

DECLARATION

I hereby declare that this thesis titled: A mathematical model for the transmission dynamics and control of hepatitis B virus, is a collection of my original research work and it has not been presented for any other qualification anywhere. Information from other sources (published or unpublished) has been duly acknowledged.

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CERTIFICATION

The thesis titled: A mathematical model for the transmission dynamics and control of hepatitis B virus, by ABDULRAHMAN, Sirajo (PhD/SSSE/2008/246) meets the regulations governing the award of the degree of Doctor of Philosophy (PhD) of Federal University of Technology, Minna and it is approved for its contribution to scientific knowledge and literary presentation.

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DEDICATION

This thesis is dedicated to the memory of my Guardians: Late Bima of Enagi Alhaji Abdulrahman Hassan Enagi and Hajiya Fatima Kakajummai. May the Almighty Allah forgive and reward them with eternal bliss.

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ABSTRACT

In the work, we developed and analyzed a deterministic model for the transmission dynamics and control of hepatitis B virus (HBV) in a population with vital dynamics (birth and death removal rates are not equal), incorporating vertical transmission, sexual maturity and the effects of condom usage and vaccination coverage which are influenced by public enlightenment campaign. We obtained the effective basic reproduction number R_c which can be used to control the transmission of the disease and hence, established the conditions for local and global stability of the disease-free equilibrium. Bifurcation analysis was carried out using centre manifold theory which revealed a forward bifurcation for the model. Furthermore, sensitivity analysis carried out shows that birth and death removal rates, HBV sexual transmission probability per contact rate and average total sexual contacts rate are highly sensitive parameters that affect the transmission dynamics of HBV in any population. This means that vaccination, condom usage, reduced-average total sexual contacts and birth rates are good strategies that can lead to controlling HBV. Numerical simulations validated the analytical results and further revealed that HBV vaccination of new births only at any coverage rate cannot bring about disease control. But vaccination of sexually active susceptible individuals at about 30% coverage rate can put the disease under control. Two feasible solutions to HBV infection problem in Nigeria were obtained as the present control strategy is not effective enough to even reduce the disease morbidity.

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GLOSSARY OF ABBREVIATIONS

Anti-HBs	Hepatitis B surface antibody
CDC	Centre for Disease Control
CHB	Chronic Hepatitis B
CIA	Central Intelligence Agency
EASL	European Association for the Study of the Liver
GAVI	Global Alliance for Vaccines and Immunization
HBcAg	Hepatitis B core Antigen
HBeAg	Hepatitis B e antigen
HBsAg	Hepatitis B surface antigen
HBV	Hepatitis B virus
HCC	Hepatocellular Carcinoma
IDU	Injecting Drug Users
SA	Sensitivity Analysis
SOGHIN	Society for Gastroenterology and Hepatology in Nigeria
UNICEF	United Nations Children's Fund
WHO	World Health Organization

GLOSSARY OF SYMBOLS

$S_U(t)$	Susceptible individuals under fifteen (15) years of age at time t
$S_F(t)$	Susceptible individuals at or above fifteen (15) years of age at time t
$V(t)$	Vaccinated individuals at time t
$A_U(t)$	Acutely infected individuals under fifteen (15) years of age at time t
$A_F(t)$	Acutely infected individuals at or above fifteen (15) years of age at time t
$C_U(t)$	Chronically infected individuals under fifteen (15) years of age at time t
$C_F(t)$	Chronically infected individuals at or above fifteen (15) years of age at time t
$R(t)$	Removed individuals due to recovery from infection at time t
b	Natural birth rate of humans
μ	Death removal rate of humans
δ_A	HBV-induced death rate of A_U and A_F
δ_C	HBV-induced death rate of C_F
x	Prevalence rate of HBV in the population
ξ	Average number of sexual contacts per year
y	Average number of sexual partners per year
c	Average total sexual contacts and thus, $c = \xi y$
p	HBV-Sexual transmission probability per contact rate and therefore pc is the effective sexual contact rate in the absence of control measure (rate that leads to infection in the absence of any control measure)
η	Modification parameter associated with reduced sexual transmission rate by chronic individuals
ε_c	Condom efficacy
τ_c	Condom usage rate which is enhanced by public enlightenment campaign and

- therefore $\varepsilon_c \tau_c$ is the effective condom usage rate
- ε_ρ Vaccine efficacy
- τ_b Vaccination coverage rate at birth which is enhanced by public enlightenment campaign and therefore $\rho_b = \varepsilon_\rho \tau_b$ is the effective immunization rate at birth
- τ_U Vaccination coverage rate not at birth for S_U which is enhanced by public enlightenment campaign and therefore $\rho_U = \varepsilon_\rho \tau_U$ is the effective immunization rate for S_U but not at birth
- τ_F Vaccination coverage rate for S_F which is enhanced by public enlightenment campaign and therefore $\rho_F = \varepsilon_\rho \tau_F$ is the effective immunization rate for S_F
- ω Loss (waning) rate of vaccination immunity
- θ Proportion of HBV-positive birth
- ϕ Modification parameter associated with reduced HBV-positive birth by C_F
- σ_S Rate of moving from S_U to S_F
- σ_A Rate of moving from acutely infected classes to chronically infected / removed classes
- σ_C Rate of moving from C_U to C_F
- φ_U Proportion of A_U which progresses to C_U , while $(1 - \varphi_U)$ is the proportion removed and therefore $\gamma_U = \sigma_A (1 - \varphi_U)$ is the recovery rate from A_U to R
- φ_F Proportion of A_F which progresses to C_F , while $(1 - \varphi_F)$ is the proportion removed and therefore $\gamma_F = \sigma_A (1 - \varphi_F)$ is the recovery rate from A_F to R
- γ_C Rate of recovery from C_F to R

$$b_A = b\theta(1 - \varepsilon_\rho \tau_b)$$

$$b_C = b\theta\phi(1 - \varepsilon_\rho \tau_b)$$

$$\alpha_A = pc(1 - \varepsilon_c \tau_c)$$

$$\alpha_C = pc\eta(1 - \varepsilon_c \tau_c)$$

$$K_1 = (\sigma_S + \rho_U + \mu)$$

$$K_2 = (\rho_F + \mu)$$

$$K_3 = (\omega + \mu)$$

$$K_4 = (\sigma_A + \mu + \delta_A)$$

$$K_5 = (\sigma_C + \mu)$$

$$K_6 = (\gamma_C + \mu + \delta_C)$$

$$\Omega = \left\{ \begin{array}{l} \left(\begin{array}{l} S_U \\ S_F \\ V \\ A_U \\ A_F \\ C_U \\ C_F \\ R \end{array} \right) \in \mathbb{R}_+^8 \end{array} \right\} \left. \begin{array}{l} S_U \geq 0, \\ S_F \geq 0, \\ V \geq 0, \\ A_U \geq 0, \\ A_F \geq 0, \\ C_U \geq 0, \\ C_F \geq 0, \\ R \geq 0, \\ S_U + S_F + V + A_U + A_F + C_U + C_F + R \leq N \end{array} \right\}$$