116 | .0204 |  |
| .02 | .311 | .636 | .708 | .0163 | .0294 |  |
| .03 | .154 | .386 | .416 | .0185 | .0344 |  |
| .04 | .081 | .275 | .286 | .0197 | .0376 |  |
| .05 | .043 | .221 | .225 | .0203 | .0400 |  |
| .06 | .024 | .193 | .195 | .0206 | .0421 |  |
| .07 | .013 | .179 | .180 | .0208 | .0439 |  |
| .08 | .007 | .172 | .172 | .0209 | .0457 |  |
| .09 | .004 | .169 | .169 | .0209 | .0474 |  |
| .10 | .002 | .167 | .167 | .0209 | .0491 |  |
| .11 | .001 | .166 | .166 | .0210 | .0507 |  |
| .12 | .001 | .165 | .165 | .0210 | .0524 |  |
| .13 | .000 | .165 | .165 | .0210 | .0540 |  |
| .14 | .000 | .165 | .165 | .0210 | .0557 |  |
| .15 | .000 | .165 | .165 | .0210 | .0573 |  |
| .16 | .000 | .165 | .165 | .0210 | .0590 |  |
| .17 | .000 | .165 | .165 | .0210 | .0606 |  |
| .18 | .000 | .165 | .165 | .0210 | .0623 |  |
| .19 | .000 | .165 | .165 | .0210 | .0639 |  |
| .20 | .000 | .165 | .165 | .0210 | .0656 |  |
| .21 | .000 | .165 | .165 | .0210 | .0672 |  |
| .22 | .000 | .165 | .165 | .0210 | .0689 |  |
| .23 | .000 | .165 | .165 | .0210 | .0705 |  |
| .24 | .000 | .165 | .165 | .0210 | .0721 |  |
| .25 | .000 | .165 | .165 | .0210 | .0738 |  |
| .26 | .000 | .165 | .165 | .0210 | .0754 |  |
| .27 | .000 | .165 | .165 | .0210 | .0771 |  |
| .28 | .000 | .165 | .165 | .0210 | .0787 |  |
|  |  |  |  |  |  |  |


| DIAMETER $=0.00009855 \mathrm{~m}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| PARTICLE TRAJECTORY |  |  |  |  |  |
| Time | Velocity | (m/s) |  | Distan | (m) |
| (S) | Horizontal | Vertica | Result | Horizon | 1 Vertica |
| . 00 | 1.828 | 3.166 | 3.656 | . 0000 | . 0000 |
| . 01 | . 971 | 1.755 | 2.005 | . 0133 | . 0235 |
| . 02 | . 573 | 1.111 | 1.250 | . 0210 | . 0377 |
| . 03 | . 348 | . 754 | . 830 | . 0254 | . 0468 |
| . 04 | . 222 | . 560 | . 603 | . 0282 | . 0533 |
| . 05 | . 145 | . 446 | . 470 | . 0300 | . 0582 |
| . 06 | . 097 | . 379 | . 391 | . 0312 | . 0623 |
| . 07 | . 066 | . 338 | . 344 | . 0320 | . 0659 |
| . 08 | . 045 | . 312 | . 315 | . 0326 | . 0691 |
| . 09 | . 031 | . 295 | . 297 | . 0329 | . 0722 |
| . 10 | . 021 | . 285 | . 286 | . 0332 | . 0751 |
| . 11 | . 015 | . 278 | . 278 | . 0334 | . 0779 |
| . 12 | . 010 | . 273 | . 274 | . 0335 | . 0806 |
| . 13 | . 007 | . 271 | . 271 | . 0336 | . 0834 |
| . 14 | . 005 | . 269 | . 269 | . 0336 | . 0860 |
| . 15 | . 003 | . 267 | . 267 | . 0337 | . 0887 |
| . 16 | . 002 | . 267 | . 267 | . 0337 | . 0914 |
| . 17 | . 002 | . 266 | . 266 | . 0337 | . 0941 |
| . 18 | . 001 | . 266 | . 266 | . 0337 | . 0967 |
| . 19 | . 001 | . 265 | . 265 | . 0338 | . 0994 |
| . 20 | . 001 | . 265 | . 265 | . 0338 | . 1020 |
| . 21 | . 000 | . 265 | . 265 | . 0338 | . 1047 |
| . 22 | . 000 | . 265 | . 265 | . 0338 | . 1073 |
| . 23 | . 000 | . 265 | . 265 | . 0338 | . 1100 |
| . 24 | . 000 | . 265 | . 265 | . 0338 | . 1126 |
| . 25 | . 000 | . 265 | . 265 | . 0338 | . 1153 |
| . 26 | . 000 | . 265 | . 265 | . 0338 | . 1179 |
| . 27 | . 000 | . 265 | . 265 | . 0338 | . 1206 |
| . 28 | . 000 | . 265 | . 265 | . 0338 | . 1232 |
| . 29 | . 000 | . 265 | . 265 | . 0338 | . 1259 |
| . 30 | . 000 | . 265 | . 265 | . 0338 | . 1285 |
| . 31 | . 000 | . 265 | . 265 | . 0338 | . 1312 |
| . 32 | . 000 | . 265 | . 265 | . 0338 | . 1338 |
| . 33 | . 000 | . 265 | . 265 | . 0338 | . 1365 |
| . 34 | . 000 | . 265 | . 265 | . 0338 | . 1391 |
| . 35 | . 000 | . 265 | . 265 | . 0338 | . 1418 |
| . 36 | . 000 | . 265 | . 265 | . 0338 | . 1444 |
| . 37 | . 000 | . 265 | . 265 | . 0338 | . 1471 |
| . 38 | . 000 | . 265 | . 265 | . 0338 | . 1497 |
| . 39 | . 000 | . 265 | . 265 | . 0338 | . 1524 |
| . 40 | . 000 | . 265 | . 265 | . 0338 | . 1550 |
| . 41 | . 000 | . 265 | . 265 | . 0338 | . 1577 |
| . 42 | . 000 | . 265 | . 265 | . 0338 | . 1603 |

## DIAMETER $=\mathbf{0 . 0 0 0 1 3 6 5 5} \mathrm{m}$ PARTICLE TRAJECTORY

| Time | Velocity (m/s) |  |  | Distance (m) |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| (S) Horizontal Vertical Resultant | Horizontal Vertical |  |  |  |  |
| .00 | 1.828 | 3.166 | 3.656 | .0000 | .0000 |
| .01 | 1.227 | 2.206 | 2.525 | .0150 | .0264 |
| .02 | .867 | 1.642 | 1.856 | .0253 | .0454 |
| .03 | .640 | 1.296 | 1.446 | .0328 | .0600 |
| .04 | .492 | 1.083 | 1.189 | .0384 | .0718 |
| .05 | .379 | .921 | .996 | .0427 | .0819 |
| .06 | .280 | .764 | .814 | .0460 | .0902 |
| .07 | .216 | .676 | .709 | .0485 | .0974 |
| .08 | .169 | .616 | .639 | .0504 | .1038 |
| .09 | .133 | .572 | .588 | .0519 | .1097 |
| .10 | .105 | .539 | .549 | .0531 | .1153 |
| .11 | .083 | .513 | .520 | .0540 | .1205 |
| .12 | .066 | .494 | .498 | .0547 | .1256 |
| .13 | .052 | .479 | .481 | .0553 | .1304 |
| .14 | .041 | .467 | .469 | .0558 | .1352 |
| .15 | .033 | .459 | .460 | .0561 | .1398 |
| .16 | .026 | .452 | .453 | .0564 | .1443 |
| .17 | .021 | .447 | .447 | .0567 | .1488 |
| .18 | .016 | .443 | .443 | .0569 | .1533 |
| .19 | .013 | .440 | .440 | .0570 | .1577 |
| .20 | .010 | .438 | .438 | .0571 | .1621 |
| .21 | .008 | .436 | .436 | .0572 | .1665 |
| .22 | .007 | .435 | .435 | .0573 | .1708 |
| .23 | .005 | .434 | .434 | .0573 | .1752 |
| .24 | .004 | .433 | .433 | .0574 | .1795 |
| .25 | .003 | .433 | .433 | .0574 | .1838 |
| .26 | .003 | .432 | .432 | .0575 | .1881 |
| .27 | .002 | .432 | .432 | .0575 | .1925 |
| .28 | .002 | .432 | .432 | .0575 | .1968 |
| .29 | .001 | .432 | .432 | .0575 | .2011 |
| .30 | .001 | .431 | .431 | .0575 | .2054 |
| .31 | .001 | .431 | .431 | .0575 | .2097 |
| .32 | .001 | .431 | .431 | .0576 | .2140 |
| .33 | .001 | .431 | .431 | .0576 | .2184 |
| .34 | .000 | .431 | .431 | .0576 | .2227 |
| .35 | .000 | .431 | .431 | .0576 | .2270 |
| .36 | .000 | .431 | .431 | .0576 | .2313 |
| .37 | .000 | .431 | .431 | .0576 | .2356 |
| .38 | .000 | .431 | .431 | .0576 | .2399 |
| .39 | .000 | .431 | .431 | .0576 | .2442 |
| .3 |  |  |  |  |  |


| .40 | .000 | .431 | .431 | .0576 | .2485 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .41 | .000 | .431 | .431 | .0576 | .2528 |
| .42 | .000 | .431 | .431 | .0576 | .2572 |
| .43 | .000 | .431 | .431 | .0576 | .2615 |
| .44 | .000 | .431 | .431 | .0576 | .2658 |
| .45 | .000 | .431 | .431 | .0576 | .2701 |
| .46 | .000 | .431 | .431 | .0576 | .2744 |
| .47 | .000 | .431 | .431 | .0576 | .2787 |
| .48 | .000 | .431 | .431 | .0576 | .2830 |
| .49 | .000 | .431 | .431 | .0576 | .2873 |
| .50 | .000 | .431 | .431 | .0576 | .2916 |
| .51 | .000 | .431 | .431 | .0576 | .2959 |
| .52 | .000 | .431 | .431 | .0576 | .3003 |
| .53 | .000 | .431 | .431 | .0576 | .3046 |
| .54 | .000 | .431 | .431 | .0576 | .3089 |
| .55 | .000 | .431 | .431 | .0576 | .3132 |
| .56 | .000 | .431 | .431 | .0576 | .3175 |
| .57 | .000 | .431 | .431 | .0576 | .3218 |
| .58 | .000 | .431 | .431 | .0576 | .3261 |
| .59 | .000 | .431 | .431 | .0576 | .3304 |
| .60 | .000 | .431 | .431 | .0576 | .3347 |
| .61 | .000 | .431 | .431 | .0576 | .3390 |
| .62 | .000 | .431 | .431 | .0576 | .3434 |
| .63 | .000 | .431 | .431 | .0576 | .3477 |
| .64 | .000 | .431 | .431 | .0576 | .3520 |
| .65 | .000 | .431 | .431 | .0576 | .3563 |
| .66 | .000 | .431 | .431 | .0576 | .3606 |
| .67 | .000 | .431 | .431 | .0576 | .3649 |
| .68 | .000 | .431 | .431 | .0576 | .3692 |
| .69 | .000 | .431 | .431 | .0576 | .3735 |

## DIAMETER $=\mathbf{0 . 0 0 0 2 1 0 9 5 m}$ <br> PARTICLE TRAJECTORY

Time Velocity (m/s) Distance (m)

| (S) |  |  |  |  | Horizontal |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Vertical | Resultant | Horizontal Vertical |  |  |  |
| .00 | 1.828 | 3.166 | 3.656 | .0000 | .0000 |
| .01 | 1.482 | 2.656 | 3.042 | .0165 | .0290 |
| .02 | 1.221 | 2.278 | 2.585 | .0299 | .0536 |
| .03 | 1.019 | 1.990 | 2.236 | .0411 | .0748 |
| .04 | .859 | 1.767 | 1.964 | .0504 | .0936 |
| .05 | .729 | 1.591 | 1.750 | .0584 | .1103 |
| .06 | .624 | 1.451 | 1.580 | .0651 | .1255 |
| .07 | .536 | 1.340 | 1.443 | .0709 | .1395 |
| .08 | .464 | 1.250 | 1.333 | .0759 | .1524 |
| .09 | .403 | 1.177 | 1.245 | .0802 | .1645 |


|  |  |  |  |  | .0840 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .10 | .352 | 1.119 | 1.173 | .1760 |  |
| .11 | .308 | 1.072 | 1.115 | .0873 | .1869 |
| .12 | .270 | 1.033 | 1.068 | .0902 | .1974 |
| .13 | .238 | 1.002 | 1.030 | .0927 | .2076 |
| .14 | .210 | .977 | .999 | .0949 | .2175 |
| .15 | .186 | .957 | .975 | .0969 | .2272 |
| .16 | .165 | .940 | .954 | .0987 | .2366 |
| .17 | .146 | .927 | .938 | .1002 | .2460 |
| .18 | .130 | .916 | .925 | .1016 | .2552 |
| .19 | .116 | .907 | .915 | .1028 | .2643 |
| .20 | .103 | .900 | .906 | .1039 | .2733 |
| .21 | .092 | .894 | .899 | .1049 | .2823 |
| .22 | .082 | .890 | .893 | .1058 | .2912 |
| .23 | .073 | .886 | .889 | .1065 | .3001 |
| .24 | .065 | .883 | .885 | .1072 | .3089 |
| .25 | .058 | .880 | .882 | .1078 | .3178 |
| .26 | .052 | .878 | .880 | .1084 | .3266 |
| .27 | .046 | .877 | .878 | .1089 | .3353 |
| .28 | .041 | .875 | .876 | .1093 | .3441 |
| .29 | .037 | .874 | .875 | .1097 | .3528 |
| .30 | .033 | .873 | .874 | .1101 | .3616 |
| .31 | .029 | .873 | .873 | .1104 | .3703 |
| .32 | .026 | .872 | .873 | .1106 | .3790 |
| .33 | .023 | .872 | .872 | .1109 | .3878 |
| .34 | .021 | .871 | .872 | .1111 | .3965 |
| .35 | .019 | .871 | .871 | .1113 | .4052 |
| .36 | .017 | .871 | .871 | .1115 | .4139 |
| .37 | .015 | .871 | .871 | .1116 | .4226 |
| .38 | .013 | .870 | .871 | .1118 | .4313 |
| .39 | .012 | .870 | .870 | .1119 | .4400 |
| .40 | .011 | .870 | .870 | .1120 | .4487 |
| .41 | .010 | .870 | .870 | .1121 | .4574 |
| .42 | .008 | .870 | .870 | .1122 | .4661 |
| .43 | .008 | .870 | .870 | .1123 | .4748 |
| .44 | .007 | .870 | .870 | .1124 | .4835 |
| .45 | .006 | .870 | .870 | .1124 | .4922 |
| .46 | .005 | .870 | .870 | .1125 | .5009 |
| .47 | .005 | .870 | .870 | .1125 | .5096 |
| .48 | .004 | .870 | .870 | .1126 | .5183 |
| .49 | .004 | .870 | .870 | .1126 | .5270 |
| .50 | .003 | .870 | .870 | .1127 | .5357 |
| .51 | .003 | .870 | .870 | .1127 | .5444 |
| .52 | .003 | .870 | .870 | .1127 | .5531 |
| .53 | .002 | .870 | .870 | .1127 | .5618 |
| .55 | .002 | .870 | .870 | .1128 | .5705 |
|  |  | .870 | .870 | .1128 | .5792 |


| .56 | .002 | .870 | .870 | .1128 | .5879 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .57 | .002 | .870 | .870 | .1128 | .5966 |
| .58 | .001 | .870 | .870 | .1128 | .6053 |
| .59 | .001 | .870 | .870 | .1129 | .6140 |
| .60 | .001 | .870 | .870 | .1129 | .6227 |
| .61 | .001 | .870 | .870 | .1129 | .6314 |
| .62 | .001 | .870 | .870 | .1129 | .6401 |
| .63 | .001 | .870 | .870 | .1129 | .6488 |
| .64 | .001 | .870 | .870 | .1129 | .6575 |
| .65 | .001 | .870 | .870 | .1129 | .6662 |
| .66 | .001 | .870 | .870 | .1129 | .6749 |
| .67 | .001 | .870 | .870 | .1129 | .6836 |
| .68 | .000 | .870 | .870 | .1129 | .6922 |
| .69 | .000 | .870 | .870 | .1129 | .7009 |
| .70 | .000 | .870 | .870 | .1129 | .7096 |
| .71 | .000 | .870 | .870 | .1129 | .7183 |
| .72 | .000 | .870 | .870 | .1129 | .7270 |
| .73 | .000 | .870 | .870 | .1129 | .7357 |
| .74 | .000 | .870 | .870 | .1129 | .7444 |
| .75 | .000 | .870 | .870 | .1129 | .7531 |
| .76 | .000 | .870 | .870 | .1129 | .7618 |
| .77 | .000 | .870 | .870 | .1130 | .7705 |
| .78 | .000 | .870 | .870 | .1130 | .7792 |
| .79 | .000 | .870 | .870 | .1130 | .7879 |
| .80 | .000 | .870 | .870 | .1130 | .7966 |
| .81 | .000 | .870 | .870 | .1130 | .8053 |
| .82 | .000 | .870 | .870 | .1130 | .8140 |
| .83 | .000 | .870 | .870 | .1130 | .8227 |
| .84 | .000 | .870 | .870 | .1130 | .8314 |
| .85 | .000 | .870 | .870 | .1130 | .8401 |
| .86 | .000 | .870 | .870 | .1130 | .8488 |
| .87 | .000 | .870 | .870 | .1130 | .8575 |
| .88 | .000 | .870 | .870 | .1130 | .8662 |
| .89 | .000 | .870 | .870 | .1130 | .8749 |
| .90 | .000 | .870 | .870 | .1130 | .8836 |
| .91 | .000 | .870 | .870 | .1130 | .8923 |
| .92 | .000 | .870 | .870 | .1130 | .9010 |
| .93 | .000 | .870 | .870 | .1130 | .9097 |
| .94 | .000 | .870 | .870 | .1130 | .9184 |
| .95 | .000 | .870 | .870 | .1130 | .9271 |
| .96 | .000 | .870 | .870 | .1130 | .9358 |
| .97 | .000 | .870 | .870 | .1130 | .9445 |
| .98 | .000 | .870 | .870 | .1130 | .9532 |
| .99 | .000 | .870 | .870 | .1130 | .9619 |
| 1.00 | .000 | .870 | .870 | .1130 | .9706 |
| 1.01 | .000 | .870 | .870 | .1130 | .9793 |
|  |  |  |  |  |  |


| 1.02 | .000 | .870 | .870 | .1130 | .9880 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 1.03 | .000 | .870 | .870 | .1130 | .9966 |
| 1.04 | .000 | .870 | .870 | .1130 | 1.0053 |
| 1.05 | .000 | .870 | .870 | .1130 | 1.0140 |
| 1.06 | .000 | .870 | .870 | .1130 | 1.0227 |
| 1.07 | .000 | .870 | .870 | .1130 | 1.0314 |
| 1.08 | .000 | .870 | .870 | .1130 | 1.0401 |
| 1.09 | .000 | .870 | .870 | .1130 | 1.0488 |
| 1.10 | .000 | .870 | .870 | .1130 | 1.0575 |
| 1.11 | .000 | .870 | .870 | .1130 | 1.0662 |
| 1.12 | .000 | .870 | .870 | .1130 | 1.0749 |
| 1.13 | .000 | .870 | .870 | .1130 | 1.0836 |
| 1.14 | .000 | .870 | .870 | .1130 | 1.0923 |
| 1.15 | .000 | .870 | .870 | .1130 | 1.1010 |
| 1.16 | .000 | .870 | .870 | .1130 | 1.1097 |
| 1.17 | .000 | .870 | .870 | .1130 | 1.1184 |
| 1.18 | .000 | .870 | .870 | .1130 | 1.1271 |
| 1.19 | .000 | .870 | .870 | .1130 | 1.1358 |
| 1.20 | .000 | .870 | .870 | .1130 | 1.1445 |
| 1.21 | .000 | .870 | .870 | .1130 | 1.1532 |
| 1.22 | .000 | .870 | .870 | .1130 | 1.1619 |
| 1.23 | .000 | .870 | .870 | .1130 | 1.1706 |
| 1.24 | .000 | .870 | .870 | .1130 | 1.1793 |
| 1.25 | .000 | .870 | .870 | .1130 | 1.1880 |
| 1.26 | .000 | .870 | .870 | .1130 | 1.1967 |
| 1.27 | .000 | .870 | .870 | .1130 | 1.2054 |
| 1.28 | .000 | .870 | .870 | .1130 | 1.2141 |
| 1.29 | .000 | .870 | .870 | .1130 | 1.2228 |
| 1.30 | .000 | .870 | .870 | .1130 | 1.2315 |
| 1.31 | .000 | .870 | .870 | .1130 | 1.2402 |
| 1.32 | .000 | .870 | .870 | .1130 | 1.2489 |

## DIAMETER $=\mathbf{0 . 0 0 0 4 1 2 8 0} \mathrm{m}$ PARTICLE TRAJECTORY

| Time | Velocity (m/s) |  |  | Distance (m) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (S) Horizontal Vertical Resultant | Horizontal Vertical |  |  |  |  |
| .00 | 1.828 | 3.166 | 3.656 | .0000 | .0000 |
| .01 | 1.688 | 3.018 | 3.458 | .0176 | .0309 |
| .02 | 1.562 | 2.887 | 3.282 | .0338 | .0604 |
| .03 | 1.448 | 2.771 | 3.127 | .0488 | .0887 |
| .04 | 1.345 | 2.668 | 2.988 | .0628 | .1159 |
| .05 | 1.251 | 2.577 | 2.865 | .0758 | .1421 |
| .06 | 1.166 | 2.496 | 2.755 | .0879 | .1675 |
| .07 | 1.088 | 2.424 | 2.657 | .0991 | .1921 |
| .08 | 1.017 | 2.359 | 2.569 | .1096 | .2160 |
| .09 | .951 | 2.301 | 2.490 | .1195 | .2393 |
| .10 | .890 | 2.250 | 2.419 | .1287 | .2620 |
| .11 | .834 | 2.203 | 2.356 | .1373 | .2843 |
| .12 | .783 | 2.162 | 2.299 | .1454 | .3061 |
| .13 | .735 | 2.124 | 2.248 | .1530 | .3275 |
| .14 | .690 | 2.090 | 2.201 | .1601 | .3486 |
| .15 | .649 | 2.060 | 2.160 | .1668 | .3693 |
| .16 | .610 | 2.033 | 2.122 | .1731 | .3898 |
| .17 | .574 | 2.008 | 2.088 | .1790 | .4100 |
| .18 | .540 | 1.985 | 2.058 | .1846 | .4300 |
| .19 | .509 | 1.965 | 2.030 | .1898 | .4497 |
| .20 | .480 | 1.947 | 2.005 | .1947 | .4693 |
| .21 | .452 | 1.930 | 1.982 | .1994 | .4887 |
| .22 | .426 | 1.915 | 1.962 | .2038 | .5079 |
| .23 | .402 | 1.901 | 1.944 | .2079 | .5270 |
| .24 | .379 | 1.889 | 1.927 | .2118 | .5459 |
| .25 | .358 | 1.878 | 1.912 | .2155 | .5647 |
| .26 | .338 | 1.868 | 1.898 | .2190 | .5835 |
| .27 | .319 | 1.858 | 1.886 | .2223 | .6021 |
| .28 | .301 | 1.850 | 1.874 | .2254 | .6206 |
| .29 | .284 | 1.842 | 1.864 | .2283 | .6391 |
| .30 | .269 | 1.835 | 1.855 | .2311 | .6575 |
| .31 | .254 | 1.829 | 1.846 | .2337 | .6758 |
| .32 | .240 | 1.823 | 1.839 | .2361 | .6941 |
| .33 | .226 | 1.818 | 1.832 | .2385 | .7123 |
| .34 | .214 | 1.813 | 1.826 | .2407 | .7304 |
| .35 | .202 | 1.809 | 1.820 | .2428 | .7485 |
| .36 | .191 | 1.805 | 1.815 | .2447 | .7666 |
| .37 | .181 | 1.801 | 1.810 | .2466 | .7846 |
| .38 | .171 | 1.798 | 1.806 | .2483 | .8026 |
| .39 | .161 | 1.795 | 1.802 | .2500 | .8206 |
| .40 | .153 | 1.792 | 1.798 | .2516 | .8385 |
| .41 | .144 | 1.789 | 1.795 | .2530 | .8564 |


| .42 | .136 | 1.787 | 1.792 | .2545 | .8743 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .43 | .129 | 1.785 | 1.789 | .2558 | .8921 |
| .44 | .122 | 1.783 | 1.787 | .2570 | .9100 |
| .45 | .115 | 1.781 | 1.785 | .2582 | .9278 |
| .46 | .109 | 1.779 | 1.783 | .2593 | .9456 |
| .47 | .103 | 1.778 | 1.781 | .2604 | .9634 |
| .48 | .097 | 1.777 | 1.779 | .2614 | .9812 |
| .49 | .092 | 1.775 | 1.778 | .2623 | .9989 |
| .50 | .087 | 1.774 | 1.776 | .2632 | 1.0167 |
| .51 | .082 | 1.773 | 1.775 | .2641 | 1.0344 |
| .52 | .078 | 1.772 | 1.774 | .2649 | 1.0521 |
| .53 | .074 | 1.771 | 1.773 | .2656 | 1.0699 |
| .54 | .070 | 1.771 | 1.772 | .2664 | 1.0876 |
| .55 | .066 | 1.770 | 1.771 | .2670 | 1.1053 |
| .56 | .062 | 1.769 | 1.770 | .2677 | 1.1230 |
| .57 | .059 | 1.769 | 1.770 | .2683 | 1.1407 |
| .58 | .056 | 1.768 | 1.769 | .2689 | 1.1583 |
| .59 | .053 | 1.768 | 1.768 | .2694 | 1.1760 |
| .60 | .050 | 1.767 | 1.768 | .2699 | 1.1937 |
| .61 | .047 | 1.767 | 1.767 | .2704 | 1.2114 |
| .62 | .045 | 1.766 | 1.767 | .2709 | 1.2290 |
| .63 | .042 | 1.766 | 1.766 | .2713 | 1.2467 |
| .64 | .040 | 1.766 | 1.766 | .2717 | 1.2643 |
| .65 | .038 | 1.765 | 1.766 | .2721 | 1.2820 |
| .66 | .036 | 1.765 | 1.765 | .2725 | 1.2997 |
| .67 | .034 | 1.765 | 1.765 | .2728 | 1.3173 |
| .68 | .032 | 1.765 | 1.765 | .2731 | 1.3349 |
| .69 | .030 | 1.764 | 1.765 | .2735 | 1.3526 |
| .70 | .029 | 1.764 | 1.764 | .2737 | 1.3702 |
| .71 | .027 | 1.764 | 1.764 | .2740 | 1.3879 |
| .72 | .026 | 1.764 | 1.764 | .2743 | 1.4055 |
| .73 | .024 | 1.764 | 1.764 | .2745 | 1.4231 |
| .74 | .023 | 1.763 | 1.764 | .2748 | 1.4408 |
| .75 | .022 | 1.763 | 1.763 | .2750 | 1.4584 |
| .76 | .020 | 1.763 | 1.763 | .2752 | 1.4760 |
| .77 | .019 | 1.763 | 1.763 | .2754 | 1.4937 |

## INITIAL VELOCITY $=4.478 \mathrm{~m} / \mathrm{s}$ SLURRY FLOW RATE= $8.64 \mathrm{e}-3 \mathrm{Kg} / \mathrm{s}$

| DIAMETER= $\mathbf{0 . 0 0 0 0 2 7} \mathbf{m}$ |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| PARTICLE |  |  |  |  |  |
| TRAJECTORY |  |  |  |  |  |
| Time | Velocity |  | $(\mathrm{m} / \mathrm{s}$ ) | Distance (m) |  |
| (S) Horizontal Vertical | Resultant | Horizontal Vertical |  |  |  |
| .00 | 2.239 | 3.878 | 4.478 | .0000 | .0000 |
| .01 | .017 | .052 | .055 | .0036 | .0064 |
| .02 | .000 | .024 | .024 | .0036 | .0067 |
| .03 | .000 | .023 | .023 | .0036 | .0069 |
| .04 | .000 | .023 | .023 | .0036 | .0072 |
| .05 | .000 | .023 | .023 | .0036 | .0074 |
| .06 | .000 | .023 | .023 | .0036 | .0076 |

## DIAMETER $=\mathbf{0 . 0 0 0 0 3 4 6 5} \mathrm{m}$ <br> PARTICLE TRAJECTORY

| Time | Velocity $(\mathrm{m} / \mathrm{s})$ |  |  | Distance (m) |  |
| :--- | ---: | ---: | ---: | :---: | :---: |
| (S) Horizontal Vertical Resultant | Horizontal Vertical |  |  |  |  |
| .00 | 2.239 | 3.878 | 4.478 | .0000 | .0000 |
| .01 | .092 | .192 | .213 | .0060 | .0106 |
| .02 | .007 | .050 | .051 | .0063 | .0116 |
| .03 | .001 | .040 | .040 | .0064 | .0120 |
| .04 | .000 | .039 | .039 | .0064 | .0124 |
| .05 | .000 | .039 | .039 | .0064 | .0128 |
| .06 | .000 | .039 | .039 | .0064 | .0132 |
| .07 | .000 | .039 | .039 | .0064 | .0135 |
| .08 | .000 | .039 | .039 | .0064 | .0139 |

## DIAMETER $=\mathbf{0 . 0 0 0 0 4 4 6 0} \mathrm{m}$ PARTICLE TRAJECTORY

| Time | Velocity $(\mathrm{m} / \mathrm{s})$ |  |  | Distance $(\mathrm{m})$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| (S) Horizontal | Vertical | Resultant | Horizontal Vertical |  |  |
| .00 | 2.239 | 3.878 | 4.478 | .0000 | .0000 |
| .01 | .252 | .480 | .542 | .0085 | .0150 |
| .02 | .048 | .140 | .147 | .0097 | .0176 |
| .03 | .010 | .080 | .081 | .0100 | .0186 |
| .04 | .002 | .067 | .068 | .0100 | .0193 |
| .05 | .000 | .065 | .065 | .0100 | .0200 |
| .06 | .000 | .064 | .064 | .0100 | .0206 |
| .07 | .000 | .064 | .064 | .0100 | .0213 |
| .08 | .000 | .064 | .064 | .0100 | .0219 |
| .09 | .000 | .064 | .064 | .0100 | .0226 |
| .10 | .000 | .064 | .064 | .0100 | .0232 |
| .11 | .000 | .064 | .064 | .0100 | .0238 |
| .12 | .000 | .064 | .064 | .0100 | .0245 |

## DIAMETER $=\mathbf{0 . 0 0 0 0 5 7 4 0} \mathrm{m}$

 PARTICLE TRAJECTORY| Time | Velocity $(\mathrm{m} / \mathrm{s})$ |  |  | Distance $(\mathrm{m})$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| (S) |  |  |  |  |  |
| .00 | 2.239 | 3.878 | 4.478 | .0000 | .0000 |
| .01 | .494 | .908 | 1.034 | .0111 | .0196 |
| .02 | .156 | .345 | .379 | .0140 | .0252 |
| .03 | .056 | .185 | .194 | .0150 | .0277 |
| .04 | .021 | .133 | .134 | .0153 | .0293 |
| .05 | .008 | .115 | .115 | .0155 | .0305 |
| .06 | .003 | .108 | .108 | .0155 | .0316 |
| .07 | .001 | .106 | .106 | .0155 | .0327 |
| .08 | .000 | .105 | .105 | .0155 | .0337 |
| .09 | .000 | .104 | .104 | .0156 | .0348 |
| .10 | .000 | .104 | .104 | .0156 | .0358 |
| .11 | .000 | .104 | .104 | .0156 | .0369 |
| .12 | .000 | .104 | .104 | .0156 | .0379 |
| .13 | .000 | .104 | .104 | .0156 | .0389 |
| .14 | .000 | .104 | .104 | .0156 | .0400 |
| .15 | .000 | .104 | .104 | .0156 | .0410 |
| .16 | .000 | .104 | .104 | .0156 | .0421 |
| .17 | .000 | .104 | .104 | .0156 | .0431 |
| .18 | .000 | .104 | .104 | .0156 | .0442 |
| .19 | .000 | .104 | .104 | .0156 | .0452 |

## DIAMETER $=\mathbf{0 . 0 0 0 0 7 4 4 5} \mathrm{m}$ PARTICLE TRAJECTORY

| Time | Velocity (m/s) |  |  | Distance (m) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (S) Horizontal | Vertical | Resultant | Horizontal Vertical |  |  |
| .00 | 2.239 | 3.878 | 4.478 | .0000 | .0000 |
| .01 | .821 | 1.485 | 1.697 | .0138 | .0242 |
| .02 | .355 | .710 | .794 | .0192 | .0345 |
| .03 | .173 | .417 | .452 | .0217 | .0399 |
| .04 | .090 | .289 | .303 | .0230 | .0433 |
| .05 | .048 | .228 | .233 | .0237 | .0459 |
| .06 | .026 | .197 | .199 | .0240 | .0480 |
| .07 | .014 | .181 | .182 | .0242 | .0499 |
| .08 | .008 | .173 | .173 | .0243 | .0516 |
| .09 | .004 | .169 | .169 | .0244 | .0533 |
| .10 | .002 | .167 | .167 | .0244 | .0550 |
| .11 | .001 | .166 | .166 | .0245 | .0567 |
| .12 | .001 | .165 | .165 | .0245 | .0583 |
| .13 | .000 | .165 | .165 | .0245 | .0600 |
| .14 | .000 | .165 | .165 | .0245 | .0616 |


| .15 | .000 | .165 | .165 | .0245 | .0633 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .16 | .000 | .165 | .165 | .0245 | .0649 |
| .17 | .000 | .165 | .165 | .0245 | .0666 |
| .18 | .000 | .165 | .165 | .0245 | .0682 |
| .19 | .000 | .165 | .165 | .0245 | .0699 |
| .20 | .000 | .165 | .165 | .0245 | .0715 |
| .21 | .000 | .165 | .165 | .0245 | .0732 |
| .22 | .000 | .165 | .165 | .0245 | .0748 |
| .23 | .000 | .165 | .165 | .0245 | .0764 |
| .24 | .000 | .165 | .165 | .0245 | .0781 |
| .25 | .000 | .165 | .165 | .0245 | .0797 |
| .26 | .000 | .165 | .165 | .0245 | .0814 |
| .27 | .000 | .165 | .165 | .0245 | .0830 |
| .28 | .000 | .165 | .165 | .0245 | .0847 |

## DIAMETER $=\mathbf{0 . 0 0 0 0 9 8 5 5} \mathrm{m}$ PARTICLE TRAJECTORY

| Time | Velocity (m/s) |  |  | Distance (m) |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| (S) Horizontal | Vertical | Resultant | Horizontal Vertical |  |  |
| .00 | 2.239 | 3.878 | 4.478 | .0000 | .0000 |
| .01 | 1.129 | 2.028 | 2.321 | .0160 | .0280 |
| .02 | .676 | 1.292 | 1.458 | .0247 | .0441 |
| .03 | .394 | .830 | .919 | .0298 | .0544 |
| .04 | .250 | .606 | .655 | .0330 | .0615 |
| .05 | .162 | .473 | .500 | .0350 | .0668 |
| .06 | .108 | .395 | .409 | .0364 | .0711 |
| .07 | .073 | .348 | .355 | .0373 | .0748 |
| .08 | .050 | .318 | .322 | .0379 | .0781 |
| .09 | .034 | .299 | .301 | .0383 | .0812 |
| .10 | .023 | .287 | .288 | .0386 | .0841 |
| .11 | .016 | .280 | .280 | .0387 | .0870 |
| .12 | .011 | .275 | .275 | .0389 | .0897 |
| .13 | .008 | .271 | .271 | .0390 | .0925 |
| .14 | .005 | .269 | .269 | .0390 | .0952 |
| .15 | .004 | .268 | .268 | .0391 | .0978 |
| .16 | .003 | .267 | .267 | .0391 | .1005 |
| .17 | .002 | .266 | .266 | .0391 | .1032 |
| .18 | .001 | .266 | .266 | .0391 | .1058 |
| .19 | .001 | .265 | .265 | .0392 | .1085 |
| .20 | .001 | .265 | .265 | .0392 | .1111 |
| .21 | .000 | .265 | .265 | .0392 | .1138 |
| .22 | .000 | .265 | .265 | .0392 | .1165 |
| .23 | .000 | .265 | .265 | .0392 | .1191 |
| .24 | .000 | .265 | .265 | .0392 | .1218 |


| .25 | .000 | .265 | .265 | .0392 | .1244 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .26 | .000 | .265 | .265 | .0392 | .1271 |
| .27 | .000 | .265 | .265 | .0392 | .1297 |
| .28 | .000 | .265 | .265 | .0392 | .1324 |
| .29 | .000 | .265 | .265 | .0392 | .1350 |
| .30 | .000 | .265 | .265 | .0392 | .1377 |
| .31 | .000 | .265 | .265 | .0392 | .1403 |
| .32 | .000 | .265 | .265 | .0392 | .1430 |
| .33 | .000 | .265 | .265 | .0392 | .1456 |
| .34 | .000 | .265 | .265 | .0392 | .1483 |
| .35 | .000 | .265 | .265 | .0392 | .1509 |
| .36 | .000 | .265 | .265 | .0392 | .1536 |
| .37 | .000 | .265 | .265 | .0392 | .1562 |
| .38 | .000 | .265 | .265 | .0392 | .1589 |
| .39 | .000 | .265 | .265 | .0392 | .1615 |
| .40 | .000 | .265 | .265 | .0392 | .1642 |
| .41 | .000 | .265 | .265 | .0392 | .1668 |
| .42 | .000 | .265 | .265 | .0392 | .1695 |
| .43 | .000 | .265 | .265 | .0392 | .1721 |

## DIAMETER $=\mathbf{0 . 0 0 0 1 3 6 5 5} \mathrm{m}$ <br> PARTICLE TRAJECTORY

| Time | Velocity $(\mathrm{m} / \mathrm{s})$ |  |  | Distance $(\mathrm{m})$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| (S) |  |  |  | Horizontal | Vertical |
| .00 | 2.239 | 3.878 | 4.478 | .0000 | .0000 |
| .01 | 1.457 | 2.604 | 2.984 | .0181 | .0318 |
| .02 | 1.006 | 1.880 | 2.132 | .0302 | .0539 |
| .03 | .728 | 1.444 | 1.617 | .0388 | .0703 |
| .04 | .549 | 1.175 | 1.296 | .0451 | .0833 |
| .05 | .430 | 1.008 | 1.096 | .0500 | .0941 |
| .06 | .318 | .829 | .888 | .0537 | .1033 |
| .07 | .241 | .714 | .754 | .0565 | .1110 |
| .08 | .188 | .643 | .670 | .0586 | .1178 |
| .09 | .148 | .593 | .611 | .0603 | .1239 |
| .10 | .116 | .554 | .567 | .0616 | .1297 |
| .11 | .092 | .525 | .533 | .0626 | .1351 |
| .12 | .073 | .503 | .508 | .0634 | .1402 |
| .13 | .058 | .486 | .489 | .0641 | .1451 |
| .14 | .046 | .473 | .475 | .0646 | .1499 |
| .15 | .036 | .463 | .464 | .0650 | .1546 |
| .16 | .029 | .455 | .456 | .0653 | .1592 |
| .17 | .023 | .449 | .450 | .0656 | .1637 |
| .18 | .018 | .445 | .445 | .0658 | .1682 |
| .19 | .014 | .442 | .442 | .0659 | .1726 |
| .20 | .012 | .439 | .439 | .0661 | .1770 |


| .21 | .009 | .437 | .437 | .0662 | .1814 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .22 | .007 | .436 | .436 | .0663 | .1857 |
| .23 | .006 | .435 | .435 | .0663 | .1901 |
| .24 | .005 | .434 | .434 | .0664 | .1944 |
| .25 | .004 | .433 | .433 | .0664 | .1988 |
| .26 | .003 | .433 | .433 | .0664 | .2031 |
| .27 | .002 | .432 | .432 | .0665 | .2074 |
| .28 | .002 | .432 | .432 | .0665 | .2117 |
| .29 | .001 | .432 | .432 | .0665 | .2161 |
| .30 | .001 | .432 | .432 | .0665 | .2204 |
| .31 | .001 | .431 | .431 | .0665 | .2247 |
| .32 | .001 | .431 | .431 | .0665 | .2290 |
| .33 | .001 | .431 | .431 | .0665 | .2333 |
| .34 | .000 | .431 | .431 | .0666 | .2376 |
| .35 | .000 | .431 | .431 | .0666 | .2419 |
| .36 | .000 | .431 | .431 | .0666 | .2463 |
| .37 | .000 | .431 | .431 | .0666 | .2506 |
| .38 | .000 | .431 | .431 | .0666 | .2549 |
| .39 | .000 | .431 | .431 | .0666 | .2592 |
| .40 | .000 | .431 | .431 | .0666 | .2635 |
| .41 | .000 | .431 | .431 | .0666 | .2678 |
| .42 | .000 | .431 | .431 | .0666 | .2721 |
| .43 | .000 | .431 | .431 | .0666 | .2764 |
| .44 | .000 | .431 | .431 | .0666 | .2807 |
| .45 | .000 | .431 | .431 | .0666 | .2850 |
| .46 | .000 | .431 | .431 | .0666 | .2894 |
| .47 | .000 | .431 | .431 | .0666 | .2937 |
| .48 | .000 | .431 | .431 | .0666 | .2980 |
| .49 | .000 | .431 | .431 | .0666 | .3023 |
| .50 | .000 | .431 | .431 | .0666 | .3066 |
| .51 | .000 | .431 | .431 | .0666 | .3109 |
| .52 | .000 | .431 | .431 | .0666 | .3152 |
| .53 | .000 | .431 | .431 | .0666 | .3195 |
| .54 | .000 | .431 | .431 | .0666 | .3238 |
| .55 | .000 | .431 | .431 | .0666 | .3281 |
| .56 | .000 | .431 | .431 | .0666 | .3325 |
| .57 | .000 | .431 | .431 | .0666 | .3368 |
| .58 | .000 | .431 | .431 | .0666 | .3411 |
| .59 | .000 | .431 | .431 | .0666 | .3454 |
| .60 | .000 | .431 | .431 | .0666 | .3497 |
| .61 | .000 | .431 | .431 | .0666 | .3540 |
| .62 | .000 | .431 | .431 | .0666 | .3583 |
| .64 | .000 | .431 | .431 | .0666 | .3626 |
| .66 | .000 | .431 | .431 | .0666 | .3669 |
|  | .431 | .431 | .0666 | .3712 |  |
| .431 | .431 | .0666 | .3756 |  |  |

## DIAMETER $=\mathbf{0 . 0 0 0 2 1 0 9 5 m}$

 PARTICLE TRAJECTORY| Time | Velocity (m/s) |  |  | Distance (m) |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| (S) Horizontal | Vertical | Resultant | Horizontal | Vertical |  |
| .00 | 2.239 | 3.878 | 4.478 | .0000 | .0000 |
| .01 | 1.779 | 3.168 | 3.633 | .0200 | .0350 |
| .02 | 1.443 | 2.659 | 3.025 | .0360 | .0640 |
| .03 | 1.190 | 2.281 | 2.573 | .0491 | .0886 |
| .04 | .993 | 1.994 | 2.227 | .0600 | .1100 |
| .05 | .837 | 1.770 | 1.958 | .0691 | .1287 |
| .06 | .711 | 1.594 | 1.746 | .0768 | .1455 |
| .07 | .608 | 1.454 | 1.576 | .0834 | .1607 |
| .08 | .523 | 1.342 | 1.441 | .0890 | .1747 |
| .09 | .452 | 1.252 | 1.332 | .0939 | .1877 |
| .10 | .393 | 1.180 | 1.243 | .0981 | .1998 |
| .11 | .343 | 1.121 | 1.172 | .1018 | .2113 |
| .12 | .300 | 1.073 | 1.115 | .1050 | .2223 |
| .13 | .264 | 1.035 | 1.068 | .1078 | .2328 |
| .14 | .232 | 1.004 | 1.030 | .1103 | .2430 |
| .15 | .205 | .978 | .999 | .1125 | .2529 |
| .16 | .181 | .958 | .975 | .1144 | .2626 |
| .17 | .161 | .941 | .955 | .1161 | .2720 |
| .18 | .143 | .928 | .938 | .1176 | .2814 |
| .19 | .127 | .917 | .925 | .1190 | .2906 |
| .20 | .113 | .908 | .915 | .1202 | .2997 |
| .21 | .100 | .901 | .906 | .1212 | .3088 |
| .22 | .089 | .895 | .899 | .1222 | .3177 |
| .23 | .080 | .890 | .894 | .1230 | .3267 |
| .24 | .071 | .886 | .889 | .1238 | .3355 |
| .25 | .063 | .883 | .885 | .1244 | .3444 |
| .26 | .057 | .881 | .882 | .1250 | .3532 |
| .27 | .050 | .879 | .880 | .1256 | .3620 |
| .28 | .045 | .877 | .878 | .1260 | .3708 |
| .29 | .040 | .876 | .876 | .1265 | .3795 |
| .30 | .036 | .874 | .875 | .1269 | .3883 |
| .31 | .032 | .874 | .874 | .1272 | .3970 |
| .32 | .029 | .873 | .873 | .1275 | .4058 |
| .33 | .026 | .872 | .873 | .1278 | .4145 |
| .34 | .023 | .872 | .872 | .1280 | .4232 |
| .35 | .020 | .871 | .872 | .1282 | .4319 |
| .36 | .018 | .871 | .871 | .1284 | .4406 |
| .37 | .016 | .871 | .871 | .1286 | .4494 |
| .38 | .015 | .871 | .871 | .1287 | .4581 |
| .39 | .013 | .870 | .871 | .1289 | .4668 |
| .40 | .012 | .870 | .870 | .1290 | .4755 |
|  |  |  |  |  |  |


| .41 | .010 | .870 | .870 | .1291 | .4842 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .42 | .009 | .870 | .870 | .1292 | .4929 |
| .43 | .008 | .870 | .870 | .1293 | .5016 |
| .44 | .007 | .870 | .870 | .1294 | .5103 |
| .45 | .007 | .870 | .870 | .1294 | .5190 |
| .46 | .006 | .870 | .870 | .1295 | .5277 |
| .47 | .005 | .870 | .870 | .1296 | .5364 |
| .48 | .005 | .870 | .870 | .1296 | .5451 |
| .49 | .004 | .870 | .870 | .1297 | .5538 |
| .50 | .004 | .870 | .870 | .1297 | .5625 |
| .51 | .003 | .870 | .870 | .1297 | .5712 |
| .52 | .003 | .870 | .870 | .1298 | .5799 |
| .53 | .003 | .870 | .870 | .1298 | .5886 |
| .54 | .002 | .870 | .870 | .1298 | .5973 |
| .55 | .002 | .870 | .870 | .1298 | .6060 |
| .56 | .002 | .870 | .870 | .1299 | .6146 |
| .57 | .002 | .870 | .870 | .1299 | .6233 |
| .58 | .002 | .870 | .870 | .1299 | .6320 |
| .59 | .001 | .870 | .870 | .1299 | .6407 |
| .60 | .001 | .870 | .870 | .1299 | .6494 |
| .61 | .001 | .870 | .870 | .1299 | .6581 |
| .62 | .001 | .870 | .870 | .1299 | .6668 |
| .63 | .001 | .870 | .870 | .1300 | .6755 |
| .64 | .001 | .870 | .870 | .1300 | .6842 |
| .65 | .001 | .870 | .870 | .1300 | .6929 |
| .66 | .001 | .870 | .870 | .1300 | .7016 |
| .67 | .001 | .870 | .870 | .1300 | .7103 |
| .68 | .000 | .870 | .870 | .1300 | .7190 |
| .69 | .000 | .870 | .870 | .1300 | .7277 |
| .70 | .000 | .870 | .870 | .1300 | .7364 |
| .71 | .000 | .870 | .870 | .1300 | .7451 |
| .72 | .000 | .870 | .870 | .1300 | .7538 |
| .73 | .000 | .870 | .870 | .1300 | .7625 |
| .74 | .000 | .870 | .870 | .1300 | .7712 |
| .75 | .000 | .870 | .870 | .1300 | .7799 |
| .76 | .000 | .870 | .870 | .1300 | .7886 |
| .77 | .000 | .870 | .870 | .1300 | .7973 |
| .78 | .000 | .870 | .870 | .1300 | .8060 |
| .79 | .000 | .870 | .870 | .1300 | .8147 |
| .80 | .000 | .870 | .870 | .1300 | .8234 |
| .81 | .000 | .870 | .870 | .1300 | .8321 |
| .82 | .000 | .870 | .870 | .1300 | .8408 |
| .83 | .000 | .870 | .870 | .1300 | .8495 |
| .84 | .000 | .870 | .870 | .1300 | .8582 |
| .85 | .000 | .870 | .870 | .1300 | .8669 |
| .86 | .000 | .870 | .870 | .1300 | .8756 |
|  |  |  |  |  |  |


| .87 | .000 | .870 | .870 | .1300 | .8843 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .88 | .000 | .870 | .870 | .1300 | .8930 |
| .89 | .000 | .870 | .870 | .1300 | .9017 |
| .90 | .000 | .870 | .870 | .1300 | .9104 |
| .91 | .000 | .870 | .870 | .1300 | .9190 |
| .92 | .000 | .870 | .870 | .1300 | .9277 |
| .93 | .000 | .870 | .870 | .1300 | .9364 |
| .94 | .000 | .870 | .870 | .1300 | .9451 |
| .95 | .000 | .870 | .870 | .1300 | .9538 |
| .96 | .000 | .870 | .870 | .1300 | .9625 |
| .97 | .000 | .870 | .870 | .1300 | .9712 |
| .98 | .000 | .870 | .870 | .1300 | .9799 |
| .99 | .000 | .870 | .870 | .1300 | .9886 |
| 1.00 | .000 | .870 | .870 | .1300 | .9973 |
| 1.01 | .000 | .870 | .870 | .1300 | 1.0060 |
| 1.02 | .000 | .870 | .870 | .1300 | 1.0147 |
| 1.03 | .000 | .870 | .870 | .1300 | 1.0234 |
| 1.04 | .000 | .870 | .870 | .1300 | 1.0321 |
| 1.05 | .000 | .870 | .870 | .1300 | 1.0408 |
| 1.06 | .000 | .870 | .870 | .1300 | 1.0495 |
| 1.07 | .000 | .870 | .870 | .1300 | 1.0582 |
| 1.08 | .000 | .870 | .870 | .1300 | 1.0669 |
| 1.09 | .000 | .870 | .870 | .1300 | 1.0756 |
| 1.10 | .000 | .870 | .870 | .1300 | 1.0843 |
| 1.11 | .000 | .870 | .870 | .1300 | 1.0930 |
| 1.12 | .000 | .870 | .870 | .1300 | 1.1017 |
| 1.13 | .000 | .870 | .870 | .1300 | 1.1104 |
| 1.14 | .000 | .870 | .870 | .1300 | 1.1191 |
| 1.15 | .000 | .870 | .870 | .1300 | 1.1278 |
| 1.16 | .000 | .870 | .870 | .1300 | 1.1365 |
| 1.17 | .000 | .870 | .870 | .1300 | 1.1452 |
| 1.18 | .000 | .870 | .870 | .1300 | 1.1539 |
| 1.19 | .000 | .870 | .870 | .1300 | 1.1626 |
| 1.20 | .000 | .870 | .870 | .1300 | 1.1713 |
| 1.21 | .000 | .870 | .870 | .1300 | 1.1800 |
| 1.22 | .000 | .870 | .870 | .1300 | 1.1887 |
| 1.23 | .000 | .870 | .870 | .1300 | 1.1974 |
| 1.24 | .000 | .870 | .870 | .1300 | 1.2061 |
| 1.25 | .000 | .870 | .870 | .1300 | 1.2148 |
| 1.26 | .000 | .870 | .870 | .1300 | 1.2234 |
| 1.27 | .000 | .870 | .870 | .1300 | 1.2321 |
|  | 0 |  |  |  |  |

## DIAMETER $=\mathbf{0 . 0 0 0 4 1 2 8 0} \mathrm{m}$

## PARTICLE TRAJECTORY

| Time | Velocity (m/s) |  |  | Distance $(\mathrm{m})$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| (S) Horizontal | Vertical | Resultant | Horizontal | Vertical |  |
| .00 | 2.239 | 3.878 | 4.478 | .0000 | .0000 |
| .01 | 2.051 | 3.646 | 4.183 | .0214 | .0376 |
| .02 | 1.884 | 3.443 | 3.924 | .0411 | .0730 |
| .03 | 1.735 | 3.264 | 3.697 | .0592 | .1065 |
| .04 | 1.601 | 3.107 | 3.495 | .0758 | .1384 |
| .05 | 1.481 | 2.968 | 3.317 | .0912 | .1687 |
| .06 | 1.372 | 2.845 | 3.159 | .1055 | .1978 |
| .07 | 1.274 | 2.736 | 3.019 | .1187 | .2257 |
| .08 | 1.185 | 2.640 | 2.893 | .1310 | .2526 |
| .09 | 1.104 | 2.553 | 2.782 | .1424 | .2785 |
| .10 | 1.030 | 2.476 | 2.682 | .1531 | .3037 |
| .11 | .962 | 2.407 | 2.592 | .1630 | .3281 |
| .12 | .899 | 2.346 | 2.512 | .1723 | .3518 |
| .13 | .842 | 2.291 | 2.440 | .1810 | .3750 |
| .14 | .788 | 2.241 | 2.376 | .1892 | .3977 |
| .15 | .739 | 2.196 | 2.318 | .1968 | .4198 |
| .16 | .694 | 2.156 | 2.265 | .2040 | .4416 |
| .17 | .652 | 2.120 | 2.218 | .2107 | .4630 |
| .18 | .612 | 2.087 | 2.175 | .2170 | .4840 |
| .19 | .576 | 2.058 | 2.137 | .2230 | .5047 |
| .20 | .542 | 2.031 | 2.102 | .2286 | .5252 |
| .21 | .510 | 2.007 | 2.071 | .2338 | .5454 |
| .22 | .480 | 1.985 | 2.042 | .2388 | .5653 |
| .23 | .452 | 1.965 | 2.017 | .2434 | .5851 |
| .24 | .426 | 1.947 | 1.993 | .2478 | .6046 |
| .25 | .402 | 1.931 | 1.972 | .2520 | .6240 |
| .26 | .379 | 1.916 | 1.953 | .2559 | .6433 |
| .27 | .358 | 1.903 | 1.936 | .2595 | .6624 |
| .28 | .337 | 1.890 | 1.920 | .2630 | .6813 |
| .29 | .318 | 1.879 | 1.906 | .2663 | .7002 |
| .30 | .300 | 1.869 | 1.893 | .2694 | .7189 |
| .31 | .284 | 1.860 | 1.881 | .2723 | .7376 |
| .32 | .268 | 1.851 | 1.871 | .2751 | .7561 |
| .33 | .253 | 1.844 | 1.861 | .2777 | .7746 |
| .34 | .239 | 1.837 | 1.852 | .2801 | .7930 |
| .35 | .226 | 1.830 | 1.844 | .2825 | .8113 |
| .36 | .213 | 1.825 | 1.837 | .2846 | .8296 |
| .37 | .201 | 1.819 | 1.830 | .2867 | .8478 |
| .38 | .190 | 1.814 | 1.824 | .2887 | .8660 |
| .39 | .180 | 1.810 | 1.819 | .2905 | .8841 |
| .170 | 1.806 | 1.814 | .2923 | .9022 |  |
|  |  |  |  |  |  |


| .41 | .161 | 1.802 | 1.809 | .2939 | .9202 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .42 | .152 | 1.799 | 1.805 | .2955 | .9382 |
| .43 | .144 | 1.796 | 1.801 | .2970 | .9562 |
| .44 | .136 | 1.793 | 1.798 | .2984 | .9741 |
| .45 | .128 | 1.790 | 1.795 | .2997 | .9921 |
| .46 | .121 | 1.788 | 1.792 | .3009 | 1.0099 |
| .47 | .115 | 1.786 | 1.789 | .3021 | 1.0278 |
| .48 | .108 | 1.784 | 1.787 | .3032 | 1.0457 |
| .49 | .103 | 1.782 | 1.785 | .3043 | 1.0635 |
| .50 | .097 | 1.780 | 1.783 | .3053 | 1.0813 |
| .51 | .092 | 1.779 | 1.781 | .3062 | 1.0991 |
| .52 | .087 | 1.777 | 1.779 | .3071 | 1.1169 |
| .53 | .082 | 1.776 | 1.778 | .3080 | 1.1346 |
| .54 | .078 | 1.775 | 1.777 | .3088 | 1.1524 |
| .55 | .073 | 1.774 | 1.775 | .3095 | 1.1701 |
| .56 | .069 | 1.773 | 1.774 | .3102 | 1.1879 |
| .57 | .066 | 1.772 | 1.773 | .3109 | 1.2056 |
| .58 | .062 | 1.771 | 1.772 | .3115 | 1.2233 |
| .59 | .059 | 1.770 | 1.771 | .3121 | 1.2410 |
| .60 | .055 | 1.770 | 1.771 | .3127 | 1.2587 |
| .61 | .052 | 1.769 | 1.770 | .3133 | 1.2764 |
| .62 | .050 | 1.768 | 1.769 | .3138 | 1.2941 |
| .63 | .047 | 1.768 | 1.769 | .3142 | 1.3118 |
| .64 | .044 | 1.767 | 1.768 | .3147 | 1.3295 |
| .65 | .042 | 1.767 | 1.767 | .3151 | 1.3471 |
| .66 | .040 | 1.767 | 1.767 | .3155 | 1.3648 |
| .67 | .038 | 1.766 | 1.767 | .3159 | 1.3825 |
| .68 | .036 | 1.766 | 1.766 | .3163 | 1.4001 |
| .69 | .034 | 1.765 | 1.766 | .3166 | 1.4178 |
| .70 | .032 | 1.765 | 1.765 | .3170 | 1.4354 |
| .71 | .030 | 1.765 | 1.765 | .3173 | 1.4531 |
| .72 | .028 | 1.765 | 1.765 | .3176 | 1.4707 |
| .73 | .027 | 1.764 | 1.765 | .3178 | 1.4884 |

## SLURRY FLOWRATE $=9.78 \mathrm{e}-3 \mathrm{Kg} / \mathrm{s}$ INITIAL VELOCITY= $\mathbf{5 . 1 7 1} \mathbf{m} / \mathrm{s}$

## DIAMETER $=0.000027 \mathrm{~m}$ PARTICLE TRAJECTORY

| Time |  | Velocity (m/s) |  | Distance (m) |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (S) Horizonta |  | Veti | R | - | Vert |
| . 00 | 2.586 | 4.478 | 5.171 | . 0000 | . 0000 |
| . 01 | . 018 | . 054 | . 057 | . 0040 | . 0071 |
| . 02 | . 000 | . 024 | . 024 | . 0040 | . 0074 |
| . 03 | . 000 | . 023 | . 023 | . 0040 | . 0076 |
| . 04 | . 000 | . 023 | . 023 | . 0040 | . 0078 |
| 05 | . 000 | . 023 | . 023 | . 0040 | . 0081 |

## DIAMETER=0.00003465m <br> PARTICLE TRAJECTORY <br> Time Velocity (m/s) Distance (m)

(S) Horizontal Vertical Resultant Horizontal Vertical

| .00 | 2.586 | 4.478 | 5.171 | .0000 | .0000 |
| :--- | :--- | :--- | :--- | :--- | :--- |

. 01.097 . 200.222 . 0065 . 0115

| .02 | .007 | .051 | .052 | .0069 |
| :--- | :--- | :--- | :--- | :--- |

. 03 . 001.040 . 040 . 0069 . 0129

| .04 | .000 | .039 | .039 | .0069 |
| :--- | :--- | :--- | :--- | :--- |

. 05 . 000.039 . 039 . 0069 . 0137

| .06 | .000 | .039 | .039 | .0069 | .0141 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .07 | .000 | .039 | .039 | .0069 | .0145 |

. 08 . 000.039 .039 . 0069.0149

DIAMETER $=\mathbf{0 . 0 0 0 0 4 4 6 0} \mathrm{m}$
PARTICLE TRAJECTORY

Time Velocity (m/s) Distance (m)
(S) Horizontal Vertical Resultant Horizontal Vertical

| .00 | 2.586 | 4.478 | 5.171 | .0000 | .0000 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .01 | .268 | .506 | .573 | .0093 | .0164 |
| .02 | .050 | .144 | .152 | .0106 | .0192 |
| .03 | .011 | .081 | .082 | .0109 | .0202 |
| .04 | .002 | .068 | .068 | .0109 | .0209 |
| .05 | .001 | .065 | .065 | .0109 | .0216 |
| .06 | .000 | .064 | .064 | .0109 | .0222 |
| .07 | .000 | .064 | .064 | .0109 | .0229 |
| .08 | .000 | .064 | .064 | .0109 | .0235 |
| .09 | .000 | .064 | .064 | .0109 | .0242 |
| .10 | .000 | .064 | .064 | .0109 | .0248 |
| .11 | .000 | .064 | .064 | .0109 | .0254 |
| .12 | .000 | .064 | .064 | .0109 | .0261 |

## DIAMETER $=0.00005740 \mathrm{~m}$ PARTICLE TRAJECTORY

| Time | Velocity (m/s) |  |  | Distance (m) |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| (S) Horizontal | Vertical Resultant | Horizontal | Vertical |  |  |

## DIAMETER $=\mathbf{0 . 0 0 0 0 7 4 4 5 m}$ PARTICLE TRAJECTORY

| Time |  | ocity |  | Dista |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (S) | izon | Vertic | esult | orizo | Ver |
| . 00 | 2.586 | 4.478 | 5.171 | . 0000 | . 0000 |
| . 01 | . 923 | 1.663 | 1.902 | . 0155 | . 0272 |
| . 02 | . 386 | . 763 | . 855 | . 0215 | . 0384 |
| . 03 | . 187 | . 439 | . 477 | . 0242 | . 0442 |
| . 04 | . 097 | . 300 | . 315 | . 0256 | . 0478 |
| . 05 | . 052 | . 233 | . 238 | . 0263 | . 0504 |
| . 06 | . 028 | . 199 | . 201 | . 0267 | . 0525 |
| . 07 | . 015 | . 183 | . 183 | . 0269 | . 0544 |
| . 08 | . 008 | . 174 | . 174 | . 0270 | . 0562 |
| . 09 | . 005 | . 169 | . 169 | . 0271 | . 0579 |
| . 10 | . 003 | . 167 | . 167 | . 0271 | . 0596 |
| . 11 | . 001 | . 166 | . 166 | . 0271 | . 0613 |
| . 12 | . 001 | . 165 | . 165 | . 0271 | . 0629 |
| . 13 | . 000 | . 165 | . 165 | . 0271 | . 0646 |
| . 14 | . 000 | . 165 | . 165 | . 0271 | . 0662 |


| .15 | .000 | .165 | .165 | .0272 | .0679 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .16 | .000 | .165 | .165 | .0272 | .0695 |
| .17 | .000 | .165 | .165 | .0272 | .0712 |
| .18 | .000 | .165 | .165 | .0272 | .0728 |
| .19 | .000 | .165 | .165 | .0272 | .0744 |
| .20 | .000 | .165 | .165 | .0272 | .0761 |
| .21 | .000 | .165 | .165 | .0272 | .0777 |
| .22 | .000 | .165 | .165 | .0272 | .0794 |
| .23 | .000 | .165 | .165 | .0272 | .0810 |
| .24 | .000 | .165 | .165 | .0272 | .0827 |
| .25 | .000 | .165 | .165 | .0272 | .0843 |
| .26 | .000 | .165 | .165 | .0272 | .0860 |
| .27 | .000 | .165 | .165 | .0272 | .0876 |
| .28 | .000 | .165 | .165 | .0272 | .0893 |

## DIAMETER $=0.00009855 \mathrm{~m}$ PARTICLE TRAJECTORY

| Time | Velocity $(\mathrm{m} / \mathrm{s})$ |  |  | Distance (m) |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| (S) |  |  |  |  |  |
| Horizontal | Vertical | Resultant | Horizontal Vertical |  |  |
| .00 | 2.586 | 4.478 | 5.171 | .0000 | .0000 |
| .01 | 1.257 | 2.249 | 2.576 | .0181 | .0318 |
| .02 | .735 | 1.392 | 1.574 | .0277 | .0493 |
| .03 | .429 | .888 | .986 | .0334 | .0605 |
| .04 | .271 | .640 | .695 | .0368 | .0680 |
| .05 | .175 | .493 | .523 | .0390 | .0736 |
| .06 | .116 | .407 | .423 | .0404 | .0781 |
| .07 | .078 | .355 | .363 | .0414 | .0819 |
| .08 | .053 | .323 | .327 | .0420 | .0852 |
| .09 | .036 | .302 | .304 | .0425 | .0884 |
| .10 | .025 | .289 | .290 | .0428 | .0913 |
| .11 | .017 | .281 | .281 | .0430 | .0942 |
| .12 | .012 | .275 | .276 | .0431 | .0969 |
| .13 | .008 | .272 | .272 | .0432 | .0997 |
| .14 | .006 | .269 | .269 | .0433 | .1024 |
| .15 | .004 | .268 | .268 | .0433 | .1051 |
| .16 | .003 | .267 | .267 | .0434 | .1077 |
| .17 | .002 | .266 | .266 | .0434 | .1104 |
| .18 | .001 | .266 | .266 | .0434 | .1131 |
| .19 | .001 | .266 | .266 | .0434 | .1157 |
| .20 | .001 | .265 | .265 | .0434 | .1184 |
| .21 | .000 | .265 | .265 | .0434 | .1210 |
| .22 | .000 | .265 | .265 | .0434 | .1237 |
| .23 | .000 | .265 | .265 | .0434 | .1263 |
| .24 | .000 | .265 | .265 | .0434 | .1290 |


| .25 | .000 | .265 | .265 | .0434 | .1316 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .26 | .000 | .265 | .265 | .0434 | .1343 |
| .27 | .000 | .265 | .265 | .0434 | .1369 |
| .28 | .000 | .265 | .265 | .0434 | .1396 |
| .29 | .000 | .265 | .265 | .0434 | .1422 |
| .30 | .000 | .265 | .265 | .0434 | .1449 |
| .31 | .000 | .265 | .265 | .0434 | .1475 |
| .32 | .000 | .265 | .265 | .0434 | .1502 |
| .33 | .000 | .265 | .265 | .0434 | .1528 |
| .34 | .000 | .265 | .265 | .0434 | .1555 |
| .35 | .000 | .265 | .265 | .0434 | .1581 |
| .36 | .000 | .265 | .265 | .0434 | .1608 |
| .37 | .000 | .265 | .265 | .0434 | .1634 |
| .38 | .000 | .265 | .265 | .0434 | .1661 |
| .39 | .000 | .265 | .265 | .0434 | .1687 |
| .40 | .000 | .265 | .265 | .0434 | .1714 |
| .41 | .000 | .265 | .265 | .0434 | .1740 |
| .42 | .000 | .265 | .265 | .0434 | .1767 |
| .43 | .000 | .265 | .265 | .0434 | .1793 |

## DIAMETER $=\mathbf{0 . 0 0 0 1 3 6 5 5} \mathrm{m}$

 PARTICLE TRAJECTORYTime Velocity (m/s) Distance (m)
(S) Horizontal Vertical Resultant Horizontal Vertical

| .00 | 2.586 | 4.478 | 5.171 | .0000 | .0000 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| .01 | 1.642 | 2.923 | 3.353 | .0206 | .0362 |
| .02 | 1.116 | 2.068 | 2.350 | .0342 | .0607 |
| .03 | .796 | 1.559 | 1.750 | .0436 | .0787 |
| .04 | .593 | 1.246 | 1.380 | .0505 | .0926 |
| .05 | .459 | 1.053 | 1.149 | .0557 | .1040 |
| .06 | .348 | .883 | .949 | .0598 | .1137 |
| .07 | .260 | .744 | .788 | .0628 | .1218 |
| .08 | .201 | .663 | .693 | .0651 | .1288 |
| .09 | .158 | .607 | .627 | .0668 | .1351 |
| .10 | .124 | .565 | .579 | .0682 | .1410 |
| .11 | .098 | .534 | .543 | .0694 | .1465 |
| .12 | .078 | .509 | .515 | .0702 | .1517 |
| .13 | .061 | .491 | .494 | .0709 | .1567 |
| .14 | .049 | .476 | .479 | .0715 | .1615 |
| .15 | .039 | .465 | .467 | .0719 | .1662 |
| .16 | .031 | .457 | .458 | .0722 | .1708 |
| .17 | .024 | .451 | .452 | .0725 | .1753 |
| .18 | .019 | .446 | .447 | .0727 | .1798 |
| .19 | .015 | .442 | .443 | .0729 | .1843 |
| .20 | .012 | .440 | .440 | .0731 | .1887 |
| .21 | .010 | .438 | .438 | .0732 | .1931 |


| .22 | .008 | .436 | .436 | .0733 | .1974 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .23 | .006 | .435 | .435 | .0733 | .2018 |
| .24 | .005 | .434 | .434 | .0734 | .2061 |
| .25 | .004 | .433 | .433 | .0734 | .2105 |
| .26 | .003 | .433 | .433 | .0735 | .2148 |
| .27 | .002 | .432 | .432 | .0735 | .2191 |
| .28 | .002 | .432 | .432 | .0735 | .2234 |
| .29 | .002 | .432 | .432 | .0735 | .2278 |
| .30 | .001 | .432 | .432 | .0735 | .2321 |
| .31 | .001 | .431 | .431 | .0735 | .2364 |
| .32 | .001 | .431 | .431 | .0736 | .2407 |
| .33 | .001 | .431 | .431 | .0736 | .2450 |
| .34 | .001 | .431 | .431 | .0736 | .2493 |
| .35 | .000 | .431 | .431 | .0736 | .2536 |
| .36 | .000 | .431 | .431 | .0736 | .2580 |
| .37 | .000 | .431 | .431 | .0736 | .2623 |
| .38 | .000 | .431 | .431 | .0736 | .2666 |
| .39 | .000 | .431 | .431 | .0736 | .2709 |
| .40 | .000 | .431 | .431 | .0736 | .2752 |
| .41 | .000 | .431 | .431 | .0736 | .2795 |
| .42 | .000 | .431 | .431 | .0736 | .2838 |
| .43 | .000 | .431 | .431 | .0736 | .2881 |
| .44 | .000 | .431 | .431 | .0736 | .2924 |
| .45 | .000 | .431 | .431 | .0736 | .2967 |
| .46 | .000 | .431 | .431 | .0736 | .3011 |
| .47 | .000 | .431 | .431 | .0736 | .3054 |
| .48 | .000 | .431 | .431 | .0736 | .3097 |
| .49 | .000 | .431 | .431 | .0736 | .3140 |
| .50 | .000 | .431 | .431 | .0736 | .3183 |
| .51 | .000 | .431 | .431 | .0736 | .3226 |
| .52 | .000 | .431 | .431 | .0736 | .3269 |
| .53 | .000 | .431 | .431 | .0736 | .3312 |
| .54 | .000 | .431 | .431 | .0736 | .3355 |
| .55 | .000 | .431 | .431 | .0736 | .3398 |
| .56 | .000 | .431 | .431 | .0736 | .3442 |
| .57 | .000 | .431 | .431 | .0736 | .3485 |
| .58 | .000 | .431 | .431 | .0736 | .3528 |
| .59 | .000 | .431 | .431 | .0736 | .3571 |
| .60 | .000 | .431 | .431 | .0736 | .3614 |
| .61 | .000 | .431 | .431 | .0736 | .3657 |
| .62 | .000 | .431 | .431 | .0736 | .3700 |
| .63 | .000 | .431 | .431 | .0736 | .3743 |
| .64 | .000 | .431 | .431 | .0736 | .3786 |
| .65 | .000 | .431 | .431 | .0736 | .3829 |
| .67 | .000 | .431 | .431 | .0736 | .3873 |
| .000 | .431 | .431 | .0736 | .3916 |  |


| DIAMETER=0.00021095m |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| PARTICLE | TRAJECTORY |  |  |  |  |
| Time | Velocity |  |  |  |  |
| (m/s) | Distance (m) |  |  |  |  |
| (S) Horizontal | Vertical | Resultant | Horizontal | Vertical |  |
| .00 | 2.586 | 4.478 | 5.171 | .0000 | .0000 |
| .01 | 2.020 | 3.586 | 4.116 | .0229 | .0400 |
| .02 | 1.619 | 2.962 | 3.376 | .0409 | .0726 |
| .03 | 1.323 | 2.509 | 2.836 | .0556 | .0999 |
| .04 | 1.096 | 2.168 | 2.429 | .0676 | .1232 |
| .05 | .918 | 1.906 | 2.116 | .0777 | .1435 |
| .06 | .776 | 1.702 | 1.871 | .0861 | .1615 |
| .07 | .661 | 1.540 | 1.676 | .0933 | .1777 |
| .08 | .567 | 1.412 | 1.521 | .0994 | .1924 |
| .09 | .489 | 1.308 | 1.396 | .1047 | .2060 |
| .10 | .423 | 1.225 | 1.296 | .1092 | .2186 |
| .11 | .369 | 1.158 | 1.215 | .1132 | .2305 |
| .12 | .322 | 1.103 | 1.149 | .1166 | .2418 |
| .13 | .282 | 1.059 | 1.096 | .1196 | .2526 |
| .14 | .248 | 1.023 | 1.053 | .1223 | .2630 |
| .15 | .219 | .994 | 1.018 | .1246 | .2731 |
| .16 | .193 | .971 | .990 | .1267 | .2829 |
| .17 | .171 | .952 | .967 | .1285 | .2925 |
| .18 | .152 | .936 | .948 | .1301 | .3020 |
| .19 | .135 | .924 | .933 | .1315 | .3113 |
| .20 | .120 | .913 | .921 | .1328 | .3204 |
| .21 | .107 | .905 | .911 | .1339 | .3295 |
| .22 | .095 | .899 | .904 | .1349 | .3386 |
| .23 | .085 | .893 | .897 | .1358 | .3475 |
| .24 | .075 | .889 | .892 | .1366 | .3564 |
| .25 | .067 | .885 | .888 | .1374 | .3653 |
| .26 | .060 | .882 | .884 | .1380 | .3741 |
| .27 | .053 | .880 | .882 | .1386 | .3829 |
| .28 | .048 | .878 | .879 | .1391 | .3917 |
| .29 | .043 | .876 | .877 | .1395 | .4005 |
| .30 | .038 | .875 | .876 | .1399 | .4093 |
| .31 | .034 | .874 | .875 | .1403 | .4180 |
| .32 | .030 | .873 | .874 | .1406 | .4267 |
| .33 | .027 | .873 | .873 | .1409 | .4355 |
| .34 | .024 | .872 | .872 | .1411 | .4442 |
| .35 | .022 | .872 | .872 | .1414 | .4529 |
| .36 | .019 | .871 | .872 | .1416 | .4616 |
| .37 | .017 | .871 | .871 | .1418 | .4703 |
| .38 | .015 | .871 | .871 | .1419 | .4790 |
| .39 | .014 | .871 | .871 | .1421 | .4878 |
| .40 | .012 | .870 | .870 | .1422 | .4965 |
| 7 | .870 | .870 | .1423 | .5052 |  |


|  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .42 | .010 | .870 | .870 | .1424 | .5139 |
| .43 | .009 | .870 | .870 | .1425 | .5226 |
| .44 | .008 | .870 | .870 | .1426 | .5313 |
| .46 | .007 | .870 | .870 | .1427 | .5400 |
| .006 | .870 | .870 | .1427 | .5487 |  |
| .48 | .006 | .870 | .870 | .1428 | .5574 |
| .49 | .005 | .870 | .870 | .1428 | .5661 |
| .50 | .004 | .870 | .870 | .1429 | .5748 |
| .51 | .004 | .870 | .870 | .1429 | .5835 |
| .52 | .003 | .870 | .870 | .1430 | .5922 |
| .53 | .003 | .870 | .870 | .1430 | .6096 |
| .54 | .003 | .870 | .870 | .1431 | .6183 |
| .55 | .002 | .870 | .870 | .1431 | .6269 |
| .56 | .002 | .870 | .870 | .1431 | .6356 |
| .57 | .002 | .870 | .870 | .1431 | .6443 |
| .58 | .002 | .870 | .870 | .1431 | .6530 |
| .59 | .001 | .870 | .870 | .1432 | .6617 |
| .60 | .001 | .870 | .870 | .1432 | .6704 |
| .61 | .001 | .870 | .870 | .1432 | .6791 |
| .62 | .001 | .870 | .870 | .1432 | .6878 |
| .63 | .001 | .870 | .870 | .1432 | .6965 |
| .64 | .001 | .870 | .870 | .1432 | .7052 |
| .65 | .001 | .870 | .870 | .1432 | .7139 |
| .66 | .001 | .870 | .870 | .1432 | .7226 |
| .67 | .001 | .870 | .870 | .1432 | .7313 |
| .68 | .001 | .870 | .870 | .1432 | .7400 |
| .69 | .000 | .870 | .870 | .1432 | .7487 |
| .70 | .000 | .870 | .870 | .1432 | .7574 |
| .71 | .000 | .870 | .870 | .1433 | .7661 |
| .72 | .000 | .870 | .870 | .1433 | .7748 |
| .73 | .000 | .870 | .870 | .1433 | .7835 |
| .74 | .000 | .870 | .870 | .1433 | .7922 |
| .75 | .000 | .870 | .870 | .1433 | .8009 |
| .76 | .000 | .870 | .870 | .1433 | .8096 |
| .77 | .000 | .870 | .870 | .1433 | .8183 |
| .78 | .000 | .870 | .870 | .1433 | .8270 |
| .79 | .000 | .870 | .870 | .1433 | .8357 |
| .80 | .000 | .870 | .870 | .1433 | .8444 |
| .81 | .000 | .870 | .870 | .1433 | .8531 |
| .82 | .000 | .870 | .870 | .1433 | .8618 |
| .83 | .000 | .870 | .870 | .1433 | .8705 |
| .84 | .000 | .870 | .870 | .1433 | .8792 |
| .85 | .000 | .870 | .870 | .1433 | .8879 |
| .86 | .000 | .870 | .870 | .1433 | .8966 |
| .87 | .000 | .870 | .870 | .1433 | .9053 |
|  |  |  |  |  |  |


| .88 | .000 | .870 | .870 | .1433 | .9140 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .89 | .000 | .870 | .870 | .1433 | .9227 |
| .90 | .000 | .870 | .870 | .1433 | .9313 |
| .91 | .000 | .870 | .870 | .1433 | .9400 |
| .92 | .000 | .870 | .870 | .1433 | .9487 |
| .93 | .000 | .870 | .870 | .1433 | .9574 |
| .94 | .000 | .870 | .870 | .1433 | .9661 |
| .95 | .000 | .870 | .870 | .1433 | .9748 |
| .96 | .000 | .870 | .870 | .1433 | .9835 |
| .97 | .000 | .870 | .870 | .1433 | .9922 |
| .98 | .000 | .870 | .870 | .1433 | 1.0009 |
| .99 | .000 | .870 | .870 | .1433 | 1.0096 |
| 1.00 | .000 | .870 | .870 | .1433 | 1.0183 |
| 1.01 | .000 | .870 | .870 | .1433 | 1.0270 |
| 1.02 | .000 | .870 | .870 | .1433 | 1.0357 |
| 1.03 | .000 | .870 | .870 | .1433 | 1.0444 |
| 1.04 | .000 | .870 | .870 | .1433 | 1.0531 |
| 1.05 | .000 | .870 | .870 | .1433 | 1.0618 |
| 1.06 | .000 | .870 | .870 | .1433 | 1.0705 |
| 1.07 | .000 | .870 | .870 | .1433 | 1.0792 |
| 1.08 | .000 | .870 | .870 | .1433 | 1.0879 |
| 1.09 | .000 | .870 | .870 | .1433 | 1.0966 |
| 1.10 | .000 | .870 | .870 | .1433 | 1.1053 |
| 1.11 | .000 | .870 | .870 | .1433 | 1.1140 |
| 1.12 | .000 | .870 | .870 | .1433 | 1.1227 |
| 1.13 | .000 | .870 | .870 | .1433 | 1.1314 |
| 1.14 | .000 | .870 | .870 | .1433 | 1.1401 |
| 1.15 | .000 | .870 | .870 | .1433 | 1.1488 |
| 1.16 | .000 | .870 | .870 | .1433 | 1.1575 |
| 1.17 | .000 | .870 | .870 | .1433 | 1.1662 |
| 1.18 | .000 | .870 | .870 | .1433 | 1.1749 |
| 1.19 | .000 | .870 | .870 | .1433 | 1.1836 |
| 1.20 | .000 | .870 | .870 | .1433 | 1.1923 |
| 1.21 | .000 | .870 | .870 | .1433 | 1.2010 |
| 1.22 | .000 | .870 | .870 | .1433 | 1.2097 |
| 1.23 | .000 | .870 | .870 | .1433 | 1.2184 |
| 1.24 | .000 | .870 | .870 | .1433 | 1.2271 |
| 1.25 | .000 | .870 | .870 | .1433 | 1.2357 |
| 1.26 | .000 | .870 | .870 | .1433 | 1.2444 |
| 1.27 | .000 | .870 | .870 | .1433 | 1.2531 |
|  |  |  |  |  |  |

## DIAMETER $=\mathbf{0 . 0 0 0 4 1 2 8 0} \mathrm{m}$

 PARTICLE TRAJECTORY| Time | Velocity ( $\mathrm{m} / \mathrm{s}$ ) |  |  | Distance (m) |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| (S) Horizontal | Vertical | Resultant | Horizontal | Vertical |  |
| .00 | 2.586 | 4.478 | 5.171 | .0000 | .0000 |
| .01 | 2.354 | 4.170 | 4.788 | .0247 | .0432 |
| .02 | 2.150 | 3.903 | 4.455 | .0472 | .0835 |
| .03 | 1.969 | 3.669 | 4.164 | .0677 | .1214 |
| .04 | 1.809 | 3.465 | 3.909 | .0866 | .1570 |
| .05 | 1.666 | 3.285 | 3.683 | .1040 | .1907 |
| .06 | 1.538 | 3.127 | 3.484 | .1200 | .2228 |
| .07 | 1.423 | 2.987 | 3.308 | .1348 | .2533 |
| .08 | 1.319 | 2.863 | 3.152 | .1485 | .2826 |
| .09 | 1.225 | 2.753 | 3.013 | .1612 | .3106 |
| .10 | 1.139 | 2.655 | 2.889 | .1730 | .3377 |
| .11 | 1.061 | 2.568 | 2.778 | .1840 | .3638 |
| .12 | .990 | 2.490 | 2.679 | .1942 | .3891 |
| .13 | .924 | 2.420 | 2.591 | .2038 | .4136 |
| .14 | .864 | 2.358 | 2.511 | .2128 | .4375 |
| .15 | .809 | 2.302 | 2.440 | .2211 | .4608 |
| .16 | .758 | 2.251 | 2.376 | .2289 | .4835 |
| .17 | .711 | 2.206 | 2.318 | .2363 | .5058 |
| .18 | .667 | 2.165 | 2.266 | .2432 | .5277 |
| .19 | .626 | 2.129 | 2.219 | .2496 | .5491 |
| .20 | .589 | 2.095 | 2.177 | .2557 | .5703 |
| .21 | .553 | 2.065 | 2.138 | .2614 | .5911 |
| .22 | .521 | 2.038 | 2.104 | .2668 | .6116 |
| .23 | .490 | 2.014 | 2.072 | .2718 | .6318 |
| .24 | .462 | 1.991 | 2.044 | .2766 | .6519 |
| .25 | .435 | 1.971 | 2.018 | .2811 | .6717 |
| .26 | .410 | 1.953 | 1.995 | .2853 | .6913 |
| .27 | .386 | 1.936 | 1.974 | .2893 | .7107 |
| .28 | .364 | 1.921 | 1.955 | .2930 | .7300 |
| .29 | .344 | 1.907 | 1.938 | .2966 | .7491 |
| .30 | .324 | 1.894 | 1.922 | .2999 | .7682 |
| .31 | .306 | 1.883 | 1.908 | .3031 | .7870 |
| .32 | .289 | 1.873 | 1.895 | .3060 | .8058 |
| .33 | .273 | 1.863 | 1.883 | .3088 | .8245 |
| .34 | .257 | 1.854 | 1.872 | .3115 | .8431 |
| .35 | .243 | 1.847 | 1.863 | .3140 | .8616 |
| .36 | .230 | 1.839 | 1.854 | .3163 | .8800 |
| .37 | .217 | 1.833 | 1.846 | .3186 | .8984 |
| .38 | .205 | 1.827 | 1.838 | .3207 | .9167 |
| .39 | .194 | 1.821 | 1.832 | .3227 | .9349 |
| .40 | .183 | 1.816 | 1.826 | .3246 | .9531 |
|  |  |  |  |  |  |


| .41 | .173 | 1.812 | 1.820 | .3263 | .9712 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| .42 | .163 | 1.808 | 1.815 | .3280 | .9893 |
| .43 | .154 | 1.804 | 1.810 | .3296 | 1.0074 |
| .44 | .146 | 1.800 | 1.806 | .3311 | 1.0254 |
| .45 | .138 | 1.797 | 1.802 | .3325 | 1.0434 |
| .46 | .130 | 1.794 | 1.799 | .3339 | 1.0614 |
| .47 | .123 | 1.791 | 1.796 | .3351 | 1.0793 |
| .48 | .117 | 1.789 | 1.793 | .3363 | 1.0972 |
| .49 | .110 | 1.787 | 1.790 | .3375 | 1.1151 |
| .50 | .104 | 1.785 | 1.788 | .3385 | 1.1329 |
| .51 | .099 | 1.783 | 1.785 | .3396 | 1.1508 |
| .52 | .093 | 1.781 | 1.783 | .3405 | 1.1686 |
| .53 | .088 | 1.779 | 1.782 | .3414 | 1.1864 |
| .54 | .083 | 1.778 | 1.780 | .3423 | 1.2042 |
| .55 | .079 | 1.777 | 1.778 | .3431 | 1.2219 |
| .56 | .074 | 1.775 | 1.777 | .3438 | 1.2397 |
| .57 | .070 | 1.774 | 1.776 | .3446 | 1.2575 |
| .58 | .067 | 1.773 | 1.775 | .3453 | 1.2752 |
| .59 | .063 | 1.772 | 1.774 | .3459 | 1.2929 |
| .60 | .060 | 1.772 | 1.773 | .3465 | 1.3106 |
| .61 | .056 | 1.771 | 1.772 | .3471 | 1.3283 |
| .62 | .053 | 1.770 | 1.771 | .3476 | 1.3461 |
| .63 | .050 | 1.769 | 1.770 | .3482 | 1.3638 |
| .64 | .048 | 1.769 | 1.769 | .3487 | 1.3814 |
| .65 | .045 | 1.768 | 1.769 | .3491 | 1.3991 |
| .66 | .043 | 1.768 | 1.768 | .3496 | 1.4168 |
| .67 | .040 | 1.767 | 1.768 | .3500 | 1.4345 |
| .68 | .038 | 1.767 | 1.767 | .3504 | 1.4521 |
| .69 | .036 | 1.766 | 1.767 | .3507 | 1.4698 |
| .70 | .034 | 1.766 | 1.766 | .3511 | 1.4875 |

## APPENDIX B

Program drawn up to solve the equations of motion of droplet trajectories.

```
C PROGRAM TRAMODEL
C THIS IS A PROGRAM FOR PARTICLE TRAJECTORY
    REAL X(1000),Y(1000),T(1000),M1,D(20)
    REAL V3(50),V4(50),V5(5),V6(5),V7(5),V8(5),U2(20)
    REAL V1(20),V2(20)
    REAL AA(20),AB(20),AC(20),AD(20)
    REAL BA(20),BB(20),BC(20),BD(20)
    REAL UR(1000),VH(1000),VV(1000)
    INTEGER N1,N2,N3,J,K,K1,K2,K3,K4,K5
    OPEN (1,FILE='TRAMODAT.DAT')
    ANGLE=60.0
    ANGLE=ANGLE*0.0174533
    G=9.81456
    R1=1.2
    M1=0.0000176
    X(1)=0.0
    Y(1)=0.0
    T(1)=0.0
    D(7)=0.000027
    D(8)=0.00003465
    D(9)=0.0000446
    D(10)=0.0000574
    D(11)=0.00007445
    D(12)=0.00009855
    D(13)=0.00013655
    D(14)=0.00021095
    D(15)=0.0004128
    N1=8
    N2=10
    N3=7
    DO 50 I=1,N1
    READ(1,*)BA(I),AA(I),BD(I),AD(I)
    5 0 ~ C O N T I N U E ~
    DO }60\textrm{I}=1,\textrm{N}
    READ(1,*)BB(I),AB(I)
    6 0 \text { CONTINUE}
    DO 70 I=1,N3
    READ(1,*)BC(I),AC(I)
7 0 ~ C O N T I N U E ~
    READ (1,*)R2,UE
    READ(1,*) JSTART,JSTOP
    72 DO 500 JA=JSTART,JSTOP
```

```
    U=UE
    R4=R1*U*D(JA)/M1
    U2(1)=U
    V1(1)=U*SIN(ANGLE/2)
    V2(1)=U*COS(ANGLE/2)
    VH(1)=V1(1)
    VV(1)=V2(1)
    UR(1)=U2(1)
    CALL INTAPO(R4,C,AA,BA,AB,BB,AC,BC,AD,BD,N1,N2,N3)
    K2=1
    K3=1
    K4=10
    K5=1
    K=1
    V3(1)=V1(1)
    V4(1)=V2(1)
    U2(1)=U
    DO 300 J=1,5000
    K1=1
    M3=1
130 V5(K1)=-(3.*R1*C*V3(K)*U/(4.*R2*D(JA)))*0.001
    V6(K1)=(G*((R2-R1)/R2)-(3.*R1*C*V4(K)*U/(4.*R2*D(JA))))*0.001
    IF(K1.LT.2)GOTO 180
    V7(M3)=(V5(1)+V5(K1))/2
    V8(M3)=(V6(1)+V6(K1))/2
    IF (M3.GE.2) GOTO 150
    K=2
    V3(K)=V3(1)+V7(M3)
    V4(K)=V4(1)+V8(M3)
    U=SQRT(V3(K)**2+V4(K)**2)
    R4=R1*U*D(JA)/M1
    CALL INTAPO(R4,C,AA,BA,AB,BB,AC,BC,AD,BD,N1,N2,N3)
    M3=M3+1
    GO TO 200
    150 IF (ABS(V7(1)-V7(2)).LE.0.00001.AND.ABS(V8(1)-
V8(2)).LE.0.00001)GO
2TO 220
    K=2
    V3(K)=V3(1)+V7(M3)
    V4(K)=V4(1)+V8(M3)
    U=SQRT(V3(K)**2+V4(K)**2)
    R4=R1*U*D(JA)/M1
    CALL INTAPO(R4,C,AA,BA,AB,BB,AC,BC,AD,BD,N1,N2,N3)
    V7(1)=V7(2)
    V8(1)=V8(2)
    M3=2
```

GOTO 200
$180 \mathrm{~K}=2$
$\mathrm{V} 4(\mathrm{~K})=\mathrm{V} 4(\mathrm{~K}-1)+\mathrm{V} 6(\mathrm{~K} 1)$
$\mathrm{V} 3(\mathrm{~K})=\mathrm{V} 3(\mathrm{~K}-1)+\mathrm{V} 5(\mathrm{~K} 1)$
$\mathrm{U}=\mathrm{SQRT}\left(\mathrm{V} 3(\mathrm{~K})^{* *} 2+\mathrm{V} 4(\mathrm{~K})^{* *} 2\right)$
R4=R1*U*D(JA)/M1
CALL INTAPO(R4,C,AA,BA,AB,BB,AC,BC,AD,BD,N1,N2,N3)
$200 \mathrm{~K} 1=2$
GOTO 130
$220 \mathrm{~K} 2=\mathrm{K} 2+1$
$\mathrm{V} 1(\mathrm{~K} 2)=\mathrm{V} 3(\mathrm{~K})$
$\mathrm{V} 2(\mathrm{~K} 2)=\mathrm{V} 4(\mathrm{~K})$
$\mathrm{U} 2(\mathrm{~K} 2)=\mathrm{SQRT}\left(\left(\mathrm{V} 1(\mathrm{~K} 2)^{* *} 2+\mathrm{V} 2(\mathrm{~K} 2)^{* *} 2\right)\right)$
$\mathrm{K}=1$
$\mathrm{V} 3(\mathrm{~K})=\mathrm{V} 1(\mathrm{~K} 2)$
$\mathrm{V} 4(\mathrm{~K})=\mathrm{V} 2(\mathrm{~K} 2)$
IF (K3.EQ.K4)GO TO 250
K3 3 K3+1
GO TO 300
$250 \mathrm{~K} 3=1$
K5=K5+1
CALL INTEGA(V1,V2,K2,H1,H2)
$\mathrm{X}(\mathrm{K} 5)=\mathrm{X}(\mathrm{K} 5-1)+\mathrm{H} 1$
$Y(K 5)=Y(K 5-1)+H 2$
$\mathrm{T}(\mathrm{K} 5)=\mathrm{T}(\mathrm{K} 5-1)^{*} 0.01$
$\mathrm{VH}(\mathrm{K} 5)=\mathrm{V} 1(\mathrm{~K} 2)$
$\mathrm{VV}(\mathrm{K} 5)=\mathrm{V} 2(\mathrm{~K} 2)$
UR(K5) $=\mathrm{U} 2(\mathrm{~K} 2)$
IF (X(K5).EQ.X(K5-1)) GO TO 495
IF (Y(K5).EQ.Y(K5-1)) GO TO 495
IF (X(K5).GT.0.6096.OR.Y(K5).GT.1.5)GO TO 490
$\mathrm{V} 1(1)=\mathrm{V} 1(\mathrm{~K} 2)$
$\mathrm{V} 2(1)=\mathrm{V} 2(\mathrm{~K} 2)$
$\mathrm{U} 2(1)=\mathrm{U} 2(\mathrm{~K} 2)$
K2=1
300 CONTINUE
490 K5=K5-1
495 WRITE $(2,4500)$
4500 FORMAT(38X,'PARTICLE
TRAJECTORY'//,,20X,'TIME',12X,'VELOCITY (M/S
1)',16X,'DISTANCE (M)'/,21X,'(S)',4X,'HORIZONTAL VERTICAL RESULTAN

2T',5X,'HORIZONTAL VERTICAL'/)
DO $600 \mathrm{JJ}=1, \mathrm{~K} 5$
WRITE(2,5000)T(JJ), VH(JJ),VV(JJ),UR(JJ),X(JJ),Y(JJ)
600 CONTINUE

## 500 CONTINUE

5000 FORMAT(20X,F4.2,3F11.3,F15.4,F11.4)
650 CLOSE (1)
CLOSE (2)
END
SUBROUTINE
INTAPO(R4,C,AA,BA,AB,BB,AC,BC,AD,BD,N1,N2,N3)
C INTERPOLATION SECTION
REAL $\mathrm{AA}(20), \mathrm{AB}(20), \mathrm{AC}(20), \mathrm{AD}(20)$
REAL BA(20), BB(20), $\mathrm{BC}(20), \mathrm{BD}(20)$
REAL P1(20),P2(20),R3,P3,C
INTEGER N,N1,N2,N3,J1,J2
P3 $=0.0$
IF (R4.LE.0.3)GO TO 500
IF (R4.LE.10.0)GO TO 510
IF (R4.LE.1000.0)GO TO 520
IF (R4.LE.30000.0)GO TO 530
IF (R4.LE. 1000000.0)GO TO 540
$500 \mathrm{C}=24.0 / \mathrm{R} 4$
GO TO 580
$510 \mathrm{~N}=\mathrm{N} 1$
DO $515 \mathrm{~J} 1=1, \mathrm{~N}$
P 1 (J1) $=\mathrm{BA}(\mathrm{J} 1)$
$\mathrm{P} 2(\mathrm{~J} 1)=\mathrm{AA}(\mathrm{J} 1)$
515 CONTINUE
GO TO 550
520 N=N2
DO $525 \mathrm{~J} 1=1, \mathrm{~N}$
$\mathrm{P} 1(\mathrm{~J} 1)=\mathrm{BB}(\mathrm{J} 1)$
$\mathrm{P} 2(\mathrm{~J} 1)=\mathrm{AB}(\mathrm{J} 1)$
525 CONTINUE
GO TO 550
$530 \mathrm{~N}=\mathrm{N} 3$
DO $535 \mathrm{~J} 1=1, \mathrm{~N}$
P1(J1) $=\mathrm{BC}(\mathrm{J} 1)$
P2(J1)=AC(J1)
535 CONTINUE GO TO 550
$540 \mathrm{~N}=\mathrm{N} 1$
DO $545 \mathrm{~J} 1=1, \mathrm{~N}$
$\mathrm{P} 1(\mathrm{~J} 1)=\mathrm{BD}(\mathrm{J} 1)$
P 2 (J1) $=\mathrm{AD}$ (J1)
545 CONTINUE
550 DO $570 \mathrm{~J} 1=1, \mathrm{~N}$
$\mathrm{R} 3=\mathrm{P} 2(\mathrm{~J} 1)$
DO $560 \mathrm{~J} 2=1, \mathrm{~N}$

```
    IF (J1.EQ.J2) GO TO 560
    R3=R3*(R4-P1(J2))/(P1(J1)-P1(J2))
560 CONTINUE
    P3=P3+R3
570 CONTINUE
    C=P3/R4**2
580 CONTINUE
    RETURN
    END
    SUBROUTINE INTEGA(V1,V2,K2,H1,H2)
    REAL V1(20),V2(20)
    REAL FA(20),FB(20)
    INTEGER N4,N5,N6,N7,K6,J6
    K6=1
    DO 700 J3=1,K2
    FA(K6)=V1(J3)
    FB(K6)=V2(J3)
    K6=K6+1
700 CONTINUE
    K6=K6-1
    H=0.001
    N4=K6-1
    N5=N4/2
    H1=0
    H2=0
    N6=1
    IF((N4-N5*2).EQ.0)GO TO }80
    H1=3.*H/8.*(FA(1)+3.*FA(2)+3.*FA(3)+FA(4))
    H2=3.*H/8.*(FB(1)+3.*FB(2)+3.*FB(3)+FB(4))
    N6=4
800 H1 =H1+H/3.*(FA(N6)+4.*FA(N6+1)+FA(K6))
    H2=H2+H/3.*(FB(N6)+4.*FB(N6+1)+FB(K6))
    N6=N6+2
    IF(N6.EQ.K6) GO TO 900
    N7=K6-2
    DO 850 J6=N6,N7,2
    H1=H1+H/3.*(2.*FA(J6)+4.*FA(J6+1))
    H2=H2+H/3.*(2.*FB(J6)+4.*FB(J6+1))
850 CONTINUE
900 RETURN
    END
```


## APPENDIX C

TABLE OF Re VS CRe ${ }^{2}$

| Re | $\mathrm{CRe}^{2}$ |
| :--- | :--- |
| 0.5 | 12.4 |
| 0.7 | 17.9 |
| 1.0 | 26.5 |
| 2 | 57.6 |
| 3 | 93.7 |
| 5 | 173 |
| 7 | 265 |
| 10 | 410 |
| 20 | $1.02 \mathrm{e}+3$ |
| 30 | $1.80 \mathrm{e}+3$ |
| 50 | $3.75 \mathrm{e}+3$ |
| 70 | $6.23 \mathrm{e}+3$ |
| 100 | $10.7 \mathrm{e}+3$ |
| 200 | $30.8 \mathrm{e}+3$ |
| 300 | $58.5 \mathrm{e}+3$ |
| 500 | $138 \mathrm{e}+3$ |
| 700 | $245 \mathrm{e}+3$ |
| 1000 | $460 \mathrm{e}+3$ |
| 2000 | $1.68 \mathrm{e}+6$ |
| 3000 | $3.60 \mathrm{e}+6$ |
| 5000 | $9.60 \mathrm{e}+6$ |
| 7000 | $19.1 \mathrm{e}+6$ |
| 10000 | $40.5 \mathrm{e}+6$ |
| 20000 | $180 \mathrm{e}+6$ |
| 30000 | $426 \mathrm{e}+6$ |
| 50000 | $1.23 \mathrm{e}+9$ |
| 70000 | $2.45 \mathrm{e}+9$ |
| 100000 | $4.8 \mathrm{e}+9$ |
| 200000 | $16.8 \mathrm{e}+9$ |
| 300000 | $18.0 \mathrm{e}+9$ |
| 400000 | $13.4 \mathrm{e}+9$ |
| 600000 | $36.0 \mathrm{e}+9$ |
| 1000000 | $130 \mathrm{e}+9$ |

