MATHEMATICAL MODELLING FOR THE TRANSMISSION DYNAMICS OF LYMPHATIC FILARIASIS AND MALARIA CO-INFECTION

BY

UDOM, Collins Francis
MTech/SPS/2018/8908

DEPARTMENT OF MATHEMATICS

FEDERAL UNIVERSITY OF TECHNOLOGY

MINNA

NOVEMBER, 2022

ABSTRACT

In this research, a mathematical model for the transmission dynamics of Lymphatic Filariasis and Malaria co-infection in the presence of treatment was formulated, using Bed net, insecticide and chemoprevention as control. It was assumed that the susceptible individual and vector can get infected with Malaria, lymphatic filariasis and co-infection when there is an interaction with any of the five infectious classes: acute stage or chronic stage of Lymphatic filariasis, malaria, both Lymphatic filariasis and malaria co-infected individuals and the infected vectors. The basic reproductive number was obtained using the next generation matrix approach. The Jacobian stability technique and Castillo-Chavez method were used to establish the Local and global stabilities of the Disease free equilibrium state respectively. The stability analysis showed that the Lyphatic filariasis and Malaria co-infection can be eradicated from the entire population when $R_c \leq 1$ but will continue to persevere within the population when $R_c > 1$. The model was solved analytically using Adomian decomposition method, the stability analysis was verified with graphs using Maple 15. The result showed that use of bednet and insecticides have significant impact on the Susceptible, infected malaria and the vector compartments, but treatment have effect on the infected human compartments. It is therefore recommended that every susceptible individual get and make use of bednet and insecticide always. Those who are acutely and chronically infected with Filariasis should get early medical attention.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER ONE

1.0 INTRODUCTION

1.1 Background to the Study

History has shown that the dream of eradicating some parasitic diseases like filariasis and malaria has not been fully realized. Instead, these two old endemic debilitating parasitic diseases are totally neglected most especially filariasis in some parts of developing countries. Malaria and filariasis are the two vector-borne diseases that account for the largest global burdens of mortality and morbidity, respectively. More than half the world's population is at risk of at least one of these diseases (Tersoo and Gweryina, 2014).

Lymphatic filariasis was derived from the latin word 'filarial' it is a vector borne disease of Human caused by *Wuchereria banerofti*, Brugia and skin dwelling *Onchocerca volvulus* (Ottesen, 1984). Among these three species, *Wuchereria banerofti* is the most common and account for 90% infection globally (Anosike *et al*., 2005). Over 120 million people are infected with filariasis, claiming over 40 million people with Africa accounting for 40% of all the global cases (Lenhart *et al.,* 2007). The lymph system maintains the body fluid balance and carry a clear fluid called lymph (water) towards the heart that helps the body get rid of toxins, waste and unwanted materials. People with long term infections with filariasis are clinically asymptomatic and recurrent bacterial infections in some lymphedema patients leads to elephantiasis (Dreyer *et al*., 2000).

Despite the increase on the literacy level of pathology of lymphatic filariasis and the efforts made to treat this disease with diethylcarbamazine and albendazole, filariasis continue to pose a major public health threat in tropical and sub-tropical regions. This is because the disease is more prevalent in the region with higher incidence of poverty (Tan, 2003) and making it a disease of the poor and most often serves as an indicator of underdevelopment (WHO, 1993). It is also noted that areas with higher prevalence of malaria and filariasis have poor environmental and settlement planning and other activities that favour the breeding reservoirs for mosquito vectors (Haddix and Kestler, 2000). Coincidentally, both malaria and filariasis have feverish symptoms. In rural areas, it is often hard to distinguish drug-resistant falciparium malaria and periodic fever due to filariasis infection (Ojiako and Onyeze, 2009).

Malaria was first discovered centuries ago by the Chinese in 2700 BC (CDC, 2011). However it was in the 1800's when Ross made his ground breaking discoveries that led to our understanding of the mechanism behind malaria infections. There are estimated 300-600 million clinical episodes of malaria and approximately 1-2 million results in deaths worldwide of which 90% occur in tropical Sahara (Hay *et al*., 2004). Vector-borne malaria disease like filariasis is transmitted to the humans through a bite from an infected female Anopheles mosquito. The female Anopheles mosquito gets infected when it bites a person carrying the malaria or filariasis parasite. Malaria is caused by the protozoan parasite called plasmodium. There are four species of the plasmodium parasite namely *Plasmodium falciparium*, *Plasmodium vivax*, *Plasmodium ovale* and *Plasmodium malairae*; of the four species, *Plasmodium falciparium* is the most virulent, lethal and responsible for the majority of morbidity and mortality due to malaria (Burke, 2010). Apart from the four species mentioned above, simian malaria, *Plasmodium inul*, *Plasmodium knowlesi* and *Plasmodium cynomolgi* are also known to cause the disease in humans (Balbir *et al*., 2004). The malaria parasite passes through human blood then into the liver where it develops. After completing its development, it goes back to the blood stream and a person develops symptoms of malaria. The most clinical symptoms a patient may experience after malaria infection include headache, aching muscles, stomachache, loss of appetite, nausea, vomiting, back pain and increased sweating (Bupa, 2009).

Individuals most vulnerable to malaria are children under the age of 5 years. This is attributed to their weaker immunity. Aside from children, pregnant women are also heavily affected, with resultant effects on maternal health and birth outcomes. Malaria like filariasis has physiological impact to the body ranging from severe anemia, fits, spleen enlargement, cerebral malaria, multi-organ failure or death. There is developing geographical distribution of these diseases in large areas of Africa, Asia and the Americas. Historically, there is evidence that efforts to control malaria have in adversely resulted in the interruption of transmission of lymphatic filariasis in some areas, such as Solomon Islands (Webber, 1977). The dangers of malaria and filariasis can be controlled or treated by the effective scaling up coverage of insecticide treated mosquito nets or combination of drugs known as artemisinin-based combination therapies as well as implementing indoor residual spraying (ACTs) (Whitty *et al*., 2007). Moreso, the Global Programme to Eliminate Lymphatic Filariasis (GPELF) is currently targeting elimination of the disease through annual mass drug administration (MDA) of abendazole with either diethylcarbamazine (DEC) or invermectin. This has been widely acclaimed to be one of the successful public health programmes and is expected to block transmission of filariasis in endemic countries by 2020 (WHO, 2005). Whilst there is enormous and abundant literature on the mathematical models for communicable diseases, there is growing interest in the dynamics of co-infection of HIV and malaria (Chunky, 2012), malaria and TB (Expeditho *et al*., 2009), malaria and meningitis (Lawi *et al*., 2011), still no or very little literature is available on the mathematical models on filariasis and malaria co-infection, due to the little work done in modeling of lymphatic filariasis (Bhunu and Mushayabasa, 2012) as compared to malaria.

1.2 Statement of the Research Problem

Statistics has showed that the dream of eradicating some parasitic disease like lymphatic filariasis and malaria has not been fully achieved; instead this two old disease have been neglected or totally ignored especially lymphatic filariasis (also known as elephantiasis). Lymphatic filariasis and malaria are two vector borne diseases that account for the largest morbidity and mortality rate globally. It is in light of this that the present study sought to develop and analyse a mathematical model for the transmission of Lymphatic filariasis and malaria co-infection.

1.3 Motivation of the study

Nigeria is said to be the most lymphatic filariasis (elephantiasis) endemic country. It also accounted for 94% of all malaria cases and death in the world (CDC, 2019). In view of the above, the Author is motivated by the need to provide valuable information that will help policy makers in the fight against lymphatic filariasis and malaria co infection in Nigeria and the world at large.

1.4 Justification of the Study

Lymphatic filariasis and Malaria are two vector born disease that account for the largest global burden of morbidity and mortality respectively. Filariasis is a leading cause of permanent and long term disability, loss of work, productivity, they cause direct or indirect economic loss and functional impairment (CDC, 2010). Government and health workers will find the research useful for both short and long term planning in the fight against the co infection of lymphatic filariasis and malaria. The Thesis may also help mathematicians and research scientists to further develop models to help public health professionals to make better strategies for controlling and eradicating the disease. Hence this justifies the study.

1.5 Scope and Limitation of the Study

This work covers the formulation and analysis of mathematical model for the transmission dynamics of lymphatic filariasis and Malaria co infection through mosquito bite.

1.6 Aim and Objectives

The aim of this work is to develop and analyse a mathematical model for the transmission dynamics of Lymphatic filariasis and malaria co-infection

The objectives are to:

i. formulate a mathematical model of Lymphatic and Malaria co-infection;
ii. investigate the positivity and feasible region of the model's solution;
iii. obtain the disease free equilibrium state of the model;
iv. compute the effective reproduction number of the model;
v. obtain the conditions for local and global stability of the disease-free equilibrium (DFE) state;
vi. solve the model equations using Adomian Decomposition Method.
vii. carry out the numerical simulation of the model using Maple 15 software and Nigeria demographic data.

1.7 Definition of terms

Mathematical Model: Any description of a system using mathematical concepts and languages is called a mathematical model.

Acute stage infection: An infection that is brief, intense or short term.

Chronic stage infection: An infection often low intensity but lasts for a long term.

Population: A collection of people of similar characteristic in the same region or geographical area, in a given time.

Susceptible Individual: A person who is free from lymphatic filariasis and malaria but may be infected if exposed to the disease.

Epidemiology: the study of the occurrence and distribution of health related states or events in specified population, including the study of the determinant influencing such state and application of this knowledge to control the health problems.

CHAPTER TWO

2.0 LITERATURE REVIEW

2.1 Mathematical Model of Lymphatic Filariasis and Malaria Co-Infection

The earliest mathematical model describing the lymphatic Filariasis (LF) dynamics was proposed by S. Subramainian in 1998 where he focused on the prediction and control strategies using LYMFASIM simulation program (Subramainian *et al*., 1998). Since then, a few other researchers have contributed to formulate mathematical models to study the prevention, control and transmission dynamics of LF.

Hiroyuki *et al*. (2002) formulated a mathematical model for the transmission of LF and its applications. They constructed a stochastic transmission model for LF caused by Wuchereria, analyzed its prevalence using computed simulations, aimed at evaluating the effect of vector control in the context of Pondicherry (india).

Supriatna *et al*. (2009) formulated a mathematical model to investigate long term effects of the LF medical treatment in Indonesia. In formulating their model the population was divided into five (5) components, namely: the Susceptible human S_h , Infected carrier A, Infected chronic K, Susceptible mosquito $S_{v,}$ Infected mosquito $I_{v,}$ they analyzed the model to find a condition for the existence and stability of the endemic equilibrium, which shows that it exists and stable if the basic reproductive number is greater than one. They also showed that if the level of screening is sufficiently large, the current medical treatment strategy will be able to reduce the long term level of incidence.

Bhunu and Mushayabasa (2012) developed a mathematical model for the transmission dynamics of Lymphatic filariasis with treatment for those displaying elephantiasis symptoms, they determined the reproductive number and equilibria for the model and analysis of the reproductive number suggested that treatment will somehow contributes to a reduction in Lymphatic Filariasis (LF) cases but it didn't show the magnitude of the reduction.

Other researchers such as Swaminathan *et al*. (2008); Norman (2000), formulated mathematical model for Lymphatic filariasis. Oguntolu (2019) developed a mathematical model to study the transmission dynamics of LF which improved on other previous studies of LF. The model had compartments for those undergoing treatment since the treatment would take six years before recovery for chronic cases, class of susceptible individuals taking drug, vector control using bed net and insecticides, drugs administration to both infected class with symptoms and without symptoms. The existence and uniqueness of the model was found, stabilities analyses were also obtained.

Li-Ming *et al.* (2013) modeled a deterministic model with variable human population for the transmission dynamics of malaria disease, which allows transmission by the recovered humans. The model revealed the presence of the phenomenon of backward bifurcation, where a stable disease-free equilibrium coexists with one or more stable endemic equilibria when the associated reproduction number is less than unity. This phenomenon may arise due to the reinfection of host individuals who recovered from the disease. The model in an asymptotical constant population is also investigated. This results in a model with mass action incidence. A complete global analysis of the model with mass action incidence was given, which revealed that the global dynamics of malaria disease with reinfection is completely determined by the associated reproduction number.

Traore *et al.* (2017) formulated a mathematical model of non-autonomous ordinary differential equations describing the dynamics of malaria transmission with age structure for the vector population. The biting rate of mosquitoes was considered as a positive periodic function which depends on climatic factors. The basic reproduction ratio of the model was obtained and they showed that it was the threshold parameter between the extinction and the persistence of the disease. Thus, by applying the theorem of comparison and the theory of uniform persistence, they proved that if the basic reproduction ratio is less than 1, then the disease-free equilibrium is globally asymptotically stable and if it is greater than 1, then there exists at least one positive periodic solution.

Olumuyiwa and Hammed (2019) formulated a malaria mathematical model by incorporating four control strategies: insecticide-treated bednets control, infected humans treatment control, sterile mosquitoes technique control and use of control on pregnant women and newborn births. It also explains the various stages of the disease jointly in humans and mosquitoes as well as the treatment of both asymptomatic and infectious humans. Preventive measures were developed to control the spread of disease. Forward-backward fourth-order Runge-Kutta method (Sweep method) was used to see the spread of disease and how to eradicate the disease. This was based on the fact that these measures are deployed adequately using control tools and without control tools respectively.

Hannah *et al*. (2013) developed a modeling framework incorporating the specifics of malaria-LF co-infection to investigate how the transmission of each infection is altered for a range of possible interaction scenarios. They found out that a control strategy that reduces LF transmission via mass drug administration, for example could potentially increase malaria prevalence. Their work illustrated the potential perverse effects of targeting just one infection and emphasises the need to take into account co-endemic diseases when designing control programmes. The developed modelling framework can provide the basis for exploring the mix of options for joint control of these infections.

Oluwatayo and Valery (2019) formulated a mathematical nonlinear model system of equations describing the dynamics of the co-interaction between malaria and filariasis epidemic affecting the susceptible host population of pregnant women in the tropics was formulated. The basic reproduction number R_0 of the co-epidemic model was obtained, and it was shown that it was the threshold parameter between the extinction and persistence of the co-epidemic disease. If $R_0 < 1$, then the disease-free steady state is both locally and globally asymptotically stable resulting in the disease dying out of the host. Also, if $R_0 > 1$, the disease lingers on. The center manifold theory was used to show that the unique endemic equilibrium was locally asymptotically stable. However, variations in the parameter values involved in the model build up would bring about appropriate control measures to curtail the spread of the co-epidemic disease.

Gweryina and Tersoo (2014) formulated and analyzed a deterministic model for malaria-filariasis co-infection with chemoprevention and treatment. They proved that the disease-free equilibrium was globally asymptotically stable. They also showed that the bifurcation analysis of the endemic state was subcritical. Furthermore, results from numerical simulation suggests that high chemotherapy and treatment hold great promise for helping to stem the tide of new malaria and filariasis infections. The model was divided into eight (8) compartment of six (6) Human compartment and two vector compartment. The following differential equations were formed.

$$
\left.
\begin{aligned}
\frac{S_h}{dt} &= \pi A_h - \left(K_{ma} + K_f\right)S_h - \left(\mu_h + \delta\right)S_h + \tau T \\
\frac{S_2}{dt} &= (1\text{-}\pi)A_h + \delta_1 S_h + \left(K_{ma} + K_f\right)S_h \text{-}(\in \sigma_h + \gamma + \mu_h)S_2 \\
\frac{I_m}{dt} &= \in \sigma_h S_2 \text{-}\theta K_f I_m \text{-}(\psi + \phi_1 + \mu_h)I_m \\
\frac{I_f}{dt} &= \psi I_m \text{-}\rho K_{ma} I_f \text{-}(\gamma + \phi_2 + \mu_h)I_f \\
\frac{I_{mf}}{dt} &= \theta K_f I_m + \rho K_{ma} I_f \text{-}(\phi_3 + \nu\phi + \eta\gamma + \mu_h)I_{mf} \\
\frac{\mathrm{T}}{dt} &= \nu\phi I_{mf} + (\phi_1 I_m + \phi_2 I_f + \phi_3 I_{mf})\text{-}(\mu_h + \tau)T \\
\frac{S_v}{dt} &= \mathrm{A}_v \text{-}K_v S_v \text{-}\mu_v S_v \\
\frac{E_v}{dt} &= K_v S_v \text{-} \sigma_v E_v \text{-}\mu_v E_v \\
\frac{I_v}{dt} &= \sigma_v E_v \text{-}\mu_v I_v
\end{aligned}
\right\} \qquad (2.1)
$$

Considering all the cited literature above, a new mathematical model and lymphatic filariasis and Malaria co infection dynamics incorporating relevant features is formulated using a system of ordinary differential equations. It is hoped that the results of this research work will be found useful and eventually be added to the existing literature. The model is an improvement on the literatures cited above as it considers:

i. The class undergoing treatment since the treatment will take six years for Lymphatic filariasis

ii. Chemoprevention Class consisting of susceptible individual taking drugs

iii. Acute and chronic stage of Lymphatic filariasis infected individuals

iv. Vector control (using bed-net and insecticide)

v. Drug administration to both the infected class showing symptoms and not showing symptoms of Lymphatic filariasis, Malaria infected inviduals and lymphatic and Malaria co infection

2.2 Effective Reproduction Number (R_c)

One of the most useful threshold parameters used in the study of stability of equilibria is the effective reproduction number R_c. According to Diekmann and Heesterbeek (2000), Murray (2002), the basic reproduction number R_0 is defined as the average number of infected people generated by single infectious person in an entirely susceptible population.

If $R_c < 1$, then on average, an infected individual produces less than one new infected individual during its entire period of infectiousness and the disease can be controlled. Thus, the disease free equilibrium is locally asymptotically stable. Conversely, if $R_c > 1$, then each infected individual produces, on average more than one new infection (i.e. epidemic occurs). Hence, the disease free equilibrium is unstable and disease invasion is possible. $R_c = 1$ is a threshold below which the generation of secondary cases is insufficient to maintain the infection with human community (Diekmann *et al.*,1990).

The next generation approach described by Van Den and Watmough (2002) is a widely accepted method used to compute the basic reproduction number R_0. Other publications by Castillo-Chavez *et al*. (2007) were devoted to the calculation of basic reproduction number R_0 for different models of various diseases. Using the approach described by Van Den and Watmough, we obtained the effective reproduction number R_c which is the largest eigenvalue (spectral radius ρ) of the next generation matrix FV^{-1} i.e. $R_c = \rho(FV^{-1})$ as follows;

$V_i^+(x)$ is the rate of transfer of individuals into compartment i by every means except the epidemic.

$V_i^-(x)$ is the transfer of individuals out of compartment i.

$V_i = V_i^-(x) - V_i^+(x)$

Given the DFE, R_c is calculated thus:

$$F = \frac{\partial F_i}{\partial x_j}(E_0) \quad (2.2)$$

$$V = \frac{\partial V_i}{\partial x_j}(E_0) \qquad 1 \leq i; j \leq m \quad (2.3)$$

With *F* being non-negative and *V*, a non-singular *M*-matrix.

Thus $R_c = \rho(FV^{-1})$

2.3 Global Stability of Disease Free Equilibrium

In global stability of equilibrium, the restrictions on initial conditions are removed and the global asymptotic stability property requires that for all initial conditions, solutions approach the equilibrium. Several techniques including Castillo- Chavez' global stability theorem have been used to determine the global stability of disease free equilibrium.

2.4 Castillo-Chavez global stability theorem

Consider a model system written in the form

$$\frac{dX}{dt} = F(X, Z) \quad (2.4)$$

$$\frac{dX}{dt} = G(X, Z); \qquad G(X, 0) = 0 \quad (2.5)$$

Where $X \in \Re^m$ denotes (its component), the number of uninfected individuals and $Z \in \Re^n$ denotes (its component) the number of infected individuals including those undergoing treatment, etc; $E^0 = \left(N^0, 0\right)$ denotes the disease free equilibrium of the system.

Assume the conditions H_1 and H_2 below;

$$\left(H_1\right) \text{ for } \frac{dX}{dt} = F\left(X, 0\right), X^* \text{ is globally asymptotically stable (GAS)} \tag{2.6}$$

$$\left(H_2\right) = G\left(X, Z\right) = AZ - \hat{G}\left(X, Z\right), \hat{G}\left(X, Z\right) \geq 0 \text{ for } \left(X, Z\right) \in \Gamma \tag{2.7}$$

Where $A = \frac{\partial G}{\partial Z}\left(X^*, 0\right)$ is an M- matrix (the off diagonal elements of A are non-negative) and Γ is the region where the model makes biological sense. If the model equations satisfy the two conditions (2.6) and (2.7) then the disease free equilibrium $E^0 = \left(N^0, 0\right)$ is globally asymptotically stable.

2.5 Adomian Decomposition Method

At the beginning of the 1980, George Adomian developed a very powerful method called Adomian decomposition method for solving linear and nonlinear functional equations.

The Adomian decomposition (ADM) involves separating the equation under consideration into linear and nonlinear parts. The linear operator representing the linear part of the equation is inverted and the linear operator is then applied to the equation. Any given conditions are taken into consideration. The nonlinear part is decomposed into a series of what is known as Adomian Polynomials. The method generates a solution in the form of a series whose terms are obtained by a recursive relationship using the Adomian Polynomials. A brief outline of the method is given as follows;

Consider a differential equation in general form

$$G(y) = g \tag{2.8}$$

This can be written in operator form as

$$Ly + Ry + Ny = g \tag{2.9}$$

Where L is a linear operator acting on y which is easily invertible, R is a linear operator for remainder of the linear part, and N is a nonlinear operator representing the nonlinear term in G. For convenience, L is usually taken as the highest derivative.

Applying the inverse operator L^{-1} on both sides of equation (2.9) gives

$$L^{-1}Ly = L^{-1}g - L^{-1}Ry - L^{-1}Ny \tag{2.10}$$

L^{-1} is the integration since G is taken as a nonlinear differential operator and L is linear. That is, L^{-1} is an nth integral of y for nth order differential equation, where $n \in Z$. Equation (2.10) becomes

$$y(t) = f(t) - L^{-1}Ry - L^{-1}Ny \tag{2.11}$$

where $f(t)$ is the function obtained by integrating g and applying the initial or boundary conditions.

The unknown function is assumed to be an infinite series of the form

$$y(t) = \sum_{n=0}^{\infty} y_n \tag{2.12}$$

we let

$$y_0 = f(t) \tag{2.13}$$

And the remaining terms are obtained by a recursive relationship. This relationship is found by decomposing the nonlinear terms into a series of what is called Adomian polynomial, P_n (Biazar and Babalion, 2005).

The nonlinear term is written as

$$Ny(t) = \sum_{n=0}^{\infty} P_n \tag{2.14}$$

In order to obtain P_n, a grouping parameter, λ is introduced. The following series are established

$$y(\lambda) = \sum_{n=0}^{\infty} \lambda^n y_n \tag{2.15}$$

$$Ny(t) = \sum_{n=0}^{\infty} \lambda^n P_n \tag{2.16}$$

Substituting equation (2.12), (2.13), (2.14) into equation (2.11) gives

$$y(t) = y_0 - L^{-1}\sum_{n=0}^{\infty} Ry_n - L^{-1}\sum_{n=0}^{\infty} P_n \tag{2.17}$$

Where P_n can be obtain from

$$P_n = \frac{1}{n!}\left[\frac{d^n}{d\lambda^n} Ny(\lambda)\right]_{\lambda=0} \tag{2.18}$$

The recursive relation is obtained to be

$$y_0 = f(t) \tag{2.19}$$

$$y_{n+1} = -L^{-1}\sum_{n=0}^{\infty} Ry_n - L^{-1}\sum_{n=0}^{\infty} P_n \tag{2.20}$$

The Adomian decomposition method (ADM) produces series that is absolutely and uniformly convergent (El-Kalla, 2008).

CHAPTER THREE

3.0 MATERIAL AND METHODS

3.1 Development of the Model

A mathematical model provides a framework within which we can communicate an understanding of the spread of disease in human population, both in space and time. In this chapter we developed and analyzed a mathematical model to study the transmission dynamics of Lymphatic filariasis and malaria co-infection.

The model incorporates relevant features such as chemoprevention, vector control (using bed net and insecticide), the recovery class, and the infected classes of LF with symptoms and without symptoms.

The human population of size $N_h(t)$ is subdivided based on Lymphatic filariasis and malaria status into the following subpopulations: Susceptible human $(S_h(t))$, chemoprevention class $(V_h(t))$, infected human $(I_{hal}(t))$ not showing sign of Lymphatic filariasis, infected human $(I_{hcl}(t))$ showing sign of Lymphatic filariasis, infected human with malaria symptoms $(I_m(t))$, infected human with Lymphatic filariasis and malaria symptoms $(I_{lm}(t))$, Recovered human $(T_h(t))$ from $(I_{hal}(t),\ I_{hcl}(t), I_m(t), I_{lm}(t))$. Thus, the total human population is given by:

$$N_h(t) = S_h(t) + I_{hal}(t) + I_{hcl}(t) + I_m(t) + I_{lm}(t) + T_h(t) \tag{3.1}$$

The mosquito population is divided into the following subgroups: non-carrier vector (mosquitoes) $(S_v(t))$ and carrier vector (mosquitoes) $(I_v(t))$, so the total mosquito population is given by

$$N_v(t) = S_v(t) + I_v(t) \tag{3.2}$$

The mosquitoes and Human beings are recruited into their corresponding susceptible populations at rate Λ_v and Λ_h respectively. Mosquito experiences natural death rate μ_v and death by insecticide at a rate δ_v which is proportional to the number in each mosquito class. Similarly human beings experience natural death rate at the rate μ_h and death rate due to malaria at rate δ_m ,which is proportional to the number in each human class.

The mosquito ingests microfilarias or malaria parasite or both when it bites a human who is infected with filariasis or malaria or both. This filariasis could be at the acute stage or chronic stage in the human at the rate.

$$\lambda_h = \beta_h \sigma_v \frac{\left(I_{hal} + I_{hcl} + I_m + I_{lm}\right)}{N_h} \tag{3.3}$$

where,

β_h is the probability that a bite by susceptible mosquito on an infected human will transfer infections to the mosquito and σ_v is the average number of bites given to humans by each mosquito per unit time.

For filariasis infection, upon getting infected, the susceptible mosquitoes entered the infected class $I_v(t)$. Microfilariae passes through mosquito gut into hemocoel and develop into filariform juvenites. filariform juveniles escapes from mosquito's proboscis when the insect is feeding and penetrates wound structure of a human being at rates.

$$\frac{\beta_l \sigma_v (1-\theta) I_v}{N_v} \text{ and } \frac{\beta_{lm} \sigma_v (1-\theta) I_v}{N_v} \tag{3.4}$$

Similarly, humans get infected with malaria when there is a bite from infected mosquitoes at the rate of

$$\frac{\beta_{lm}\sigma_v(1-\theta)I_v}{N_v}, \frac{\beta_m\sigma_v(1-\theta)I_v}{N_v} \text{ and } \frac{\beta_m\sigma_v I_v}{N_v} \tag{3.5}$$

$\frac{\beta_{lm}\sigma_v(1-\theta)I_v}{N_v}$ is the rate of co-infection.

Effective biting interaction between S_h and I_v or V_h and I_v result in the movement of individual from S_h to I_{hal}, I_{hal} to I_{hcl}, S_h to I_m, S_h to I_{lm}, or V_h to I_m while a similar interaction between I_{hal}, I_{hcl}, I_m, I_{lm} and S_v leads to the flow of individuals of S_v into I_v.

The schematic representation of the model is given below in Figure 3.1

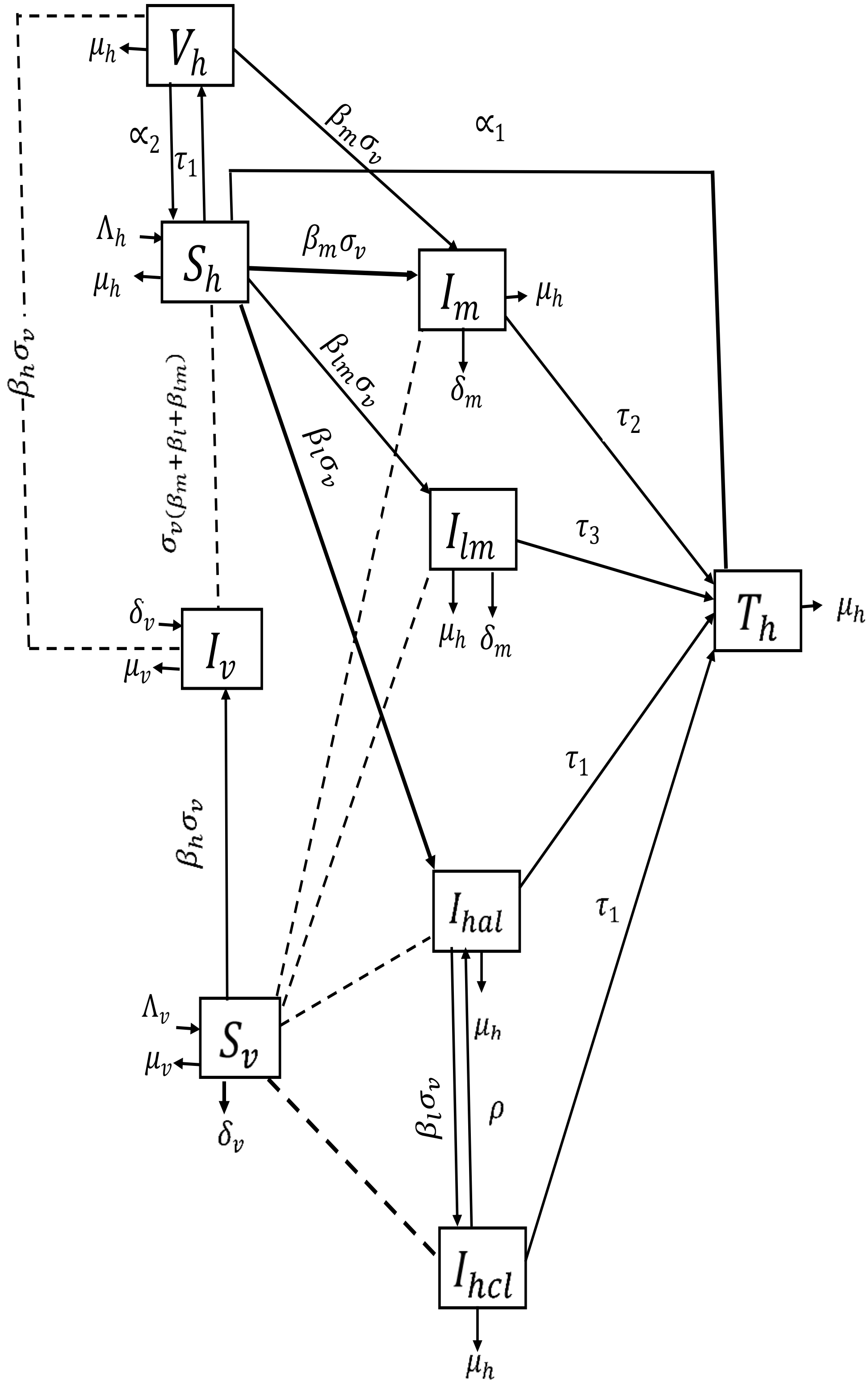

Figure 3.1 Schematic Diagram of the Model

Based on the model development described in section 3.1 and the schematic diagram in Figure 3.1 the following model equations are derived.

$$\frac{dS_h}{dt} = \Lambda_h - \sigma_v (1-\theta)(\beta_m + \beta_l + \beta_{lm}) \frac{I_v S_h}{N_v} - (\tau_1 + \mu_h) S_h + \alpha_1 T_h + \alpha_2 V_h \quad (3.6)$$

$$\frac{dV_h}{dt} = \tau_1 S_h - (\mu_h + \alpha_2) V_h - \beta_m \sigma_v \frac{I_v V_h}{N_v} \quad (3.7)$$

$$\frac{dI_{hal}}{dt} = \beta_l \sigma_v (1-\theta) \frac{I_v S_h}{N_v} - \beta_l \sigma_v (1-\theta) \frac{I_v I_{hal}}{N_v} - (\mu_h + \tau_1 + \rho) I_{hal} \quad (3.8)$$

$$\frac{dI_{hcl}}{dt} = \beta_l \sigma_v (1-\theta) \frac{I_v I_{hal}}{N_v} + \rho I_{hal} - (\mu_h + \tau_1) I_{hcl} \quad (3.9)$$

$$\frac{dI_m}{dt} = \beta_m \sigma_v (1-\theta) \frac{I_v S_h}{N_v} - (\tau_2 + \delta_m + \mu_h) I_m + \beta_m \sigma_v \frac{I_v V_h}{N_v} \quad (3.10)$$

$$\frac{dI_{lm}}{dt} = \beta_{lm} \sigma_v (1-\theta) \frac{I_v S_h}{N_v} - (\tau_3 + \delta_m + \mu_h) I_{lm} \quad (3.11)$$

$$\frac{dT_h}{dt} = \tau_2 I_m + \tau_1 (I_{hal} + I_{hcl}) + \tau_3 I_{lm} - (\alpha_1 + \mu_h) T_h \quad (3.12)$$

$$\frac{dS_v}{dt} = \Lambda_v - \beta_h \sigma_v \frac{(I_{hal} + I_{hcl} + I_{lm} + I_m) S_v}{N_h} - (\mu_v + \delta_v) S_v \quad (3.13)$$

$$\frac{dI_v}{dt} = \beta_h \sigma_v \frac{(I_{hal} + I_{hcl} + I_{lm} + I_m)}{N_h} S_v - (\mu_v + \delta_v) I_v \quad (3.14)$$

and summing (3.6) to (3.12) and (3.13) - (3.14) gives

$$\frac{dN_h}{dt} = \Lambda_h - \mu_h N_h \quad (3.15)$$

$$\frac{dN_v}{dt} = \Lambda_v - k_7 N_v \quad (3.16)$$

Table 3.1 Definition of variable/Parameter of the model

Variable	Description
$S_h(t)$	A class of susceptible Human
$V_h(t)$	A class of susceptible individuals taking drugs (chemoprevention) at time (t)
$I_{hal}(t)$	A class of infected acute stage of LF (not showing sign of Lymphatic filariasis) at time (t)
$I_{hcl}(t)$	A class of infected-chronic stage of LF at time (t)
$I_m(t)$	A class of malaria infected human at time (t)
$I_{lm}(t)$	A class of Lymphatic filariasis and malaria co-infected humans at time t
$T_h(t)$	Recovered human population at time t
$S_v(t)$	Susceptible mosquitoes at time t
$I_v(t)$	Infected mosquitoes at time t
β_h	The infectivity of an infection malaria, Lymphatic filariasis and co-infection humans defined as the probability that a bite by a susceptible mosquitoes on an infected human with transfer the infection to the mosquito
β_m	The infectivity of the mosquito, define as the probability that a bite by an infected mosquito on a susceptible human will transfer malaria infection to the Human
β_l	The infectivity of the mosquito, defined as the probability that a bite by an infected mosquito on a susceptible human will transfer Lymphatic filariasis infection to the Human
β_{lm}	The infectivity of the mosquito, defined as the probability that a bite by an infected mosquito on a susceptible human will transfer Lymphatic filariasis infection to the Human
σ_v	The main biting rate of mosquitoes, define as the average number of bites given to humans by each mosquito per unit time
Λ_h	Recruitment rate of the human population
Λ_v	Recruitment rate of the mosquito population
θ	Proportion of the susceptible population using mosquito net and insecticide
μ_h	Natural death rate for the human population
μ_v	Natural death rate for the mosquito population

δ_m	Death rate due to malaria infection
ρ	Rate of progression of human from $I_{hal}(t)$ to $I_{hcl}(t)$
δ_v	Mosquitoes death rate due to the use of insecticide
τ_1	Treatment rate for Lymphatic filariasis infected individuals
τ_2	Treatment rate for malaria infected individuals
τ_3	Treatment rate for Lymphatic filariasis and malaria co-infected individuals
α_1	Progression rate at which malaria, Lymphatic filariasis and co-infected Lymphatic filariasis maleness full recovered human after treatment move to susceptible class
α_2	Rate at which the treatment immunity wanes off

The assumptions below were considered in constructing the model

1. Recruitment into the susceptible population is at constant rate.
2. There is no vertical transmission of the diseases.
3. Treatment of Lymphatic filariasis is taking to be for 8years.

Let

$$k_1 = (\mu_h + \tau_1) \tag{3.17}$$

$$k_2 = (\mu_h + \alpha_2) \tag{3.18}$$

$$k_3 = (\mu_h + \tau_1 + \rho) \tag{3.19}$$

$$k_4 = (\tau_2 + \delta_m + \mu_h) \tag{3.20}$$

$$k_5 = (\tau_3 + \delta_m + \mu_h) \tag{3.21}$$

$$k_6 = (\alpha_1 + \mu_h) \tag{3.22}$$

$$k_7 = \left(\mu_v + \delta_v\right) \tag{3.23}$$

Thus, the equation (3.6) – (3.14) becomes

$$\frac{dS_h}{dt} = \Lambda_h - \sigma_v\left(1-\theta\right)\left(\beta_m + \beta_l + \beta_{lm}\right)\frac{I_v S_h}{N_v} - k_1 S_h - \alpha_1 T_h + \alpha_2 V_h \tag{3.24}$$

$$\frac{dV_h}{dt} = \tau_1 S_h - k_2 V_h - \beta_m \sigma_v \frac{I_v V_h}{N_v} \tag{3.25}$$

$$\frac{dI_{hal}}{dt} = \beta_l \sigma_v \left(1-\theta\right)\frac{I_v S_h}{N_v} - \beta_l \sigma_v \left(1-\theta\right)\frac{I_v I_{ha}}{N_v} - k_3 I_{ha} \tag{3.26}$$

$$\frac{dI_{hcl}}{dt} = \beta_l \sigma_v \left(1-\theta\right)\frac{I_v I_{ha}}{N_v} + \rho I_{hal} - k_1 I_{hc} \tag{3.27}$$

$$\frac{dI_m}{dt} = \beta_m \sigma_v \left(1-\theta\right)\frac{I_v S_h}{N_v} - k_4 I_m + \beta_m \sigma_v \frac{I_v V_h}{N_v} \tag{3.28}$$

$$\frac{dI_{lm}}{dt} = \beta_{lm} \sigma_v \left(1-\theta\right)\frac{I_v S_h}{N_v} - k_5 I_{lm} \tag{3.29}$$

$$\frac{dT_h}{dt} = \tau_2 I_m + \tau_1\left(I_{ha} + I_{hc}\right) + \tau_3 I_{lm} - k_6 T_h \tag{3.30}$$

$$\frac{dS_v}{dt} = \Lambda_v - \beta_h \sigma_v \frac{\left(I_{hal} + I_{hcl} + I_{lm} + I_m\right)}{N_h} S_v - k_7 S_v \tag{3.31}$$

$$\frac{dI_v}{dt} = \beta_h \sigma_v \frac{\left(I_{hal} + I_{hcl} + I_{lm} + I_m\right)}{N_h} S_v - k_7 I_v \tag{3.32}$$

3.2 Basic Properties of the Model

3.2.1 Feasible region

Theorem 3.1: The system (3.24) – (3.32) has solutions, which are contained in the feasible region Ω

Proof:

Assuming there is no disease induced death

Let $\Gamma = \left(S_h, V_h, I_{hal}, I_{hcl}, I_m, I_{lm} + T_h\right) \in R_+^7$ and

$\psi = \left(S_v, I_v\right) \in R_+^2$ be any solution of the system with non-negative initial conditions, then adding the equation of the system (3.24) - (3.30) and (3.31) - (3.32) we have

$$\frac{dS_h}{dt} + \frac{dV_h}{dt} + \frac{dI_{hcl}}{dt} + \frac{dI_m}{dt} + \frac{dI_{lm}}{dt} + \frac{dT_h}{dt}$$

$$= \Lambda_h - \mu_h \left(S_h, V_h, I_{hal}, I_{hcl}, I_m, I_{lm} + T_h\right) \tag{3.33}$$

$$\frac{dN_h}{dt} = \Lambda_h - \mu_h N_h \tag{3.34}$$

$$\frac{dN_h}{dt} + \mu_h N_h = \Lambda_h \tag{3.35}$$

$$IF = e^{\mu_h t} \tag{3.36}$$

Multiplying equation (3.35) with its integrating factor, gives

$$\frac{dN_h}{dt} e^{\mu_h t} + \mu_h N_h e^{\mu_h t} \leq \Lambda_h e^{\mu_h t} \tag{3.37}$$

$$\frac{d}{dt}\left(N_h e^{\mu_h t}\right) \leq \Lambda_h e^{\mu_h t} \tag{3.38}$$

Integrating equation (3.38)

$$N_h e^{\mu_h t} \leq \int \Lambda_h e^{\mu_h t} dt + C \tag{3.39}$$

Dividing equation (3.39) by $e^{\mu_h t}$

$$N_h(t) \le \frac{\Lambda_h}{\mu_h} + Ce^{-\mu_h t} \tag{3.40}$$

Using the initial condition; $t = 0$, $N_h(0) = N_{h(0)}$ (3.41)

$$N_h(0) - \frac{\Lambda_h}{\mu_h} \le C \tag{3.42}$$

$$N_h(t) \le \frac{\Lambda_h}{\mu_h} + \left(N_h(0) - \frac{\Lambda_h}{\mu_h}\right)e^{-\mu_h t} \tag{3.43}$$

Taking the limit as $t \to \infty$ (3.44)

$$N_h \le \frac{\Lambda_h}{\mu_h} \tag{3.45}$$

This implies that as t $\to \infty$, the total population of human approaches $\frac{\Lambda_h}{\mu_h}$

Also

$$\frac{dS_v}{dt} + \frac{dI_v}{dt} = \Lambda_v - (\mu_v + \delta_v)(S_v + I_v) \tag{3.46}$$

Recall $k_7 = (\mu_v + \delta_v)$

$$\frac{dS_v}{dt} + \frac{dI_v}{dt} = \Lambda_v - k_7(S_v + I_v) \tag{3.47}$$

$$\frac{dN_v}{dt} = \Lambda_v - k_7 N_v \tag{3.48}$$

$$\frac{dN_v}{dt} + k_7 N_v = \Lambda_v \tag{3.49}$$

$$IF = \ell^{k_7 t} \tag{3.50}$$

Multiplying equation (3.49) with its integrating factor, gives

$$\frac{dN_v}{dt}e^{k_7 t} + k_7 N_v e^{k_7 t} \le \Lambda_v e^{k_7 t} \tag{3.51}$$

$$\frac{d}{dt}\left(N_v e^{k_7 t}\right) \le \Lambda_v e^{k_7 t} \tag{3.52}$$

Integrating equation (3.52), gives

$$N_v e^{k_7 t} \le \int \Lambda_v e^{k_7 t} dt + C \tag{3.53}$$

Diving equation (3.53) by $e^{-k_7 t}$

$$N_v(t) \le \frac{\Lambda_v}{k_7} + Ce^{-k_7 t} \tag{3.54}$$

Using the initial condition

$$t = 0,\ N_v(0) = N_v(0) \tag{3.55}$$

$$N_v(0) - \frac{\Lambda_v}{k_7} \le C \tag{3.56}$$

$$N_v(t) \le \frac{\Lambda_v}{k_7} + \left(N_v(0) - \frac{\Lambda_v}{k_7}\right)e^{-k_7 t} \tag{3.57}$$

Taking the limit as t $\rightarrow \infty$ (3.58)

$$N_v \le \frac{\Lambda_v}{k_7} \tag{3.59}$$

This implies that as t $\rightarrow \infty$, the total population of the vector approaches $\frac{\Lambda_v}{k_7}$

Therefore, the considered region of the system (3.24) – (3.30) and (3.31) –(3.32) is

$$\Omega = \Gamma \times \psi = \left\{ \begin{array}{l} \left(S_h, V_h, I_{hal}, I_{hcl}, I_m, I_{lm}, T_h\right) \in R_+^7 : \\ \left(S_h, V_h, I_{hal}, I_{hcl}, I_m, I_{lm}, T_h\right) \le \frac{\Lambda_h}{\mu_h} \\ S_h \ge 0, V_h \ge 0, I_{hal} \ge 0, I_{hcl} \ge 0, I_m \ge 0, I_{lm} \ge 0, T_h \ge 0 \\ \left(S_v, I_v\right) \in R_+^2 : S_v + I_v \le \frac{\Lambda_v}{k_7}, S_v \ge 0, I_v \ge 0 \end{array} \right\} \tag{3.60}$$

As t $\rightarrow \infty$, the vector field points to the interior of Ω of the total population of the human and vector $\left(N_h \text{ and } N_v\right)$ approaches $\frac{\Lambda_h}{\mu_7}$ and $\frac{\Lambda_v}{k_7}$ respectively, where $\frac{\Lambda_h}{\mu_7}$ and $\frac{\Lambda_v}{k_7}$ are the upper bonds. Hence, the solutions of the model enter the region Γ and ψ at any time t. therefore, the solution of equation (3.24) to (3.32) is positively invariant and hence in the feasible region Ω, the model remains epidemiologically meaningful and mathematically well posed. It suffices to consider the dynamics of the model (3.24) – (3.32) in Ω (Hethcote, 2000).

3.2.2 Positivity of the solutions

In this subsection, since the model consist of both human and vector classes we employed the technique of Friedman and Lungu (2013) to demonstrate the equation (3.18 – 3.26) is positively invariant and well posed. We consider the system (3.18 – 3.26) in matrix form.

$$\begin{pmatrix} \dot{S}_h \\ \dot{V}_h \\ \dot{I}_{hal} \\ \dot{I}_{hcl} \\ \dot{I}_m \\ \dot{I}_{cm} \\ \dot{T}_h \\ \dot{S}_v \\ \dot{I}_v \end{pmatrix} = \begin{pmatrix} -k_1 & \alpha_2 & 0 & 0 & 0 & 0 & \alpha_1 & 0 & 0 \\ \tau_1 S_h & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho & -k_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k_5 & 0 & 0 & 0 \\ 0 & 0 & \tau_1 & \tau_1 & \tau_2 & \tau_3 & -k_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_7-\omega_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \omega_5 & -k_7 \end{pmatrix} \begin{pmatrix} \dot{S}_h \\ \dot{V}_h \\ \dot{I}_{hal} \\ \dot{I}_{hcl} \\ \dot{I}_m \\ \dot{I}_{cm} \\ \dot{T}_h \\ \dot{S}_v \\ \dot{I}_v \end{pmatrix} + \begin{pmatrix} \Lambda_h - \omega_1 I_v S_h \\ -\frac{\beta_m \sigma_v}{N_v} I_v V_h \\ \omega_2 I_v S_h - \omega_2 I_v I_{hal} \\ \omega_2 I_v I_{hal} \\ \frac{\beta_m \sigma_v}{N_v} I_v V_h + \omega_3 I_v S_h \\ \omega_4 I_v S_h \\ 0 \\ \Lambda_v \\ 0 \end{pmatrix} \quad (3.61)$$

where

$$\omega_1 = \frac{\sigma_v (1-\theta)(\beta_m + \beta_l + \beta_{lm})}{N_v}, \quad \omega_2 = \frac{\beta_l \sigma_v (1-\theta)}{N_v}$$

$$\omega_3 = \frac{\beta_m \sigma_v (1-\theta)}{N_v}, \omega_4 = \frac{\beta_{lm} \sigma_v (1-\theta)}{N_v}, \omega_5 = \frac{\beta_h \sigma_v (I_{hal} + I_{hcl} + I_m + I_{lm})}{N_h}$$

Which in compact form can be written as

$$\frac{dx_i}{dt} = f(x,t) = A_i x_i + H(x_i) \tag{3.62}$$

For $x = (x_1, x_2, ..., x_n)$ where $x_1 = S_h$, $x_2 = V_h$, $x_3 = I_{hal}$, $x_4 = I_{hcl}$, $x_5 = I_m$, $x_6 = I_{lm}$

$x_7 = T_h$, $x_8 = S_v$, $x_9 = I_v$ and $(.)'$ denote transpose. Then the equation (3.18)-(3.26) can be written as

$$\frac{dx_i}{dt} = f(x,t) \tag{3.63}$$

For $x = (x_1, x_2, ..., x_n)'$. one can easily show that equation (3.24)-(3.32) satisfies the differential inequality

$$\frac{dx_i}{dt} \le A_i x_i + \sum_{v=1}^{n} C_{ij} x_i + \varepsilon \tag{3.64}$$

For $i = 1, 2, 3, ..., n$ with $C_{ij} \ge 0$ and $t > 0$ if $x_i(0) \ge \varepsilon$

For $i = 1, 2, 3, ..., n$ then $x_i(t) \ge \varepsilon$ for all $t \ge o$

Proof

Assuming without loss of generality that $\varepsilon > 0$. The case $\varepsilon = 0$ is trivial through approximation of the equation (3.18)-(3.26) with a sequence $\varepsilon = \varepsilon_k$, which converges to zero as k goes to infinity.

Suppose now that $x_i(0) \ge \varepsilon > 0$, for $1 \le i \le n$ does not hold, then there exist $t_0 > 0$ such that $x_i(t) \ge 0$ for $1 \le i \le n$, $0 \le t > t_0$ and $x_i(t_0) = 0$ for at least one i, say $i = i_0$. Then $x(i_0)$ is a decreasing function such that $\frac{dx_{i_0}}{dt}(t_0) \le 0$

From the differential $\frac{dx_{i_0}}{dt} \geq A_i x_{i_0} + \sum_{j=1}^{n} C_{ij} x_{i_0} + \varepsilon$ which is a contradiction. Thus, if

$x_i(0) \geq \varepsilon$ For $i = 1, 2, 3, ..., n$ then $x_i(t) \geq 0$ for all $t \geq 0$. Since this hold hence

we've showed that the Equations are all positive.

3.3 Equilibrium State of the Model

At equilibrium, let

$$\frac{dS_h}{dt} = \frac{dV_h}{dt} = \frac{dI_{hal}}{dt} = \frac{dI_{hcl}}{dt} = \frac{dI_m}{dt} = \frac{dI_{lm}}{dt} = \frac{dT_h}{dt} = \frac{dS_v}{dt} = \frac{dI_v}{dt} = 0 \qquad (3.65)$$

And at any arbitrary equilibrium state, let

$$E^* = \begin{pmatrix} S_h \\ V_h \\ I_{hal} \\ I_{hcl} \\ I_m \\ I_{lm} \\ T_h \\ S_v \\ I_v \end{pmatrix} = \begin{pmatrix} S_h^* \\ V_h^* \\ I_{hal}^* \\ I_{hcl}^* \\ I_m^* \\ I_{lm}^* \\ T_h^* \\ S_v^* \\ I_v^* \end{pmatrix} \qquad (3.66)$$

Then the steady states of (3.24) – (3.32) satisfy the following algebraic system.

$$\Lambda_h - \sigma_v (1-\theta)(\beta_m + \beta_l + \beta_{lm}) \frac{I_v^* S_h^*}{N_v} - k_1 S_h^* - \alpha_1 T_h^* + \alpha_2 V_h^* = 0 \qquad (3.67)$$

$$\tau_1 S_h^* - k_2 V_h^* - \beta_m \sigma_v \frac{I_v^* V_h^*}{N_v} = 0 \qquad (3.68)$$

$$\beta_l \sigma_v (1-\theta) \frac{I_v^* S_h^*}{N_v} - \beta_l \sigma_v (1-\theta) \frac{I_v^* I_{hal}^*}{N_v} - k_3 I_{hal}^* = 0 \qquad (3.69)$$

$$\beta_l \sigma_v (1-\theta) \frac{I_v^* I_{hal}^*}{N_v} + \rho I_{hal}^* - k_1 I_{hcl}^* = 0 \qquad (3.70)$$

$$\beta_m \sigma_v (1-\theta) \frac{I_v^* S_h^*}{N_v} - k_4 I_m^* + \beta_m \sigma_v \frac{I_v^* V_h^*}{N_v} = 0 \tag{3.71}$$

$$\beta_{lm} \sigma_v (1-\theta) \frac{I_v^* S_h^*}{N_v} - k_5 I_{lm}^* = 0 \tag{3.72}$$

$$\tau_2 I_m^* + \tau_1 \left(I_{hal}^* + I_{hcl}^* \right) + \tau_3 I_{lm}^* - k_6 T_h^* = 0 \tag{3.73}$$

$$\Lambda_v - \beta_h \sigma_v \frac{\left(I_{hal}^* + I_{hcl}^* + I_{lm}^* + I_m^* \right)}{N_h} S_v^* - k_7 S_v^* = 0 \tag{3.74}$$

$$\beta_h \sigma_v \frac{\left(I_{hal}^* + I_{hcl}^* + I_{lm}^* + I_m^* \right)}{N_h} S_v^* - k_7 I_v^* = 0 \tag{3.75}$$

From equation (3.68)

$$\tau_1 S_h^* - k_2 V_h^* - \beta_m \sigma_v \frac{I_v^* V_h^*}{N_v} = 0 \tag{3.76}$$

$$\tau_1 S_h^* - \left(k_2 - \beta_m \sigma_v \frac{I_v^*}{N_v} \right) V_h^* = 0 \tag{3.77}$$

$$\tau_1 S_h^* = k_2 - \beta_m \sigma_v \frac{I_v^*}{N_v} \tag{3.78}$$

$$N_v \tau_1 S_h^* = \left(N_v k_2 - \beta_m \sigma_v I_v^* \right) V_h^* = 0 \tag{3.79}$$

$$V_h^* = \frac{N_v \tau_1 S_h^*}{N_v k_2 - \beta_m \sigma_v I_v^*} \tag{3.80}$$

From equation (3.72)

$$\beta_{lm} \sigma_v (1-\theta) \frac{I_v^* S_h^*}{N_v} - k_5 I_{lm}^* = 0 \tag{3.81}$$

$$\beta_{lm} \sigma_v (1-\theta) \frac{I_v^* S_h^*}{N_v} = k_5 I_{lm}^* \tag{3.82}$$

$$I_{lm}^* = \frac{\beta_{lm} \sigma_v (1-\theta) I_v^* S_h^*}{N_v k_5} \tag{3.83}$$

From equation (3.69)

$$\beta_l \sigma_v (1-\theta)\frac{I_v^* S_h^*}{N_v} - \beta_l \sigma_v (1-\theta)\frac{I_v^* I_{hal}^*}{N_v} - k_3 I_{hal}^* = 0 \tag{3.84}$$

$$\beta_l \sigma_v (1-\theta)\frac{I_v^* S_h^*}{N_v} = \left(\frac{\beta_l \sigma_v (1-\theta) I_v^* + N_v k_3}{N_v}\right) I_{hal}^* \tag{3.85}$$

$$I_{hal}^* = \left(\frac{\beta_l \sigma_v (1-\theta) I_v^* S_h^*}{\beta_l \sigma_v (1-\theta) I_v^* + k_3 N_v}\right) \tag{3.86}$$

From equation (3.70)

$$\beta_l \sigma_v (1-\theta) \frac{I_v^* I_{hal}^*}{N_v} + \rho I_{hal}^* - k_1 I_{hcl}^* = 0 \tag{3.87}$$

$$\left(\frac{\beta_l \sigma_v (1-\theta) I_v^* + \rho N_v}{N_v} \right) I_{hal}^* - k_1 I_{hcl}^* = 0 \tag{3.88}$$

$$\left(\frac{\beta_l \sigma_v (1-\theta) I_v^* + \rho N_v}{N_v} \right) \left(\frac{\beta_l \sigma_v (1-\theta) I_v^* S_h^*}{\beta_l \sigma_v (1-\theta) I_v^* + k_3 N_v} \right) = k_1 I_{hcl}^* \tag{3.89}$$

$$I_{hcl}^* = \left(\frac{\left(\beta_l \sigma_v (1-\theta) I_v^* + \rho N_v\right)\left(\beta_l \sigma_v (1-\theta) I_v^* S_h^*\right)}{k_1 N_v \left(\beta_l \sigma_v (1-\theta) I_v^* + k_3 N_v\right)} \right) \tag{3.90}$$

$$I_{hcl}^* = \left(\frac{\beta_l \sigma_v (1-\theta) I_v^* S_h^* + \beta_l \sigma_v (1-\theta) \rho N_v S_h^*}{k_1 N_v \left(\beta_l \sigma_v (1-\theta) I_v^* + k_3 N_v\right)} \right) \tag{3.91}$$

From equation (3.71)

$$\beta_m \sigma_v (1-\theta) \frac{I_v^* S_h^*}{N_v} - k_4 I_m^* + \beta_m \sigma_v \frac{I_v^* V_h^*}{N_v} = 0 \tag{3.92}$$

$$\beta_m \sigma_v (1-\theta) \frac{I_v^* S_h^*}{N_v} + \beta_m \sigma_v \frac{I_v^* V_h^*}{N_v} = k_4 I_m^* \tag{3.93}$$

$$\left(\frac{\beta_m \sigma_v (1-\theta) S_h^* + \beta_m \sigma_v V_h^*}{N_v} \right) I_v^* = k_4 I_m^* \tag{3.94}$$

$$I_m^* = \left(\frac{\beta_m \sigma_v (1-\theta) S_h^* + \beta_m \sigma_v V_h^*}{N_v k_4} \right) I_v^* \tag{3.95}$$

Substituting equation (3.83), (3.86), (3.91) and (3. 95) in equation (3.96)

$$\beta_h \sigma_v \frac{(I_{hal}, I_{hcl}, I_m, I_{lm}) S_v^*}{N_h} - k_7 I_v^* = 0 \tag{3.96}$$

$$\frac{\beta_h \sigma_v}{N_h} \left(\begin{array}{l} \left(\frac{\beta_l \sigma_v (1-\theta) I_v^* S_h^*}{\beta_l \sigma_v (1-\theta) I_v^* + k_3 N_v} \right) + \left(\frac{\left(\beta_l \sigma_v (1-\theta)\right)^2 I_v^{*2} S_h^* + \beta_l \sigma_v (1-\theta) \rho N_v S_h^*}{k_1 N_v \left(\beta_l \sigma_v (1-\theta) I_v^* + k_3 N_v\right)} \right) I_v^* \\ + \left(\frac{\beta_{lm} \sigma_v (1-\theta) S_h^*}{N_v k_5} \right) I_v^* + \left(\frac{\beta_m \sigma_v (1-\theta) S_h^* + \beta_m \sigma_v V_h^*}{N_v k_4} \right) I_v^* \end{array} \right) S_v^* - k_7 I_v^* = 0 \tag{3.97}$$

$$\frac{\beta_h\sigma_v}{N_h}\left(\begin{array}{l}\frac{\left(\beta_l\sigma_v(1-\theta)S_h^*\right)}{\beta_l\sigma_v(1-\theta)I_v^*-k_3N_v}+\frac{\left(\beta_l\sigma_v(1-\theta)\right)^2 I_v^*S_h^*+\beta_l\sigma_v(1-\theta)\rho N_vS_h^*}{k_1N_v\left(\beta_l\sigma_v(1-\theta)I_v^*+k_3N_v\right)}\\+\frac{\beta_{lm}\sigma_v(1-\theta)S_h^*}{N_vk_5}+\frac{\beta_m\sigma_v(1-\theta)S_h^*+\beta_m\sigma_vV_h^*}{N_vk_4}\end{array}\right)I_v^*S_v^*-k_7I_v^*=0 \tag{3.98}$$

$$\left(\frac{\beta_h\sigma_v}{N_h}\left(\begin{array}{l}\frac{\beta_l\sigma_v(1-\theta)S_h^*}{\beta_l\sigma_v(1-\theta)I_v^*-k_3N_v}+\frac{\left(\beta_l\sigma_v(1-\theta)\right)^2 I_v^*S_h^*+\beta_l\sigma_v(1-\theta)\rho N_vS_h^*}{k_1N_v\left(\beta_l\sigma_v(1-\theta)I_v^*-k_3N_v\right)}\\+\frac{\beta_{lm}\sigma_v(1-\theta)S_h^*}{N_vk_5}+\frac{\beta_m\sigma_v(1-\theta)S_h^*+\beta_m\sigma_vV_h^*}{N_vk_4}\end{array}\right)-k_7\right)I_v^*=0 \tag{3.99}$$

This implies

$$I_v^*=0 \tag{3.100}$$

Or

$$\left(\frac{S_v^*\beta_h\sigma_v}{N_h}\left(\begin{array}{l}\frac{\beta_l\sigma_v(1-\theta)S_h^*}{\beta_l\sigma_v(1-\theta)I_v^*-k_3N_v}+\frac{\left(\beta_l\sigma_v(1-\theta)\right)^2 I_v^*S_h^*+\beta_l\sigma_v(1-\theta)\rho N_vS_h^*}{k_1N_v\left(\beta_l\sigma_v(1-\theta)I_v^*-k_3N_v\right)}\\+\frac{\beta_{lm}\sigma_v(1-\theta)S_h^*}{N_vk_5}+\frac{\beta_m\sigma_v(1-\theta)S_h^*+\beta_m\sigma_vV_h^*}{N_vk_4}\end{array}\right)-k_7\right)=0 \tag{3.101}$$

Substituting the value of $I_v^*=0$ in equation (3.83)

$$-k_5I_{lm}^*=0 \;\Rightarrow\; I_{lm}^*=0 \tag{3.102}$$

Substituting equation (3.100) in equation (3.95)

$$-k_4I_m^*=0 \;\Rightarrow\; I_m^*=0 \tag{3.103}$$

Substituting equation (3.100) in equation (3.86)

$$-k_3I_{hal}^*=0 \;\Rightarrow\; I_{hal}^*=0 \tag{3.104}$$

Substituting equation (3.100) and (3.104) in equation (3.91) we get

$$-k_1I_{hcl}^*=0 \;\Rightarrow\; I_{hcl}^*=0 \tag{3.105}$$

Substituting equation (3.102), (3.103), (3.104) and (3.105) in equation (3.73) we get

$$-k_1T_h^*=0 \;\Rightarrow\; T_h^*=0 \tag{3.106}$$

Substituting equation (3.102), (3.103), (3.104) and (3.105) in equation (3.74) we get

$$\Lambda_v - k_7 S_v^* = 0 \Rightarrow \Lambda_v = k_7 S_v^* \tag{3.107}$$

$$S_v^* = \frac{\Lambda_v}{k_7} \tag{3.108}$$

Substituting equation (3.100) in equation (3.68) we get

$$\tau_1 S_h^* - k_2 V_h^* = 0 \tag{3.109}$$

$$\tau_1 S_h^* = k_2 V_h^* \tag{3.110}$$

$$V_h^* = \frac{\tau_1 S_h^*}{k_2} \tag{3.111}$$

Substituting equation (3.100), (3.106) and (3.111) in equation (3.67) we get

$$\Lambda_h - k_1 S_h^* + \frac{\alpha_2 \tau_1 S_h^*}{k_2} = 0 \tag{3.112}$$

$$\Lambda_h = \left(\frac{k_1 k_2 - \alpha_2 \tau_1}{k_2} \right) S_h^* \tag{3.113}$$

$$\Rightarrow S_h^* = \frac{\Lambda_h k_2}{k_1 k_2 - \alpha_2 \tau_1} \tag{3.114}$$

$$V_h^* = \frac{\tau_1 \left(\frac{\Lambda_h k_2}{k_1 k_2 - \alpha_2 \tau_1} \right)}{k_2} \tag{3.115}$$

$$V_h^* = \frac{\tau_1 \left(\Lambda_h k_2 \right)}{k_2 \left(k_1 k_2 - \alpha_2 \tau_1 \right)} \tag{3.116}$$

From (3.99) the disease free equilibrium state exists if

$$I_v^* = 0 \tag{3.117}$$

And the endemic equilibrium state exist if

$$\left(\frac{S_v^* \beta_h \sigma_v}{N_h} \left(\begin{array}{l} \frac{\beta_l \sigma_v (1-\theta) S_h^*}{\beta_l \sigma_v (1-\theta) I_v^* - k_3 N_v} + \frac{\left(\beta_l \sigma_v (1-\theta)\right)^2 I_v^* S_h^* + \beta_l \sigma_v (1-\theta) \rho N_v S_h^*}{k_1 N_v \left(\beta_l \sigma_v (1-\theta) I_v^* - k_3 N_v \right)} \\ + \frac{\beta_{lm} \sigma_v (1-\theta) S_h^*}{N_v k_5} + \frac{\beta_m \sigma_v (1-\theta) S_h^* + \beta_m \sigma_v V_h^*}{N_v k_4} \end{array} \right) - k_7 \right) = 0 \tag{3.118}$$

3.4 Disease Free Equilibrium State DFE (E^0)

Disease free equilibrium state are steady state solutions where this is no infection. Thus all the infected disease will be zero and their entire population will comprise of susceptible individual and susceptible vector.

At the disease free equilibrium state, let

$$\begin{pmatrix} S_h \\ V_h \\ I_{hal} \\ I_{hcl} \\ I_m \\ I_{lm} \\ T_h \\ S_v \\ I_v \end{pmatrix} = \begin{pmatrix} S_h^{o} \\ V_h^{o} \\ I_{hal}^{o} \\ I_{hcl}^{o} \\ I_m^{o} \\ I_{lm}^{o} \\ T_h^{o} \\ S_v^{o} \\ I_v^{o} \end{pmatrix} \tag{3.119}$$

Thus, from (3.100), (3.102), (3.103), (3.104), (3.105), (3.106), (3.108), (3.114) and (3.116) the disease free equilibrium state is given by

$$\begin{pmatrix} S_h^{o} \\ V_h^{o} \\ I_{hal}^{o} \\ I_{hcl}^{o} \\ I_m^{o} \\ I_{lm}^{o} \\ T_h^{o} \\ S_v^{o} \\ I_v^{o} \end{pmatrix} = \begin{pmatrix} \dfrac{\Lambda_h k_2}{k_1 k_2 - \alpha_2 \tau_1} \\ \dfrac{\tau_1 \Lambda_h}{k_1 k_2 - \alpha_2 \tau_1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \dfrac{\Lambda_h}{k_7} \\ 0 \end{pmatrix} \tag{3.120}$$

3.5 Basic Reproductive Number R_c

In the model equation (3.2) to (3.10), the infectious compartments includes

I_{hal}, I_{hcl}, I_m, I_{lm}, I_v and the expected secondary infections depends on these classes.

The rate of appearance of new infections in compartments *i* is given by the matrix.

The Jacobean matrix of F evaluated at the disease free equilibrium point is given by

$F = \left(\dfrac{\partial f_i \left(E^0 \right)}{\partial x_i} \right)$ $x_j = I_{hal}, I_{hcl}, I_m, I_{lm}, I_v$ for $j = 1, 2, 3, 4$ and E^0 is the disease free equilibrium.

$$F = \begin{pmatrix} 0 & 0 & 0 & 0 & \dfrac{\beta_l \sigma_v (1-\theta) S_h^*}{N_v} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{\beta_m \sigma_v (1-\theta) S_h + \beta_m \sigma_v V_h}{N_v} \\ 0 & 0 & 0 & 0 & \dfrac{\beta_{lm} \sigma_v (1-\theta) S_h^*}{N_v} \\ \dfrac{\beta_h \sigma_v S_v}{N_h} & \dfrac{\beta_h \sigma_v S_v}{N_h} & \dfrac{\beta_h \sigma_v S_v}{N_h} & \dfrac{\beta_h \sigma_v S_v}{N_h} & 0 \end{pmatrix} \quad (3.121)$$

and

$$V = \begin{pmatrix} k_3 & 0 & 0 & 0 & 0 \\ -\rho & k_1 & 0 & 0 & 0 \\ 0 & 0 & k_4 & 0 & 0 \\ 0 & 0 & 0 & k_5 & 0 \\ 0 & 0 & 0 & 0 & k_7 \end{pmatrix} \quad (3.122)$$

$$V^{-1} = \begin{pmatrix} \frac{1}{k_3} & 0 & 0 & 0 & 0 \\ \frac{\rho}{k_3 k_1} & \frac{1}{k_1} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{k_4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k_5} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{k_7} \end{pmatrix} \tag{3.123}$$

$$FV^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\beta_l \sigma_v (1-\theta) S_h^*}{N_v} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta_m \sigma_v (1-\theta) S_h^* + \beta_m \sigma_v V_h^*}{N_v} \\ 0 & 0 & 0 & 0 & \frac{\beta_{lm} \sigma_v (1-\theta) S_h^*}{N_v} \\ \frac{\beta_h \sigma_v S_v^*}{N_h} & \frac{\beta_h \sigma_v S_v^*}{N_h} & \frac{\beta_h \sigma_v S_v^*}{N_h} & \frac{\beta_h \sigma_v S_v^*}{N_h} & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{k_3} & 0 & 0 & 0 & 0 \\ \frac{\rho}{k_3 k_1} & \frac{1}{k_1} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{k_4} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{k_5} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{k_7} \end{pmatrix} \tag{3.124}$$

$$FV^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & \frac{\beta_l \sigma_v (1-\theta) S_h^*}{k_7 N_v} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta_m \sigma_v (1-\theta) S_h^*}{k_7 N_v} + \frac{\beta_m \sigma_v V_h^*}{k_7 N_v} \\ 0 & 0 & 0 & 0 & \frac{\beta_{lm} \sigma_v (1-\theta) S_h^*}{k_7 N_v} \\ \frac{\beta_h \sigma_v S_v^*}{k_3 N_h} + \frac{\beta_h \sigma_v S_v^* \rho}{k_1 k_3} & \frac{\beta_h \sigma_v S_v^*}{k_1 N_h} & \frac{\beta_h \sigma_v S_v^*}{k_4 N_h} & \frac{\beta_h \sigma_v S_v^*}{k_5 N_h} & 0 \end{pmatrix} \tag{3.125}$$

From (3.125), we calculate the eigenvalues to determine the effective reproduction number R_c by taking the spectral radius (dominant eigenvalue) of the matrix FV^{-1}. Computing $|A-\lambda I| = 0$, we have;

$$\left|FV^{-1}-\lambda I\right| = \begin{vmatrix} -\lambda & 0 & 0 & 0 & \frac{\beta_l\sigma_v(1-\theta)S_h}{k_7N_v} \\ 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & -\lambda & 0 & \frac{\beta_m\sigma_v(1-\theta)S_h}{k_7N_v}+\frac{\beta_m\sigma_v V_h}{k_7N_v} \\ 0 & 0 & 0 & -\lambda & \frac{\beta_{lm}\sigma_v(1-\theta)S_h}{k_7N_v} \\ \frac{\beta_h\sigma_v S_v}{k_1k_3N_h}+\frac{\beta_h\sigma_v S_v\rho}{k_1k_3} & \frac{\beta_h\sigma_v S_v}{k_7N_h} & \frac{\beta_h\sigma_v S_v}{k_4N_h} & \frac{\beta_h\sigma_v S_v}{k_5N_h} & -\lambda \end{vmatrix} = 0 \tag{3.126}$$

This implies

$$\left.\begin{aligned} \lambda_1 &= 0 \\ \lambda_2 &= 0 \\ \lambda_3 &= 0 \\ \lambda_4 &= 0 \end{aligned}\right\} \tag{3.127}$$

$$\lambda_5 = \sqrt{\frac{\beta_h\beta_{lm}\sigma_v\sigma_v k_1k_2k_3k_4(1-\theta)+\beta_h\beta_m\sigma_v\sigma_v k_1k_2k_3k_5(1-\theta)+\beta_h\beta_m\sigma_v\sigma_v\tau_1k_1k_3k_4+\beta_h\beta_l\sigma_v\sigma_v k_2k_4k_5(1-\theta)(k_1+\rho)}{k_1k_3k_4k_5k_7(k_2+\tau_1)}} \tag{3.128}$$

$$\lambda_6 = -\sqrt{\frac{\beta_h\beta_{lm}\sigma_v\sigma_v k_1k_2k_3k_4(1-\theta)+\beta_h\beta_m\sigma_v\sigma_v k_1k_2k_3k_5(1-\theta)+\beta_h\beta_m\sigma_v\sigma_v\tau_1k_1k_3k_4+\beta_h\beta_l\sigma_v\sigma_v k_2k_4k_5(1-\theta)(k_1+\rho)}{k_1k_3k_4k_5k_7(k_2+\tau_1)}} \tag{3.129}$$

Clearly, we can see that λ_5 is the dominant eigenvalues.

$$R_c = \sqrt{\frac{\beta_h\beta_{lm}\sigma_v\sigma_v k_1k_2k_3k_4(1-\theta)+\beta_h\beta_m\sigma_v\sigma_v k_1k_2k_3k_5(1-\theta)+\beta_h\beta_m\sigma_v\sigma_v\tau_1k_1k_3k_4+\beta_h\beta_l\sigma_v\sigma_v k_2k_4k_5(1-\theta)(k_1+\rho)}{k_1k_3k_4k_5k_7(k_2+\tau_1)}} \tag{3.130}$$

R_c is the average number of secondary infectious case that an infectious individual with Lymphatic filariasis and Malaria co-infection would produce in a totally susceptible population.

3.6 Local Stability of Disease Free Equilibrium State

The Disease free equilibrium E^0 of the model equation (3.61) - (3.69) is locally asymptotically stable (LAS) of R_c<1.

Proof: Linearization of the model (3.24) - (3.32) at any arbitrary equilibrium point

(E^0) gives the Jacobian

$$J\left(E^*\right)=\begin{pmatrix} -c_1 & \alpha_2 & 0 & 0 & 0 & 0 & \alpha_1 & 0 & -c_2 \\ \tau_1 & -c_3 & 0 & 0 & 0 & 0 & 0 & 0 & -c_4 \\ c_5 & 0 & -c_6 & 0 & 0 & 0 & 0 & 0 & c_7 \\ 0 & 0 & c_8 & -k_1 & 0 & 0 & 0 & 0 & c_9 \\ c_{10} & c_{11} & 0 & 0 & -k_4 & 0 & 0 & 0 & c_{12} \\ c_{13} & 0 & 0 & 0 & 0 & -k_5 & 0 & 0 & c_{14} \\ 0 & 0 & \tau_1 & \tau_1 & \tau_2 & \tau_3 & k_6 & 0 & 0 \\ 0 & 0 & -c_{15} & -c_{15} & -c_{15} & -c_{15} & 0 & -c_{16} & 0 \\ 0 & 0 & c_{15} & c_{15} & c_{15} & c_{15} & 0 & c_{17} & -k_7 \end{pmatrix} \tag{3.131}$$

where

$$\left.\begin{aligned}
&c_1=\sigma_v\left(1-\theta\right)\left(\beta_m+\beta_l+\beta_{lm}\right)\frac{I_v}{N_v}+k_1, c_2=\sigma_v\left(1-\theta\right)\left(\beta_m+\beta_l+\beta_{lm}\right)\frac{S_h}{N_v}\\
&c_3=k_2+\beta_m\sigma_v\frac{I_v}{N_v}, c_4=\beta_m\sigma_v\frac{V_h}{N_v}, c_5=\beta_l\sigma_v\left(1-\theta\right)\frac{I_v}{N_v}\\
&c_6=\beta_l\sigma_v\left(1-\theta\right)\frac{I_v}{N_v}+k_3, c_7=\beta_l\sigma_v\left(1-\theta\right)\frac{S_h}{N_v}+\beta_l\sigma_v\left(1-\theta\right)\frac{I_{hal}}{N_v}\\
&c_8=\beta_l\sigma_v\left(1-\theta\right)\frac{I_v}{N_v}+\rho, c_9=\beta_l\sigma_v\left(1-\theta\right)\frac{I_{hal}}{N_v}\\
&c_{10}=\beta_m\sigma_v\left(1-\theta\right)\frac{I_v}{N_v}, c_{11}=\beta_m\sigma_v\frac{I_v}{N_v}\\
&c_{12}=\beta_m\sigma_v\left(1-\theta\right)\frac{S_h}{N_v}+\beta_m\sigma_v\frac{V_h}{N_v}, c_{13}=\beta_{lm}\sigma_v\left(1-\theta\right)\frac{I_v}{N_v}\\
&c_{14}=\beta_{lm}\sigma_v\left(1-\theta\right)\frac{S_h}{N_v}, c_{15}=\beta_h\sigma_v\frac{S_v}{N_h}\\
&c_{16}=\beta_h\sigma_v\frac{\left(I_{hal}+I_{hcl}+I_m+I_{lm}\right)}{N_h}+k_7\\
&c_{17}=\beta_h\sigma_v\frac{\left(I_{hal}+I_{hcl}+I_m+I_{lm}\right)}{N_h}
\end{aligned}\right\} \tag{3.132}$$

We evaluate the Jacobian of the system at disease free equilibrium to determine the local stability of the system. We obtain

$$
J\left(E^0\right)=\begin{pmatrix}
-k_1 & \alpha_2 & 0 & 0 & 0 & 0 & \alpha_1 & 0 & -\sigma_v\left(1-\theta\right)\left(\beta_m+\beta_l+\beta_{lm}\right)\frac{S_h{}^0}{N_v} \\
\tau_1 & -k_2 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_m\sigma_v\frac{V_h^0}{N_v^0} \\
0 & 0 & -k_3 & 0 & 0 & 0 & 0 & 0 & \beta_l\sigma_v\left(1-\theta\right)\frac{S_h^0}{N_v^0} \\
0 & 0 & \rho & -k_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -k_4 & 0 & 0 & 0 & \beta_m\sigma_v\left(1-\theta\right)\frac{S_h^0}{N_v^0}+\beta_m\sigma_v\frac{V_h^0}{N_v^0} \\
0 & 0 & 0 & 0 & 0 & -k_5 & 0 & 0 & \beta_{lm}\sigma_v\left(1-\theta\right)\frac{S_h^0}{N_v^0} \\
0 & 0 & \tau_1 & \tau_1 & \tau_2 & \tau_3 & -k_6 & 0 & 0 \\
0 & 0 & -\beta_h\sigma_v\frac{S_v^0}{N_h^0} & -\beta_h\sigma_v\frac{S_v^0}{N_h^0} & -\beta_h\sigma_v\frac{S_v^0}{N_h^0} & -\beta_h\sigma_v\frac{S_v^0}{N_h^0} & 0 & -k_7 & 0 \\
0 & 0 & \beta_h\sigma_v\frac{S_v^0}{N_h^0} & \beta_h\sigma_v\frac{S_v^0}{N_h^0} & \beta_h\sigma_v\frac{S_v^0}{N_h^0} & \beta_h\sigma_v\frac{S_v^0}{N_h^0} & 0 & 0 & -k_7
\end{pmatrix} \tag{3.133}
$$

Using elementary row transformation (as in Abdulrahman *et al*., 2013) the Matrix (3.133) becomes

$$
J\left(E^0\right)=\begin{pmatrix}
-k_1 & \alpha_2 & 0 & 0 & 0 & 0 & \alpha_1 & 0 & -\frac{\sigma_v\left(1-\theta\right)\left(\beta_m+\beta_l+\beta_{lm}\right)\Lambda_h k_2 k_7}{\Lambda_v(k_1k_2-\alpha_2\tau_1)} \\
0 & -\frac{k_1k_2-\alpha_2\tau_1}{k_1} & 0 & 0 & 0 & 0 & \frac{\alpha_1\tau_1}{k_1} & 0 & -\frac{\Lambda_h k_7\tau_1(\beta_m k_1\sigma_v+\sigma_v\left(1-\theta\right)\left(\beta_m+\beta_l+\beta_{lm}\right)k_2)}{k_1\Lambda_v(k_1k_2-\alpha_2\tau_1)} \\
0 & 0 & -k_3 & 0 & 0 & 0 & 0 & 0 & \frac{\beta_l\sigma_v\left(1-\theta\right)\Lambda_h k_2k_7}{\Lambda_v(k_1k_2-\alpha_2\tau_1)} \\
0 & 0 & 0 & -k_1 & 0 & 0 & 0 & 0 & \frac{\rho\beta_l\sigma_v\left(1-\theta\right)\Lambda_h k_2k_7}{k_3\Lambda_v(k_1k_2-\alpha_2\tau_1)} \\
0 & 0 & 0 & 0 & -k_4 & 0 & 0 & 0 & \frac{\Lambda_h\beta_m k_7(\sigma_v\left(1-\theta\right)k_2+\sigma_v\tau_1)}{\Lambda_v(k_1k_2-\alpha_2\tau_1)} \\
0 & 0 & 0 & 0 & 0 & -k_5 & 0 & 0 & \frac{\beta_{lm}\sigma_v\left(1-\theta\right)\Lambda_h k_2k_7}{\Lambda_v(k_1k_2-\alpha_2\tau_1)} \\
0 & 0 & \tau_1 & 0 & 0 & 0 & -k_6 & 0 & \frac{k_7\Lambda_h(\tau_3\beta_{lm}\sigma_v\left(1-\theta\right)k_1k_2k_3k_4+z_1+z_2)}{k_1k_3\Lambda_v(k_1k_2-\alpha_2\tau_1)k_4k_5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_7 & -\frac{\sigma_v\beta_h\Lambda_h(\beta_{lm}\sigma_v\left(1-\theta\right)k_1k_2k_3k_4+z_3+z_4)}{\Lambda_h(\tau_1+k_2)k_1k_3k_4k_5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sigma_v\beta_h\Lambda_h(\beta_{lm}\sigma_v\left(1-\theta\right)k_1k_2k_4+z_3+z_4+k_1k_3k_4k_5k_7(k_2+\tau_1))}{\Lambda_h(\tau_1+k_2)k_1k_3k_4k_5}
\end{pmatrix} \tag{3.134}
$$

The characteristic equation of the upper triangular Jacobian is

$$
J\left(E^0\right)=\begin{pmatrix}
-k_1-\lambda & \alpha_2 & 0 & 0 & 0 & 0 & \alpha_1 & 0 & -\frac{\sigma_v\left(1-\theta\right)\left(\beta_m+\beta_l+\beta_{lm}\right)\Lambda_h k_2 k_7}{\Lambda_v(k_1k_2-\alpha_2\tau_1)} \\
0 & -\frac{k_1k_2-\alpha_2\tau_1}{k_1}-\lambda & 0 & 0 & 0 & 0 & \frac{\alpha_1\tau_1}{k_1} & 0 & -\frac{\Lambda_h k_7\tau_1(\beta_m k_1\sigma_v+\sigma_v\left(1-\theta\right)\left(\beta_m+\beta_l+\beta_{lm}\right)k_2)}{k_1\Lambda_v(k_1k_2-\alpha_2\tau_1)} \\
0 & 0 & -k_3-\lambda & 0 & 0 & 0 & 0 & 0 & \frac{\beta_l\sigma_v\left(1-\theta\right)\Lambda_h k_2k_7}{\Lambda_v(k_1k_2-\alpha_2\tau_1)} \\
0 & 0 & 0 & -k_1-\lambda & 0 & 0 & 0 & 0 & \frac{\rho\beta_l\sigma_v\left(1-\theta\right)\Lambda_h k_2k_7}{k_3\Lambda_v(k_1k_2-\alpha_2\tau_1)} \\
0 & 0 & 0 & 0 & -k_4-\lambda & 0 & 0 & 0 & \frac{\Lambda_h\beta_m k_7(\sigma_v\left(1-\theta\right)k_2+\sigma_v\tau_1)}{\Lambda_v(k_1k_2-\alpha_2\tau_1)} \\
0 & 0 & 0 & 0 & 0 & -k_5-\lambda & 0 & 0 & \frac{\beta_{lm}\sigma_v\left(1-\theta\right)\Lambda_h k_2k_7}{\Lambda_v(k_1k_2-\alpha_2\tau_1)} \\
0 & 0 & \tau_1 & 0 & 0 & 0 & -k_6-\lambda & 0 & \frac{k_7\Lambda_h(\tau_3\beta_{lm}\sigma_v\left(1-\theta\right)k_1k_2k_3k_4+z_1+z_2)}{k_1k_3\Lambda_v(k_1k_2-\alpha_2\tau_1)k_4k_5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -k_7-\lambda & -\frac{\sigma_v\beta_h\Lambda_h(\beta_{lm}\sigma_v\left(1-\theta\right)k_1k_2k_3k_4+z_3+z_4)}{\Lambda_h(\tau_1+k_2)k_1k_3k_4k_5} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sigma_v\beta_h\Lambda_h(\beta_{lm}\sigma_v\left(1-\theta\right)k_1k_2k_4+z_3+z_4+k_1k_3k_4k_5k_7(k_2+\tau_1))}{\Lambda_h(\tau_1+k_2)k_1k_3k_4k_5}-\lambda
\end{pmatrix}
\tag{3.135}
$$

where

$$
\left.\begin{aligned}
z_1 &= \sigma_v\left(1-\theta\right)\rho\beta_l k_2k_4k_5\tau_1+\sigma_v\left(1-\theta\right)\beta_l k_1k_2k_4k_5\tau_1 \\
z_2 &= \sigma_v\left(1-\theta\right)\beta_m k_1k_2k_3k_5\tau_2+\beta_m k_1k_3k_5\sigma_v\tau_1\tau_2 \\
z_3 &= \sigma_v(\sigma_v\left(1-\theta\right)\rho\beta_l k_2k_4k_5+\sigma_v\left(1-\theta\right)\beta_l k_1k_2k_4k_5) \\
z_4 &= \sigma_v\left(1-\theta\right)\beta_m k_1k_2k_3k_5+\beta_m k_1k_3k_5\sigma_v\tau_1
\end{aligned}\right\}
\tag{3.136}
$$

Therefore, the eigenvalues are

$$\lambda_1 = -k_1 < 0 \tag{3.137}$$

$$
\left.\begin{aligned}
\lambda_2 &= -\frac{k_1k_2-\alpha_2\tau_1}{k_1} \\
&-\left(\frac{(\mu_h+\tau_1)(\mu_h+\alpha_2)-\alpha_2\tau_1}{k_1}\right) < 0 \\
&= = -\left(\frac{\mu_h^2+\mu_h\alpha_2+\mu_h\tau_1+\alpha_2\tau_1-\alpha_2\tau_1}{k_1}\right) < 0 \\
&= -\left(\frac{\mu_h^2+\mu_h\alpha_2+\mu_h\tau_1}{k_1}\right) < 0
\end{aligned}\right\}
\tag{3.138}
$$

$$\lambda_2 = -\left(\frac{\mu_h^2+\mu_h\alpha_2+\mu_h\tau_1}{k_1}\right) < 0 \tag{3.139}$$

$$\lambda_3 = -k_3 < 0 \tag{3.140}$$

$$\lambda_4 = -k_1 < 0 \tag{3.141}$$

$$\lambda_5 = -k_4 < 0 \tag{3.142}$$

$$\lambda_6 = -k_5 < 0 \tag{3143}$$

$$\lambda_7 = -k_6 < 0 \tag{3.144}$$

$$\lambda_8 = -k_7 < 0 \tag{3.145}$$

$$\lambda_9 = \frac{\sigma_v \beta_h \Lambda_h (\beta_{lm} \sigma_v (1-\theta) k_1 k_2 k_4 + z_3 + z_4 - k_1 k_3 k_4 k_5 k_7 (k_2 + \tau_1))}{\Lambda_h (\tau_1 + k_2) k_1 k_3 k_4 k_5 k_7} \tag{3.146}$$

For λ_9 to be negative, then

$$\frac{\sigma_v \beta_h (\beta_{lm} \sigma_v (1-\theta) k_1 k_2 k_4 + z_3 + z_4 - k_1 k_3 k_4 k_5 k_7 (k_2 + \tau_1))}{(\tau_1 + k_2) k_1 k_3 k_4 k_5 k_7} < 0 \tag{3.147}$$

i.e

$$\sigma_v \beta_h \beta_{lm} \sigma_v (1-\theta) k_1 k_2 k_4 + \sigma_v \beta_h z_3 + \sigma_v \beta_h z_4 < \sigma_v \beta_h k_1 k_3 k_4 k_5 k_7 (k_2 + \tau_1) \tag{3.148}$$

$$\frac{\sigma_v \beta_h \beta_{lm} \sigma_v (1-\theta) k_1 k_2 k_4 + \sigma_v \beta_h \left(\sigma_v (\sigma_v (1-\theta) \rho \beta_l k_2 k_4 k_5 + \sigma_v (1-\theta) \beta_l k_1 k_2 k_4 k_5)\right) + \sigma_v \beta_h z_4}{\sigma_v \beta_h k_1 k_3 k_4 k_5 k_7 (k_2 + \tau_1)} < 1$$

(3.149)

R_c <1

This implies that $\lambda_9 < 0$ if $R_c < 1$

Hence, the disease free equilibrium E^0 of the equation (3.24) - (3.32) is locally asymptotically stable (LAS), if $R_c < 1$

3.7 Global Stability of Disease Free Equilibrium (DFE)

Theorem: The D.F.E (E^0) of the model system is globally asymptotically stable (GAS) in the feasible region Ω if $R_C \le 1$

Proof: To establish the global stability of the D.F.E, the two conditions for the global stability of D.F.E as in (Castillo-Chavez *et al*, 2007) for $R_C < 1$ was used for the model system.

We can write the model system as:

Let $X = (S_h, V_h, T_h, S_v)$ and $Z = (I_{hal}, I_{hcl}, I_m, I_{lm}, I_v)$ and writing the model equation (3.61) – (3.69) in the form $\frac{dX}{dt} = F(X, Z) and \frac{dZ}{dt} = G(X, Z)$

where $I_{hal} = I_{hcl} = I_m = I_{lm} = I_v = 0$

With *F(X, Z)* being the RHS of $\dot{S}_h, \dot{V}_h, \dot{T}_h, \dot{S}_v$ and *G(X,Z)* the RHS $\dot{I}_{hal}, \dot{I}_{hcl}, \dot{I}_m, \dot{I}_{lm}, \dot{I}_v$

Next we consider the reduced system $\frac{dX}{dt} = F(X, Z)$ given as

$$\left.\begin{aligned}
\dot{S}_h &= \Lambda_h - \sigma_v(1-\theta)(\beta_m + \beta_l + \beta_{lm})\frac{I_v S_h}{N_v} - k_1 S_h - \alpha_1 T_h + \alpha_2 V_h \\
\dot{V}_h &= \tau_1 S_h - k_2 V_h - \beta_m \sigma_v \frac{I_v V_h}{N_v} \\
\dot{T}_h &= \tau_2 I_m + \tau_1(I_{hal} + I_{hcl}) + \tau_3 I_{lm} - k_6 T_h \\
\dot{S}_v &= \Lambda_v - \beta_h \sigma_v \frac{(I_{hal} + I_{hcl} + I_{lm} + I_m)}{N_h} S_v - k_7 S_v
\end{aligned}\right\} \qquad (3.150)$$

Let

$$X^* = (S_h^*, V_h^*, T_h^*, S_v^*) = \left(\frac{\Lambda_h k_2}{k_1 k_2 - \alpha_2 \tau_1}, \frac{\tau_1 \Lambda_h}{k_1 k_2 - \alpha_2 \tau_1}, 0, \frac{\Lambda_v}{k_7} \right) \qquad (3.151)$$

be the equilibrium point of the reduced system (3.150), we now show that X^* is globally stable by solving equation (3.150) and taking limit as $t \to \infty$ Solving for $S_h(t)$ gives

$$S_h(t) = \Lambda_h - k_1 S_h + a_2 V_h \tag{3.152}$$

$$S_h(t) + k_1 S_h = \Lambda_h + a_2 V_h \tag{3.153}$$

Multiplying by its integrating factor $e^{k_1 t}$

$$S_h(t) e^{k_1 t} + k_1 S_h e^{k_1 t} = (\Lambda_h + a_2 V_h) e^{k_1 t} \tag{3.154}$$

Integrating and diving by $e^{k_1 t}$ gives

$$\int \frac{d}{dt}\left[S_h e^{k_1 t}\right] = \int (\Lambda_h + a_2 V_h) e^{k_1 t} dt \tag{3.155}$$

$$S_h e^{k_1 t} = \left(\frac{\Lambda_h + a_2 V_h}{k_1}\right) e^{k_1 t} + C \tag{3.156}$$

$$S_h(t) = \left(\frac{\Lambda_h + a_2 V_h}{k_1}\right) + C e^{-k_1 t} \tag{3.157}$$

When $t = 0$, the equation becomes,

$$C = S_h(0) - \left(\frac{\Lambda_h + a_2 V_h}{k_1}\right) \tag{3.158}$$

Substituting equation (3.158) into equation (3.156)

$$S_h(t) = \left(\frac{\Lambda_h + a_2 V_h}{k_1}\right) - \left(\frac{\Lambda_h + a_2 V_h}{k_1}\right) e^{-k_1 t} + S_h(0) e^{-k_1 t} \tag{3.159}$$

Solving for V_h gives

$$V_h(t) = \tau_1 S_h - k_2 V_h \tag{3.160}$$

$$V_h\,(t) + k_2 V_h = \tau_1 S_h \tag{3.161}$$

Multiplying by its integrating factor $e^{k_2 t}$

$$V_h\,(t)e^{k_2 t} + k_2 V_h e^{k_2 t} = \tau_1 S_h e^{k_2 t} \tag{3.162}$$

Integrating and diving by $e^{k_2 t}$ gives

$$\int \frac{d}{dt}\left[V_h e^{k_2 t}\right] = \int \tau_1 S_h e^{k_2 t} dt \tag{3.163}$$

$$V_h\, e^{k_2 t} = \left(\frac{\tau_1 S_h}{k_2}\right) e^{k_2 t} + C \tag{3.164}$$

$$V_h\,(t) = \left(\frac{\tau_1 S_h}{k_2}\right) + Ce^{-k_2 t} \tag{3.165}$$

When t = 0, the equation becomes,

$$C = V_h\,(0) - \left(\frac{\tau_1 S_h}{k_2}\right) \tag{3.166}$$

Substituting equation (3.166) into equation (3.164)

$$V_h\,(t) = \frac{\tau_1 S_h}{k_2} - \frac{\tau_1 S_h}{k_2} e^{-k_2 t} + V_h\,(0)e^{-k_2 t} \tag{3.167}$$

Solving for T_h gives

$$T_h\,(t) = T_h\,(0)e^{-k_6 t} \tag{3.168}$$

Solving for S_v gives

$$S_v\,(t) = \Lambda_v - k_7 S_v \tag{3.169}$$

$$S_v\,(t) + k_6 S_v = \Lambda_v \tag{3.170}$$

Multiplying by its integrating factor $e^{k_7 t}$

$$S_v(t)e^{k_7 t} + k_7 S_v e^{k_7 t} = \Lambda_v e^{k_7 t} \tag{3.171}$$

Integrating and diving by $e^{k_7 t}$ gives

$$\int \frac{d}{dt}\left[S_v e^{k_7 t}\right] = \int \Lambda_v e^{k_7 t} dt \tag{3.172}$$

$$S_v e^{k_7 t} = \left(\frac{\Lambda_v}{k_7}\right) e^{k_7 t} + C \tag{3.173}$$

$$S_v(t) = \left(\frac{\Lambda_v}{k_7}\right) + Ce^{-k_7 t} \tag{3.174}$$

When t = 0, the equation becomes,

$$C = S_v(0) - \left(\frac{\Lambda_v}{k_7}\right) \tag{3.175}$$

Substituting equation (3.175) into equation (3.173)

$$S_v(t) = \frac{\Lambda_v}{k_7} - \frac{\Lambda_v}{k_7} e^{-k_7 t} + S_v(0) e^{-k_7 t} \tag{3.176}$$

$$S_h^\circ(t) + V_h^\circ(t) + T_h^\circ(t) \rightarrow N_h^\circ(t) \text{ as } t \rightarrow 0$$

and

$$S_v^\circ(t) \rightarrow N_v^\circ(t) \text{ as } t \rightarrow 0$$

As $t \rightarrow \infty$

$$S_h^*(t) \rightarrow \frac{N_h^\circ(t) + \alpha_2 V_h}{k_1}$$

$$V_h^* \rightarrow \frac{\tau_1 S_h}{k_1}$$

$T_h^* \to 0$

$S_h^* \to \dfrac{\wedge_h}{\kappa_7}$

The asymptotic dynamics are independent of initial condition.

Hence, the solution is globally Asymptotically stable.

Also it is required to show that $G(X,Z)$ satisfied the two stated conditions

(i) $G(X,0)=0$ and

(ii) $G(X,Z)=D_z G(X^*,0)Z - G^*(X,Z), G^*(X,Z) \geq 0$

where

$$\left(X^*,0\right)=\left(\frac{\wedge_h k_2}{k_1k_2-\alpha_2\tau_1},\frac{\wedge_h \tau_1}{k_1k_2-\alpha_2\tau_1},0,0,0,0,0,\frac{\wedge_v}{k_7},0\right) \tag{3.177}$$

$D_z G(X^*,0)$ is the Jacobian of $G(X,Z)$ taken with respect to the infected classes evaluated at $\left(X^*,0\right)$

$$G(X,Z)=\begin{pmatrix} \dfrac{\beta_1\sigma_v(1-\theta)I_vS_h}{N_v}-\dfrac{\beta_1\sigma_v(1-\theta)I_vI_{hal}}{N_v}-k_3I_{hal} \\ \dfrac{\beta_1\sigma_v(1-\theta)I_vI_{hal}}{N_v}+\rho I_{hal}-k_1I_{hcl} \\ \dfrac{\beta_m\sigma_v(1-\theta)I_vS_h}{N_v}-\dfrac{\beta_m\sigma_vI_vV_h}{N_v}-k_4I_m \\ \dfrac{\beta_{lm}\sigma_v(1-\theta)I_vS_h}{N_v}-k_5I_{lm} \\ \dfrac{\beta_h\sigma_v\left(I_{hal}+I_{hcl}+I_m+I_{lm}\right)S_v}{N_h}-k_7I_v \end{pmatrix} \tag{3.178}$$

$$
D_z G\left(X^*,0\right)=\begin{pmatrix} -\frac{\beta_l\sigma_v(1-\theta)\mathrm{I}_v}{N_v}-k_3 & 0 & 0 & 0 & \frac{\beta_l\sigma_v(1-\theta)\mathrm{S}_h}{N_v}-\frac{\beta_l\sigma_v(1-\theta)\mathrm{I}_{hal}}{N_v} \\ \frac{\beta_l\sigma_v(1-\theta)\mathrm{I}_v}{N_v}+\rho & -k_1 & 0 & 0 & \frac{\beta_l\sigma_v(1-\theta)\mathrm{I}_{hal}}{N_v} \\ 0 & 0 & -k_4 & 0 & \frac{\beta_m\sigma_v(1-\theta)\mathrm{S}_h}{N_v}+\frac{\beta_m\sigma_v V_h}{N_v} \\ 0 & 0 & 0 & -k_5 & \frac{\beta_{lm}\sigma_v(1-\theta)\mathrm{S}_h}{N_v} \\ \frac{\beta_h\sigma_v S_v}{N_h} & \frac{\beta_h\sigma_v S_v}{N_h} & \frac{\beta_h\sigma_v S_v}{N_h} & \frac{\beta_h\sigma_v S_v}{N_h} & -k_7 \end{pmatrix} \qquad (3.179)
$$

$$
G^*\left(X,\mathrm{Z}\right)=\begin{pmatrix} \frac{\beta_1\sigma_v\left(1-\theta\right)I_v S_h^*}{N_v^*}\left(1-\frac{N_v^* S_h}{S_h^* N_v}\right)-\frac{\beta_1\sigma_v\left(1-\theta\right)I_v I_{hal}}{N_v} \\ \frac{\beta_1\sigma_v\left(1-\theta\right)I_v I_{hal}}{N_v} \\ 0 \\ 0 \\ \frac{\beta_1\sigma_v S_h^*}{N_h^*}\left(I_{hal}+I_{hcl}+I_m+I_{lm}\right)\left(1-\frac{N_v^* S_h}{S_h^* N_v}\right) \end{pmatrix} \qquad (3.180)
$$

Since we have

$$
S_v^*=\frac{\wedge_v}{k_7}, I_v=0, I_{hal}=0, \mathrm{N}_v=\frac{\wedge_v}{k_7}
$$

$$
\mathrm{N}_v=S_h+v_h, \mathrm{N}_h\le N_h^*, S_h=\frac{\wedge_h k_1}{k_1k_2-\alpha_2\tau_1}
$$

If the human population is at equilibrium, we have

$$
\left(1-\frac{N_v^* S_h}{S_h^* N_v}\right)>0, \left(1-\frac{N_v^* S_h}{S_h^* N_v}\right)>0
$$

Thus $G^*\left(X,\mathrm{Z}\right)\ge 0,$ therefore, the DFE is globally asymptotically stable.

3.8 Solution of the Model via Adomian Decomposition Method

3.8.1 Solution of the model equation using adomian decomposition method

Consider equation (3.2) through (3.10) with respect to and applying the initial condition

$S_h(0)=S_{h0}$, $V_h(0)=V_{h0}$, $I_{hal}(0)=I_{hal0}$, $I_{hcl}(0)=I_{hcl0}$, $I_m(0)=I_{m0}$, $I_{lm}(0)=I_{lm0}$,

$T_h(0)=T_{h0}$, $S_v(0)=S_{v0}$, $I_v(0)=I_{v0}$

Integrating both sides of (3.6) to (3.14) with respect to t and applying the initial conditions gives

$$S_h(t)=S_{h0}+\Lambda_h t-\frac{\sigma_v(1-\theta)\beta_m}{N_v}\int_0^t I_v S_h dt-\frac{\sigma_v(1-\theta)\beta_l}{N_v}\int_0^t I_v S_h dt-\frac{\sigma_v(1-\theta)\beta_{lm}}{N_v}\int_0^t I_v S_h dt$$

$$-(\tau_1+\mu_h)\int_0^t S_h dt+\alpha_1\int_0^t T_h dt+\alpha_2\int_0^t V_h dt \quad (3.181)$$

$$\tau_1\int_0^t S_h dt-(\mu_h+\alpha_2)\int_0^t V_h dt-\frac{\sigma_v\beta_m}{N_v}\int_0^t I_v V_h dt \quad (3.182)$$

$$I_{hal}(t)=\frac{\sigma_v(1-\theta)\beta_l}{N_v}\int_0^t I_v S_h dt-\frac{\sigma_v(1-\theta)\beta_l}{N_v}\int_0^t I_v I_{hal} dt-(\mu_h+\tau_1+\rho)\int_0^t I_{hal} dt+I_{hal0} \quad (3.183)$$

$$I_{hcl}(t)=\frac{\sigma_v(1-\theta)\beta_l}{N_v}\int_0^t I_v I_{hal} dt+\rho\int_0^t I_{hal} dt-(\mu_h+\tau_1)\int_0^t I_{hcl} dt+I_{hcl0} \quad (3.184)$$

$$I_m(t)=\frac{\sigma_v(1-\theta)\beta_m}{N_v}\int_0^t I_v S_h dt-(\tau_2+\delta_m+\mu_h)\int_0^t I_m dt+\frac{\sigma_v\beta_m}{N_v}\int_0^t I_v V_h dt+I_{m0} \quad (3.185)$$

$$I_{lm}(t)=\frac{\sigma_v(1-\theta)\beta_{lm}}{N_v}\int_0^t I_v S_h dt-(\tau_3+\delta_m+\mu_h)\int_0^t I_{lm} dt+I_{lm0} \quad (3.186)$$

$$T_h(t)=T_{h0}+\tau_2\int_0^t I_m dt+\tau_1\int_0^t I_{hal} dt+\tau_1\int_0^t I_{hcl} dt+\tau_3\int_0^t I_{lm} dt+$$
$$(\alpha_1+\mu_h)\int_0^t T_h dt \quad (3.187)$$

$$S_v(t) = S_{v0} + \Lambda_h t - \frac{\sigma_v \beta_h}{N_h}\int_0^t I_{hal} S_v dt - \frac{\sigma_v \beta_h}{N_h}\int_0^t I_{hcl} S_v dt - \frac{\sigma_v \beta_h}{N_h}\int_0^t I_m S_v dt$$

$$+ \frac{\sigma_v \beta_h}{N_h}\int_0^t I_{lm} S_v dt - (\mu_h + \delta_v)\int_0^t S_v dt \qquad (3.188)$$

$$I_v(t) = I_{v0} + \frac{\sigma_v \beta_h}{N_h}\int_0^t I_{hal} S_v dt + \frac{\sigma_v \beta_h}{N_h}\int_0^t I_{hcl} S_v dt + \frac{\sigma_v \beta_h}{N_h}\int_0^t I_m S_v dt + \frac{\sigma_v \beta_h}{N_h}\int_0^t I_{lm} S_v dt$$

$$-(\mu_v + \delta_v)\int_0^t I_v dt \qquad (3.189)$$

Using Adomian decomposition method, the solution of equation (3.181) through (3.189) are given as the series of the form

$$\begin{aligned}
&S_h = \sum_{n=0}^{\infty} S_{hn}, \quad V_h = \sum_{n=0}^{\infty} V_{hn}, \quad I_{hal} = \sum_{n=0}^{\infty} I_{haln} \\
&I_m = \sum_{n=0}^{\infty} I_{mn}, \quad I_{lm} = \sum_{n=0}^{\infty} I_{lmn}, \quad T_h = \sum_{n=0}^{\infty} T_{hn} \\
&S_v = \sum_{n=0}^{\infty} S_{vn}, \quad I_v = \sum_{n=0}^{\infty} I_{vn}
\end{aligned} \qquad (3.190)$$

And also, the integrands in equation (3.181) through (3.189) are expressed as

$$\begin{aligned}
&A = I_v S_h, \quad B = I_v V_h, \quad C = I_v I_{hal}, \quad D = I_{hcl} S_v, \\
&E = I_{hcl} S_v, \quad F = I_m S_v, \quad G = I_{lm} S_v, \quad J = I_{hal}, \\
&K = I_{hcl}, \quad L = I_m, \quad M = I_{lm}, \quad N = I_v, \\
&O = S_h, \quad P = S_v, \quad Q = T_h, \quad R = V_h,
\end{aligned} \qquad (3.191)$$

The linear and nonlinear operators in equation (3.191) are decomposed in series form as

$$\begin{bmatrix} A=\sum_{n=0}^{\infty}A_n, & B=\sum_{n=0}^{\infty}B_n, & C=\sum_{n=0}^{\infty}C_n, & D=\sum_{n=0}^{\infty}D_n, \\ E=\sum_{n=0}^{\infty}E_n, & F=\sum_{n=0}^{\infty}F_n, & G=\sum_{n=0}^{\infty}G_n, & J=\sum_{n=0}^{\infty}J_n, \\ K=\sum_{n=0}^{\infty}K_n, & L=\sum_{n=0}^{\infty}L_n, & M=\sum_{n=0}^{\infty}M_n, & N=\sum_{n=0}^{\infty}N_n, \\ O=\sum_{n=0}^{\infty}O_n, & P=\sum_{n=0}^{\infty}P_n, & Q=\sum_{n=0}^{\infty}Q_n, & R=\sum_{n=0}^{\infty}R_n, \end{bmatrix} \tag{3.192}$$

Where $A_n, B_n, C_n, D_n, E_n, F_n, G_n, J_n, K_n, L_n, M_n, N_n, O_n, P_n, Q_n$ are Adomian polynomials. Substituting equation (3.181) through (3.189) into equation (3.191) gives

$$\sum_{n=0}^{\infty}S_{hn} = S_{h0} + \Lambda_h t - \frac{\sigma_v(1-\theta)\beta_m}{N_v}\int_0^t \sum_{n=0}^{\infty}A_n dt - \frac{\sigma_v(1-\theta)\beta_l}{N_v}\int_0^t \sum_{n=0}^{\infty}A_n dt - \frac{\sigma_v(1-\theta)\beta_{lm}}{N_v}\int_0^t \sum_{n=0}^{\infty}A_n dt$$

$$-(\tau_1+\mu_h)\int_0^t \sum_{n=0}^{\infty}O_n dt + \alpha_1\int_0^t \sum_{n=0}^{\infty}Q_n dt + \alpha_2\int_0^t \sum_{n=0}^{\infty}R_n dt \tag{3.193}$$

$$\sum_{n=0}^{\infty}V_{hn} = V_{h0} + \tau_1\int_0^t \sum_{n=0}^{\infty}O_n dt - (\mu_h+\alpha_2)\int_0^t \sum_{n=0}^{\infty}R_n dt - \frac{\sigma_v\beta_m}{N_v}\int_0^t \sum_{n=0}^{\infty}B_n dt \tag{3.194}$$

$$\sum_{n=0}^{\infty}I_{haln} = I_{hal0} + \frac{\sigma_v(1-\theta)\beta_l}{N_v}\int_0^t \sum_{n=0}^{\infty}A_n dt - \frac{\sigma_v(1-\theta)\beta_l}{N_v}\int_0^t \sum_{n=0}^{\infty}C_n dt$$

$$-(\mu_h+\tau_1+\rho)\int_0^t \sum_{n=0}^{\infty}J_n dt \tag{3.195}$$

$$\sum_{n=0}^{\infty}I_{hcln} = I_{hcl0} + \frac{\sigma_v(1-\theta)\beta_l}{N_v}\int_0^t \sum_{n=0}^{\infty}C_n dt + \rho\int_0^t \sum_{n=0}^{\infty}J_n dt - (\mu_h+\tau_1)\int_0^t \sum_{n=0}^{\infty}K_n dt \tag{3.196}$$

$$\sum_{n=0}^{\infty}I_{mn} = I_{m0} + \frac{\sigma_v(1-\theta)\beta_m}{N_v}\int_0^t \sum_{n=0}^{\infty}A_n dt - (\tau_2+\delta_m+\mu_h)\int_0^t \sum_{n=0}^{\infty}L_n dt$$

$$+\frac{\sigma_v\beta_m}{N_v}\int_0^t \sum_{n=0}^{\infty}B_n dt \tag{3.197}$$

$$\sum_{n=0}^{\infty}I_{lmn} = I_{lm0} + \frac{\sigma_v(1-\theta)\beta_{lm}}{N_v}\int_0^t \sum_{n=0}^{\infty}A_n dt - (\tau_3+\delta_m+\mu_h)\int_0^t \sum_{n=0}^{\infty}M_n dt \tag{3.198}$$

$$\sum_{n=0}^{\infty} T_{hn} = T_{h0} + \tau_2 \int_0^t \sum_{n=0}^{\infty} L_n dt + \tau_1 \int_0^t \sum_{n=0}^{\infty} J_n dt + \tau_1 \int_0^t \sum_{n=0}^{\infty} K_n dt + \tau_3 \int_0^t \sum_{n=0}^{\infty} M_n dt$$

$$+(\alpha_1 + \mu_h) \int_0^t \sum_{n=0}^{\infty} Q_n d\chi \qquad (3.199)$$

$$\sum_{n=0}^{\infty} S_{vn} = S_{v0} + \Lambda_h t - \frac{\sigma_v \beta_h}{N_h} \int_0^t \sum_{n=0}^{\infty} D_n dt - \frac{\sigma_v \beta_h}{N_h} \int_0^t \sum_{n=0}^{\infty} E_n dt - \frac{\sigma_v \beta_h}{N_h} \int_0^t \sum_{n=0}^{\infty} F_n dt$$

$$+\frac{\sigma_v \beta_h}{N_h} \int_0^t \sum_{n=0}^{\infty} G_n dt - (\mu_h + \delta_v) \int_0^t \sum_{n=0}^{\infty} P_n dt \qquad (3.200)$$

$$\sum_{n=0}^{\infty} I_{vn} = I_{v0} + \frac{\sigma_v \beta_h}{N_h} \int_0^t \sum_{n=0}^{\infty} D_n dt + \frac{\sigma_v \beta_h}{N_h} \int_0^t \sum_{n=0}^{\infty} E_n dt + \frac{\sigma_v \beta_h}{N_h} \int_0^t \sum_{n=0}^{\infty} F_n dt + \frac{\sigma_v \beta_h}{N_h} \int_0^t \sum_{n=0}^{\infty} G_n dt$$

$$-(\mu_v + \delta_v) \int_0^t \sum_{n=0}^{\infty} N_n dt \qquad (3.201)$$

Equation (3.193) through (3.201) can be written as

$$\sum_{n=0}^{\infty} S_{hn} = S_{h0} + \Lambda_h t - \frac{\sigma_v (1-\theta)}{N_v} (\beta_m + \beta_l + \beta_{lm}) \sum_{n=0}^{\infty} \int_0^t A_n dt - (\tau_1 + \mu_h) \sum_{n=0}^{\infty} \int_0^t O_n dt$$

$$+\alpha_1 \sum_{n=0}^{\infty} \int_0^t Q_n d\chi + \alpha_2 \sum_{n=0}^{\infty} \int_0^t R_n d\chi \qquad (3.202)$$

$$\sum_{n=0}^{\infty} V_{hn} = V_{h0} + \tau_1 \sum_{n=0}^{\infty} \int_0^t O_n dt - (\mu_h + \alpha_2) \sum_{n=0}^{\infty} \int_0^t R_n dt - \frac{\sigma_v \beta_m}{N_v} \sum_{n=0}^{\infty} \int_0^t B_n dt \qquad (3.203)$$

$$\sum_{n=0}^{\infty} I_{haln} = I_{hal0} + \frac{\sigma_v (1-\theta) \beta_l}{N_v} \sum_{n=0}^{\infty} \int_0^t A_n dt - \frac{\sigma_v (1-\theta) \beta_l}{N_v} \sum_{n=0}^{\infty} \int_0^t C_n dt$$

$$-(\mu_h + \tau_1 + \rho) \sum_{n=0}^{\infty} \int_0^t J_n dt \qquad (3.204)$$

$$\sum_{n=0}^{\infty} I_{hcln} = I_{hcl0} + \frac{\sigma_v (1-\theta) \beta_l}{N_v} \sum_{n=0}^{\infty} \int_0^t C_n dt + \rho \sum_{n=0}^{\infty} \int_0^t J_n dt - (\mu_h + \tau_1) \sum_{n=0}^{\infty} \int_0^t K_n dt \qquad (3.205)$$

$$\sum_{n=0}^{\infty} I_{mn} = I_{m0} + \frac{\sigma_v (1-\theta)\beta_m}{N_v} \sum_{n=0}^{\infty} \int_0^t A_n dt - (\tau_2 + \delta_m + \mu_h) \sum_{n=0}^{\infty} \int_0^t L_n dt$$

$$+ \frac{\sigma_v \beta_m}{N_v} \sum_{n=0}^{\infty} \int_0^t B_n dt \tag{3.206}$$

$$\sum_{n=0}^{\infty} I_{lmn} = I_{lm0} + \frac{\sigma_v (1-\theta)\beta_{lm}}{N_v} \sum_{n=0}^{\infty} \int_0^t A_n dt - (\tau_3 + \delta_m + \mu_h) \sum_{n=0}^{\infty} \int_0^t M_n dt \tag{3.207}$$

$$\sum_{n=0}^{\infty} T_{hn} = T_{h0} + \tau_2 \sum_{n=0}^{\infty} \int_0^t L_n dt + \tau_1 \sum_{n=0}^{\infty} \int_0^t J_n dt + \tau_1 \sum_{n=0}^{\infty} \int_0^t K_n dt + \tau_3 \sum_{n=0}^{\infty} \int_0^t M_n dt$$

$$-(\alpha_1 + \mu_h) \sum_{n=0}^{\infty} \int_0^t Q_n dt \tag{3.208}$$

$$\sum_{n=0}^{\infty} S_{vn} = S_{v0} + \Lambda_h t - \frac{\sigma_v \beta_h}{N_h} \sum_{n=0}^{\infty} \int_0^t D_n dt - \frac{\sigma_v \beta_h}{N_h} \sum_{n=0}^{\infty} \int_0^t E_n dt - \frac{\sigma_v \beta_h}{N_h} \sum_{n=0}^{\infty} \int_0^t F_n dt$$

$$- \frac{\sigma_v \beta_h}{N_h} \sum_{n=0}^{\infty} \int_0^t G_n dt - (\mu_h + \delta_v) \sum_{n=0}^{\infty} \int_0^t P_n dt \tag{3.209}$$

$$\sum_{n=0}^{\infty} I_{vn} = I_{v0} + \frac{\sigma_v \beta_h}{N_h} \sum_{n=0}^{\infty} \int_0^t D_n dt + \frac{\sigma_v \beta_h}{N_h} \sum_{n=0}^{\infty} \int_0^t E_n dt + \frac{\sigma_v \beta_h}{N_h} \sum_{n=0}^{\infty} \int_0^t F_n dt + \frac{\sigma_v \beta_h}{N_h} \sum_{n=0}^{\infty} \int_0^t G_n dt$$

$$-(\mu_v + \delta_v) \sum_{n=0}^{\infty} \int_0^t N_n dt \tag{3.210}$$

From equation (3.202) through (3.210) we define the following scheme

$$\left.\begin{array}{ll} S_{h0}=S_{h0}+\Lambda_h t & T_{h0}=T_{h0} \\ I_{hal0}=I_{hal0} & S_{v0}=S_{v0}+\Lambda_h t \\ I_{hcl0}=I_{hcl0} & I_{v0}=I_{v0} \\ I_{m0}=I_{m0} & V_{h0}=V_{h0} \\ I_{lm0}=I_{lm0} & \end{array}\right\} \tag{3.211}$$

$$S_{hn+1}=-\frac{\sigma_v(1-\theta)}{N_v}(\beta_m+\beta_l+\beta_{lm})\int_0^t A_n dt-(\tau_1+\mu_h)\int_0^t O_n dt+\alpha_1\int_0^t Q_n dt$$
$$+\alpha_2\int_0^t R_n dt \tag{3.212}$$

$$V_{hn+1}=\tau_1\int_0^t O_n dt-(\mu_h+\alpha_2)\int_0^t R_n dt-\frac{\sigma_v\beta_m}{N_v}\int_0^t B_n dt \tag{3.213}$$

$$I_{haln+1}=\frac{\sigma_v(1-\theta)\beta_l}{N_v}\int_0^t A_n dt-\frac{\sigma_v(1-\theta)\beta_l}{N_v}\int_0^t C_n dt-(\mu_h+\tau_1+\rho)\int_0^t J_n dt \tag{3.214}$$

$$I_{hcln+1}=\frac{\sigma_v(1-\theta)\beta_l}{N_v}\int_0^t C_n dt+\rho\int_0^t J_n dt-(\mu_h+\tau_1)\int_0^t K_n dt \tag{3.215}$$

$$I_{mn+1}=\frac{\sigma_v(1-\theta)\beta_m}{N_v}\int_0^t A_n dt-(\tau_2+\delta_m+\mu_h)\int_0^t L_n dt+\frac{\sigma_v\beta_m}{N_v}\int_0^t B_n dt \tag{3.216}$$

$$I_{lmn+1}=\frac{\sigma_v(1-\theta)\beta_{lm}}{N_v}\int_0^t A_n dt-(\tau_3+\delta_m+\mu_h)\int_0^t M_n dt \tag{3.217}$$

$$T_{hn+1}=\tau_2\int_0^t L_n dt+\tau_1\int_0^t J_n dt+\tau_1\int_0^t K_n dt+\tau_3\int_0^t M_n dt-(\alpha_1+\mu_h)\int_0^t Q_n dt \tag{3.218}$$

$$S_{vn+1}=-\frac{\sigma_v\beta_h}{N_h}\int_0^t D_n dt-\frac{\sigma_v\beta_h}{N_h}\int_0^t E_n dt-\frac{\sigma_v\beta_h}{N_h}\int_0^t F_n dt-\frac{\sigma_v\beta_h}{N_h}\int_0^t G_n dt$$
$$-(\mu_h+\delta_v)\int_0^t P_n dt \tag{3.219}$$

$$I_{vn+1} = \frac{\sigma_v \beta_h}{N_h} \int_0^t D_n dt + \frac{\sigma_v \beta_h}{N_h} \int_0^t E_n dt + \frac{\sigma_v \beta_h}{N_h} \int_0^t F_n dt + \frac{\sigma_v \beta_h}{N_h} \int_0^t G_n dt$$

$$-(\mu_v + \delta_v) \int_0^t N_n dt \qquad (3.220)$$

Using the Algorithm in (3.191), the Adomian polynomials in (3.211) are computed as

$$\left. \begin{aligned} A_0 &= I_{v0} S_{ho} \\ A_1 &= I_{v0} S_{h1} + I_{v1} S_{h0} \\ A_2 &= I_{v0} S_{h2} + I_{v1} S_{h1} + I_{v2} S_{ho} \end{aligned} \right\} \qquad (3.221)$$

$$\left. \begin{aligned} B_0 &= I_{v0} V_{ho} \\ B_1 &= I_{v0} V_{h1} + I_{v1} V_{ho} \\ B_2 &= I_{v0} V_{h2} + I_{v1} V_{h1} + I_{v2} V_{ho} \end{aligned} \right\} \qquad (3.222)$$

$$\left. \begin{aligned} C_0 &= I_{v0} I_{halo} \\ C_1 &= I_{v0} I_{hal1} + I_{v1} I_{hal0} \\ C_2 &= I_{v0} I_{hal2} + I_{v1} I_{hal1} + I_{v2} I_{halo} \end{aligned} \right\} \qquad (3.223)$$

$$\left. \begin{aligned} D_0 &= I_{hal0} S_{vo} \\ D_1 &= I_{hal0} S_{v1} + I_{hal1} S_{vo} \\ D_2 &= I_{hal0} S_{v2} + I_{hal1} S_{v1} + I_{hal2} S_{vo} \end{aligned} \right\} \qquad (3.224)$$

$$\left. \begin{aligned} E_0 &= I_{hcl0} S_{vo} \\ E_1 &= I_{hcl0} S_{v1} + I_{hcl1} S_{vo} \\ E_2 &= I_{hal0} S_{v2} + I_{hcl1} S_{v1} + I_{hcl2} S_{vo} \end{aligned} \right\} \qquad (3.225)$$

$$\left. \begin{aligned} F_0 &= I_{m0} S_{vo} \\ F_1 &= I_{m0} S_{v1} + I_{m1} S_{vo} \\ F_2 &= I_{m0} S_{v2} + I_{m1} S_{v1} + I_{m2} S_{vo} \end{aligned} \right\} \qquad (3.226)$$

$$\left.\begin{aligned} G_0 &= I_{lm0}S_{vo} \\ G_1 &= I_{lm0}S_{v1} + I_{lm1}S_{vo} \\ G_2 &= I_{lm0}S_{v2} + I_{lm1}S_{v1} + I_{lm2}S_{vo} \end{aligned}\right\} \quad (3.227)$$

$$\left.\begin{aligned} J_0 &= I_{hal0} \\ J_1 &= I_{hal1} \\ J_2 &= I_{hal2} \end{aligned}\right\} \quad (3.228)$$

$$\left.\begin{aligned} K_0 &= I_{hcl0} \\ K_1 &= I_{hcl1} \\ K_2 &= I_{hcl2} \end{aligned}\right\} \quad (3.229)$$

$$\left.\begin{aligned} L_0 &= I_{m0} \\ L_1 &= I_{m1} \\ L_2 &= I_{m2} \end{aligned}\right\} \quad (3.230)$$

$$\left.\begin{aligned} M_0 &= I_{lm0} \\ M_1 &= I_{lm1} \\ M_2 &= I_{lm2} \end{aligned}\right\} \quad (3.231)$$

$$\left.\begin{aligned} N_0 &= I_{v0} \\ N_1 &= I_{v1} \\ N_2 &= I_{v2} \end{aligned}\right\} \quad (3.232)$$

$$\left.\begin{aligned} O_0 &= S_{h0} \\ O_1 &= S_{h1} \\ O_2 &= S_{h2} \end{aligned}\right\} \quad (3.233)$$

$$\left.\begin{aligned} P_0 &= S_{v0} \\ P_1 &= S_{v1} \\ P_2 &= S_{v2} \end{aligned}\right\} \quad (3.234)$$

$$\left.\begin{aligned} Q_0 &= T_{h0} \\ Q_1 &= T_{h1} \\ Q_2 &= T_{h2} \end{aligned}\right\} \tag{3.235}$$

$$\left.\begin{aligned} R_0 &= V_{h0} \\ R_1 &= V_{h1} \\ R_2 &= V_{h2} \end{aligned}\right\} \tag{3.236}$$

For $n=0$, equation (3.212) gives

$$S_{h1} = -\frac{\sigma_v(1-\theta)}{N_v}(\beta_m + \beta_l + \beta_{lm})\int_0^t A_0 dt - (\tau_1 + \mu_h)\int_0^t O_0 dt + \alpha_1 \int_0^t Q_0 dt$$

$$+\alpha_2 \int_0^t R_0 dt \tag{4.237}$$

Substituting equation (3.221) through (3.236) into equation (3.237) gives

$$S_{h1} = -\frac{\sigma_v(1-\theta)}{N_v}(\beta_m + \beta_l + \beta_{lm})\int_0^t I_{v0} S_{h0} dt - (\tau_1 + \mu_h)\int_0^t S_{h0} dt + \alpha_1 \int_0^t T_{h0} dt$$
$$+\alpha_2 \int_0^t V_{h0} dt \tag{3.238}$$

Substituting equation (3.211) into equation (3.237) gives

$$S_{h1} = -\frac{\sigma_v(1-\theta)}{N_v}(\beta_m + \beta_l + \beta_{lm})\int_0^t (S_{h0} + \Lambda_h t) I_{v0} dt - (\tau_1 + \mu_h)\int_0^t (S_{h0} + \Lambda_h t) dt + \alpha_1 \int_0^t T_{h0} dt$$

$$+\alpha_2 \int_0^t V_{h0} dt \tag{3.239}$$

$$S_{h1} = -\frac{\sigma_v(1-\theta)}{N_v}(\beta_m + \beta_l + \beta_{lm})\int_0^t (S_{h0} I_{v0} + \Lambda_h t I_{v0}) dt - (\tau_1 + \mu_h)\int_0^t (S_{h0} + \Lambda_h t) dt + \alpha_1 \int_0^t T_{h0} dt$$

$$+\alpha_2 \int_0^t V_{h0} dt \tag{3.240}$$

$$= -\frac{\sigma_v(1-\theta)}{N_v}(\beta_m+\beta_l+\beta_{lm})\int_0^t (S_{h0}I_{v0}+\Lambda_h t I_{v0})dt-(\tau_1+\mu_h)\int_0^t (S_{h0}+\Lambda_h t)dt+\alpha_1\int_0^t T_{h0}dt$$

$$+\alpha_2\int_0^t V_{h0}dt \qquad (3.241)$$

Integrating and collecting like terms gives

$$S_{h1}=\left(-\frac{\sigma_v(1-\theta)}{N_v}(\beta_m+\beta_l+\beta_{lm})S_{h0}I_{v0}-(\tau_1+\mu_h)S_{h0}+\alpha_1 T_{h0}+V_{h0}\right)t$$

$$-\frac{\Lambda_h}{2}\left(\frac{\sigma_v(1-\theta)}{N_v}(\beta_m+\beta_l+\beta_{lm})-(\tau_1+\mu_h)\right)t^2 \qquad (3.242)$$

for $n=0$ in equation (3.213) gives

$$V_{h1}=\tau_1\int_0^t 0_0 dt-(\mu_h+\alpha_2)\int_0^t R_0 dt-\frac{\beta_m\sigma_v}{N_v}\int_0^t \beta_0 dt \qquad (3.243)$$

$$=\tau_1\int_0^t S_{h0}dt-(\mu_h+\alpha_2)\int_0^t V_{h0}dt-\frac{\beta_m\sigma_v}{N_v}\int_0^t I_{v0}V_{h0}dt \qquad (3.244)$$

$$=\tau_1\int_0^t (S_{h0}+\Lambda_h t)dt-(\mu_h+\alpha_2)\int_0^t V_{h0}dt-\frac{\beta_m\sigma_v}{N_v}\int_0^t I_{v0}V_{h0}dt \qquad (3.245)$$

Integrating and collecting like terms gives

$$V_{h1}=\left(\tau_1 S_{h0}-(\mu_h+\alpha_2)V_{h0}-\frac{\beta_m\sigma_v}{N_v}I_{v0}V_{h0}\right)t-\frac{\tau_1\Lambda_h t^2}{2} \qquad (3.246)$$

for $n=0$ in equation (3.214) gives

$$I_{hal1}=\frac{\beta_l\sigma_v(1-\theta)}{N_v}\int_0^t A_0 dt-\frac{\beta_l\sigma_v(1-\theta)}{N_v}\int_0^t C_0 dt-(\mu_h+\tau_1+\rho)\int_0^t J_0 dt \qquad (3.247)$$

$$=\frac{\beta_l\sigma_v\left(1-\theta\right)}{N_v}\int_0^t I_{v0}S_{h0}dt-\frac{\beta_l\sigma_v\left(1-\theta\right)}{N_v}\int_0^t I_{v0}I_{hal0}dt-\left(\mu_h+\tau_1+\rho\right)\int_0^t I_{hal0}dt \quad (3.248)$$

$$=\frac{\beta_l\sigma_v\left(1-\theta\right)}{N_v}\int_0^t\left(S_{h0}+\Lambda_h t\right)I_{v0}dt-\frac{\beta_l\sigma_v\left(1-\theta\right)}{N_v}\int_0^t I_{v0}I_{hal0}dt-\left(\mu_h+\tau_1+\rho\right)\int_0^t I_{hal0}dt \quad (3.\ 249)$$

$$I_{hal1}=\left(\frac{\beta_l\sigma_v\left(1-\theta\right)}{N_v}S_{h0}I_{v0}-\frac{\beta_l\sigma_v\left(1-\theta\right)}{N_v}I_{v0}I_{hal0}-\left(\mu_h+\tau_1+\rho\right)I_{hal0}\right)t$$

$$+\frac{\beta_l\sigma_v\left(1-\theta\right)}{N_v}\left(\frac{\Lambda_h I_{v0}}{2}\right)t^2 \quad (3.250)$$

for $n=0$ in equation (3.215) gives

$$I_{hcl1}=\frac{\beta_l\sigma_v\left(1-\theta\right)}{N_v}\int_0^t C_0dt+\rho\int_0^t J_0dt-\left(\mu_h+\tau_1\right)\int_0^t K_0dt \quad (3.251)$$

$$=\frac{\beta_l\sigma_v\left(1-\theta\right)}{N_v}\int_0^t I_{v0}I_{hal0}dt+\rho\int_0^t I_{hal0}dt-\left(\mu_h+\tau_1\right)\int_0^t I_{hcl0}dt \quad (3.252)$$

Integrating and collecting like terms gives

$$I_{hcl1}=\left(\frac{\beta_l\sigma_v\left(1-\theta\right)}{N_v}\left(I_{v0}I_{hal0}+\frac{\rho N_v I_{hal0}}{\beta_l\sigma_v\left(1-\theta\right)}-\frac{I_{hcl0}N_v\left(\mu_h+\tau_1\right)}{\beta_l\sigma_v\left(1-\theta\right)}\right)\right)t \quad (3.253)$$

for $n=0$ in equation (3.216) gives

$$I_{m1}=\frac{\beta_m\sigma_v\left(1-\theta\right)}{N_v}\int_0^t A_0dt-\left(\tau_2+\mu_h+\delta_m\right)\int_0^t L_0dt+\frac{\beta_m\sigma_v}{N_v}\int_0^t B_0dt \quad (3.254)$$

$$=\frac{\beta_m\sigma_v\left(1-\theta\right)}{N_v}\int_0^t I_{v0}S_{h0}dt-\left(\tau_2+\mu_h+\delta_m\right)\int_0^t I_{m0}dt+\frac{\beta_m\sigma_v}{N_v}\int_0^t I_{v0}V_{h0}dt \quad (3.255)$$

$$=\frac{\beta_m\sigma_v(1-\theta)}{N_v}\int_0^t(S_{h0}+\Lambda_h t)I_{v0}dt-(\tau_2+\mu_h+\delta_m)\int_0^t I_{m0}dt+\frac{\beta_m\sigma_v}{N_v}\int_0^t I_{v0}V_{h0}dt \quad (3.256)$$

Integrating and collecting like terms gives;

$$I_{m1}=\frac{\beta_m\sigma_v(1-\theta)}{N_v}\left(S_{h0}I_{v0}-\frac{(\tau_2+\mu_h+\delta_m)N_vI_{m0}}{\beta_m\sigma_v(1-\theta)}+\frac{I_{v0}V_{h0}}{(1-\theta)}\right)t$$

$$+\left(\frac{\beta_m\sigma_v(1-\theta)}{N_v}\frac{I_{v0}\Lambda_h}{2}\right)t^2 \quad (3.257)$$

for $n=0$ in equation (3.217) gives

$$I_{lm1}=\frac{\beta_{lm}\sigma_v(1-\theta)}{N_v}\int_0^t A_0dt-(\tau_3+\mu_h+\delta_m)\int_0^t M_0dt \quad (3.258)$$

$$=\frac{\beta_{lm}\sigma_v(1-\theta)}{N_v}\int_0^t I_{v0}S_{h0}dt-(\tau_3+\mu_h+\delta_m)\int_0^t I_{lm0}dt \quad (3.259)$$

$$=\frac{\beta_{lm}\sigma_v(1-\theta)}{N_v}\int_0^t(S_{h0}+\Lambda_h t)I_{v0}dt-(\tau_3+\mu_h+\delta_m)\int_0^t I_{lm0}dt \quad (3.260)$$

Integrating and collecting like terms gives

$$I_{lm1}=\frac{\beta_{lm}\sigma_v(1-\theta)}{N_v}\left(S_{h0}I_{v0}-\frac{(\tau_3+\mu_h+\delta_m)N_vI_{lm0}}{\beta_{lm}\sigma_v(1-\theta)}\right)t+\frac{\beta_{lm}\sigma_v(1-\theta)\Lambda_h t^2}{2N_v} \quad (3.261)$$

for $n=0$ in equation (3.218) gives

$$T_{h1}=\tau_2\int_0^t L_0dt+\tau_1\int_0^t J_0dt+\tau_1\int_0^t K_0dt+\tau_3\int_0^t M_0dt-(\alpha_1+\mu_h)\int_0^t Q_0dt \quad (3.262)$$

$$=\tau_2\int_0^t I_{m0}dt+\tau_1\int_0^t I_{hal0}dt+\tau_1\int_0^t I_{hcl0}dt+\tau_3\int_0^t I_{lm0}dt-(\alpha_1+\mu_h)\int_0^t T_{h0}dt \quad (3.263)$$

Integrating and collecting like terms gives

$$T_{h1} = \left(\tau_2 I_{m0} + \tau_1 \left(I_{hal0} + I_{hcl0}\right) + \tau_3 I_{lm0} - \left(\alpha_1 + \mu_h\right) T_{h0}\right) t \quad (3.264)$$

for $n = 0$ in equation (3.219) gives

$$S_{v1} = -\frac{\beta_h \sigma_v}{N_h} \int_0^t D_0 dt - \frac{\beta_h \sigma_v}{N_h} \int_0^t E_0 dt - \frac{\beta_h \sigma_v}{N_h} \int_0^t F_0 dt - \frac{\beta_h \sigma_v}{N_h} \int_0^t G_0 dt - \left(\mu_h + \delta_v\right) \int_0^t P_0 dt \quad (3.265)$$

$$= -\frac{\beta_h \sigma_v}{N_h} \int_0^t I_{hal0} S_{v0} dt - \frac{\beta_h \sigma_v}{N_h} \int_0^t I_{hcl0} S_{v0} dt - \frac{\beta_h \sigma_v}{N_h} \int_0^t I_{m0} S_{v0} dt - \frac{\beta_h \sigma_v}{N_h} \int_0^t I_{lm0} S_{v0} dt - \left(\mu_h + \delta_v\right) \int_0^t S_{v0} dt \quad (3.266)$$

$$= -\frac{\beta_h \sigma_v}{N_h} \int_0^t \left(S_{v0} + \Lambda_v t\right) I_{hal0} dt - \frac{\beta_h \sigma_v}{N_h} \int_0^t \left(S_{v0} + \Lambda_v t\right) I_{hcl0} dt - \frac{\beta_h \sigma_v}{N_h} \int_0^t \left(S_{v0} + \Lambda_v t\right) I_{m0} dt \quad (3.267)$$

$$-\frac{\beta_h \sigma_v}{N_h} \int_0^t \left(S_{v0} + \Lambda_v t\right) I_{lm0} dt - \left(\mu_v + \delta_m\right) \int_0^t \left(S_{v0} + \Lambda_v t\right) dt \quad (3.268)$$

$$S_{v1} = -\frac{\beta_h \sigma_v}{N_h} \left(S_{v0} I_{hal0} + S_{v0} I_{hcl0} + S_{v0} I_{m0} + S_{v0} I_{lm0} + \left(\mu_v + \delta_v\right) S_{v0}\right) t \quad (3.269)$$

$$-\frac{\beta_h \sigma_v}{2N_h} \left(\Lambda_v I_{hal0} + \Lambda_v I_{hcl0} + \Lambda_v I_{m0} + \Lambda_v I_{lm0} + \frac{2\left(\mu_v + \delta_m\right) N_h \Lambda_v}{\beta_h \sigma_v}\right) t^2 \quad (3.270)$$

for $n = 0$ in equation (3.220) gives

$$I_{v1} = \frac{\beta_h \sigma_v}{N_h} \int_0^t D_0 dt + \frac{\beta_h \sigma_v}{N_h} \int_0^t E_0 dt + \frac{\beta_h \sigma_v}{N_h} \int_0^t F_0 dt + \frac{\beta_h \sigma_v}{N_h} \int_0^t G_0 dt - \left(\mu_v + \delta_v\right) \int_0^t N_0 dt \quad (3.271)$$

$$= \frac{\beta_h \sigma_v}{N_h} \int_0^t I_{hal0} S_{v0} dt + \frac{\beta_h \sigma_v}{N_h} \int_0^t I_{hcl0} S_{v0} dt + \frac{\beta_h \sigma_v}{N_h} \int_0^t I_{m0} S_{v0} dt + \frac{\beta_h \sigma_v}{N_h} \int_0^t I_{lm0} S_{v0} dt - \left(\mu_v + \delta_m\right) \int_0^t I_{v0} dt \quad (3.272)$$

$$= \frac{\beta_h \sigma_v}{N_h} \int_0^t \left(S_{v0} + \Lambda_v t\right) I_{hal0} dt + \frac{\beta_h \sigma_v}{N_h} \int_0^t \left(S_{v0} + \Lambda_v t\right) I_{hcl0} dt + \frac{\beta_h \sigma_v}{N_h} \int_0^t \left(S_{v0} + \Lambda_v t\right) I_{m0} dt$$

$$+ \frac{\beta_h \sigma_v}{N_h} \int_0^t \left(S_{v0} + \Lambda_v t\right) I_{lm0} dt - \left(\mu_v + \delta_m\right) \int_0^t I_{v0} dt \quad (3.273)$$

$$I_{v1} = \frac{\beta_h \sigma_v}{N_h}\left(S_{v0} I_{hal0} + S_{v0} I_{hcl0} + S_{v0} I_{m0} + S_{v0} I_{lm0} - \frac{\left(\mu_v + \delta_m\right) N_h I_{v0}}{\beta_h \sigma_v} \right) t$$

$$+ \frac{\beta_h \sigma_v}{2 N_h}\left(\Lambda_v I_{hal0} + \Lambda_v I_{hcl0} + \Lambda_v I_{m0} + \Lambda_v I_{lm0} + \right) t^2 \qquad (3.274)$$

for $n=1$ in equation (3.212) gives

$$S_{h2} = -\frac{\sigma_v (1-\theta)}{N_v}\left(\beta_m + \beta_l + \beta_{lm}\right)\int_0^t A_1 dt - \left(\tau_1 + \mu_h\right)\int_0^t O_1 dt + \alpha_1 \int_0^t Q_1 dt + \alpha_2 \int_0^t R_1 dt \qquad (3.275)$$

$$S_{h2} = -\frac{\sigma_v (1-\theta)}{N_v}\left(\beta_m + \beta_l + \beta_{lm}\right)\int_0^t \left(I_{v0} S_{h1} + I_{v1} S_{h0}\right) dt - \left(\tau_1 + \mu_h\right)\int_0^t S_{h1} dt$$

$$+ \alpha_1 \int_0^t T_{h1} dt + \alpha_2 \int_0^t V_{h1} dt \qquad (3.276)$$

$$\begin{aligned}
\Bigg(& -\frac{1}{N_v}\Big(\sigma_v (1-\theta)\, \beta_m \Big(\frac{1}{3}\, I_{v0} \Big(-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_m I_{v0} \Lambda_h}{N_v} - \frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l I_{v0} \Lambda_h}{N_v} \\
& - \frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm} I_{v0} \Lambda_h}{N_v} - \frac{1}{2}\,(\mu_h + \tau_1)\,\Lambda_h \Big) + \frac{1}{3}\Big(\frac{\beta_h \sigma_v I_{hal0} S_{v0}}{N_h} \\
& + \frac{\beta_h \sigma_v I_{hcl0} S_{v0}}{N_h} + \frac{\beta_h \sigma_v I_{m0} S_{v0}}{N_h} + \frac{\beta_h \sigma_v I_{lm0} S_{v0}}{N_h} - (\mu_v + \delta_v)\, I_{v0} \Big) \Lambda_h \\
& + \frac{1}{3}\Big(\frac{1}{2}\,\frac{\beta_h \sigma_v I_{hal0} \Lambda_v}{N_h} + \frac{1}{2}\,\frac{\beta_h \sigma_v I_{hcl0} \Lambda_v}{N_h} + \frac{1}{2}\,\frac{\beta_h \sigma_v I_{m0} \Lambda_v}{N_h} \\
& + \frac{1}{2}\,\frac{\beta_h \sigma_v I_{lm0} \Lambda_v}{N_h} \Big) S_{h0} \Big) \Big) - \frac{1}{N_v}\Big(\sigma_v (1-\theta)\, \beta_l \Big(\frac{1}{3}\, I_{v0} \Big(\\
& -\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_m I_{v0} \Lambda_h}{N_v} - \frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l I_{v0} \Lambda_h}{N_v} - \frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm} I_{v0} \Lambda_h}{N_v} \\
& - \frac{1}{2}\,(\mu_h + \tau_1)\,\Lambda_h \Big) + \frac{1}{3}\Big(\frac{\beta_h \sigma_v I_{hal0} S_{v0}}{N_h} + \frac{\beta_h \sigma_v I_{hcl0} S_{v0}}{N_h} + \frac{\beta_h \sigma_v I_{m0} S_{v0}}{N_h} \\
& + \frac{\beta_h \sigma_v I_{lm0} S_{v0}}{N_h} - (\mu_v + \delta_v)\, I_{v0} \Big) \Lambda_h + \frac{1}{3}\Big(\frac{1}{2}\,\frac{\beta_h \sigma_v I_{hal0} \Lambda_v}{N_h} + \frac{1}{2}\,\frac{\beta_h \sigma_v I_{hcl0} \Lambda_v}{N_h} \\
& + \frac{1}{2}\,\frac{\beta_h \sigma_v I_{m0} \Lambda_v}{N_h} + \frac{1}{2}\,\frac{\beta_h \sigma_v I_{lm0} \Lambda_v}{N_h} \Big) S_{h0} \Big) \Big) - \frac{1}{N_v}\Big(\sigma_v (1-\theta)\, \beta_{lm} \Big(\frac{1}{3}\, I_{v0} \Big(\\
& -\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_m I_{v0} \Lambda_h}{N_v} - \frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l I_{v0} \Lambda_h}{N_v} - \frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm} I_{v0} \Lambda_h}{N_v} \\
& - \frac{1}{2}\,(\mu_h + \tau_1)\,\Lambda_h \Big) + \frac{1}{3}\Big(\frac{\beta_h \sigma_v I_{hal0} S_{v0}}{N_h} + \frac{\beta_h \sigma_v I_{hcl0} S_{v0}}{N_h} + \frac{\beta_h \sigma_v I_{m0} S_{v0}}{N_h} \\
& + \frac{\beta_h \sigma_v I_{lm0} S_{v0}}{N_h} - (\mu_v + \delta_v)\, I_{v0} \Big) \Lambda_h + \frac{1}{3}\Big(\frac{1}{2}\,\frac{\beta_h \sigma_v I_{hal0} \Lambda_v}{N_h} + \frac{1}{2}\,\frac{\beta_h \sigma_v I_{hcl0} \Lambda_v}{N_h} \\
& + \frac{1}{2}\,\frac{\beta_h \sigma_v I_{m0} \Lambda_v}{N_h} + \frac{1}{2}\,\frac{\beta_h \sigma_v I_{lm0} \Lambda_v}{N_h} \Big) S_{h0} \Big) \Big) - (\mu_h + \tau_1)\Big(-\frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_m I_{v0} \Lambda_h}{N_v} \\
& - \frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_l I_{v0} \Lambda_h}{N_v} - \frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_{lm} I_{v0} \Lambda_h}{N_v} - \frac{1}{6}\,(\mu_h + \tau_1)\,\Lambda_h \Big) \\
& - \frac{1}{6}\,\alpha_2 \tau_1 \Lambda_h \Bigg) t^3
\end{aligned}$$

$$
\left(-\frac{1}{N_v}\left(\sigma_v(1-\theta)\beta_m\left(\frac{1}{2}I_{v0}\left(-\frac{\sigma_v(1-\theta)\beta_m I_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_l I_{v0}S_{h0}}{N_v}\right.\right.\right.\right.
$$

$$
\left.-\frac{\sigma_v(1-\theta)\beta_{lm}I_{v0}S_{h0}}{N_v}-(\mu_h+\tau_1)S_{h0}+\alpha_1 T_{h0}-\alpha_2 V_{h0}\right)+\frac{1}{2}\left(\frac{\beta_h\sigma_v I_{hal0}S_{v0}}{N_h}\right.
$$

$$
\left.\left.\left.+\frac{\beta_h\sigma_v I_{hcl0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{m0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)I_{v0}\right)S_{h0}\right)\right)
$$

$$
-\frac{1}{N_v}\left(\sigma_v(1-\theta)\beta_l\left(\frac{1}{2}I_{v0}\left(-\frac{\sigma_v(1-\theta)\beta_m I_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_l I_{v0}S_{h0}}{N_v}\right.\right.\right.
$$

$$
\left.-\frac{\sigma_v(1-\theta)\beta_{lm}I_{v0}S_{h0}}{N_v}-(\mu_h+\tau_1)S_{h0}+\alpha_1 T_{h0}-\alpha_2 V_{h0}\right)+\frac{1}{2}\left(\frac{\beta_h\sigma_v I_{hal0}S_{v0}}{N_h}\right.
$$

$$
\left.\left.\left.+\frac{\beta_h\sigma_v I_{hcl0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{m0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)I_{v0}\right)S_{h0}\right)\right)
$$

$$
-\frac{1}{N_v}\left(\sigma_v(1-\theta)\beta_{lm}\left(\frac{1}{2}I_{v0}\left(-\frac{\sigma_v(1-\theta)\beta_m I_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_l I_{v0}S_{h0}}{N_v}\right.\right.\right.
$$

$$
\left.-\frac{\sigma_v(1-\theta)\beta_{lm}I_{v0}S_{h0}}{N_v}-(\mu_h+\tau_1)S_{h0}+\alpha_1 T_{h0}-\alpha_2 V_{h0}\right)+\frac{1}{2}\left(\frac{\beta_h\sigma_v I_{hal0}S_{v0}}{N_h}\right.
$$

$$
\left.\left.\left.+\frac{\beta_h\sigma_v I_{hcl0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{m0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)I_{v0}\right)S_{h0}\right)\right)-(\mu_h
$$

$$
+\tau_1)\left(-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_m I_{v0}S_{h0}}{N_v}-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_l I_{v0}S_{h0}}{N_v}\right.
$$

$$
\left.-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_{lm}I_{v0}S_{h0}}{N_v}-\frac{1}{2}(\mu_h+\tau_1)S_{h0}+\frac{1}{2}\alpha_1 T_{h0}-\frac{1}{2}\alpha_2 V_{h0}\right)
$$

$$
+\frac{1}{2}\alpha_1\left(\tau_2 I_{m0}+\tau_1 I_{hal0}+\tau_1 I_{hcl0}+\tau_3 I_{lm0}-(\mu_h+\alpha_1)T_{h0}\right)-\alpha_2\left(\frac{1}{2}\tau_1 S_{h0}\right.
$$

$$
\left.\left.-\frac{1}{2}(\mu_h+\alpha_2)V_{h0}-\frac{1}{2}\frac{\beta_m\sigma_v I_{v0}V_{h0}}{N_v}\right)\right)t^2
$$

(3.277)

for $n=1$ in equation (3.213) gives

$$
V_{h2}=\tau_1\int_0^t 0_1 dt-(\mu_h+\alpha_2)\int_0^t R_1 dt-\frac{\beta_m\sigma_v}{N_v}\int_0^t B_1 dt \qquad (3.278)
$$

$$= \tau_1 \int_0^t S_{h1} dt - (\mu_h + \alpha_2) \int_0^t V_{h1} dt - \frac{\beta_m \sigma_v}{N_v} \int_0^t (I_{v0} V_{h1} + I_{v1} V_{ho}) dt \tag{3.279}$$

$$\left(\tau_1 \left(-\frac{1}{6} \frac{\sigma_v (1-\theta) \beta_m I_{v0} \Lambda_h}{N_v} - \frac{1}{6} \frac{\sigma_v (1-\theta) \beta_l I_{v0} \Lambda_h}{N_v} - \frac{1}{6} \frac{\sigma_v (1-\theta) \beta_{lm} I_{v0} \Lambda_h}{N_v} \right.\right.$$
$$\left. - \frac{1}{6} (\mu_h + \tau_1) \Lambda_h \right) - \frac{1}{6} (\mu_h + \alpha_2) \tau_1 \Lambda_h - \frac{1}{N_v} \left(\beta_m \sigma_v \left(\frac{1}{6} I_{v0} \tau_1 \Lambda_h \right.\right.$$
$$+ \frac{1}{3} \left(\frac{1}{2} \frac{\beta_h \sigma_v I_{hal0} \Lambda_v}{N_h} + \frac{1}{2} \frac{\beta_h \sigma_v I_{hcl0} \Lambda_v}{N_h} + \frac{1}{2} \frac{\beta_h \sigma_v I_{m0} \Lambda_v}{N_h} \right.$$
$$\left.\left.\left.\left. + \frac{1}{2} \frac{\beta_h \sigma_v I_{lm0} \Lambda_v}{N_h} \right) V_{h0} \right) \right) \right) t^3 + \left(\tau_1 \left(-\frac{1}{2} \frac{\sigma_v (1-\theta) \beta_m I_{v0} S_{h0}}{N_v} \right.\right.$$
$$- \frac{1}{2} \frac{\sigma_v (1-\theta) \beta_l I_{v0} S_{h0}}{N_v} - \frac{1}{2} \frac{\sigma_v (1-\theta) \beta_{lm} I_{v0} S_{h0}}{N_v} - \frac{1}{2} (\mu_h + \tau_1) S_{h0} + \frac{1}{2} \alpha_1 T_{h0}$$
$$\left. - \frac{1}{2} \alpha_2 V_{h0} \right) - (\mu_h + \alpha_2) \left(\frac{1}{2} \tau_1 S_{h0} - \frac{1}{2} (\mu_h + \alpha_2) V_{h0} - \frac{1}{2} \frac{\beta_m \sigma_v I_{v0} V_{h0}}{N_v} \right)$$
$$- \frac{1}{N_v} \left(\beta_m \sigma_v \left(\frac{1}{2} I_{v0} \left(\tau_1 S_{h0} - (\mu_h + \alpha_2) V_{h0} - \frac{\beta_m \sigma_v I_{v0} V_{h0}}{N_v} \right) + \frac{1}{2} \left(\frac{\beta_h \sigma_v I_{hal0} S_{v0}}{N_h} \right.\right.\right.$$
$$\left.\left.\left.\left. + \frac{\beta_h \sigma_v I_{hcl0} S_{v0}}{N_h} + \frac{\beta_h \sigma_v I_{m0} S_{v0}}{N_h} + \frac{\beta_h \sigma_v I_{lm0} S_{v0}}{N_h} - (\mu_v + \delta_v) I_{v0} \right) V_{h0} \right) \right) \right) t^2$$

` (3.280)

for $n=1$ in equation (3.214) gives

$$I_{hal2} = \frac{\beta_l \sigma_v (1-\theta)}{N_v} \int_0^t A_1 dt - \frac{\beta_l \sigma_v (1-\theta)}{N_v} \int_0^t C_1 dt - (\mu_h + \tau_1 + \rho) \int_0^t J_1 dt \tag{3.281}$$

$$= \frac{\beta_l \sigma_v (1-\theta)}{N_v} \int_0^t (I_{v0} S_{h1} + I_{v1} S_{h0}) dt - \frac{\beta_l \sigma_v (1-\theta)}{N_v} \int_0^t (I_{v0} I_{hal1} + I_{v1} I_{hal0}) dt$$

$$-(\mu_h + \tau_1 + \rho) \int_0^t I_{hal1} dt \tag{3.282}$$

$$
\begin{aligned}
&\Bigg(\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_l\Bigg(\frac{1}{3}\,I_{v0}\Bigg(-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,\Lambda_h}{N_v}-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,\Lambda_h}{N_v} \\
&-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,\Lambda_h}{N_v}-\frac{1}{2}\,(\mu_h+\tau_1)\,\Lambda_h\Bigg)+\frac{1}{3}\,\Bigg(\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h} \\
&+\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h}-(\mu_v+\delta_v)\,I_{v0}\Bigg)\,\Lambda_h \\
&+\frac{1}{3}\,\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h} \\
&+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h}\Bigg)\,S_{h0}\Bigg)\Bigg)-\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_l\Bigg(\frac{1}{3}\,I_{hal0}\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h} \\
&+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h}\Bigg) \\
&+\frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}^2\,\Lambda_h}{N_v}\Bigg)\Bigg)-\frac{1}{6}\,\frac{(\mu_h+\tau_1+\rho)\,\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,\Lambda_h}{N_v}\Bigg)\,t^3 \\
&+\Bigg(\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_l\Bigg(\frac{1}{2}\,I_{v0}\Bigg(-\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v} \\
&-\frac{\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,S_{h0}}{N_v}-(\mu_h+\tau_1)\,S_{h0}+\alpha_1\,T_{h0}-\alpha_2\,V_{h0}\Bigg)+\frac{1}{2}\,\Bigg(\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h} \\
&+\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h}-(\mu_v+\delta_v)\,I_{v0}\Bigg)\,S_{h0}\Bigg)\Bigg) \\
&-\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_l\Bigg(\frac{1}{2}\,I_{hal0}\Bigg(\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h} \\
&+\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h}-(\mu_v+\delta_v)\,I_{v0}\Bigg)+\frac{1}{2}\,\Bigg(\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v} \\
&-(\mu_h+\tau_1+\rho)\,I_{hal0}\Bigg)\,I_{v0}\Bigg)\Bigg)-(\mu_h+\tau_1+\rho)\,\Bigg(\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v} \\
&-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v}-\frac{1}{2}\,(\mu_h+\tau_1+\rho)\,I_{hal0}\Bigg)\Bigg)\,t^2
\end{aligned}
$$

(3.283)

for $n=1$ in equation (3.215) gives

$$I_{hcl2} = \frac{\beta_l \sigma_v (1-\theta)}{N_v} \int_0^t C_1 dt + \rho \int_0^t J_1 dt - (\mu_h + \tau_1) \int_0^t K_1 dt \tag{3.284}$$

$$= \frac{\beta_l \sigma_v (1-\theta)}{N_v} \int_0^t (I_{v0} I_{hal1} + I_{v1} I_{hal0}) dt + \rho \int_0^t I_{hal1} dt - (\mu_h + \tau_1) \int_0^t I_{hcl1} dt \tag{3.285}$$

$$\begin{aligned}
&\left(\frac{1}{N_v} \left(\sigma_v (1-\theta) \beta_l \left(\frac{1}{3} I_{hal0} \left(\frac{1}{2} \frac{\beta_h \sigma_v I_{hal0} \Lambda_v}{N_h} + \frac{1}{2} \frac{\beta_h \sigma_v I_{hcl0} \Lambda_v}{N_h} + \frac{1}{2} \frac{\beta_h \sigma_v I_{m0} \Lambda_v}{N_h} \right.\right.\right.\right. \\
&\quad \left.\left.\left. + \frac{1}{2} \frac{\beta_h \sigma_v I_{lm0} \Lambda_v}{N_h} \right) + \frac{1}{6} \frac{\sigma_v (1-\theta) \beta_l I_{v0}^2 \Lambda_h}{N_v} \right)\right) + \frac{1}{6} \frac{\rho \sigma_v (1-\theta) \beta_l I_{v0} \Lambda_h}{N_v} \Bigg) t^3 \\
&\quad + \left(\frac{1}{N_v} \left(\sigma_v (1-\theta) \beta_l \left(\frac{1}{2} I_{hal0} \left(\frac{\beta_h \sigma_v I_{hal0} S_{v0}}{N_h} + \frac{\beta_h \sigma_v I_{hcl0} S_{v0}}{N_h} + \frac{\beta_h \sigma_v I_{m0} S_{v0}}{N_h} \right.\right.\right.\right. \\
&\quad \left. + \frac{\beta_h \sigma_v I_{lm0} S_{v0}}{N_h} - (\mu_v + \delta_v) I_{v0} \right) + \frac{1}{2} \left(\frac{\sigma_v (1-\theta) \beta_l I_{v0} S_{h0}}{N_v} - \frac{\sigma_v (1-\theta) \beta_l I_{v0} I_{hal0}}{N_v} \right. \\
&\quad \left.\left.\left. - (\mu_h + \tau_1 + \rho) I_{hal0} \right) I_{v0} \right)\right) + \rho \left(\frac{1}{2} \frac{\sigma_v (1-\theta) \beta_l I_{v0} S_{h0}}{N_v} - \frac{1}{2} \frac{\sigma_v (1-\theta) \beta_l I_{v0} I_{hal0}}{N_v} \right. \\
&\quad \left. - \frac{1}{2} (\mu_h + \tau_1 + \rho) I_{hal0} \right) - \frac{1}{2} (\mu_h + \tau_1) \left(\frac{\sigma_v (1-\theta) \beta_l I_{v0} I_{hal0}}{N_v} + \rho I_{hal0} - (\mu_h \right. \\
&\quad \left.\left. + \tau_1) I_{hcl0} \right) \right) t^2
\end{aligned} \tag{3.286}$$

for $n=1$ in equation (3.216) gives

$$I_{m2} = \frac{\beta_m \sigma_v (1-\theta)}{N_v} \int_0^t A_1 dt - (\tau_2 + \mu_h + \delta_m) \int_0^t L_1 dt + \frac{\beta_m \sigma_v}{N_v} \int_0^t B_1 dt \tag{3.287}$$

$$= \frac{\beta_m \sigma_v (1-\theta)}{N_v} \int_0^t (I_{v0} S_{h1} + I_{v1} S_{h0}) dt - (\tau_2 + \mu_h + \delta_m) \int_0^t I_{m1} dt$$

$$+ \frac{\beta_m \sigma_v}{N_v} \int_0^t (I_{v0} V_{h1} + I_{v1} V_{ho}) dt \tag{3.288}$$

$$\frac{1}{4}\frac{1}{N_v}\left(\sigma_v(1-\theta)\beta_m\left(\frac{1}{2}\frac{\beta_h\sigma_v \mathrm{I}_{hal0}\Lambda_v}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v \mathrm{I}_{hcl0}\Lambda_v}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v \mathrm{I}_{m0}\Lambda_v}{N_h}\right.\right.$$

$$\left.\left.+\frac{1}{2}\frac{\beta_h\sigma_v \mathrm{I}_{lm0}\Lambda_v}{N_h}\right)\Lambda_h t^4\right)+\left(\frac{1}{N_v}\left(\sigma_v(1-\theta)\beta_m\left(\frac{1}{3}\mathrm{I}_{v0}\left(\right.\right.\right.\right.$$

$$-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_m \mathrm{I}_{v0}\Lambda_h}{N_v}-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_l \mathrm{I}_{v0}\Lambda_h}{N_v}-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_{lm} \mathrm{I}_{v0}\Lambda_h}{N_v}$$

$$\left.-\frac{1}{2}(\mu_h+\tau_1)\Lambda_h\right)+\frac{1}{3}\left(\frac{\beta_h\sigma_v \mathrm{I}_{hal0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v \mathrm{I}_{hcl0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v \mathrm{I}_{m0}S_{v0}}{N_h}\right.$$

$$\left.+\frac{\beta_h\sigma_v \mathrm{I}_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)\mathrm{I}_{v0}\right)\Lambda_h+\frac{1}{3}\left(\frac{1}{2}\frac{\beta_h\sigma_v \mathrm{I}_{hal0}\Lambda_v}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v \mathrm{I}_{hcl0}\Lambda_v}{N_h}\right.$$

$$\left.\left.\left.+\frac{1}{2}\frac{\beta_h\sigma_v \mathrm{I}_{m0}\Lambda_v}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v \mathrm{I}_{lm0}\Lambda_v}{N_h}\right)S_{h0}\right)\right)$$

$$-\frac{1}{6}\frac{(\tau_2+\delta_m+\mu_h)\sigma_v(1-\theta)\beta_m \mathrm{I}_{v0}\Lambda_h}{N_v}+\frac{1}{N_v}\left(\beta_m\sigma_v\left(\frac{1}{6}\mathrm{I}_{v0}\tau_1\Lambda_h\right.\right.$$

$$+\frac{1}{3}\left(\frac{1}{2}\frac{\beta_h\sigma_v \mathrm{I}_{hal0}\Lambda_v}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v \mathrm{I}_{hcl0}\Lambda_v}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v \mathrm{I}_{m0}\Lambda_v}{N_h}\right.$$

$$\left.\left.\left.\left.+\frac{1}{2}\frac{\beta_h\sigma_v \mathrm{I}_{lm0}\Lambda_v}{N_h}\right)V_{h0}\right)\right)\right)t^3+\left(\frac{1}{N_v}\left(\sigma_v(1-\theta)\beta_m\left(\frac{1}{2}\mathrm{I}_{v0}\left(\right.\right.\right.\right.$$

$$-\frac{\sigma_v(1-\theta)\beta_m \mathrm{I}_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_l \mathrm{I}_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_{lm} \mathrm{I}_{v0}S_{h0}}{N_v}-(\mu_h$$

$$\left.+\tau_1)S_{h0}+\alpha_1 T_{h0}-\alpha_2 V_{h0}\right)+\frac{1}{2}\left(\frac{\beta_h\sigma_v \mathrm{I}_{hal0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v \mathrm{I}_{hcl0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v \mathrm{I}_{m0}S_{v0}}{N_h}\right.$$

$$\left.\left.\left.+\frac{\beta_h\sigma_v \mathrm{I}_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)\mathrm{I}_{v0}\right)S_{h0}\right)\right)-(\tau_2+\delta_m+\mu_h)\left(\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_m \mathrm{I}_{v0}S_{h0}}{N_v}\right.$$

$$\left.-\frac{1}{2}(\tau_2+\delta_m+\mu_h)\mathrm{I}_{m0}+\frac{1}{2}\frac{\beta_m\sigma_v \mathrm{I}_{v0}V_{h0}}{N_v}\right)+\frac{1}{N_v}\left(\beta_m\sigma_v\left(\frac{1}{2}\mathrm{I}_{v0}\left(\tau_1 S_{h0}-(\mu_h\right.\right.\right.$$

$$\left.+\alpha_2)V_{h0}-\frac{\beta_m\sigma_v \mathrm{I}_{v0}V_{h0}}{N_v}\right)+\frac{1}{2}\left(\frac{\beta_h\sigma_v \mathrm{I}_{hal0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v \mathrm{I}_{hcl0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v \mathrm{I}_{m0}S_{v0}}{N_h}\right.$$

$$\left.\left.\left.\left.+\frac{\beta_h\sigma_v \mathrm{I}_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)\mathrm{I}_{v0}\right)V_{h0}\right)\right)\right)t^2$$

(3.889)

for $n=1$ in equation (3.217) gives

$$I_{lm2}=\frac{\beta_{lm}\sigma_v\left(1-\theta\right)}{N_v}\int_0^t A_1 dt-\left(\tau_3+\mu_h+\delta_m\right)\int_0^t M_1 dt \tag{3.290}$$

$$=\frac{\beta_{lm}\sigma_v\left(1-\theta\right)}{N_v}\int_0^t (I_{v0}S_{h1}+I_{v1}S_{h0})dt-\left(\tau_3+\mu_h+\delta_m\right)\int_0^t I_{lm1} dt \tag{3.291}$$

$$\begin{aligned}
&\frac{1}{4}\,\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_{lm}\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h}\\
&\quad+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h}\Bigg)\Lambda_h\,t^4\Bigg)+\Bigg(\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_{lm}\Bigg(\frac{1}{3}\,I_{v0}\Bigg(\\
&\quad-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,\Lambda_h}{N_v}-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,\Lambda_h}{N_v}-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,\Lambda_h}{N_v}\\
&\quad-\frac{1}{2}\,(\mu_h+\tau_1)\,\Lambda_h\Bigg)+\frac{1}{3}\,\Bigg(\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h}\\
&\quad+\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h}-(\mu_v+\delta_v)\,I_{v0}\Bigg)\Lambda_h+\frac{1}{3}\,\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h}\\
&\quad+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h}\Bigg)S_{h0}\Bigg)\Bigg)\\
&\quad-\frac{1}{6}\,\frac{(\tau_3+\mu_h+\delta_m)\,\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,\Lambda_h}{N_v}\Bigg)t^3+\Bigg(\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_{lm}\Bigg(\frac{1}{2}\,I_{v0}\Bigg(\\
&\quad-\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,S_{h0}}{N_v}-(\mu_h\\
&\quad+\tau_1)\,S_{h0}+\alpha_1\,T_{h0}-\alpha_2\,V_{h0}\Bigg)+\frac{1}{2}\,\Bigg(\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h}\\
&\quad+\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h}-(\mu_v+\delta_v)\,I_{v0}\Bigg)S_{h0}\Bigg)\Bigg)-(\tau_3+\mu_h+\delta_m)\Bigg(\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,S_{h0}}{N_v}\\
&\quad-\frac{1}{2}\,(\tau_3+\mu_h+\delta_m)\,I_{lm0}\Bigg)\Bigg)t^2
\end{aligned} \tag{3.292}$$

for $n=1$ in equation (3.218) gives

$$T_{h2}=\tau_2\int_0^t L_1dt+\tau_1\int_0^t J_1dt+\tau_1\int_0^t K_1dt+\tau_3\int_0^t M_1dt-(\alpha_1+\mu_h)\int_0^t Q_1dt \quad (3.293)$$

$$=\tau_2\int_0^t I_{m1}dt+\tau_1\int_0^t I_{hal1}dt+\tau_1\int_0^t I_{hcl1}dt+\tau_3\int_0^t I_{lm1}dt-(\alpha_1+\mu_h)\int_0^t T_{h1}dt \quad (3.294)$$

$$\left(\frac{1}{6}\frac{\tau_2\sigma_v(1-\theta)\beta_m I_{v0}\Lambda_h}{N_v}+\frac{1}{6}\frac{\tau_1\sigma_v(1-\theta)\beta_l I_{v0}\Lambda_h}{N_v}+\frac{1}{6}\frac{\tau_3\sigma_v(1-\theta)\beta_{lm} I_{v0}\Lambda_h}{N_v}\right)t^3$$
$$+\left(\tau_2\left(\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_m I_{v0}S_{h0}}{N_v}-\frac{1}{2}(\tau_2+\delta_m+\mu_h)I_{m0}+\frac{1}{2}\frac{\beta_m\sigma_v I_{v0}V_{h0}}{N_v}\right)\right.$$
$$+\tau_1\left(\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_l I_{v0}S_{h0}}{N_v}-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_l I_{v0}I_{hal0}}{N_v}-\frac{1}{2}(\mu_h+\tau_1+\rho)I_{hal0}\right)$$
$$+\frac{1}{2}\tau_1\left(\frac{\sigma_v(1-\theta)\beta_l I_{v0}I_{hal0}}{N_v}+\rho I_{hal0}-(\mu_h+\tau_1)I_{hcl0}\right)$$
$$+\tau_3\left(\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_{lm} I_{v0}S_{h0}}{N_v}-\frac{1}{2}(\tau_3+\mu_h+\delta_m)I_{lm0}\right)-\frac{1}{2}(\mu_h+\alpha_1)\left(\tau_2 I_{m0}\right.$$
$$\left.\left.+\tau_1 I_{hal0}+\tau_1 I_{hcl0}+\tau_3 I_{lm0}-(\mu_h+\alpha_1)T_{h0}\right)\right)t^2$$

(3.295)

for $n=1$ in equation (3.219) gives

$$S_{v2}=-\frac{\beta_h\sigma_v}{N_h}\int_0^t D_1dt-\frac{\beta_h\sigma_v}{N_h}\int_0^t E_1dt-\frac{\beta_h\sigma_v}{N_h}\int_0^t F_1dt-\frac{\beta_h\sigma_v}{N_h}\int_0^t G_1dt-(\mu_h+\delta_v)\int_0^t P_1dt \quad (3.296)$$

$$=-\frac{\beta_h\sigma_v}{N_h}\int_0^t(I_{hal0}S_{v1}+I_{hal1}S_{vo})dt-\frac{\beta_h\sigma_v}{N_h}\int_0^t(I_{hcl0}S_{v1}+I_{hcl1}S_{vo})dt-\frac{\beta_h\sigma_v}{N_h}\int_0^t(I_{m0}S_{v1}+I_{m1}S_{vo})dt-\frac{\beta_h\sigma_v}{N_h}\int_0^t(I_{lm0}S_{v1}+I_{lm1}S_{vo})dt$$

$$-(\mu_h+\delta_v)\int_0^t S_{v1}dt \quad (3.297)$$

$$\Bigg(-\frac{1}{8}\,\frac{\beta_h\,\sigma_v\,\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\,\Lambda_h\,\Lambda_v}{N_h\,N_v} - \frac{1}{8}\,\frac{\beta_h\,\sigma_v\,\sigma_v(1-\theta)\,\beta_m\,\mathrm{I}_{v0}\,\Lambda_h\,\Lambda_v}{N_h\,N_v}$$

$$- \frac{1}{8}\,\frac{\beta_h\,\sigma_v\,\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\,\Lambda_h\,\Lambda_v}{N_h\,N_v}\Bigg)\,t^4 + \Bigg(-\frac{1}{N_h}\Bigg(\beta_h\,\sigma_v\Bigg(\frac{1}{3}\,\mathrm{I}_{hal0}\Bigg($$

$$-\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hal0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hcl0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{m0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{lm0}\,\Lambda_v}{N_h} - \frac{1}{2}\,(\mu_v$$

$$+\delta_v)\,\Lambda_v\Bigg) + \frac{1}{3}\Bigg(\frac{\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\,S_{h0}}{N_v} - \frac{\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\,\mathrm{I}_{hal0}}{N_v} - (\mu_h + \tau_1$$

$$+\rho)\,\mathrm{I}_{hal0}\Bigg)\Lambda_v + \frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\,\Lambda_h\,S_{v0}}{N_v}\Bigg)\Bigg) - \frac{1}{N_h}\Bigg(\beta_h\,\sigma_v\Bigg(\frac{1}{3}\,\mathrm{I}_{hcl0}\Bigg($$

$$-\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hal0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hcl0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{m0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{lm0}\,\Lambda_v}{N_h} - \frac{1}{2}\,(\mu_v$$

$$+\delta_v)\,\Lambda_v\Bigg) + \frac{1}{3}\Bigg(\frac{\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\,\mathrm{I}_{hal0}}{N_v} + \rho\,\mathrm{I}_{hal0} - (\mu_h + \tau_1)\,\mathrm{I}_{hcl0}\Bigg)\Lambda_v\Bigg)\Bigg)$$

$$-\frac{1}{N_h}\Bigg(\beta_h\,\sigma_v\Bigg(\frac{1}{3}\,\mathrm{I}_{m0}\Bigg(-\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hal0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hcl0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{m0}\,\Lambda_v}{N_h}$$

$$-\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{lm0}\,\Lambda_v}{N_h} - \frac{1}{2}\,(\mu_v + \delta_v)\,\Lambda_v\Bigg) + \frac{1}{3}\Bigg(\frac{\sigma_v(1-\theta)\,\beta_m\,\mathrm{I}_{v0}\,S_{h0}}{N_v} - (\tau_2 + \delta_m$$

$$+\mu_h)\,\mathrm{I}_{m0} + \frac{\beta_m\,\sigma_v\,\mathrm{I}_{v0}\,V_{h0}}{N_v}\Bigg)\Lambda_v + \frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_m\,\mathrm{I}_{v0}\,\Lambda_h\,S_{v0}}{N_v}\Bigg)\Bigg)$$

$$-\frac{1}{N_h}\Bigg(\beta_h\,\sigma_v\Bigg(\frac{1}{3}\,\mathrm{I}_{lm0}\Bigg(-\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hal0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hcl0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{m0}\,\Lambda_v}{N_h}$$

$$-\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{lm0}\,\Lambda_v}{N_h} - \frac{1}{2}\,(\mu_v + \delta_v)\,\Lambda_v\Bigg) + \frac{1}{3}\Bigg(\frac{\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\,S_{h0}}{N_v} - (\tau_3 + \mu_h$$

$$+\delta_m)\,\mathrm{I}_{lm0}\Bigg)\Lambda_v + \frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\,\Lambda_h\,S_{v0}}{N_v}\Bigg)\Bigg) - (\mu_v + \delta_v)\Bigg(-\frac{1}{6}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hal0}\,\Lambda_v}{N_h}$$

$$-\frac{1}{6}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hcl0}\,\Lambda_v}{N_h} - \frac{1}{6}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{m0}\,\Lambda_v}{N_h} - \frac{1}{6}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{lm0}\,\Lambda_v}{N_h} - \frac{1}{6}\,(\mu_v + \delta_v)\,\Lambda_v\Bigg)\Bigg)\,t^3$$

$$
\begin{aligned}
&+\left(-\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{2}\mathrm{I}_{hal0}\left(-\frac{\beta_h\sigma_v\mathrm{I}_{hal0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{hcl0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{m0}S_{v0}}{N_h}\right.\right.\right.\right.\\
&-\left.\frac{\beta_h\sigma_v\mathrm{I}_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)S_{v0}\right)+\frac{1}{2}\left(\frac{\sigma_v(1-\theta)\beta_l\mathrm{I}_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_l\mathrm{I}_{v0}\mathrm{I}_{hal0}}{N_v}\right.\\
&-\left.\left.\left.(\mu_h+\tau_1+\rho)\mathrm{I}_{hal0}\right)S_{v0}\right)\right)-\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{2}\mathrm{I}_{hcl0}\left(-\frac{\beta_h\sigma_v\mathrm{I}_{hal0}S_{v0}}{N_h}\right.\right.\right.\\
&-\left.\frac{\beta_h\sigma_v\mathrm{I}_{hcl0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{m0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)S_{v0}\right)\\
&+\left.\left.\frac{1}{2}\left(\frac{\sigma_v(1-\theta)\beta_l\mathrm{I}_{v0}\mathrm{I}_{hal0}}{N_v}+\rho\,\mathrm{I}_{hal0}-(\mu_h+\tau_1)\mathrm{I}_{hcl0}\right)S_{v0}\right)\right)\\
&-\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{2}\mathrm{I}_{m0}\left(-\frac{\beta_h\sigma_v\mathrm{I}_{hal0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{hcl0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{m0}S_{v0}}{N_h}\right.\right.\right.\\
&-\left.\frac{\beta_h\sigma_v\mathrm{I}_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)S_{v0}\right)+\frac{1}{2}\left(\frac{\sigma_v(1-\theta)\beta_m\mathrm{I}_{v0}S_{h0}}{N_v}-(\tau_2+\delta_m+\mu_h)\mathrm{I}_{m0}\right.\\
&+\left.\left.\left.\frac{\beta_m\sigma_v\mathrm{I}_{v0}V_{h0}}{N_v}\right)S_{v0}\right)\right)-\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{2}\mathrm{I}_{lm0}\left(-\frac{\beta_h\sigma_v\mathrm{I}_{hal0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{hcl0}S_{v0}}{N_h}\right.\right.\right.\\
&-\left.\frac{\beta_h\sigma_v\mathrm{I}_{m0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)S_{v0}\right)+\frac{1}{2}\left(\frac{\sigma_v(1-\theta)\beta_{lm}\mathrm{I}_{v0}S_{h0}}{N_v}-(\tau_3\right.\\
&+\left.\left.\left.\mu_h+\delta_m)\mathrm{I}_{lm0}\right)S_{v0}\right)\right)-(\mu_v+\delta_v)\left(-\frac{1}{2}\frac{\beta_h\sigma_v\mathrm{I}_{hal0}S_{v0}}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_v\mathrm{I}_{hcl0}S_{v0}}{N_h}\right.\\
&-\left.\left.\frac{1}{2}\frac{\beta_h\sigma_v\mathrm{I}_{m0}S_{v0}}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_v\mathrm{I}_{lm0}S_{v0}}{N_h}-\frac{1}{2}(\mu_v+\delta_v)S_{v0}\right)\right)t^2
\end{aligned}
$$

(3.298)

$$I_{v2}=\frac{\beta_h\sigma_v}{N_h}\int_0^t(I_{hal0}S_{v1}+I_{hal1}S_{vo})dt+\frac{\beta_h\sigma_v}{N_h}\int_0^t(I_{hcl0}S_{v1}+I_{hcl1}S_{vo})dt+\frac{\beta_h\sigma_v}{N_h}\int_0^t(I_{m0}S_{v1}+I_{m1}S_{vo})dt$$

$$+\frac{\beta_h\sigma_v}{N_h}\int_0^t(I_{lm0}S_{v1}+I_{lm1}S_{vo})dt-(\mu_v+\delta_m)\int_0^t I_{v1}dt$$

$$\left(\frac{1}{8}\frac{\beta_h\sigma_v\sigma_v(1-\theta)\beta_l I_{v0}\Lambda_h\Lambda_v}{N_hN_v}+\frac{1}{8}\frac{\beta_h\sigma_v\sigma_v(1-\theta)\beta_m I_{v0}\Lambda_h\Lambda_v}{N_hN_v}+\frac{1}{8}\frac{\beta_h\sigma_v\sigma_v(1-\theta)\beta_{lm} I_{v0}\Lambda_h\Lambda_v}{N_hN_v}\right)t^4$$

$$+\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{3}I_{hal0}\left(-\frac{1}{2}\frac{\beta_h\sigma_vI_{hal0}\Lambda_v}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_vI_{hcl0}\Lambda_v}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_vI_{m0}\Lambda_v}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_vI_{lm0}\Lambda_v}{N_h}-\frac{1}{2}(\mu_v+\delta_v)\Lambda_v\right)+\frac{1}{3}\left(\frac{\sigma_v(1-\theta)\beta_lI_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_lI_{v0}I_{hal0}}{N_v}-(\mu_h+\tau_1+\rho)I_{hal0}\right)\Lambda_v+\frac{1}{6}\frac{\sigma_v(1-\theta)\beta_lI_{v0}\Lambda_hS_{v0}}{N_v}\right)\right)$$

$$+\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{3}I_{hcl0}\left(-\frac{1}{2}\frac{\beta_h\sigma_vI_{hal0}\Lambda_v}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_vI_{hcl0}\Lambda_v}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_vI_{m0}\Lambda_v}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_vI_{lm0}\Lambda_v}{N_h}-\frac{1}{2}(\mu_v+\delta_v)\Lambda_v\right)+\frac{1}{3}\left(\frac{\sigma_v(1-\theta)\beta_lI_{v0}I_{hal0}}{N_v}+\rho I_{hal0}-(\mu_h+\tau_1)I_{hcl0}\right)\Lambda_v\right)\right)$$

$$+\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{3}I_{m0}\left(-\frac{1}{2}\frac{\beta_h\sigma_vI_{hal0}\Lambda_v}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_vI_{hcl0}\Lambda_v}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_vI_{m0}\Lambda_v}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_vI_{lm0}\Lambda_v}{N_h}-\frac{1}{2}(\mu_v+\delta_v)\Lambda_v\right)+\frac{1}{3}\left(\frac{\sigma_v(1-\theta)\beta_mI_{v0}S_{h0}}{N_v}-(\tau_2+\delta_m+\mu_h)I_{m0}+\frac{\beta_m\sigma_vI_{v0}V_{h0}}{N_v}\right)\Lambda_v+\frac{1}{6}\frac{\sigma_v(1-\theta)\beta_mI_{v0}\Lambda_hS_{v0}}{N_v}\right)\right)$$

$$+\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{3}\mathrm{I}_{lm0}\left(-\frac{1}{2}\frac{\beta_h\sigma_v\mathrm{I}_{hal0}\Lambda_v}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_v\mathrm{I}_{hcl0}\Lambda_v}{N_h}-\frac{1}{2}\frac{\beta_h\sigma_v\mathrm{I}_{m0}\Lambda_v}{N_h}\right.\right.\right.$$

$$-\frac{1}{2}\frac{\beta_h\sigma_v\mathrm{I}_{lm0}\Lambda_v}{N_h}-\left.\frac{1}{2}(\mu_v+\delta_v)\Lambda_v\right)+\frac{1}{3}\left(\frac{\sigma_v(1-\theta)\beta_{lm}\mathrm{I}_{v0}S_{h0}}{N_v}-(\tau_3+\mu_h\right.$$

$$\left.\left.\left.+\delta_m)\mathrm{I}_{lm0}\right)\Lambda_v+\frac{1}{6}\frac{\sigma_v(1-\theta)\beta_{lm}\mathrm{I}_{v0}\Lambda_h S_{v0}}{N_v}\right)\right)-(\mu_v+\delta_v)\left(\frac{1}{6}\frac{\beta_h\sigma_v\mathrm{I}_{hal0}\Lambda_v}{N_h}\right.$$

$$\left.+\frac{1}{6}\frac{\beta_h\sigma_v\mathrm{I}_{hcl0}\Lambda_v}{N_h}+\frac{1}{6}\frac{\beta_h\sigma_v\mathrm{I}_{m0}\Lambda_v}{N_h}+\frac{1}{6}\frac{\beta_h\sigma_v\mathrm{I}_{lm0}\Lambda_v}{N_h}\right)$$

$$\left(\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{2}\mathrm{I}_{hal0}\left(-\frac{\beta_h\sigma_v\mathrm{I}_{hal0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{hcl0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{m0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{lm0}S_{v0}}{N_h}\right.\right.\right.\right.$$

$$\left.-(\mu_v+\delta_v)S_{v0}\right)+\frac{1}{2}\left(\frac{\sigma_v(1-\theta)\beta_l\mathrm{I}_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_l\mathrm{I}_{v0}\mathrm{I}_{hal0}}{N_v}-(\mu_h+\tau_1\right.$$

$$\left.\left.\left.+\rho)\mathrm{I}_{hal0}\right)S_{v0}\right)\right)+\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{2}\mathrm{I}_{hcl0}\left(-\frac{\beta_h\sigma_v\mathrm{I}_{hal0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{hcl0}S_{v0}}{N_h}\right.\right.\right.$$

$$\left.-\frac{\beta_h\sigma_v\mathrm{I}_{m0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)S_{v0}\right)+\frac{1}{2}\left(\frac{\sigma_v(1-\theta)\beta_l\mathrm{I}_{v0}\mathrm{I}_{hal0}}{N_v}\right.$$

$$\left.\left.\left.+\rho\,\mathrm{I}_{hal0}-(\mu_h+\tau_1)\mathrm{I}_{hcl0}\right)S_{v0}\right)\right)+\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{2}\mathrm{I}_{m0}\left(-\frac{\beta_h\sigma_v\mathrm{I}_{hal0}S_{v0}}{N_h}\right.\right.\right.$$

$$\left.-\frac{\beta_h\sigma_v\mathrm{I}_{hcl0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{m0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)S_{v0}\right)$$

$$\left.\left.+\frac{1}{2}\left(\frac{\sigma_v(1-\theta)\beta_m\mathrm{I}_{v0}S_{h0}}{N_v}-(\tau_2+\delta_m+\mu_h)\mathrm{I}_{m0}+\frac{\beta_m\sigma_v\mathrm{I}_{v0}V_{h0}}{N_v}\right)S_{v0}\right)\right)$$

$$+\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{2}\mathrm{I}_{lm0}\left(-\frac{\beta_h\sigma_v\mathrm{I}_{hal0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{hcl0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v\mathrm{I}_{m0}S_{v0}}{N_h}\right.\right.\right.$$

$$\left.-\frac{\beta_h\sigma_v\mathrm{I}_{lm0}S_{v0}}{N_h}-(\mu_v+\delta_v)S_{v0}\right)+\frac{1}{2}\left(\frac{\sigma_v(1-\theta)\beta_{lm}\mathrm{I}_{v0}S_{h0}}{N_v}-(\tau_3+\mu_h\right.$$

$$\left.\left.\left.+\delta_m)\mathrm{I}_{lm0}\right)S_{v0}\right)\right)-(\mu_v+\delta_v)\left(\frac{1}{2}\frac{\beta_h\sigma_v\mathrm{I}_{hal0}S_{v0}}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v\mathrm{I}_{hcl0}S_{v0}}{N_h}\right.$$

$$\left.\left.+\frac{1}{2}\frac{\beta_h\sigma_v\mathrm{I}_{m0}S_{v0}}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v\mathrm{I}_{lm0}S_{v0}}{N_h}-\frac{1}{2}(\mu_v+\delta_v)\mathrm{I}_{v0}\right)\right)t^2$$

(3.301)

$$S_h = S_{h0} + S_{h1} + S_{h2}$$

$$\Bigg(-\frac{1}{N_v} \Bigg(\sigma_v (1-\theta)\, \beta_m \Bigg(\frac{1}{3}\, I_{v0} \Bigg(-\frac{1}{2}\, \frac{\sigma_v (1-\theta)\, \beta_m\, I_{v0}\, \Lambda_h}{N_v} - \frac{1}{2}\, \frac{\sigma_v (1-\theta)\, \beta_l\, I_{v0}\, \Lambda_h}{N_v}$$

$$- \frac{1}{2}\, \frac{\sigma_v (1-\theta)\, \beta_{lm}\, I_{v0}\, \Lambda_h}{N_v} - \frac{1}{2} \left(\mu_h + \tau_1\right) \Lambda_h \Bigg) + \frac{1}{3} \Bigg(\frac{\beta_h\, \sigma_v\, I_{hal0}\, S_{v0}}{N_h}$$

$$+ \frac{\beta_h\, \sigma_v\, I_{hcl0}\, S_{v0}}{N_h} + \frac{\beta_h\, \sigma_v\, I_{m0}\, S_{v0}}{N_h} + \frac{\beta_h\, \sigma_v\, I_{lm0}\, S_{v0}}{N_h} - \left(\mu_v + \delta_v\right) I_{v0} \Bigg) \Lambda_h$$

$$+ \frac{1}{3} \Bigg(\frac{1}{2}\, \frac{\beta_h\, \sigma_v\, I_{hal0}\, \Lambda_v}{N_h} + \frac{1}{2}\, \frac{\beta_h\, \sigma_v\, I_{hcl0}\, \Lambda_v}{N_h} + \frac{1}{2}\, \frac{\beta_h\, \sigma_v\, I_{m0}\, \Lambda_v}{N_h}$$

$$+ \frac{1}{2}\, \frac{\beta_h\, \sigma_v\, I_{lm0}\, \Lambda_v}{N_h} \Bigg) S_{h0} \Bigg) \Bigg) - \frac{1}{N_v} \Bigg(\sigma_v (1-\theta)\, \beta_l \Bigg(\frac{1}{3}\, I_{v0} \Bigg($$

$$-\frac{1}{2}\, \frac{\sigma_v (1-\theta)\, \beta_m\, I_{v0}\, \Lambda_h}{N_v} - \frac{1}{2}\, \frac{\sigma_v (1-\theta)\, \beta_l\, I_{v0}\, \Lambda_h}{N_v} - \frac{1}{2}\, \frac{\sigma_v (1-\theta)\, \beta_{lm}\, I_{v0}\, \Lambda_h}{N_v}$$

$$- \frac{1}{2} \left(\mu_h + \tau_1\right) \Lambda_h \Bigg) + \frac{1}{3} \Bigg(\frac{\beta_h\, \sigma_v\, I_{hal0}\, S_{v0}}{N_h} + \frac{\beta_h\, \sigma_v\, I_{hcl0}\, S_{v0}}{N_h} + \frac{\beta_h\, \sigma_v\, I_{m0}\, S_{v0}}{N_h}$$

$$+ \frac{\beta_h\, \sigma_v\, I_{lm0}\, S_{v0}}{N_h} - \left(\mu_v + \delta_v\right) I_{v0} \Bigg) \Lambda_h + \frac{1}{3} \Bigg(\frac{1}{2}\, \frac{\beta_h\, \sigma_v\, I_{hal0}\, \Lambda_v}{N_h} + \frac{1}{2}\, \frac{\beta_h\, \sigma_v\, I_{hcl0}\, \Lambda_v}{N_h}$$

$$+ \frac{1}{2}\, \frac{\beta_h\, \sigma_v\, I_{m0}\, \Lambda_v}{N_h} + \frac{1}{2}\, \frac{\beta_h\, \sigma_v\, I_{lm0}\, \Lambda_v}{N_h} \Bigg) S_{h0} \Bigg) \Bigg) - \frac{1}{N_v} \Bigg(\sigma_v (1-\theta)\, \beta_{lm} \Bigg(\frac{1}{3}\, I_{v0} \Bigg($$

$$-\frac{1}{2}\, \frac{\sigma_v (1-\theta)\, \beta_m\, I_{v0}\, \Lambda_h}{N_v} - \frac{1}{2}\, \frac{\sigma_v (1-\theta)\, \beta_l\, I_{v0}\, \Lambda_h}{N_v} - \frac{1}{2}\, \frac{\sigma_v (1-\theta)\, \beta_{lm}\, I_{v0}\, \Lambda_h}{N_v}$$

$$- \frac{1}{2} \left(\mu_h + \tau_1\right) \Lambda_h \Bigg) + \frac{1}{3} \Bigg(\frac{\beta_h\, \sigma_v\, I_{hal0}\, S_{v0}}{N_h} + \frac{\beta_h\, \sigma_v\, I_{hcl0}\, S_{v0}}{N_h} + \frac{\beta_h\, \sigma_v\, I_{m0}\, S_{v0}}{N_h}$$

$$+ \frac{\beta_h\, \sigma_v\, I_{lm0}\, S_{v0}}{N_h} - \left(\mu_v + \delta_v\right) I_{v0} \Bigg) \Lambda_h + \frac{1}{3} \Bigg(\frac{1}{2}\, \frac{\beta_h\, \sigma_v\, I_{hal0}\, \Lambda_v}{N_h} + \frac{1}{2}\, \frac{\beta_h\, \sigma_v\, I_{hcl0}\, \Lambda_v}{N_h}$$

$$+ \frac{1}{2}\, \frac{\beta_h\, \sigma_v\, I_{m0}\, \Lambda_v}{N_h} + \frac{1}{2}\, \frac{\beta_h\, \sigma_v\, I_{lm0}\, \Lambda_v}{N_h} \Bigg) S_{h0} \Bigg) \Bigg) - \left(\mu_h + \tau_1\right) \Bigg(-\frac{1}{6}\, \frac{\sigma_v (1-\theta)\, \beta_m\, I_{v0}\, \Lambda_h}{N_v}$$

$$- \frac{1}{6}\, \frac{\sigma_v (1-\theta)\, \beta_l\, I_{v0}\, \Lambda_h}{N_v} - \frac{1}{6}\, \frac{\sigma_v (1-\theta)\, \beta_{lm}\, I_{v0}\, \Lambda_h}{N_v} - \frac{1}{6} \left(\mu_h + \tau_1\right) \Lambda_h \Bigg)$$

$$- \frac{1}{6}\, \alpha_2\, \tau_1\, \Lambda_h \Bigg) t^3$$

$$+\left(-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_m I_{v0}\Lambda_h}{N_v}-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_l I_{v0}\Lambda_h}{N_v}-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_{lm} I_{v0}\Lambda_h}{N_v}\right.$$

$$-\frac{1}{2}(\mu_h+\tau_1)\Lambda_h-\frac{1}{N_v}\left(\sigma_v(1-\theta)\beta_m\left(\frac{1}{2}I_{v0}\left(-\frac{\sigma_v(1-\theta)\beta_m I_{v0}S_{h0}}{N_v}\right.\right.\right.$$

$$\left.-\frac{\sigma_v(1-\theta)\beta_l I_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_{lm} I_{v0}S_{h0}}{N_v}-(\mu_h+\tau_1)S_{h0}+\alpha_1 T_{h0}-\alpha_2 V_{h0}\right)$$

$$+\frac{1}{2}\left(\frac{\beta_h\sigma_v I_{hal0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{hcl0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{m0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{lm0}S_{v0}}{N_h}-(\mu_v\right.$$

$$\left.\left.\left.+\delta_v)I_{v0}\right)S_{h0}\right)\right)-\frac{1}{N_v}\left(\sigma_v(1-\theta)\beta_l\left(\frac{1}{2}I_{v0}\left(-\frac{\sigma_v(1-\theta)\beta_m I_{v0}S_{h0}}{N_v}\right.\right.\right.$$

$$\left.-\frac{\sigma_v(1-\theta)\beta_l I_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_{lm} I_{v0}S_{h0}}{N_v}-(\mu_h+\tau_1)S_{h0}+\alpha_1 T_{h0}-\alpha_2 V_{h0}\right)$$

$$+\frac{1}{2}\left(\frac{\beta_h\sigma_v I_{hal0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{hcl0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{m0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{lm0}S_{v0}}{N_h}-(\mu_v\right.$$

$$\left.\left.\left.+\delta_v)I_{v0}\right)S_{h0}\right)\right)-\frac{1}{N_v}\left(\sigma_v(1-\theta)\beta_{lm}\left(\frac{1}{2}I_{v0}\left(-\frac{\sigma_v(1-\theta)\beta_m I_{v0}S_{h0}}{N_v}\right.\right.\right.$$

$$\left.-\frac{\sigma_v(1-\theta)\beta_l I_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_{lm} I_{v0}S_{h0}}{N_v}-(\mu_h+\tau_1)S_{h0}+\alpha_1 T_{h0}-\alpha_2 V_{h0}\right)$$

$$+\frac{1}{2}\left(\frac{\beta_h\sigma_v I_{hal0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{hcl0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{m0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{lm0}S_{v0}}{N_h}-(\mu_v\right.$$

$$\left.\left.\left.+\delta_v)I_{v0}\right)S_{h0}\right)\right)-(\mu_h+\tau_1)\left(-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_m I_{v0}S_{h0}}{N_v}-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_l I_{v0}S_{h0}}{N_v}\right.$$

$$\left.-\frac{1}{2}\frac{\sigma_v(1-\theta)\beta_{lm} I_{v0}S_{h0}}{N_v}-\frac{1}{2}(\mu_h+\tau_1)S_{h0}+\frac{1}{2}\alpha_1 T_{h0}-\frac{1}{2}\alpha_2 V_{h0}\right)$$

$$+\frac{1}{2}\alpha_1\left(\tau_2 I_{m0}+\tau_1 I_{hal0}+\tau_1 I_{hcl0}+\tau_3 I_{lm0}-(\mu_h+\alpha_1)T_{h0}\right)-\alpha_2\left(\frac{1}{2}\tau_1 S_{h0}\right.$$

$$\left.\left.-\frac{1}{2}(\mu_h+\alpha_2)V_{h0}-\frac{1}{2}\frac{\beta_m\sigma_v I_{v0}V_{h0}}{N_v}\right)\right)t^2+\left(\Lambda_h-\frac{\sigma_v(1-\theta)\beta_m I_{v0}S_{h0}}{N_v}\right.$$

$$\left.-\frac{\sigma_v(1-\theta)\beta_l I_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_{lm} I_{v0}S_{h0}}{N_v}-(\mu_h+\tau_1)S_{h0}+\alpha_1 T_{h0}-\alpha_2 V_{h0}\right)t$$

$$+S_{h0}$$

(3.302)

$$V_h = V_{h0} + V_{h1} + V_{h2}$$

$$\Bigg(\tau_1\left(-\frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_m\,\mathrm{I}_{v0}\Lambda_h}{N_v} - \frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\Lambda_h}{N_v} - \frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\Lambda_h}{N_v}\right.$$

$$\left. - \frac{1}{6}\left(\mu_h + \tau_1\right)\Lambda_h\right) - \frac{1}{6}\left(\mu_h + \alpha_2\right)\tau_1\,\Lambda_h - \frac{1}{N_v}\left(\beta_m\,\sigma_v\left(\frac{1}{6}\,\mathrm{I}_{v0}\,\tau_1\,\Lambda_h\right.\right.$$

$$+ \frac{1}{3}\left(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hal0}\,\Lambda_v}{N_h} + \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hcl0}\,\Lambda_v}{N_h} + \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{m0}\,\Lambda_v}{N_h}\right.$$

$$\left.\left.\left. + \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{lm0}\,\Lambda_v}{N_h}\right)V_{h0}\right)\right)\Bigg)\,t^3 + \Bigg(\frac{1}{2}\,\tau_1\,\Lambda_h + \tau_1\left(-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_m\,\mathrm{I}_{v0}\,S_{h0}}{N_v}\right.$$

$$- \frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\,S_{h0}}{N_v} - \frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\,S_{h0}}{N_v} - \frac{1}{2}\left(\mu_h + \tau_1\right)S_{h0} + \frac{1}{2}\,\alpha_1\,T_{h0}$$

$$\left. - \frac{1}{2}\,\alpha_2\,V_{h0}\right) - \left(\mu_h + \alpha_2\right)\left(\frac{1}{2}\,\tau_1\,S_{h0} - \frac{1}{2}\left(\mu_h + \alpha_2\right)V_{h0} - \frac{1}{2}\,\frac{\beta_m\,\sigma_v\,\mathrm{I}_{v0}\,V_{h0}}{N_v}\right)$$

$$- \frac{1}{N_v}\left(\beta_m\,\sigma_v\left(\frac{1}{2}\,\mathrm{I}_{v0}\left(\tau_1\,S_{h0} - \left(\mu_h + \alpha_2\right)V_{h0} - \frac{\beta_m\,\sigma_v\,\mathrm{I}_{v0}\,V_{h0}}{N_v}\right) + \frac{1}{2}\left(\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hal0}\,S_{v0}}{N_h}\right.\right.\right.$$

$$\left.\left.\left. + \frac{\beta_h\,\sigma_v\,\mathrm{I}_{hcl0}\,S_{v0}}{N_h} + \frac{\beta_h\,\sigma_v\,\mathrm{I}_{m0}\,S_{v0}}{N_h} + \frac{\beta_h\,\sigma_v\,\mathrm{I}_{lm0}\,S_{v0}}{N_h} - \left(\mu_v + \delta_v\right)\mathrm{I}_{v0}\right)V_{h0}\right)\right)\Bigg)\,t^2$$

$$+ \left(\tau_1\,S_{h0} - \left(\mu_h + \alpha_2\right)V_{h0} - \frac{\beta_m\,\sigma_v\,\mathrm{I}_{v0}\,V_{h0}}{N_v}\right)t + V_{h0}$$

(3.303)

$$\mathrm{I}_{hal} = \mathrm{I}_{hal0} + \mathrm{I}_{hal1} + \mathrm{I}_{hal2}$$

$$
\begin{aligned}
&\frac{1}{4}\,\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_l\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h}\\
&\quad+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h}\Bigg)\Lambda_h\,t^4\Bigg)+\Bigg(\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_l\Bigg(\frac{1}{3}\,I_{v0}\Bigg(\\
&\quad-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,\Lambda_h}{N_v}-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,\Lambda_h}{N_v}-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,\Lambda_h}{N_v}\\
&\quad-\frac{1}{2}\,(\mu_h+\tau_1)\,\Lambda_h\Bigg)+\frac{1}{3}\,\Bigg(\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h}\\
&\quad+\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h}-(\mu_v+\delta_v)\,I_{v0}\Bigg)\Lambda_h+\frac{1}{3}\,\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h}\\
&\quad+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h}\Bigg)S_{h0}\Bigg)\Bigg)-\frac{1}{N_v}\Bigg(\sigma_v(1\\
&\quad-\theta)\,\beta_l\Bigg(\frac{1}{3}\,I_{hal0}\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h}\\
&\quad+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h}\Bigg)+\frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}^2\,\Lambda_h}{N_v}\Bigg)\Bigg)\\
&\quad-\frac{1}{6}\,\frac{(\mu_h+\tau_1+\rho)\,\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,\Lambda_h}{N_v}\Bigg)t^3+\Bigg(\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,\Lambda_h}{N_v}\\
&\quad+\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_l\Bigg(\frac{1}{2}\,I_{v0}\Bigg(-\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v}\\
&\quad-\frac{\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,S_{h0}}{N_v}-(\mu_h+\tau_1)\,S_{h0}+\alpha_1\,T_{h0}-\alpha_2\,V_{h0}\Bigg)+\frac{1}{2}\,\Bigg(\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h}\\
&\quad+\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h}-(\mu_v+\delta_v)\,I_{v0}\Bigg)S_{h0}\Bigg)\Bigg)\\
&\quad-\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_l\Bigg(\frac{1}{2}\,I_{hal0}\Bigg(\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h}\\
&\quad+\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h}-(\mu_v+\delta_v)\,I_{v0}\Bigg)+\frac{1}{2}\,\Bigg(\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v}\\
&\quad-(\mu_h+\tau_1+\rho)\,I_{hal0}\Bigg)I_{v0}\Bigg)\Bigg)-(\mu_h+\tau_1+\rho)\,\Bigg(\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v}\\
&\quad-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v}-\frac{1}{2}\,(\mu_h+\tau_1+\rho)\,I_{hal0}\Bigg)\Bigg)t^2+\Bigg(\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v}\\
&\quad-\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v}-(\mu_h+\tau_1+\rho)\,I_{hal0}\Bigg)t+I_{hal0}
\end{aligned}
\qquad (3.304)
$$

$$I_{hcl}=I_{hcl0}+I_{hcl1}+I_{hcl2}$$

$$\Bigg(\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_l\Bigg(\frac{1}{3}\,I_{hal0}\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h}$$

$$+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h}\Bigg)+\frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}^{2}\,\Lambda_h}{N_v}\Bigg)\Bigg)+\frac{1}{6}\,\frac{\rho\,\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,\Lambda_h}{N_v}\Bigg)t^3$$

$$+\Bigg(\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_l\Bigg(\frac{1}{2}\,I_{hal0}\Bigg(\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h}$$

$$+\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h}-(\mu_v+\delta_v)\,I_{v0}\Bigg)+\frac{1}{2}\Bigg(\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v}$$

$$-(\mu_h+\tau_1+\rho)\,I_{hal0}\Bigg)I_{v0}\Bigg)\Bigg)+\rho\Bigg(\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v}-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v}$$

$$-\frac{1}{2}\,(\mu_h+\tau_1+\rho)\,I_{hal0}\Bigg)-\frac{1}{2}\,(\mu_h+\tau_1)\Bigg(\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v}+\rho\,I_{hal0}-\big(\mu_h$$

$$+\tau_1\big)\,I_{hcl0}\Bigg)\Bigg)t^2+\Bigg(\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v}+\rho\,I_{hal0}-(\mu_h+\tau_1)\,I_{hcl0}\Bigg)t+I_{hcl0}$$

(3.305)

$$I_m=I_{m0}+I_{m1}+I_{m2}$$

$$\frac{1}{4}\,\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_m\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h}$$

$$+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h}\Bigg)\Lambda_h\,t^4\Bigg)+\Bigg(\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_m\Bigg(\frac{1}{3}\,I_{v0}\Bigg($$

$$-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,\Lambda_h}{N_v}-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,\Lambda_h}{N_v}-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,\Lambda_h}{N_v}$$

$$-\frac{1}{2}\,(\mu_h+\tau_1)\,\Lambda_h\Bigg)+\frac{1}{3}\Bigg(\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h}$$

$$+\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h}-(\mu_v+\delta_v)\,I_{v0}\Bigg)\Lambda_h+\frac{1}{3}\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h}$$

$$+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h}\Bigg)S_{h0}\Bigg)\Bigg)$$

$$-\frac{1}{6}\,\frac{(\tau_2+\delta_m+\mu_h)\,\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,\Lambda_h}{N_v}+\frac{1}{N_v}\Bigg(\beta_m\,\sigma_v\Bigg(\frac{1}{6}\,I_{v0}\,\tau_1\,\Lambda_h$$

$$+\frac{1}{3}\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h}$$

$$+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h}\Bigg)V_{h0}\Bigg)\Bigg)\Bigg)t^3$$

$$
\begin{aligned}
&+\left(\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,\Lambda_h}{N_v}+\frac{1}{N_v}\left(\sigma_v(1-\theta)\,\beta_m\left(\frac{1}{2}\,I_{v0}\left(-\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,S_{h0}}{N_v}\right.\right.\right.\right.\\
&\quad-\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,S_{h0}}{N_v}-\left(\mu_h+\tau_1\right)S_{h0}+\alpha_1\,T_{h0}-\alpha_2\,V_{h0}\Bigg)\\
&\quad+\frac{1}{2}\left(\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h}-\left(\mu_v\right.\right.\\
&\quad\left.\left.\left.\left.+\delta_v\right)I_{v0}\right)S_{h0}\right)\Bigg)-\left(\tau_2+\delta_m+\mu_h\right)\left(\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,S_{h0}}{N_v}-\frac{1}{2}\left(\tau_2+\delta_m+\mu_h\right)I_{m0}\right.\\
&\quad\left.+\frac{1}{2}\,\frac{\beta_m\,\sigma_v\,I_{v0}\,V_{h0}}{N_v}\right)+\frac{1}{N_v}\left(\beta_m\,\sigma_v\left(\frac{1}{2}\,I_{v0}\left(\tau_1\,S_{h0}-\left(\mu_h+\alpha_2\right)V_{h0}\right.\right.\right.\\
&\quad\left.-\frac{\beta_m\,\sigma_v\,I_{v0}\,V_{h0}}{N_v}\right)+\frac{1}{2}\left(\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h}\right.\\
&\quad\left.\left.\left.\left.+\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h}-\left(\mu_v+\delta_v\right)I_{v0}\right)V_{h0}\right)\right)\right)t^2+\left(\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,S_{h0}}{N_v}-\left(\tau_2+\delta_m\right.\right.\\
&\quad\left.\left.+\mu_h\right)I_{m0}+\frac{\beta_m\,\sigma_v\,I_{v0}\,V_{h0}}{N_v}\right)t+I_{m0}
\end{aligned}
$$

(3.306)

$$I_{lm}=I_{lm0}+I_{lm1}+I_{lm2}$$

$$
\begin{aligned}
&\frac{1}{4}\,\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_{lm}\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hal0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hcl0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{m0}\,\Lambda_v}{N_h}\\
&\quad+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{lm0}\,\Lambda_v}{N_h}\Bigg)\Lambda_h\,t^4\Bigg)+\Bigg(\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_{lm}\Bigg(\frac{1}{3}\,\mathrm{I}_{v0}\Bigg(\\
&\quad-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_m\,\mathrm{I}_{v0}\,\Lambda_h}{N_v}-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\,\Lambda_h}{N_v}-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\,\Lambda_h}{N_v}\\
&\quad-\frac{1}{2}\,(\mu_h+\tau_1)\,\Lambda_h\Bigg)+\frac{1}{3}\,\Bigg(\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hal0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,\mathrm{I}_{m0}\,S_{v0}}{N_h}\\
&\quad+\frac{\beta_h\,\sigma_v\,\mathrm{I}_{lm0}\,S_{v0}}{N_h}-(\mu_v+\delta_v)\,\mathrm{I}_{v0}\Bigg)\Lambda_h+\frac{1}{3}\,\Bigg(\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hal0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hcl0}\,\Lambda_v}{N_h}\\
&\quad+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{m0}\,\Lambda_v}{N_h}+\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,\mathrm{I}_{lm0}\,\Lambda_v}{N_h}\Bigg)S_{h0}\Bigg)\Bigg)\\
&\quad-\frac{1}{6}\,\frac{(\tau_3+\mu_h+\delta_m)\,\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\,\Lambda_h}{N_v}\Bigg)t^3+\Bigg(\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\,\Lambda_h}{N_v}\\
&\quad+\frac{1}{N_v}\Bigg(\sigma_v(1-\theta)\,\beta_{lm}\Bigg(\frac{1}{2}\,\mathrm{I}_{v0}\Bigg(-\frac{\sigma_v(1-\theta)\,\beta_m\,\mathrm{I}_{v0}\,S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\,S_{h0}}{N_v}\\
&\quad-\frac{\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\,S_{h0}}{N_v}-(\mu_h+\tau_1)\,S_{h0}+\alpha_1\,T_{h0}-\alpha_2\,V_{h0}\Bigg)+\frac{1}{2}\,\Bigg(\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hal0}\,S_{v0}}{N_h}\\
&\quad+\frac{\beta_h\,\sigma_v\,\mathrm{I}_{hcl0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,\mathrm{I}_{m0}\,S_{v0}}{N_h}+\frac{\beta_h\,\sigma_v\,\mathrm{I}_{lm0}\,S_{v0}}{N_h}-(\mu_v+\delta_v)\,\mathrm{I}_{v0}\Bigg)S_{h0}\Bigg)\Bigg)-(\tau_3+\mu_h\\
&\quad+\delta_m)\Bigg(\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\,S_{h0}}{N_v}-\frac{1}{2}\,(\tau_3+\mu_h+\delta_m)\,\mathrm{I}_{lm0}\Bigg)\Bigg)t^2\\
&\quad+\Bigg(\frac{\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\,S_{h0}}{N_v}-(\tau_3+\mu_h+\delta_m)\,\mathrm{I}_{lm0}\Bigg)t+\mathrm{I}_{lm0}
\end{aligned}
\qquad (3.307)
$$

$$\mathrm{T}_h=\mathrm{T}_{h0}+\mathrm{T}_{h1}+\mathrm{T}_{h2}$$

$$\left(\frac{1}{6}\,\frac{\tau_2\,\sigma_v(1-\theta)\,\beta_m\,\mathrm{I}_{v0}\Lambda_h}{N_v}+\frac{1}{6}\,\frac{\tau_1\,\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\Lambda_h}{N_v}+\frac{1}{6}\,\frac{\tau_3\,\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\Lambda_h}{N_v}\right)t^3$$
$$+\left(\tau_2\left(\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_m\,\mathrm{I}_{v0}S_{h0}}{N_v}-\frac{1}{2}\left(\tau_2+\delta_m+\mu_h\right)\mathrm{I}_{m0}+\frac{1}{2}\,\frac{\beta_m\,\sigma_v\,\mathrm{I}_{v0}\,V_{h0}}{N_v}\right)\right.$$
$$+\tau_1\left(\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}S_{h0}}{N_v}-\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\,\mathrm{I}_{hal0}}{N_v}-\frac{1}{2}\left(\mu_h+\tau_1+\rho\right)\mathrm{I}_{hal0}\right)$$
$$+\frac{1}{2}\,\tau_1\left(\frac{\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\,\mathrm{I}_{hal0}}{N_v}+\rho\,\mathrm{I}_{hal0}-\left(\mu_h+\tau_1\right)\mathrm{I}_{hcl0}\right)$$
$$+\tau_3\left(\frac{1}{2}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}S_{h0}}{N_v}-\frac{1}{2}\left(\tau_3+\mu_h+\delta_m\right)\mathrm{I}_{lm0}\right)-\frac{1}{2}\left(\mu_h+\alpha_1\right)\left(\tau_2\,\mathrm{I}_{m0}\right.$$
$$\left.\left.+\tau_1\,\mathrm{I}_{hal0}+\tau_1\,\mathrm{I}_{hcl0}+\tau_3\,\mathrm{I}_{lm0}-\left(\mu_h+\alpha_1\right)T_{h0}\right)\right)t^2+\left(\tau_2\,\mathrm{I}_{m0}+\tau_1\,\mathrm{I}_{hal0}+\tau_1\,\mathrm{I}_{hcl0}\right.$$
$$\left.+\tau_3\,\mathrm{I}_{lm0}-\left(\mu_h+\alpha_1\right)T_{h0}\right)t+T_{h0}$$

(3.308)

$\mathrm{S}_v=\mathrm{S}_{v0}+\mathrm{S}_{v1}+\mathrm{S}_{v2}$

$$\left(-\frac{1}{8}\,\frac{\beta_h\,\sigma_v\,\sigma_v(1-\theta)\,\beta_l\,\mathrm{I}_{v0}\Lambda_h\Lambda_v}{N_hN_v}-\frac{1}{8}\,\frac{\beta_h\,\sigma_v\,\sigma_v(1-\theta)\,\beta_m\,\mathrm{I}_{v0}\Lambda_h\Lambda_v}{N_hN_v}\right.$$
$$\left.-\frac{1}{8}\,\frac{\beta_h\,\sigma_v\,\sigma_v(1-\theta)\,\beta_{lm}\,\mathrm{I}_{v0}\Lambda_h\Lambda_v}{N_hN_v}\right)t^4+$$

$$\left(-\frac{1}{N_h} \left(\beta_h \sigma_v \left(\frac{1}{3} \mathrm{I}_{hal0} \left(-\frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{hal0} \Lambda_v}{N_h} - \frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{hcl0} \Lambda_v}{N_h} - \frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{m0} \Lambda_v}{N_h} \right.\right.\right.\right.$$

$$- \frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{lm0} \Lambda_v}{N_h} - \frac{1}{2} (\mu_v + \delta_v) \Lambda_v \Big) + \frac{1}{3} \left(\frac{\sigma_v (1-\theta) \beta_l \mathrm{I}_{v0} S_{h0}}{N_v} \right.$$

$$- \frac{\sigma_v (1-\theta) \beta_l \mathrm{I}_{v0} \mathrm{I}_{hal0}}{N_v} - (\mu_h + \tau_1 + \rho) \mathrm{I}_{hal0} \Big) \Lambda_v + \frac{1}{6} \frac{\sigma_v (1-\theta) \beta_l \mathrm{I}_{v0} \Lambda_h S_{v0}}{N_v} \Big) \Big)$$

$$- \frac{1}{N_h} \left(\beta_h \sigma_v \left(\frac{1}{3} \mathrm{I}_{hcl0} \left(-\frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{hal0} \Lambda_v}{N_h} - \frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{hcl0} \Lambda_v}{N_h} - \frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{m0} \Lambda_v}{N_h} \right.\right.\right.$$

$$- \frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{lm0} \Lambda_v}{N_h} - \frac{1}{2} (\mu_v + \delta_v) \Lambda_v \Big) + \frac{1}{3} \left(\frac{\sigma_v (1-\theta) \beta_l \mathrm{I}_{v0} \mathrm{I}_{hal0}}{N_v} + \rho \mathrm{I}_{hal0} - (\mu_h \right.$$

$$+ \tau_1) \mathrm{I}_{hcl0} \Big) \Lambda_v \Big) \Big) - \frac{1}{N_h} \left(\beta_h \sigma_v \left(\frac{1}{3} \mathrm{I}_{m0} \left(-\frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{hal0} \Lambda_v}{N_h} - \frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{hcl0} \Lambda_v}{N_h} \right.\right.\right.$$

$$- \frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{m0} \Lambda_v}{N_h} - \frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{lm0} \Lambda_v}{N_h} - \frac{1}{2} (\mu_v + \delta_v) \Lambda_v \Big)$$

$$+ \frac{1}{3} \left(\frac{\sigma_v (1-\theta) \beta_m \mathrm{I}_{v0} S_{h0}}{N_v} - (\tau_2 + \delta_m + \mu_h) \mathrm{I}_{m0} + \frac{\beta_m \sigma_v \mathrm{I}_{v0} V_{h0}}{N_v} \right) \Lambda_v$$

$$+ \frac{1}{6} \frac{\sigma_v (1-\theta) \beta_m \mathrm{I}_{v0} \Lambda_h S_{v0}}{N_v} \Big) \Big) - \frac{1}{N_h} \left(\beta_h \sigma_v \left(\frac{1}{3} \mathrm{I}_{lm0} \left(-\frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{hal0} \Lambda_v}{N_h} \right.\right.\right.$$

$$- \frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{hcl0} \Lambda_v}{N_h} - \frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{m0} \Lambda_v}{N_h} - \frac{1}{2} \frac{\beta_h \sigma_v \mathrm{I}_{lm0} \Lambda_v}{N_h} - \frac{1}{2} (\mu_v + \delta_v) \Lambda_v \Big)$$

$$+ \frac{1}{3} \left(\frac{\sigma_v (1-\theta) \beta_{lm} \mathrm{I}_{v0} S_{h0}}{N_v} - (\tau_3 + \mu_h + \delta_m) \mathrm{I}_{lm0} \right) \Lambda_v$$

$$+ \frac{1}{6} \frac{\sigma_v (1-\theta) \beta_{lm} \mathrm{I}_{v0} \Lambda_h S_{v0}}{N_v} \Big) \Big) - (\mu_v + \delta_v) \left(-\frac{1}{6} \frac{\beta_h \sigma_v \mathrm{I}_{hal0} \Lambda_v}{N_h} - \frac{1}{6} \frac{\beta_h \sigma_v \mathrm{I}_{hcl0} \Lambda_v}{N_h} \right.$$

$$\left. - \frac{1}{6} \frac{\beta_h \sigma_v \mathrm{I}_{m0} \Lambda_v}{N_h} - \frac{1}{6} \frac{\beta_h \sigma_v \mathrm{I}_{lm0} \Lambda_v}{N_h} - \frac{1}{6} (\mu_v + \delta_v) \Lambda_v \right) \Big) t^3$$

$$
\begin{aligned}
&+ \Bigg(-\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h} \\
&- \frac{1}{2}\,(\mu_v + \delta_v)\,\Lambda_v - \frac{1}{N_h}\Bigg(\beta_h\,\sigma_v\Bigg(\frac{1}{2}\,I_{hal0}\Bigg(-\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h} - \frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h} \\
&- \frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h} - \frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h} - (\mu_v + \delta_v)\,S_{v0}\Bigg) + \frac{1}{2}\Bigg(\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v} \\
&- \frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v} - (\mu_h + \tau_1 + \rho)\,I_{hal0}\Bigg)S_{v0}\Bigg)\Bigg) - \frac{1}{N_h}\Bigg(\beta_h\,\sigma_v\Bigg(\frac{1}{2}\,I_{hcl0}\Bigg(\\
&-\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h} - \frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h} - \frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h} - \frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h} - (\mu_v + \delta_v)\,S_{v0}\Bigg) \\
&+ \frac{1}{2}\Bigg(\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v} + \rho\,I_{hal0} - (\mu_h + \tau_1)\,I_{hcl0}\Bigg)S_{v0}\Bigg)\Bigg) \\
&- \frac{1}{N_h}\Bigg(\beta_h\,\sigma_v\Bigg(\frac{1}{2}\,I_{m0}\Bigg(-\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h} - \frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h} - \frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h} \\
&- \frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h} - (\mu_v + \delta_v)\,S_{v0}\Bigg) + \frac{1}{2}\Bigg(\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,S_{h0}}{N_v} - (\tau_2 + \delta_m + \mu_h)\,I_{m0} \\
&+ \frac{\beta_m\,\sigma_v\,I_{v0}\,V_{h0}}{N_v}\Bigg)S_{v0}\Bigg)\Bigg) - \frac{1}{N_h}\Bigg(\beta_h\,\sigma_v\Bigg(\frac{1}{2}\,I_{lm0}\Bigg(-\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h} - \frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h} \\
&- \frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h} - \frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h} - (\mu_v + \delta_v)\,S_{v0}\Bigg) + \frac{1}{2}\Bigg(\frac{\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,S_{h0}}{N_v} - (\tau_3 \\
&+ \mu_h + \delta_m)\,I_{lm0}\Bigg)S_{v0}\Bigg)\Bigg) - (\mu_v + \delta_v)\Bigg(-\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h} \\
&- \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h} - \frac{1}{2}\,(\mu_v + \delta_v)\,S_{v0}\Bigg)\Bigg)\,t^2 + \Bigg(\Lambda_v \\
&- \frac{\beta_h\,\sigma_v\,I_{hal0}\,S_{v0}}{N_h} - \frac{\beta_h\,\sigma_v\,I_{hcl0}\,S_{v0}}{N_h} - \frac{\beta_h\,\sigma_v\,I_{m0}\,S_{v0}}{N_h} - \frac{\beta_h\,\sigma_v\,I_{lm0}\,S_{v0}}{N_h} - (\mu_v + \delta_v)\,S_{v0}\Bigg)\,t \\
&+ S_{v0}
\end{aligned}
\tag{3.309}
$$

$$I_v = I_{v0} + I_{v1} + I_{v2}$$

$$\left(\frac{1}{8}\,\frac{\beta_h\,\sigma_v\,\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,\Lambda_h\,\Lambda_v}{N_h\,N_v} + \frac{1}{8}\,\frac{\beta_h\,\sigma_v\,\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,\Lambda_h\,\Lambda_v}{N_h\,N_v}\right.$$

$$\left. + \frac{1}{8}\,\frac{\beta_h\,\sigma_v\,\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,\Lambda_h\,\Lambda_v}{N_h\,N_v}\right) t^4 + \left(\frac{1}{N_h}\left(\beta_h\,\sigma_v\left(\frac{1}{3}\,I_{hal0}\left(\right.\right.\right.\right.$$

$$-\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h} - \frac{1}{2}\,(\mu_v$$

$$\left. + \delta_v)\,\Lambda_v\right) + \frac{1}{3}\left(\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,S_{h0}}{N_v} - \frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v} - (\mu_h + \tau_1\right.$$

$$\left. + \rho)\,I_{hal0}\right)\Lambda_v + \frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,\Lambda_h\,S_{v0}}{N_v}\Bigg)\Bigg) + \frac{1}{N_h}\left(\beta_h\,\sigma_v\left(\frac{1}{3}\,I_{hcl0}\left(\right.\right.\right.$$

$$-\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h} - \frac{1}{2}\,(\mu_v$$

$$\left. + \delta_v)\,\Lambda_v\right) + \frac{1}{3}\left(\frac{\sigma_v(1-\theta)\,\beta_l\,I_{v0}\,I_{hal0}}{N_v} + \rho\,I_{hal0} - (\mu_h + \tau_1)\,I_{hcl0}\right)\Lambda_v\Bigg)\Bigg)$$

$$+ \frac{1}{N_h}\left(\beta_h\,\sigma_v\left(\frac{1}{3}\,I_{m0}\left(-\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h}\right.\right.\right.$$

$$\left. - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h} - \frac{1}{2}\,(\mu_v + \delta_v)\,\Lambda_v\right) + \frac{1}{3}\left(\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,S_{h0}}{N_v} - (\tau_2 + \delta_m\right.$$

$$\left. + \mu_h)\,I_{m0} + \frac{\beta_m\,\sigma_v\,I_{v0}\,V_{h0}}{N_v}\right)\Lambda_v + \frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_m\,I_{v0}\,\Lambda_h\,S_{v0}}{N_v}\Bigg)\Bigg)$$

$$+ \frac{1}{N_h}\left(\beta_h\,\sigma_v\left(\frac{1}{3}\,I_{lm0}\left(-\frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h} - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h}\right.\right.\right.$$

$$\left. - \frac{1}{2}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h} - \frac{1}{2}\,(\mu_v + \delta_v)\,\Lambda_v\right) + \frac{1}{3}\left(\frac{\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,S_{h0}}{N_v} - (\tau_3 + \mu_h\right.$$

$$\left. + \delta_m)\,I_{lm0}\right)\Lambda_v + \frac{1}{6}\,\frac{\sigma_v(1-\theta)\,\beta_{lm}\,I_{v0}\,\Lambda_h\,S_{v0}}{N_v}\Bigg)\Bigg) - (\mu_v + \delta_v)\left(\frac{1}{6}\,\frac{\beta_h\,\sigma_v\,I_{hal0}\,\Lambda_v}{N_h}\right.$$

$$\left. + \frac{1}{6}\,\frac{\beta_h\,\sigma_v\,I_{hcl0}\,\Lambda_v}{N_h} + \frac{1}{6}\,\frac{\beta_h\,\sigma_v\,I_{m0}\,\Lambda_v}{N_h} + \frac{1}{6}\,\frac{\beta_h\,\sigma_v\,I_{lm0}\,\Lambda_v}{N_h}\right)\Bigg) t^3$$

$$
+\left(\frac{1}{2}\frac{\beta_h\sigma_v I_{hal0}\Lambda_v}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v I_{hcl0}\Lambda_v}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v I_{m0}\Lambda_v}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v I_{lm0}\Lambda_v}{N_h}\right.
$$

$$
+\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{2}I_{hal0}\left(-\frac{\beta_h\sigma_v I_{hal0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v I_{hcl0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v I_{m0}S_{v0}}{N_h}\right.\right.\right.
$$

$$
\left.-\frac{\beta_h\sigma_v I_{lm0}S_{v0}}{N_h}-\left(\mu_v+\delta_v\right)S_{v0}\right)+\frac{1}{2}\left(\frac{\sigma_v(1-\theta)\beta_l I_{v0}S_{h0}}{N_v}-\frac{\sigma_v(1-\theta)\beta_l I_{v0}I_{hal0}}{N_v}\right.
$$

$$
\left.\left.\left.-\left(\mu_h+\tau_1+\rho\right)I_{hal0}\right)S_{v0}\right)\right)+\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{2}I_{hcl0}\left(-\frac{\beta_h\sigma_v I_{hal0}S_{v0}}{N_h}\right.\right.\right.
$$

$$
\left.-\frac{\beta_h\sigma_v I_{hcl0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v I_{m0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v I_{lm0}S_{v0}}{N_h}-\left(\mu_v+\delta_v\right)S_{v0}\right)
$$

$$
\left.\left.+\frac{1}{2}\left(\frac{\sigma_v(1-\theta)\beta_l I_{v0}I_{hal0}}{N_v}+\rho I_{hal0}-\left(\mu_h+\tau_1\right)I_{hcl0}\right)S_{v0}\right)\right)
$$

$$
+\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{2}I_{m0}\left(-\frac{\beta_h\sigma_v I_{hal0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v I_{hcl0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v I_{m0}S_{v0}}{N_h}\right.\right.\right.
$$

$$
\left.-\frac{\beta_h\sigma_v I_{lm0}S_{v0}}{N_h}-\left(\mu_v+\delta_v\right)S_{v0}\right)+\frac{1}{2}\left(\frac{\sigma_v(1-\theta)\beta_m I_{v0}S_{h0}}{N_v}-\left(\tau_2+\delta_m+\mu_h\right)I_{m0}\right.
$$

$$
\left.\left.\left.+\frac{\beta_m\sigma_v I_{v0}V_{h0}}{N_v}\right)S_{v0}\right)\right)+\frac{1}{N_h}\left(\beta_h\sigma_v\left(\frac{1}{2}I_{lm0}\left(-\frac{\beta_h\sigma_v I_{hal0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v I_{hcl0}S_{v0}}{N_h}\right.\right.\right.
$$

$$
\left.-\frac{\beta_h\sigma_v I_{m0}S_{v0}}{N_h}-\frac{\beta_h\sigma_v I_{lm0}S_{v0}}{N_h}-\left(\mu_v+\delta_v\right)S_{v0}\right)+\frac{1}{2}\left(\frac{\sigma_v(1-\theta)\beta_{lm}I_{v0}S_{h0}}{N_v}-\left(\tau_3\right.\right.
$$

$$
\left.\left.\left.\left.+\mu_h+\delta_m\right)I_{lm0}\right)S_{v0}\right)\right)-\left(\mu_v+\delta_v\right)\left(\frac{1}{2}\frac{\beta_h\sigma_v I_{hal0}S_{v0}}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v I_{hcl0}S_{v0}}{N_h}\right.
$$

$$
\left.\left.+\frac{1}{2}\frac{\beta_h\sigma_v I_{m0}S_{v0}}{N_h}+\frac{1}{2}\frac{\beta_h\sigma_v I_{lm0}S_{v0}}{N_h}-\frac{1}{2}\left(\mu_v+\delta_v\right)I_{v0}\right)\right)t^2+\left(\frac{\beta_h\sigma_v I_{hal0}S_{v0}}{N_h}\right.
$$

$$
\left.+\frac{\beta_h\sigma_v I_{hcl0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{m0}S_{v0}}{N_h}+\frac{\beta_h\sigma_v I_{lm0}S_{v0}}{N_h}-\left(\mu_v+\delta_v\right)I_{v0}\right)t+I_{v0}
$$

(3.310)

CHAPTER FOUR

4.0 RESULTS AND DISCUSSION

4.1 Variable and Parameter Values Estimation

Table 4.1 Values for Population-Dependent Parameter of the Model

Variables	Values	Source
S_h	200	Assumed
V_h	180	Assumed
I_{hal}	120	Assumed
I_{hcl}	100	Assumed
I_m	80	Assumed
I_{lm}	100	Assumed
T_h	50	Assumed
S_v	500	Assumed
I_v	300	Asummed
N_h	750	Assumed
N_v	800	Assumed

Table 4.2 Values for Population-Independent Parameter of the Model

Parameter	Values	Source
β_h	0.09	(Chunky Choole,2012)
Λ_v	0.071	(Gweryina Reuben, 2014)
Λ_h	0.3	Assumed

μ_v	0.05	(Gweryina Reuben, 2014)
μ_h	0.017	(Gweryina Reuben, 2014)
β_{lm}	0.8333	(Chunky Choole,2012)
σ_v	0.125	(Lawi *et al*, 2011)
Θ	0.25	Assumed
δ_m	0.00049312	(Lawi *et al*, 2011)
δ_v	0.25	Assumed
P	0.00002797	(Bhunu and Mushayabasa, 2012)
τ_1	0.25	Assumed
τ_2	0.25	Assumed
τ_3	0.25	Assumed
α_1	0.25	Assumed
α_2	0.25	Assumed

4.2 Sensitivity Analysis for the Parameter Using Basic Reproductive Number

Sensitivity Analysis (SA) brings out the importance of the model parameter by exposing their relative effects or impact in the model of LF and Malaria co-infection. It provides an appropriate signal towards a suitable and timely intervention in curtailing the transmission of LF and Malaria co-infection. According to Powell *et al* (2005), sensitivity analysis is commonly used to determine the robustness of the model predictions to parameter values.

Sensitivity indices measures the relative changes in a variable when a parameter changes. Arriola and Hyman (2007); Chinis *et al*. (2008); Mikuchi *et al* (2012) and Abdurrahman *et al*. (2013) as it was used was what we adopt for the sensitivity analysis of this study. the normalized forward sensitivity index of the system is given by

$$S_{\psi}^{R_0} = \frac{\partial R_0}{\partial \psi} x \frac{\psi}{R_c} \text{ where } \psi \in Q = \left\{\beta_h, \beta_l, \beta_m, \beta_{lm,} \sigma_v, \tau_1, \tau_2, \tau_3, \alpha_1, \alpha_2, \rho, \theta, \delta_m, \mu_h\right\}$$

and R_c is the basic reproductive number. The values of the parameter are obtained from table 4.2, the sensitivity indices of the parameters of the basic reproductive number are calculated using Maple 15 software.

Table 4.3 Sensitivity Indices for the Respective Parameter Using the Basic Reproductive Number (R_c)

Parameter	Sensitivity Index
σ_v	1.000000000
τ_1	-0.8901600321
β_h	0.5000000000
α_2	0.4681647939
τ_2	-0.4290750780
β_{lm}	0.2358260585
β_m	0.2358260585
μ_h	0.1272809744
τ_3	0.03753791419
β_l	0.02834788317
δ_m	0.0008876188246
ρ	0.00005237279077
θ	-0.1666666667
δ_v	-0.4166666668
μ_v	-0.0833333334

From table 4.3, sensitivity indices of the parameters either has positive or negative values on the basic reproductive number R_0.. if any of the parameters with the positive sensitivity indices is high then the basic reproductive number will be high, but reduction in any any of the parameter will reduce the basic reproductive number. the parameters with negative sensitivity indices will decrease the basic reproductive number if it is high but increase the basic reproductive number if its low.

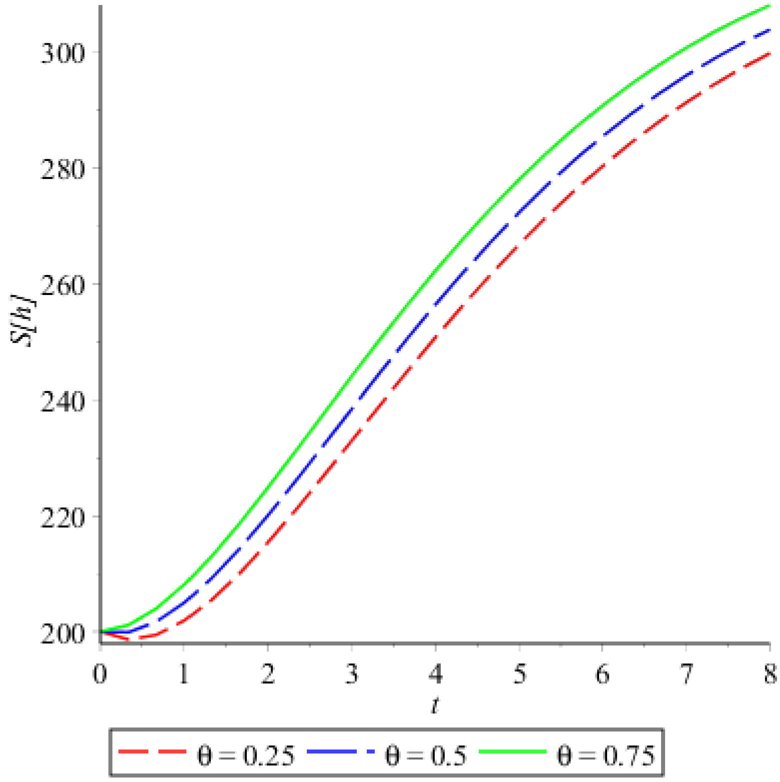

Figure 4.1 Effect of the rate of the use of bed-net and insecticides on the Susceptible population

Figure 4.1 shows the relationship between the Susceptible population who are using both bed-net and insecticides (θ), this means that S_h increases with the increase in the rate at with both bed-net and insecticides are used.

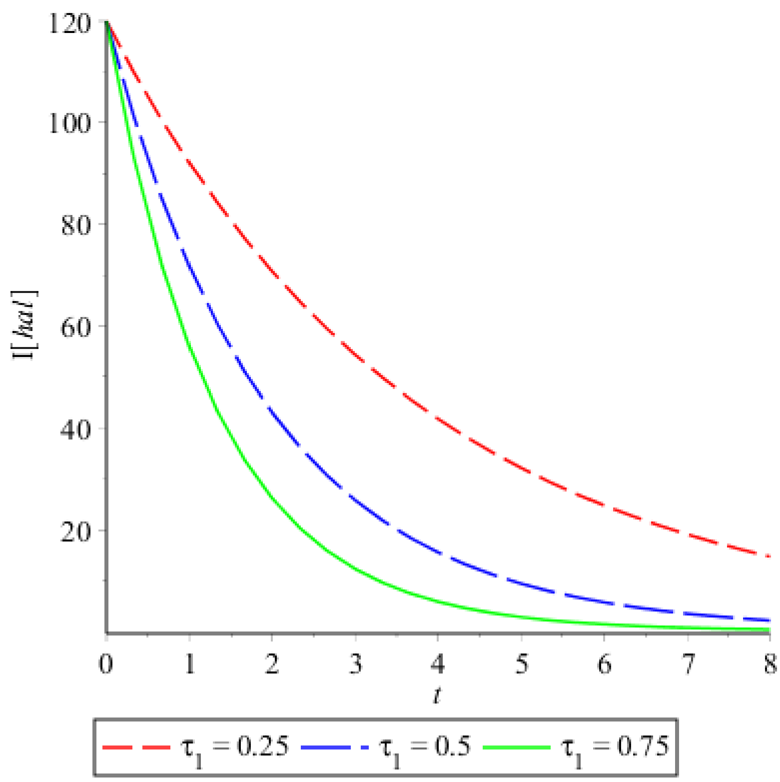

Figure 4.2 Effect of treatment of LF on the Acute stage LF population

Figure 4.2 shows the relationship between those infected with LF who are undergoing LF treatment (τ_1), the results shows that there is a high recovery rate as the treatment rate increases since the Acute Stage infected LF population reduces with increase in τ_1.

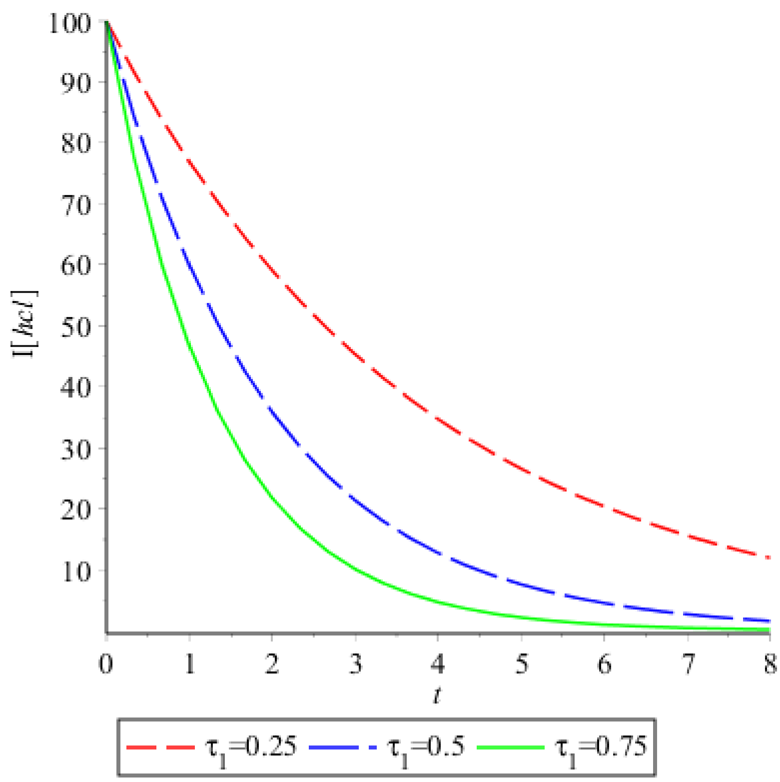

Figure 4.3: Effect of treatment of LF on the Chronic stage LF population

Figure 4.3 shows the relationship between those infected with LF who are undergoing LF treatment (τ_1), the results shows that there is a high recovery rate as the treatment rate increases as the Chronic Stage infected LF population reduces .

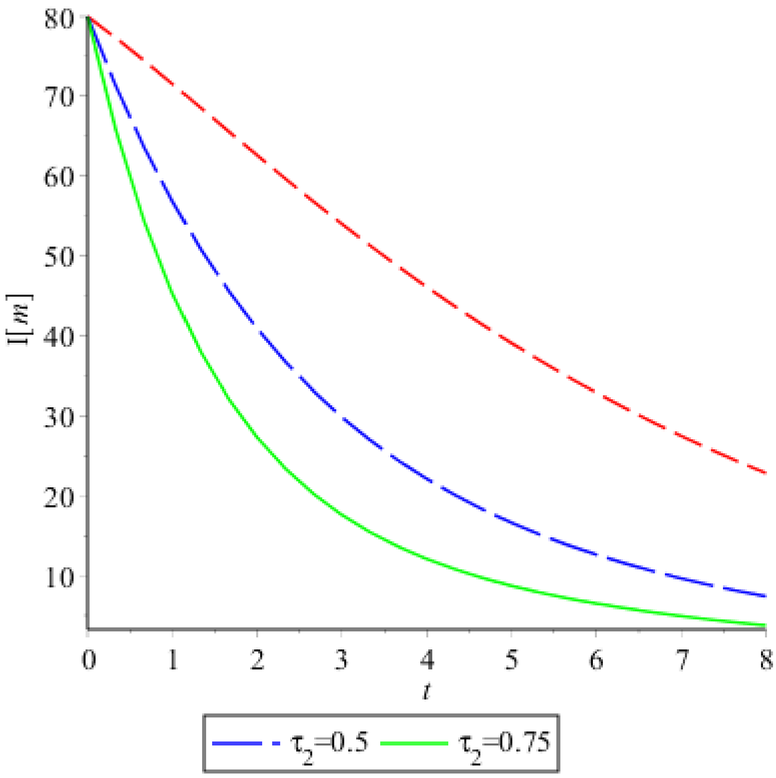

Figure 4.4 Effect of treatment of Malaria on those who are infected malaria

Figure 4.4 shows the relationship between those infected with malaria who are undergoing Malaria treatment (τ_2), the results shows that there is a high recovery rate as the treatment rate increases as the infected Malaria population reduces.

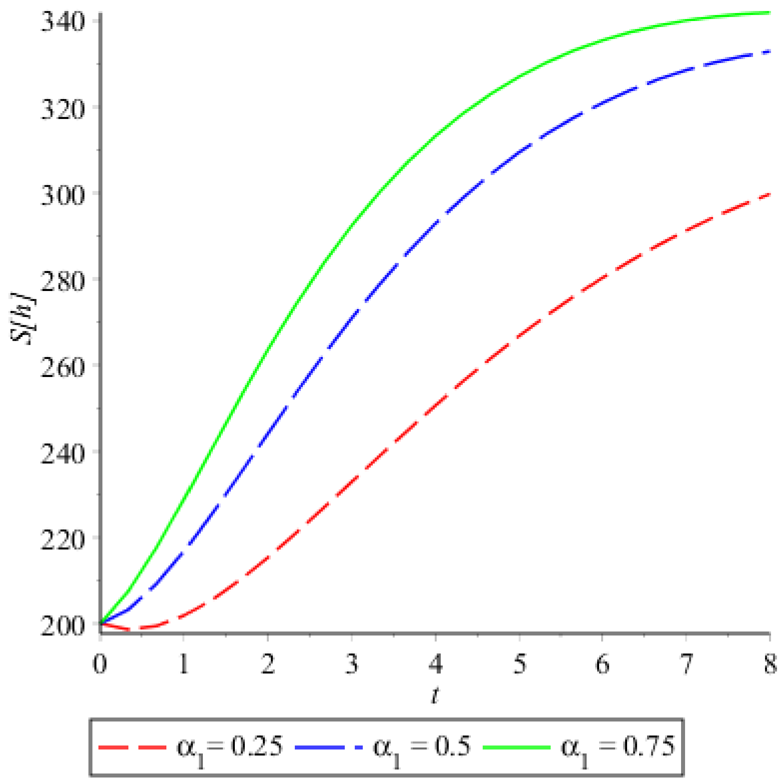

Figure 4.5 The Progression rate at which Malaria, LF and co-infected fully recovered Human move to Susceptible

Figure 4.5 The results shows that as the rate of those who are fully recovered (α_1) increases, the Susceptible population also increase.

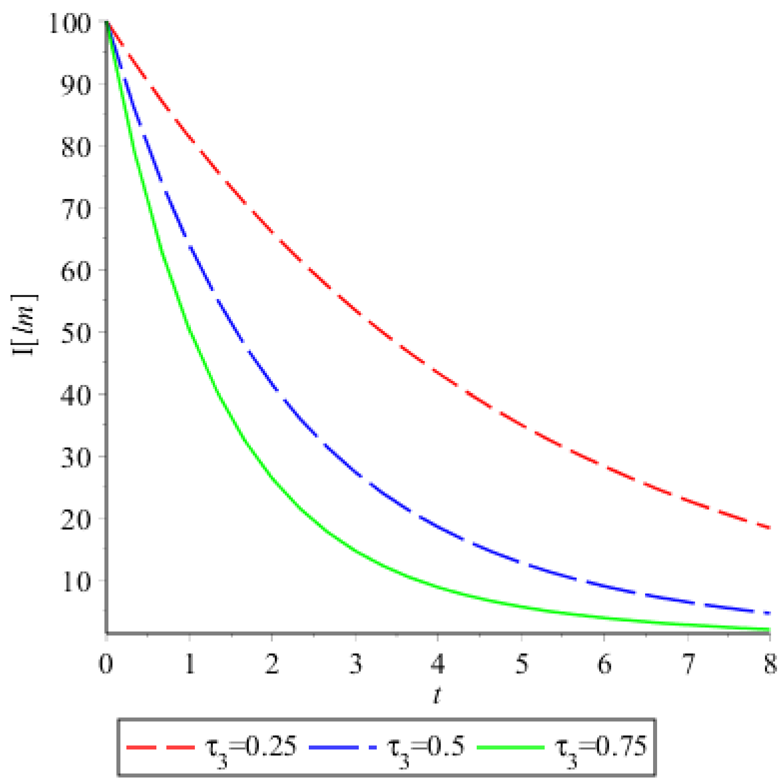

Figure 4.6 The Effect of the rate of treatment of LF and malaria co-infection

Figure 4.6 shows the relationship between those infected with LF and malaria who are undergoing treatment (τ_3), the results shows that there is a high recovery rate as the treatment rate increases as the infected LF and Malaria population reduces.

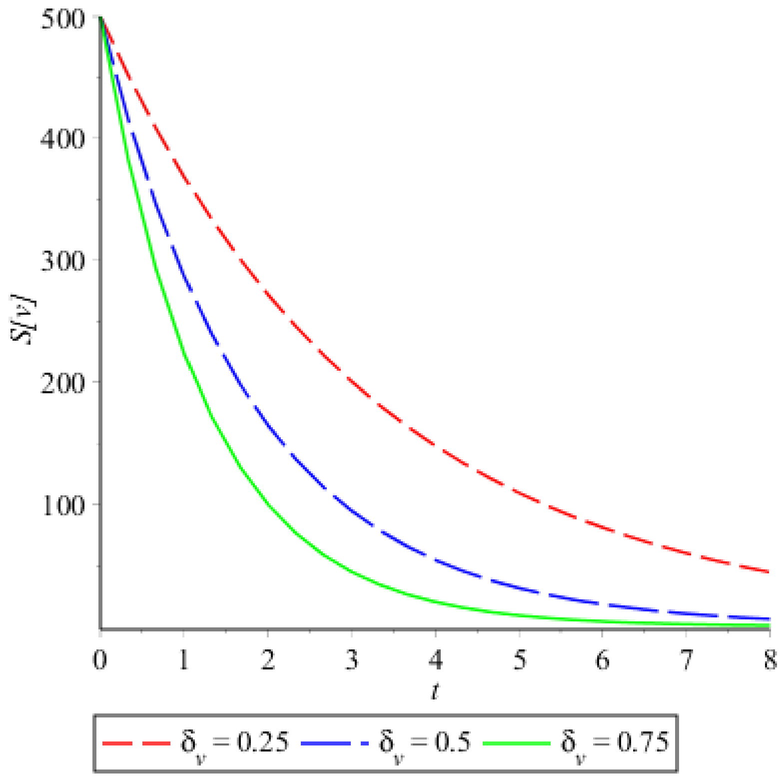

Figure 4.7 Effect of the use of insecticide on the Susceptible Vector Population

Figure 4.7 shows the relationship of using insecticide (δ_v) on the Susceptible vector population (S_v), the results shows that as the use of insecticides increases, there's reduction in the S_v population.

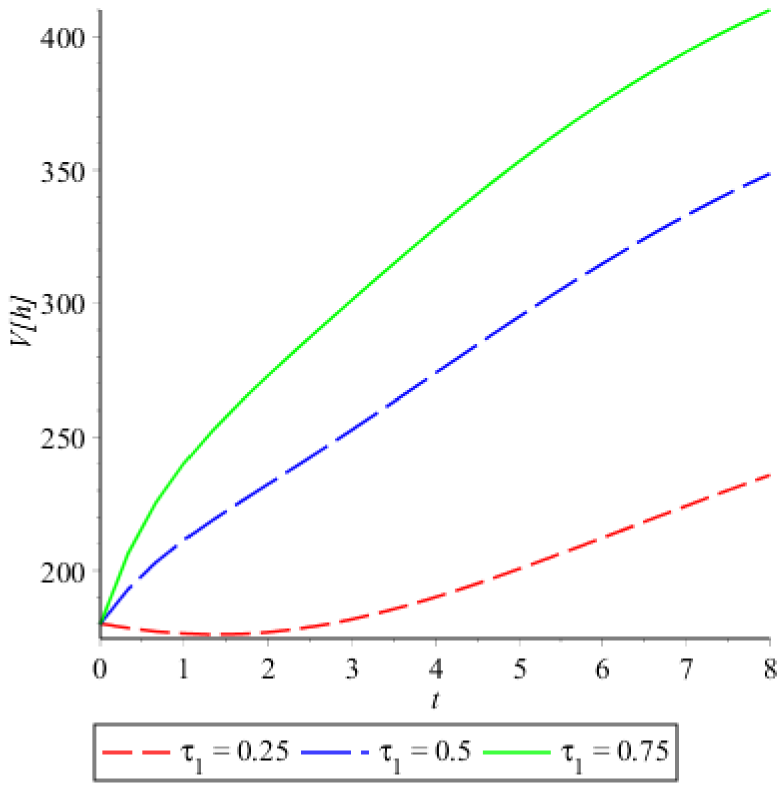

Figure 4.8 Effect of Treatment on the Chemoprevention class (V_h)

Figure 4.8 shows the relationship between the V_h who are taking drugs (τ_1), the results shows that as Treatment rate increases, there's increase in the V_h population.

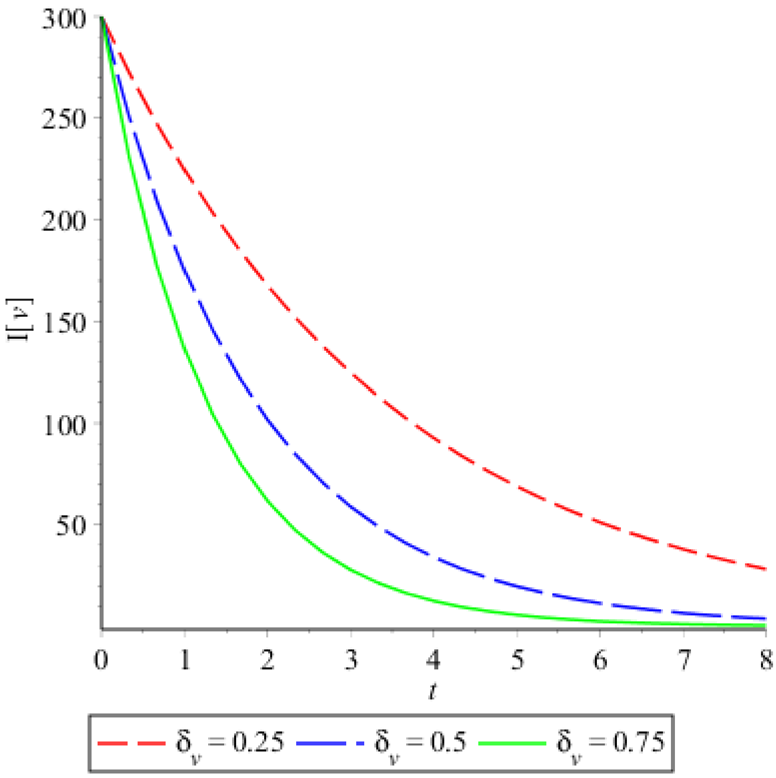

Figure 4.9 Effect of the use of insecticide on the Infected Vector Population (I_v)

Figure 4.9 shows the relationship of using insecticide (δ_v) on the infected vector population (I_v), the results shows that as the use of insecticides increases, there's reduction in the I_v population.

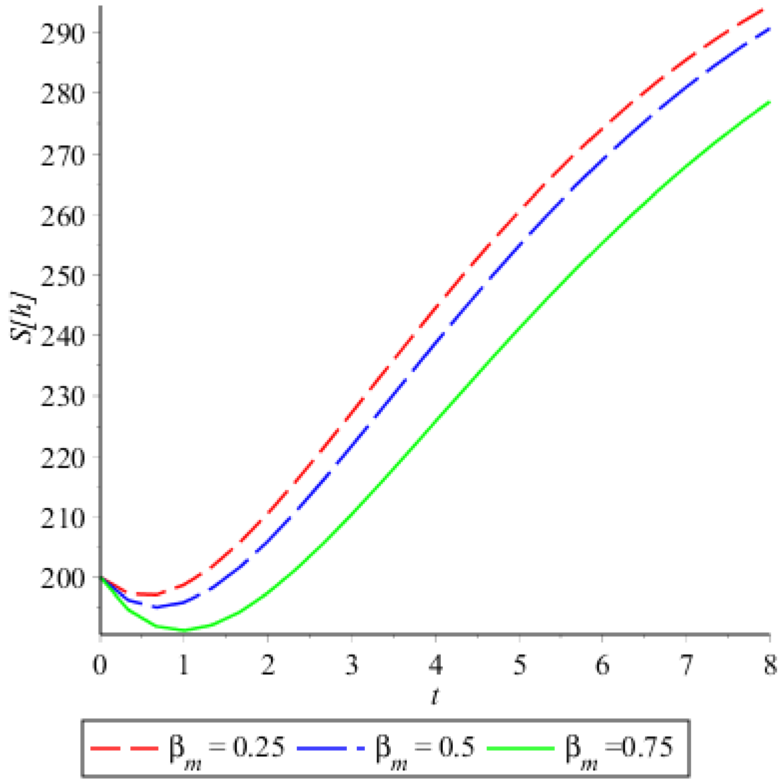

Figure 4.10: Effect of probability that a bite by an infected mosquito will transfer malaria to the susceptible human population

Figure 4.10 show the relationship of the infectivity of the mosquito, that define the probability that a bite by an infected mosquito on a susceptible human will transfer malaria infection to the Human. This implies that the more an infected mosquito bites a susceptible human, it transfer malaria thereby causing them to be infected and thus reducing the Susceptible Human population S_h .

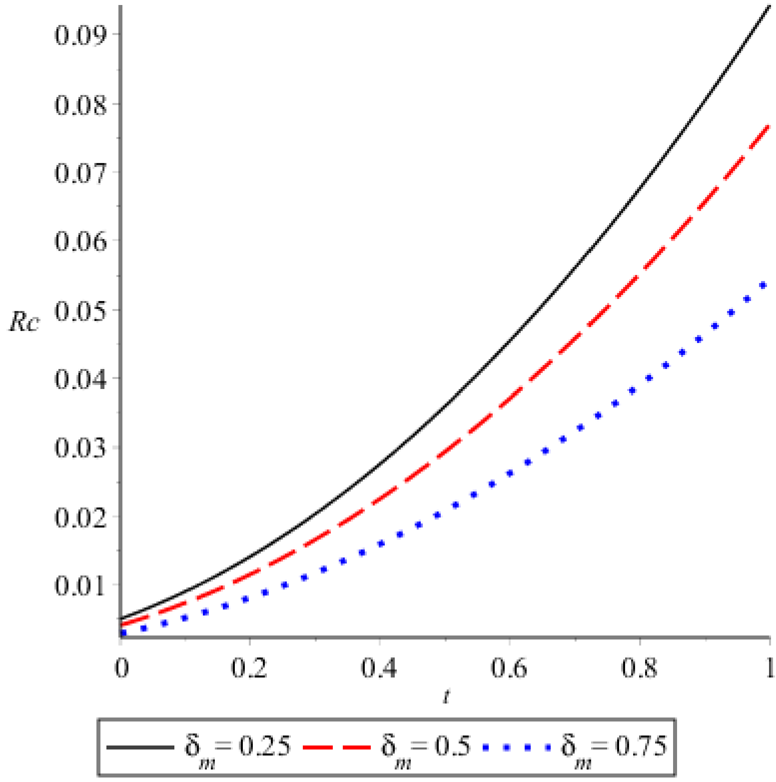

Figure 4.11: Effect of death rate of mosquito on a on the effective reproductive number R_c

Figure 4.11 shows the relationship between the effective reproduction number and the death rate of the mosquito. This means R_c decreases with increasing death rate of mosquitoes.

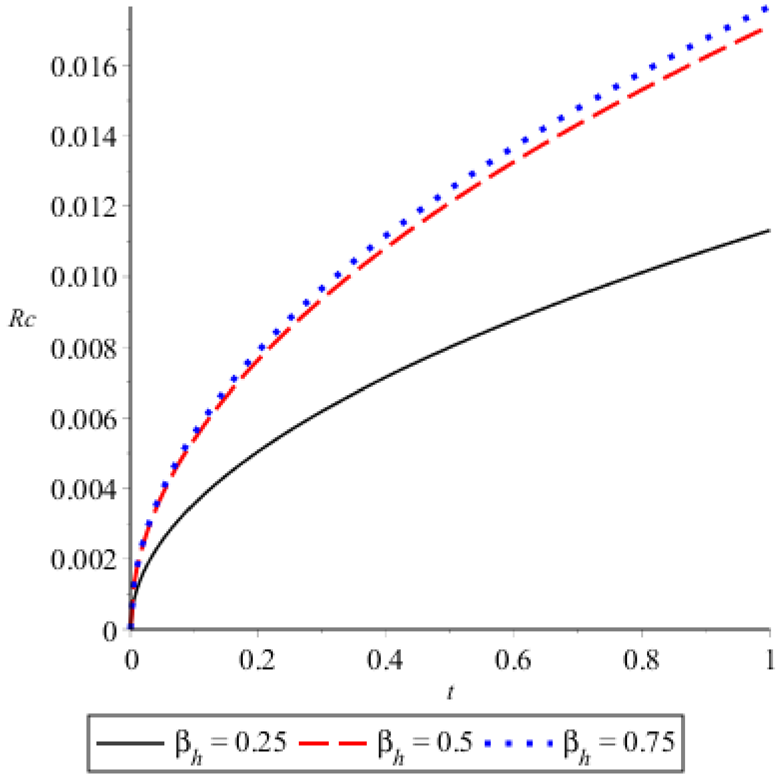

Figure 4.12: The rate at which the mosquito ingest microfilariae, malaria or both when biting a human on the effective reproductive number R_c

This shows the relationship between R_c and the rate at which the mosqitoes ingest microfilariae, malaria or both on the human who is infected β_h . this means the R_c increases with increasing in the rate at which the mosquitoes ingest microfilariae, malaria and co infection on human who is infected.

CHAPTER FIVE

5.0 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

In this study, the mathematical model of Lymphatic Filariasis and malaria co-infection was developed using a system of first order differential equations. The Positivity of the solution was obtained Using Lungu method. The Disease-Free Equilibrium state (DFE) was obtained. The effective reproduction number R_c of the model was obtained. The Disease-Free Equilibrium (DFE) was analyse for local and global stability. The result from the analysis of the DFE showed that, the DFE is locally asymptotically stable and globally asymptotically stable if $R_c < 1$. Sensitivity analysis was also conducted on the effective reproductive number. The model equations were solved using Adomian Decomposition Method (ADM). Graphical profiles were obtained from the solution of the model using Maple15.

Variables and parameters were used for analytical solution. The solutions of the model were presented graphically in order to have a better understanding of the model. Figures 4.1 to 4.9 are the different graph of the solution using the populations of Vector and human against time with different parameters of the model. Maple 15 software was used for the graphical.

Figure 4.1 shows that the high usage of bed-net and insecticide, the susceptible population increases due to the reduction in the contact rate β with Mosquitoes. Figure 4.2-4.5, shows a reduction in the population of the infected classes respectively when treated of either Malaria, LF or both LF and malaria co-infection. Figure 4.6 and 4.9 shows the effects of using insecticide (δ_v), the results shows that as the use of insecticides increases, there's reduction in the vector population both the Susceptible and infected classes respectively.

Figure 4.8 shows the relationship between the I_v who are using bed-net and insecticide (δ_v), the results shows that as the use of bed-net and insecticides increases, there's little reduction in the I_v population.

5.2 Contributions to Knowledge

i. This work has improved on the existing models of Lymphatic Filariasis and Malaria co-infection by incorporating the Acute stage and Chronic stage class with the use of both bed-net and insecticide as control measure.

ii. The system of nine ordinary differential equation was solved and validated with Adomian decomposition method in Maple software.

iii. This work has shown a possibility of a disease free equilibrium which can be globally asymptotically stable.

iv. This work has been able to establish how the infection rate (β_h) below which LF and malaria co infection can be put under control in the population.

5.3 Recommendations

Based on the findings from our study, the following recommendations were made:

i. Diethylcarbamazine(τ_1) drugs should be made affordable and accessible.

ii. Bed-net and insecticides should readily available and accessible at little or no cost.

iii. People should always consult a Doctor and not resolve into Self-medication so as to know what treatment should be carried when they see any symptoms of either malaria or LF.

REFERENCES

Abdulrahman, S., Akinwande, N. I., Awojoyegbe, O. B. & Abubakar U. Y. (2013). Sensitivity analysis of the parameters of a mathematical model of Hepatitis B virus transmission. *Universal Journal of Applied Mathematics*, 1(4), 230-240.

Anosike J. C., Nwoke B. E., Ajayi E. G., Onwuliri C. O., Okoro O. U., Oku E. E., Asor J. E., Amajuoyi O. U., Ikpeama C. A., Ogbusu F. I. & Meribe, C. O. (2005). Lymphatic filariasis among the Ezza of Ebonyi State, Eastern Nigerian. *Ann. Agric. Med*, 12(3), 181-186.

Arriola, I. & Hyman, J. (2007). Forward and adjoint sensitivity analysis with applications in dynamical system, Lecture Notes in Linear Algebra and Optimization.

Balbir S., Lee K.S., Asmand M. & Anand R. (2004). A large focus of naturally acquired Plasmodium knowlesi infections in human being, The lancet, 363(9414) 1017-1024

Bhunu C. P. & Mushayabasa S. (2012). Transmission Dynamics of Lymphatic Filariasis: A Mathematical Approach. *International Scholarly Research Network, Biomathematics*, 3(2), 19 – 76.

Biazar, J. & Babalion, M. R. (2005). A computational method for solution of the Prey and predator problem. *Applied mathematics and Computer*, 1(3), 841-847.

Bupa U. K. (2009). Malaria symptoms, causes and treatment of diseases, http: hcd2. Bupa.co.uk/fact sheets/html/malaria. Html. Accessed on 20th April, 2011.

Burke A. C. (2010). Infectious Diseases in Critical Care Medicine, Information Health Care. New York: Inc Publishers.

Castillo-Chavez, C., Feng Z. & Huang W. (2007). On the computation of R_0 and its role on global stability. Mathematical approaches for Emerging & Reemerging Infectious Disease. *The IMA volumes in Mathematics and its Applications*, 125, 229-250

Center for Disease Control and Prevention (CDC) (2010). Lymphatic Filariasis FAQs. May 20th 2010. ww.cdc.gov/parasite/lymphatic filariasis/

Center for Disease Control and Prevention (CDC) (2011). Lymphatic Filariasis FAQs. May 20th 2010. ww.cdc.gov/parasite/lymphatic filariasis/

Centre for Disease Control (CDC) (2007). World Wide Web http:www. cdc. gov/malaria/facts. htm Retrieved on 22nd April 2011.

Centre for Disease Control (CDC) (2019). World Wide Web http:www. cdc. gov/malaria/facts. htm Retrieved on 22nd April 2011.

Chitnis, N., Hyman, J. M. & Cushing, J. M. (2008). Determining Iportant Parameters in the spread of Malaria through the Sensitivity Analysis of a mathematical Model. *Bulletin of Mathematical Biology*, 34, 123- 139.

Chunky, C. K. (2012). Modelling the transmission dynamics of HIV/AIDS Malaria coinfection with treatment. M.sc (mathematical modelling) Dissertation, University of Dares Salaam.

Dickmann, O. & Heesterbeek, J. A. P. (2000). Mathematical Epidemology of infectious Diseases: Model Building, Analysis and Interpretation. *Mathematical and Computational Biology*, 13, 289 – 301.

Dickmann, O., Heesterbeek, J.A.P. & Melz, J.A.J (1990). Mathematical Epidemology of infectious Dieases and the computation of the Basic Reproductive Number R_0 in models of infectious Diseases in Hetrogeneous Populations. *Journal of Mathematical Biology*, 28, 365-38.

Dreyer, G., Noroes, J., Figueredo-Silva, J. & Piessens, W. F. (2000). Pathogenesis of Lymphatic Disease In Bancroftian Filariasis A Clinical Perspective. Parasitology Today, 544-548

El-Kalla, I. L. (2008). Convergence of Adomian method applied to a class of nonlinear integral equations, Applied Mathematics Letter, Physics and Mathematics Department, Faculty of Engineering, Mansoura University, P.O. 35516 Mansoura, Egypt.

Expeditho, M., Herieth, R. & Jean, M. T. (2009). A Mathematical Analysis of Malaria and Tuberculosis co dynamics. *AIMS*, 12(4), 827-864.

Friedman, A. & Longu, E. N. (2013). The positivity of a Mathematical Model. *Mathematical Biosciences and Engineering*, 10(3), 609 – 624.

Gweryina, R. I. & Tersoo, L. (2014). Modelling the Potential Impact of Chemotherapy and Treatment on the Dynamics of Malaria-Filariasis Co-infection. Unpublished Thesis. Department of Mathematics/Statistics/Computer Science, University of Agriculture, PMB 2373, Makurdi, Benue State, Nigeria.

Haddix, A. C. & Kestler, A. (2000). Elimnation of lymphatic filariasis: economic aspects of the disease and programmes for its elimination. *Royal Society of Tropical medicine and Hygience*, 94, 592-593.

Hannah, S.C., Manoj, G., Paul, P. E. & Edwin, M. (2013). Modelling co-infection with malaria and lymphatic filariasis, Department of Infectious Disease Epidemiology, Imperial College London, London, United Kingdom.

Hay, S. I., Guerra, C. A. Tatem, A. J. Noor, A. M. & Snow, A. M. (2004) Global distribution and population at risk of malaria past, present and future, Lancet infectious diseases, 4(6), 327 - 336.

Hethcote, W. (2000). The Mathematics of infectious Disease. *SIAM Review*, 42(4), 599-653.

Hiroyuki, I., Hirofumi, I. & Yukio, O. (2002). Mathematical Model for the Treatment of LymphaticFilariasis and its Application. *Journal of the facility of Environmental Science and technology*, 7(1), 7-16.

Lawi, G. O., Mugisha, J. Y. T. & Nomola, O. (2011). Mathematical model for malaria and Meningitis co-infection among children. *Applied Mathematical Science*, 5(47), 2337-3239

Lenhart, A., Eigege, A., Kal, A., Pam, D., Miri, E. S., Gerlong G., Oneyka J., Sambo Y., Danboyi J., Ibraham. B, E. Dahi, D. Kumbak, A. Dakul, M.Y. Jinadu, J. Umaru, F.O. Richards & Lehmann T. (2007). Contributions of different mosquito species to the transmission of lymphatic filariasis in central Nigeria: Implications for monitoring Infection by PCR mosquito pools. *Filaria. Journal*, 6(3), 75-128.

Li-Ming, C., AbidAliLashari, I., HyoJung, K., Oare, O. & Young, S. (2012). Mathematical Analysis of a Malaria Model with Partial Immunity to Reinfection. *Applied Mathematical Science*, 5(47), 237-359

Mikuchi, M. A. (2012). Sensitivity analysis of the basic reproduction number and other qualities for infectious disease models. An Unpublsihed M.Sc. thesis, Colarado state University Fort Collins, Colorado, Spring.

Murray, J. D. (2002). Mathematical Biology: An Introduction. New York: Springer. 1-576.

Norman, R. A., Chan, M.S. & Srividya, A. (2000). The development of an age structure model for describing the transmission dynamics and control of lymphatic filariasis Epidemiology and infection. *EPIFIL*, 124(3), 529-541.

Oguntolu F. A. (2019). Mathematical Modelling for the Transmission Dynamics of Lymphatic Filariasis. Unpublished Thesis, Federal University of Technology Minna, Nigeria.

Ojiako O. A. & Onyeze G. O. C. (2009). Epidemiological and biochemical studies of human lymphatic filariasis and associated parasitoses in Oguta, South-Eastern, Nigeria. *African Journal of Biochemistry Research*, 3(4), 114-119.

Olumuyiwa, J. P. & Hammed, A. O. (2019). Malaria Mathematical Model Incorporating Four Control Strategies: Insecticide-Treated Bednets Control, Unpublished Thesis Department of Mathematics/ Statistics / Computer Science, University of Ilorin, Kwara State.

Oluwatayo, M. O. & Valery, C. (2019). Local and global asymptotic behavior of malaria-filariasis co-infections in compliant and non-compliant susceptible pregnant women to antenatal medical program in the tropics. *e-Journal of Analysis and Applied Mathematics*, 2(4), 31–54.

Ottesen E. A. (1984). Filariasis & tropical eosinopliia In: Tropical and Geographical Medicine, Warrenks, Mahmoud A A F (eds), New York: McGraw-Hill Inc. 390-422.

Powell, D. R., Fair, J., LeClaire, R. J., Moore, L. M. & Thompson, D. (2005). Sensitivity analysis for an infectious disease model. International System Dynamics Conference, Boston, MA.

Subramainian, S. & Das, P. K. (1998). Modeling the epidemiology, transmission and control of lyphatic filariasis. *Annals of Tropical Medicine and Parasitology,* 96, 153-164.

Supriatna, A. K., Serviana, H. & Soewono, E. (2009). A mathematical model to investigate the Long-Term Effects of the Lymphatic filariasis Medical Treatment in Jati Samurna, West Java. *ITB Journal of Science*, 41(1), 1-14.

Swaminathan, S., Pani, S. P., Ravi, R., Krishnamoorthy, K. & Das, P. K. (2008). Mathematical models for lymphatic filariasis transmission and control: challenges and prospects. *Mode Parasites and Vectors*, 1(2), 121 – 431.

Tan, J. Z. G. (2003). The elimination of Lymphatic filariasis, a strategy for poverty alleviation and sustainable development perspectives, the Philippines. *Filaria Journal*, 2(12), 329 – 334.

Traore, B., Boureima, S. & Sado, T. (2017). A Mathematical Model of Malaria Transmission with structured vector population. *Journal of applied mathematics*, 6(7), 54 – 67.

Van Den, D. P. & Watmough J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical Biosciences*, 180, 29–48.

Van Den, D. P., Wang, L. & Zou, X. (2007). Modeling disease with latency and relapse. *Mathematical Biosciences and Engineering*, 4(2), 205-219.

Webber R. H. (1977). The natural decline of Wuchcreria banocrofti infection in a vector control situation in the Solomon Islands. *Royal Society of Tropical Medicine and Hygiene,* 71(2), 396-400

Whitty C. J., Lalloo D. M., Ustianowski A. (2007). Malaria, an update on treatment of adults in nonendemic Countria. *BMJ*, 333, 214-245

World Health Organization (WHO) (2005) Malaria and HIV interactions and the implications for public health policy, WHO, Geneva, Switzerland.

World Health Organization, (1993). World Health Organization Expert Committee on Filariasis, Lymphatic filariasis the disease and its control' fiftieth report of the World Health Organization Technical report series 821- 871.

APPENDIX

```
restart;
S[h0] := 200;
V[h0] := 180;
Z[hal0] := 120;
Z[hcl0] := 100;
Z[m0] := 80;
Z[lm0] := 100;
T[h0] := 50;
S[v0] := 500;
Z[v0] := 300;
beta[h] := 0.9e-1;
beta[v] := .5;
beta[m] := .8333;
beta[l] := .1;
beta[lm] := .8333;
N[h] := 145;
N[v] := 350;
Lambda[v] := 0.71e-1;
sigma[v] := .125;
mu[v] := 0.5e-1;
mu[h] := 0.17e-1;
delta[m] := 0.449312e-3;
Lambda[h] := .5;
delta[v] := .5;
k[1] := mu[h]+tau[1];
k[2] := mu[h]+alpha[2];
k[3] := mu[h]+tau[1]+rho;
k[4] := tau[2]+delta[m]+mu[h];
k[5] := tau[3]+mu[h]+delta[m];
k[6] := mu[h]+alpha[1];
k[7] := mu[v]+delta[v];
Y := beta[m]*sigma[v]/N[v];
z[1] := sigma[v]*(1-theta)/N[v];
z[2] := beta[h]*sigma[v]/N[h];
S[H0] := t*Lambda[h]+S[h0];
V[H0] := V[h0];
Z[HAL0] := Z[hal0];
Z[HCL0] := Z[hcl0];
Z[M0] := Z[m0];
Z[LM0] := Z[lm0];
T[H0] := T[h0];
S[V0] := t*Lambda[v]+S[v0];
Z[V0] := Z[v0];
S[H1] := -z[1]*beta[m]*(int(Z[V0]*S[H0], t = 0 .. t))-z[1]*beta[l]*(int(Z[V0]*S[H0], t = 0 .. t))-z[1]*beta[lm]*(int(Z[V0]*S[H0], t = 0 .. t))-k[1]*(int(S[H0], t = 0 .. t))+alpha[1]*(int(T[H0], t = 0 .. t))+alpha[2]*(int(V[H0], t = 0 .. t));
V[H1] := tau[1]*(int(S[H0], t = 0 .. t))-k[2]*(int(V[H0], t = 0 .. t))-Y*(int(Z[V0]*V[H0], t = 0 .. t));
```

```
Z[HAL1] := z[1]*beta[l]*(int(Z[V0]*S[H0], t = 0 .. t))-z[1]*beta[l]*(int(Z[V0]*Z[HAL0], t = 0 .. t))-k[3]*(int(Z[HAL0], t = 0 .. t));
Z[HCL1] := z[1]*beta[l]*(int(Z[V0]*Z[HAL0], t = 0 .. t))+rho*(int(Z[HAL0], t = 0 .. t))-k[1]*(int(Z[HCL0], t = 0 .. t));
Z[M1] := z[1]*beta[m]*(int(Z[V0]*S[H0], t = 0 .. t))-k[4]*(int(Z[M0], t = 0 .. t))+Y*(int(Z[V0]*V[H0], t = 0 .. t));
Z[LM1] := z[1]*beta[lm]*(int(Z[V0]*S[H0], t = 0 .. t))-k[5]*(int(Z[LM0], t = 0 .. t));
T[H1] := tau[2]*(int(Z[M0], t = 0 .. t))+tau[1]*(int(Z[HAL0], t = 0 .. t))+tau[1]*(int(Z[HCL0], t = 0 .. t))+tau[3]*(int(Z[LM0], t = 0 .. t))-k[6]*(int(T[H0], t = 0 .. t));
S[V1] := -z[2]*(int(Z[HAL0]*S[V0], t = 0 .. t))-z[2]*(int(Z[HCL0]*S[V0], t = 0 .. t))-z[2]*(int(Z[M0]*S[V0], t = 0 .. t))-z[2]*(int(Z[LM0]*S[V0], t = 0 .. t))-k[7]*(int(S[V0], t = 0 .. t));
Z[V1] := z[2]*(int(Z[HAL0]*S[V0], t = 0 .. t))+z[2]*(int(Z[HCL0]*S[V0], t = 0 .. t))+z[2]*(int(Z[M0]*S[V0], t = 0 .. t))+z[2]*(int(Z[LM0]*S[V0], t = 0 .. t))-k[7]*(int(Z[V0], t = 0 .. t));
S[H2] := collect(-z[1]*beta[m]*(int(S[H0]*Z[V1]+S[H1]*Z[V0], t = 0 .. t))-z[1]*beta[l]*(int(S[H0]*Z[V1]+S[H1]*Z[V0], t = 0 .. t))-z[1]*beta[lm]*(int(S[H0]*Z[V1]+S[H1]*Z[V0], t = 0 .. t))-k[1]*(int(S[H1], t = 0 .. t))+alpha[1]*(int(T[H1], t = 0 .. t))+alpha[2]*(int(V[H1], t = 0 .. t)), t);
V[H2] := collect(tau[1]*(int(S[H1], t = 0 .. t))-k[2]*(int(V[H1], t = 0 .. t))-Y*(int(V[H0]*Z[V1]+V[H1]*Z[V0], t = 0 .. t)), t);
Z[HAL2] := collect(z[1]*beta[l]*(int(S[H0]*Z[V1]+S[H1]*Z[V0], t = 0 .. t))-z[1]*beta[l]*(int(Z[HAL0]*Z[V1]+Z[HAL1]*Z[V0], t = 0 .. t))-k[3]*(int(Z[HAL1], t = 0 .. t)), t);
Z[HCL2] := collect(z[1]*beta[l]*(int(Z[HAL0]*Z[V1]+Z[HAL1]*Z[V0], t = 0 .. t))+rho*(int(Z[HAL1], t = 0 .. t))-k[1]*(int(Z[HCL1], t = 0 .. t)), t);
Z[M2] := collect(z[1]*beta[m]*(int(S[H0]*Z[V1]+S[H1]*Z[V0], t = 0 .. t))-k[4]*(int(Z[M1], t = 0 .. t))+Y*(int(V[H0]*Z[V1]+V[H1]*Z[V0], t = 0 .. t)), t);
Z[LM2] := collect(z[1]*beta[lm]*(int(S[H0]*Z[V1]+S[H1]*Z[V0], t = 0 .. t))-k[5]*(int(Z[LM1], t = 0 .. t)), t);
T[H2] := collect(tau[2]*(int(Z[M1], t = 0 .. t))+tau[1]*(int(Z[HAL1], t = 0 .. t))+tau[1]*(int(Z[HCL1], t = 0 .. t))+tau[3]*(int(Z[LM1], t = 0 .. t))-k[6]*(int(T[H1], t = 0 .. t)), t);
S[V2] := collect(-z[2]*(int(S[V0]*Z[HAL1]+S[V1]*Z[HAL0], t = 0 .. t))-z[2]*(int(S[V0]*Z[HCL1]+S[V1]*Z[HCL0], t = 0 .. t))-z[2]*(int(S[V0]*Z[M1]+S[V1]*Z[M0], t = 0 .. t))-z[2]*(int(S[V0]*Z[LM1]+S[V1]*Z[LM0], t = 0 .. t))-k[7]*(int(S[V1], t = 0 .. t)), t);
Z[V2] := collect(z[2]*(int(S[V0]*Z[HAL1]+S[V1]*Z[HAL0], t = 0 .. t))+z[2]*(int(S[V0]*Z[HCL1]+S[V1]*Z[HCL0], t = 0 .. t))+z[2]*(int(S[V0]*Z[M1]+S[V1]*Z[M0], t = 0 .. t))+z[2]*(int(S[V0]*Z[LM1]+S[V1]*Z[LM0], t = 0 .. t))-k[7]*(int(Z[V1], t = 0 .. t)), t);
S[H] := collect(S[H0]+S[H1]+S[H2], t);
 200 + (-0.000005213319273 + 0.000005213319273 theta) t  + (
-1.7666 (0.0003571428571 - 0.0003571428571 theta) (-1804.171995
   + 283.9178571 theta - 1500.000000 tau[1]) - 1. (0.017 + tau[1]
  ) (-1.031392857 + 0.9463928569 theta - 5.000000000 tau[1])
   + 2.5 tau[1]) t  + (-30.37687499 + 2.839178571 theta
   + 35.00000000 tau[1] - 1.7666 (0.0003571428571
   - 0.0003571428571 theta) (-7636.633010 + 5678.357140 theta
```

```
   - 30000.00000 tau[1] - 7500.000000 alpha[1]) - 1. (0.017
   + tau[1]) (24.37214286 + 18.92785714 theta
   - 100.0000000 tau[1] - 25.00000000 alpha[1]) - 0.5000000000
  alpha[1] (80. tau[2] + 220. tau[1] + 100. tau[3] - 0.8500000000
   - 50. alpha[1])) t
   + (78.74428571 + 37.85571429 theta - 200. tau[1]
   - 50. alpha[1]) t
V[H] := collect(V[H0]+V[H1]+V[H2], t);
180 + (tau[1] (-1.031392857 + 0.9463928569 theta
   - 5.000000000 tau[1]) - 3.031410714 tau[1] - 0.00001967285837)
 t  + (-45.62821428 tau[1] + tau[1] (24.37214286
   + 18.92785714 theta - 100.0000000 tau[1]
   - 25.00000000 alpha[1]) + 37.08586560) t
   + (200. tau[1] - 109.1307857) t
Z[HAL] := collect(Z[HAL0]+Z[HAL1]+Z[HAL2], t);
      /             -7                  -7      \  4
120 + \2.951046798 10   - 2.951046798 10   theta/ t  + (0.1
  (0.0003571428571 - 0.0003571428571 theta) (-1804.171995
   + 283.9178571 theta - 1500.000000 tau[1]) - 0.1
  (0.0003571428571 - 0.0003571428571 theta) (16.11549754
   - 16.07142857 theta)
   - 1. (0.017 + tau[1] + rho) (0.05357142856
   - 0.05357142856 theta)) t  + (0.1607142857
   - 0.1607142857 theta + 0.1 (0.0003571428571
   - 0.0003571428571 theta) (-7636.633010 + 5678.357140 theta
   - 30000.00000 tau[1] - 7500.000000 alpha[1]) - 0.1
  (0.0003571428571 - 0.0003571428571 theta) (-9146.394090
   - 128.5714286 theta - 18000.00000 tau[1] - 18000.00000 rho) -
  1. (0.017 + tau[1] + rho) (-0.5914285715 - 0.4285714285 theta
   - 60.00000000 tau[1] - 60.00000000 rho)) t
 + (-1.182857143 - 0.857142857 theta - 120. tau[1] - 120. rho) t
Z[HCL] := collect(Z[HCL0]+Z[HCL1]+Z[HCL2], t);
100 + (0.1 (0.0003571428571 - 0.0003571428571 theta) (16.11549754
   - 16.07142857 theta)
   + rho (0.05357142856 - 0.05357142856 theta)) t  + (0.1
  (0.0003571428571 - 0.0003571428571 theta) (-9146.394090
   - 128.5714286 theta - 18000.00000 tau[1] - 18000.00000 rho) +
  rho (-0.5914285715 - 0.4285714285 theta - 60.00000000 tau[1]
   - 60.00000000 rho) - 0.5000000000 (0.017 + tau[1]) (
-0.4142857144 - 1.285714286 theta + 120. rho - 100. tau[1])) t
   + (-0.414285714 - 1.285714286 theta + 120. rho - 100. tau[1]) t
Z[M] := collect(Z[M0]+Z[M1]+Z[M2], t);
 + (0.000002459107297 - 0.000002459107297 theta) t  + (0.8333
  (0.0003571428571 - 0.0003571428571 theta) (-1804.171995
   + 283.9178571 theta - 1500.000000 tau[1])
   - 1. (tau[2] + 0.017449312) (0.4464107143 - 0.4464107143 theta
 + 0.00001967285837 + 0.4464107143 tau[1]) t  + (-7.536325360
   - 1.339232143 theta + 0.8333 (0.0003571428571
   - 0.0003571428571 theta) (-7636.633010 + 5678.357140 theta
   - 30000.00000 tau[1] - 7500.000000 alpha[1]) - 1. (tau[2]
```

```
   + 0.017449312) (16.26563466 - 8.928214285 theta
                                        2
   - 40.00000000 tau[2]) + 8.928214284 tau[1]) t
   + (32.53126933 - 17.85642857 theta - 80. tau[2]) t
Z[LM] := collect(Z[LM0]+Z[LM1]+Z[LM2], t);
                                            4
100 + (0.000002459107297 - 0.000002459107297 theta) t  + (0.8333
  (0.0003571428571 - 0.0003571428571 theta) (-1804.171995
   + 283.9178571 theta - 1500.000000 tau[1])
   - 1. (tau[3] + 0.017449312) (0.4464107143 - 0.4464107143 theta
     3
  )) t  + (1.339232143 - 1.339232143 theta + 0.8333
  (0.0003571428571 - 0.0003571428571 theta) (-7636.633010
   + 5678.357140 theta - 30000.00000 tau[1]
   - 7500.000000 alpha[1]) - 1. (tau[3] + 0.017449312)
                                                 2
  (8.055748685 - 8.928214285 theta - 50.00000000 tau[3])) t
   + (16.11149737 - 17.85642857 theta - 100. tau[3]) t
T[H] := collect(T[H0]+T[H1]+T[H2], t);
50 + (tau[2] (0.4464107143 - 0.4464107143 theta)
   + tau[1] (0.05357142856 - 0.05357142856 theta)
                                         3
   + tau[3] (0.4464107143 - 0.4464107143 theta)) t  + (tau[2]
  (16.26563466 - 8.928214285 theta - 40.00000000 tau[2]) + tau[1]
  (-0.5914285715 - 0.4285714285 theta - 60.00000000 tau[1]
   - 60.00000000 rho) + 0.5000000000 tau[1] (-0.4142857144
   - 1.285714286 theta + 120. rho - 100. tau[1])
   + tau[3] (8.055748685 - 8.928214285 theta - 50.00000000 tau[3]
  ) - 0.5000000000 (0.017 + alpha[1]) (80. tau[2] + 220. tau[1]
                                         2
   + 100. tau[3] - 0.8500000000 - 50. alpha[1])) t  + (80. tau[2]
   + 220. tau[1] + 100. tau[3] - 0.850 - 50. alpha[1]) t
S[V] := collect(S[V0]+S[V1]+S[V2], t);
                                             4
500 + (-0.000003909989454 + 0.000003909989454 theta) t  + (
-0.03280495550 + 0.03678302692 theta + 0.0004039655171 tau[1]
                                              3
   + 0.0001468965517 tau[2] + 0.0001836206896 tau[3]) t  +
  (83.46711789 + 0.7342703202 theta + 4.267241379 tau[1]
                                      2
   + 1.551724138 tau[2] + 1.939655172 tau[3]) t  - 290.4462413 t
Z[V] := collect(Z[V0]+Z[V1]+Z[V2], t);
                                            4
300 + (0.000003909989454 - 0.000003909989454 theta) t  +
  (0.03638453883 - 0.03678302692 theta - 0.0004039655171 tau[1]
                                              3
   - 0.0001468965517 tau[2] - 0.0001836206896 tau[3]) t  +
  (37.51335708 - 0.7342703202 theta - 4.267241379 tau[1]
                                      2
   - 1.551724138 tau[2] - 1.939655172 tau[3]) t  - 149.4827587
```