## DECLARATION

I hereby declare that this thesis titled: A mathematical model for the transmission dynamics and control of hepatitis B virus, is a collection of my original research work and it has not been presented for any other qualification anywhere. Information from other sources (published or unpublished) has been duly acknowledged.

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## CERTIFICATION

The thesis titled: A mathematical model for the transmission dynamics and control of hepatitis B virus, by ABDULRAHMAN, Sirajo (PhD/SSSE/2008/246) meets the regulations governing the award of the degree of Doctor of Philosophy (PhD) of Federal University of Technology, Minna and it is approved for its contribution to scientific knowledge and literary presentation.

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## DEDICATION

This thesis is dedicated to the memory of my Guardians: Late Bima of Enagi Alhaji Abdulrahman Hassan Enagi and Hajiya Fatima Kakajummai. May the Almighty Allah forgive and reward them with eternal bliss.

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## ABSTRACT

In the work, we developed and analyzed a deterministic model for the transmission dynamics and control of hepatitis B virus (HBV) in a population with vital dynamics (birth and death removal rates are not equal), incorporating vertical transmission, sexual maturity and the effects of condom usage and vaccination coverage which are influenced by public enlightenment campaign. We obtained the effective basic reproduction number $R_C$ which can be used to control the transmission of the disease and hence, established the conditions for local and global stability of the disease-free equilibrium. Bifurcation analysis was carried out using centre manifold theory which revealed a forward bifurcation for the model. Furthermore, sensitivity analysis carried out shows that birth and death removal rates, HBV sexual transmission probability per contact rate and average total sexual contacts rate are highly sensitive parameters that affect the transmission dynamics of HBV in any population. This means that vaccination, condom usage, reduced-average total sexual contacts and birth rates are good strategies that can lead to controlling HBV. Numerical simulations validated the analytical results and further revealed that HBV vaccination of new births only at any coverage rate cannot bring about disease control. But vaccination of sexually active susceptible individuals at about 30% coverage rate can put the disease under control. Two feasible solutions to HBV infection problem in Nigeria were obtained as the present control strategy is not effective enough to even reduce the disease morbidity.

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## GLOSSARY OF ABBREVIATIONS

| | |
|---|---|
| Anti-HBs | Hepatitis B surface antibody |
| CDC | Centre for Disease Control |
| CHB | Chronic Hepatitis B |
| CIA | Central Intelligence Agency |
| EASL | European Association for the Study of the Liver |
| GAVI | Global Alliance for Vaccines and Immunization |
| HBcAg | Hepatitis B core Antigen |
| HBeAg | Hepatitis B e antigen |
| HBsAg | Hepatitis B surface antigen |
| HBV | Hepatitis B virus |
| HCC | Hepatocellular Carcinoma |
| IDU | Injecting Drug Users |
| SA | Sensitivity Analysis |
| SOGHIN | Society for Gastroenterology and Hepatology in Nigeria |
| UNICEF | United Nations Children's Fund |
| WHO | World Health Organization |

## GLOSSARY OF SYMBOLS

$S_U(t)$ Susceptible individuals under fifteen (15) years of age at time $t$

$S_F(t)$ Susceptible individuals at or above fifteen (15) years of age at time $t$

$V(t)$ Vaccinated individuals at time $t$

$A_U(t)$ Acutely infected individuals under fifteen (15) years of age at time $t$

$A_F(t)$ Acutely infected individuals at or above fifteen (15) years of age at time $t$

$C_U(t)$ Chronically infected individuals under fifteen (15) years of age at time $t$

$C_F(t)$ Chronically infected individuals at or above fifteen (15) years of age at time $t$

$R(t)$ Removed individuals due to recovery from infection at time $t$

$b$ Natural birth rate of humans

$\mu$ Death removal rate of humans

$\delta_A$ HBV-induced death rate of $A_U$ and $A_F$

$\delta_C$ HBV-induced death rate of $C_F$

$x$ Prevalence rate of HBV in the population

$\xi$ Average number of sexual contacts per year

$y$ Average number of sexual partners per year

$c$ Average total sexual contacts and thus, $c = \xi y$

$p$ HBV-Sexual transmission probability per contact rate and therefore $pc$ is the effective sexual contact rate in the absence of control measure (rate that leads to infection in the absence of any control measure)

$\eta$ Modification parameter associated with reduced sexual transmission rate by chronic individuals

$\varepsilon_c$ Condom efficacy

$\tau_c$ Condom usage rate which is enhanced by public enlightenment campaign and

therefore $\varepsilon_c \tau_c$ is the effective condom usage rate

$\varepsilon_\rho$ Vaccine efficacy

$\tau_b$ Vaccination coverage rate at birth which is enhanced by public enlightenment campaign and therefore $\rho_b = \varepsilon_\rho \tau_b$ is the effective immunization rate at birth

$\tau_U$ Vaccination coverage rate not at birth for $S_U$ which is enhanced by public enlightenment campaign and therefore $\rho_U = \varepsilon_\rho \tau_U$ is the effective immunization rate for $S_U$ but not at birth

$\tau_F$ Vaccination coverage rate for $S_F$ which is enhanced by public enlightenment campaign and therefore $\rho_F = \varepsilon_\rho \tau_F$ is the effective immunization rate for $S_F$

$\omega$ Loss (waning) rate of vaccination immunity

$\theta$ Proportion of HBV-positive birth

$\phi$ Modification parameter associated with reduced HBV-positive birth by $C_F$

$\sigma_S$ Rate of moving from $S_U$ to $S_F$

$\sigma_A$ Rate of moving from acutely infected classes to chronically infected / removed classes

$\sigma_C$ Rate of moving from $C_U$ to $C_F$

$\varphi_U$ Proportion of $A_U$ which progresses to $C_U$, while $(1-\varphi_U)$ is the proportion removed and therefore $\gamma_U = \sigma_A(1-\varphi_U)$ is the recovery rate from $A_U$ to $R$

$\varphi_F$ Proportion of $A_F$ which progresses to $C_F$, while $(1-\varphi_F)$ is the proportion removed and therefore $\gamma_F = \sigma_A(1-\varphi_F)$ is the recovery rate from $A_F$ to $R$

$\gamma_C$ Rate of recovery from $C_F$ to $R$

$$b_A = b\theta(1-\varepsilon_\rho \tau_b)$$

$$b_C = b\theta\phi(1-\varepsilon_\rho \tau_b)$$

$$\alpha_A = pc(1-\varepsilon_c \tau_c)$$

$$\alpha_C = pc\eta(1-\varepsilon_c \tau_c)$$

$$K_1 = (\sigma_S + \rho_U + \mu)$$

$$K_2 = (\rho_F + \mu)$$

$$K_3 = (\omega + \mu)$$

$$K_4 = (\sigma_A + \mu + \delta_A)$$

$$K_5 = (\sigma_C + \mu)$$

$$K_6 = (\gamma_C + \mu + \delta_C)$$

$$\Omega = \left\{ \begin{pmatrix} S_U \\ S_F \\ V \\ A_U \\ A_F \\ C_U \\ C_F \\ R \end{pmatrix} \in \mathbb{R}^8_+ \;\middle|\; \begin{array}{c} S_U \geq 0, \\ S_F \geq 0, \\ V \geq 0, \\ A_U \geq 0, \\ A_F \geq 0, \\ C_U \geq 0, \\ C_F \geq 0, \\ R \geq 0, \\ S_U + S_F + V + A_U + A_F + C_U + C_F + R \leq N \end{array} \right\}$$