

# COMPARATIVE PERFORMANCE ANALYSIS OF INTERPOLATION TECHNIQUES AND DGPS MEASURED HEIGHTS.

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## ABSTRACT

*The main objective is to evaluate heights interpolation methods, and evaluate accuracy measures from observed and predicted data to find the best fit model. This research focuses on the application of four interpolation methods (IDW, Kriging, Natural Neighbor and Spline) to estimate the unknown value from the digital elevation models (DEM). Results from this study show the interpolation using GIS techniques is effective and has a higher level of accuracy compared to conventional methods, especially in the areas with undulating terrain. The IDW interpolation method has a SD of 0.230, SE as 0.0048, MSE of 0.0542, RMSE as 0.232882 and MAE of 0.110. This was closely followed by the KG interpolation method the the NN model and finally the SP model respectively and the range of error across the different interpolation methods over the control points ranges between -0.5 to +5.7m. Station CP10 has +0.5783 while MB03 was -0.585 and CP4 has no or negligible error of 0.000000m. This study concluded that the best interpolation method was IDW while KG and NN followed. The SP model performed poorly based on this research.*

**Keywords:** *Interpolation, RMSE, MAE, Topography*

## 1.0 Introduction

Topography is a field of planetary science comprising the study of surface features of the earth, which allow viewing and studying the slope, aspect, relief, shapes and landforms on a two-dimensional map (Hu, 2013). The production of topographic maps requires the three-Dimensional coordinates (X, Y, Z) of various points on the earth surface. It is not possible to perform observation on all locations on the earth surface, hence, interpolation is required to cover those areas that are not captured during field observation. Interpolation is the procedure of predicting the value of attributes at unsampled sites from measurements made at point locations within the same area or region (Borrough, 1998). It can be used to predict unknown values for any geographic point data, such as elevation, rainfall, chemical concentrations, noise levels, and so on. Interpolation is a very important process in many Geographic Information Systems (GIS) and may be used to: provide contours for displaying data graphically; calculate some property of the surface at a given point; and change the unit of comparison when using different data structures in different layers (Elamin, 2016).

Determination of the accuracy of interpolation methods is indispensable in Geosciences. However, the target of interpolations is to create a surface that is intended to best represent observed reality thus the selected method must be assessed for accuracy (Azpurua and Dos Ramos, 2010). The selection criteria for the best algorithm should be based on the real data, the level of accuracy required, the time and the computer system available (Kamińska and Grzywna, 2014). All interpolations results contain error since they are based on algorithmic estimations of unknown values from known values. It is assumed that interpolation error will be lower in areas where the terrain is smoother and higher in areas characterized by steep and abrupt changes (Hurst, 2014). Generally, measures of forecast accuracy have been group into two as scale independent and scale dependent. Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Mean Standard Error (MSE) and Median Absolute Error (MdAE) are scale dependent meaning the scale depends on the scale of data that is they are useful when comparing different forecasting methods that are applied to the data of the same scale. On the other hand, MAPE, sMAPE, MASE are scale independent (Sungil and Heeyoung, 2016). Kamińska and Grzywna (2014) compared two spatial interpolation techniques; Radial Basis Functions (RBF) and Inverse Distance Weighting (IDW) with the goal of determining which method creates the best representation of reality for measured groundwater levels in catchment area. The results shows that Radial Basis Functions creates better representation of reality for measured groundwater levels with the

help of cross validation statistics. Tunalıoğlu (2012) reveals that interpolation methods such as Kriging and RBF give more effective solutions where the heights change suddenly and significantly, but if the height differences are relatively lower in the study area, the standard deviations give similar solutions with respect to each interpolation methods.

Zimmerman *et al.* (1999) performed a comparison on synthetic data that included IDW, Ordinary Kriging, and Universal Kriging interpolation methods and found the Kriging methods performed better than IDW. In another study, Ordinary Kriging, Spline, and IDW were all compared for a study of sample elevation points on a hill in Turkey, the results showed that the DEM with the least amount of errors came from the Spline method (Erdogan, 2009). This study was conducted at the Federal College of Education, (FCE), Yola where the landscape comprised of an undulating surface, which makes it difficult to interpolate elevation with accuracy. It is therefore important to choose the best interpolation model for the terrain in the study to reduce the error in the DEM. In this study, four interpolation methods, namely Inverse distance weighted (IDW), Natural neighbour (NN), Kriging (KG) and Spline (SP) were used for estimating unknown point elevations.

The resulting surfaces generated from various methods of interpolation have to be compared using the designated methods and extracted elevation values have to be subtracted from test points' elevation (ground control heights) in order to obtain residuals and compute descriptive statistics with the aim of pointing out the best algorithm's performance.

## 2.0 Methodology/Materials

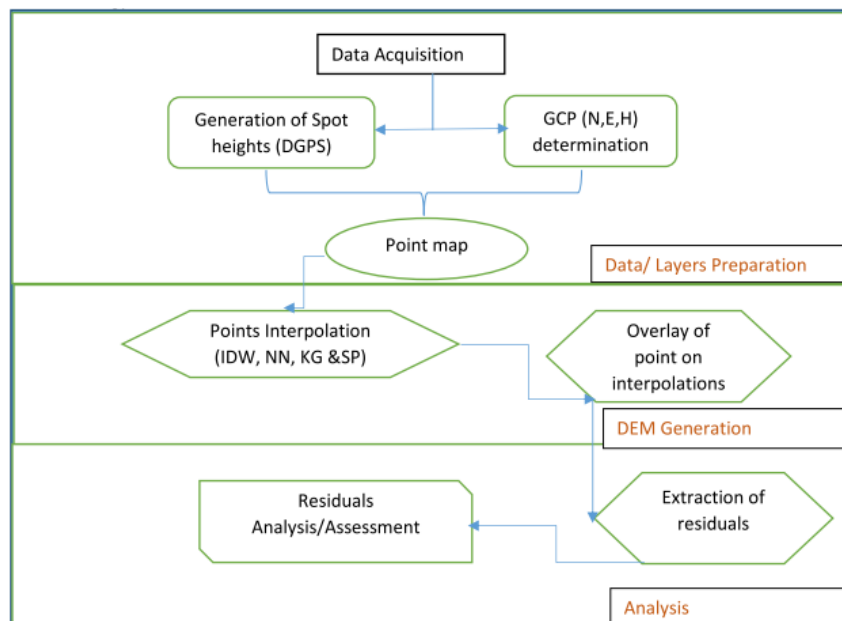


Figure 1. Flow chart of methodology

## 2.1 Study Area Description

Federal College of Education, Yola is situated in Yola the capital city of Adamawa State, North Eastern Part of Nigeria. The campus lies 1km from River Benue on the eastern site and 3km away from Chouchi river on the southern site. The campus has a total land area of 155 Hectares and is located in zone 33 of the Universal Traverse Mercator (UTM) and lies between Latitudes  $9^{\circ} 14' 52.57''\text{N}$  to  $9^{\circ} 14' 49.15''\text{N}$  & longitudes  $12^{\circ} 27' 21.79''\text{E}$  to  $12^{\circ} 28' 25.41''\text{E}$ . The land is generally undulating having up to 44m difference in height between the highest and lowest points.



Figure 2. Map of Federal College of Education (FCE), Yola

## 2.2 Data used

The Northing, Easting and Heights (N,E & H) data were obtained from DGPS survey of the study area. A total of two thousand one hundred and forty three (2,143) including twenty three (23) control points were spread across the study area. see table 1.

**Table 1. Sample of data used.**

Stn ID	Northing	Easting	Height
MTB02	1022427.84	220506.91	190.63
MTB03	1022515.84	220545.96	188.887
SP1	1022663.477	220509.868	192.508
SP2	1022640.108	220509.655	192.469
SP3	1022615.184	220509.114	192.613
SP4	1022589.907	220508.052	192.456
SP5	1022564.306	220507.954	191.948

### **2.3 Instrumentation/ software**

The Hi-Target V30 GNSS receiver along side its accessories was used in the field observation. A Hp 630 Laptop with configuration of windows 7 ultimate, intel(R) pentium(R) CPU B960@ 2.20GHz processor, 4.00GB RAM, 64 bit operating system and 450GB harddrive.

ArcGIS 10.1 software was used for the interpolation process and Microsoft excel for computing the statistics and plotting of statistical graphs.

### **2.4 Interpolation methods and Geographic Information System (GIS)**

Every data obtained by DGPS instrument N, E & H is interpreted in GIS as a point in space. The ArcGIS 10.1 software is a robust GIS tool which provides powerful geostatistical operations for optimal generation of interpolated surfaces from discrete spatial points measurements. The result of spatial interpolation process will depend mainly on the natural terrain, features and attributes. Also on the existence of a spatial dependency and on factors associated to the attribute form of modelling given by the distribution on the land (Grande, 1996).

Four different techniques of interpolation have been applied in this study namely: Inverse distance weighted (IDW), Natural neighbour (NN), Kriging (KG) and Spline (SP). These interpolation techniques are divided into two groups as Deterministic and Geostatistical. The deterministic assign values to locations based on the surrounding measured values and on specific mathematical formulae that determine the smoothness of the surface ESRI (2015) in Barthlomieij (2016) while the geostatistics are based on statistical models that include autocorrelation. This has made geostatistical not only have the capability of producing a prediction surface but also provide some measure of certainty and accuracy of the predictions.

Table 2 Interpolation techniques used

S/No	Interpolation Technique	Category
1	Inverse Distance Weighted	Deterministic
2	Natural Neighbour	“
3	Spline	“
4	Kriging	Geostatistic

#### 2.4.1 IDW interpolation method

This is a local and exact interpolation technique Miranda-Salas and Condal (2003) it only uses the sample points that are in the vicinity of each non sampled point and the result of the interpolation process reproduces exactly the values in the sample points, by this method, the value of the variable in the non sampled point is the average of the inverse distance of the values of the sampled points that are around Flores and Moreno (2005), Reinstorf *et al.* (2005) in (Romero *et al.* 2011). The IDW is a moving average (MA) technique applicable in highly variable data which used a linear wieghted combination set of sample points.

$$V = \frac{\sum_{i=1}^n \left(\frac{1}{d_i} V_i\right)}{\sum_{i=1}^n \left(\frac{1}{d_i}\right)} \quad 1$$

V= value to be estimated

$V_i$  = known value,  $d_i$ = distance from the data point to the unknown point, n = number of sample points

#### 2.4.2 Natural Neighbour (NN) interpolation method

This method finds the closest subset of input samples to a query point and applies weights to them based on proportionate areas to interpolate a value. It is also known as ‘sibson’ or ‘area stealing interpolation’ which is based on voronoi pattern for a set of separated points. It has more advantage over nearest neighbour such as ability to create a surface that is relatively smooth.

$$H_{(x,y)} = \sum_{i=1}^n w_i * H_{(x_i,y_i)} \quad 2$$

$H_{(x,y)}$  = estimated height at unknown point

$H_{(x_i, y_i)}$  = height of sample point,  $w_i$  = weight of sample followed by the area enclosed by any part of the unknown sample point.

### 2.4.3 Spline (SP) interpolation method

The Spline tool uses an interpolation method that estimates values using a mathematical function that minimizes overall surface curvature resulting in a smooth surface that passes exactly through the input points (Bartłomiej, 2016). Spline Johnson *et al.*, (2001) are interpolations that fit a function to sampled points. The algorithm uses a linear combination of a function one for each known point.

$$H_{S_o} = \sum_{i=1}^n \omega_i \phi(\|S_i - S_o\|) + \omega_{n+1} \quad 3$$

Where  $\phi$  represent the interpolation function,  $(\|S_i - S_o\|)$  the euclidean distance  $r$  between an unknown point  $S_o$  and an observed point  $S_i$ , while  $\omega_i$  are weights.

Smoothing Spline function (equalization) also assumes that there is an error in the measurement, that is in the data set that need to be locally smooth (Davidovic *et al.* 2016).

### 2.4.4 Kriging (KG) interpolation method

This interpolation technique is an advance geostatistical procedure that generates an estimated surface from a scattered set of points with  $Z$  values. Moreso than other interpolation methods supported by ArcGIS spatial analyst tool. The general Kriging model to obtain an unknown value of the regionalized variable  $Z$  in  $S_o$  location is given by

$$H_{S_o} = \mu(s) + \gamma(h) + \varepsilon \quad 4$$

Where  $\mu(s)$  is a deterministic function,  $\gamma(h)$  is a spatially correlated relationship depicted by variogram,  $h$  is the distance between sample points and  $\varepsilon$  is the random error.  $S$  indicates the points with cartesian coordinates  $X, Y$  (Romero *et al.* 2011).

## 2.5 Performance measure of interpolation techniques based on statistics analysis

### 2.5.1 Standard Deviation (SD)

This shows how much variation exist from the average that is, it measures the scatter of a given observation around the mean. A low SD indicate that the points tend to be very close to the mean while a high SD indicates that the data points are spread out over a large range of values. In addition, SD does not have any impact on the sample size. When an SD is large, the chances of standard error (SE) of the mean are likely to be larger.

### 2.5.2 Standard error (SE)

SE is very similar to SD, both are measure of spread. It is a technique for estimating the SD of a given sampling distribution. This is also termed as standard deviation of the mean. In other words, SE of the mean measures the deviation of the value of true mean of a given sample. The more the sample size the lesser the SE would be therefore, SE should be as small as possible. SE of the mean is computed thus:

$$SE_{\mu} = \frac{\partial}{\sqrt{n}} \quad 5$$

$\partial$  = sample standard deviation

$n$  = sample size or number of observations

### 2.5.3 Root mean square error (RMSE)

This is commonly use to investigate the spatial interpolation of sample points spread across a surface. Its measures the success of numeric prediction as the average error across a surface (interpolated). RMSE is also seen as the square root of the square of the difference between estimated points (interpolated) and observation points divided by the total number of the observation points.

$$\Delta_i = C_i^T - C_i^I, i = 1, 2, \dots, n \quad 6$$

$C_i^T$  = Given control points heights

$C_i^I$  = Check heights (deduced) after interpolation

$n$  = number of sample points

$$RMSE = \sqrt{\frac{\sum_{i=1}^n \Delta_i^2}{n - 1}} \quad 7$$

In orther words, RMSE is known as the square root of the MSE.

### 2.5.4 Mean Absolute Error (MAE)

Measures how far predicted values are away from obsevered values without considering direction. It basically performs two forms of computations.

- i. Sums the absolute values of the residuals
- ii. Divides by the number of observations

$$MAE = \frac{1}{n} \sum_{i=1}^n |e_i| \quad 8$$

$|e_i|$  = Absolute residuals

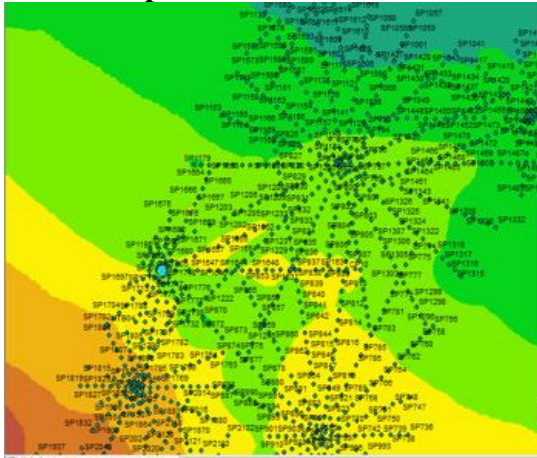


### 3.0 RESULTS AND DISCUSSION

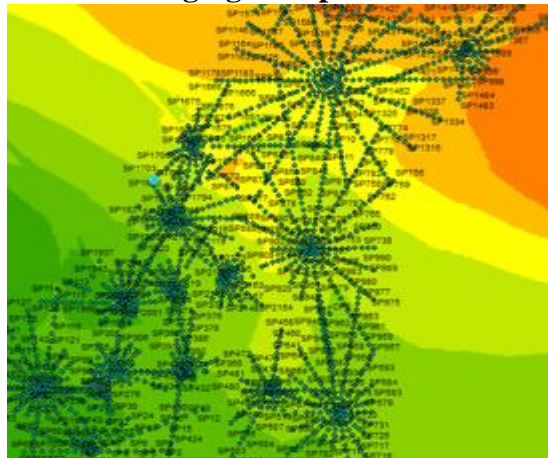
#### 3.1 Results

Four interpolation techniques (methods) have been exploited in ArcGIS 10.1 software with four results comparison. These can be seen in the figures below:

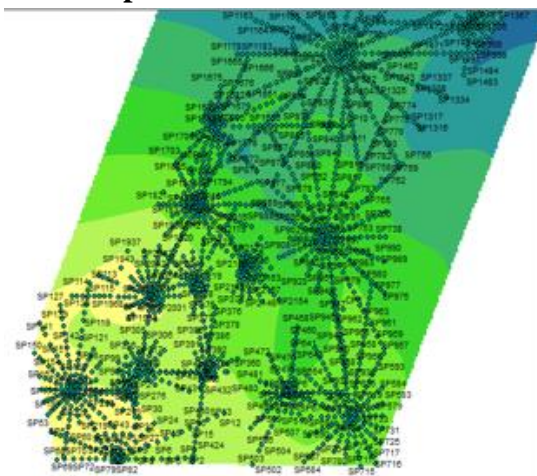
**IDW Interpolation**



**Kriging Interpolation**



**NN Interpolation**



**SP Interpolation**

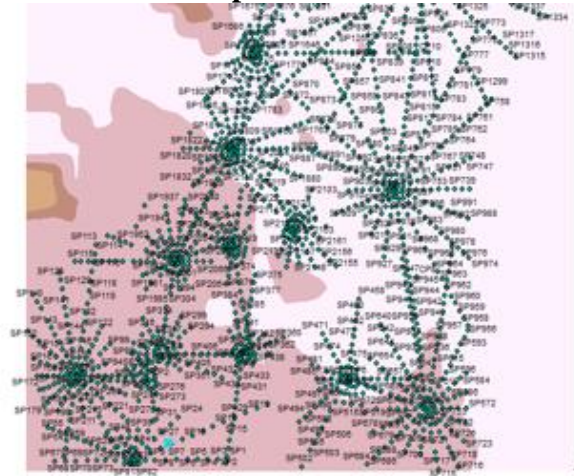
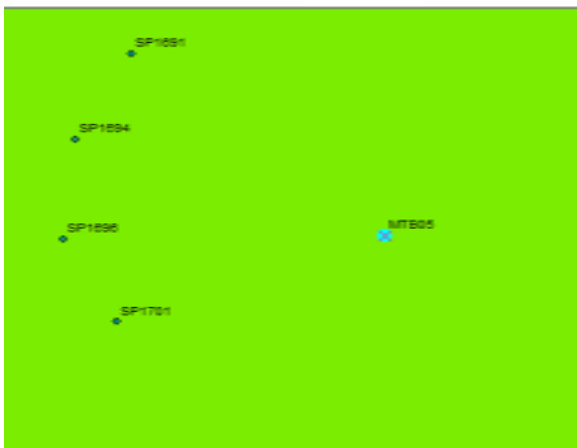


Figure 3. Four different interpolation techniques

#### 3.1.2 Process of extracting forecast values

The forecast values of all the control points were extracted from the various interpolated surfaces. This was possible through the identifier tool in ArcGIS 10.1 as shown in the following figures.

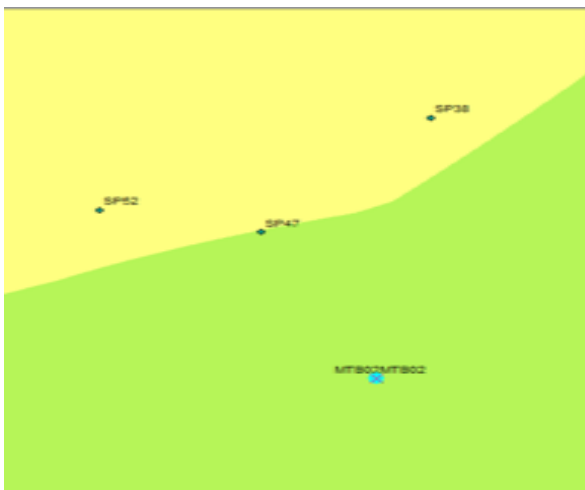
IDW



KG



NN



SP

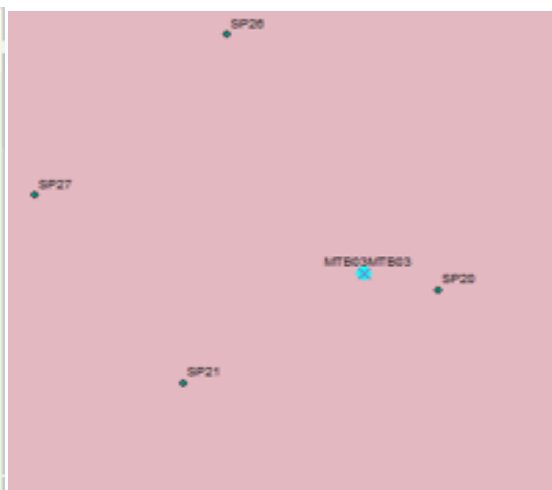


Figure 4. Process of extracting residuals

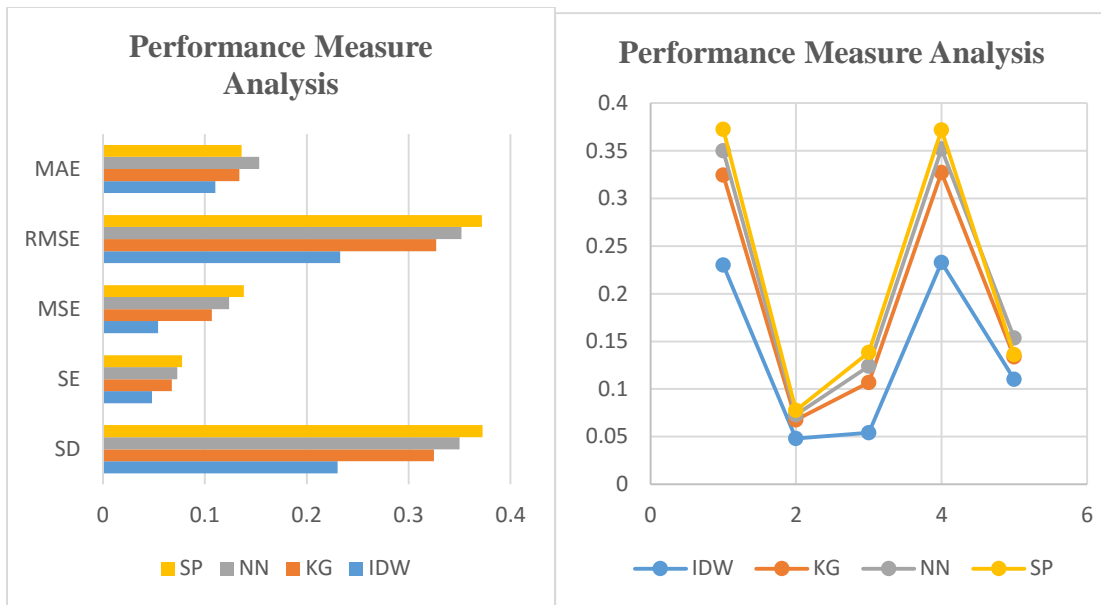
Comparing control points heights with results obtained from various interpolation methods.

This is given in the table below.

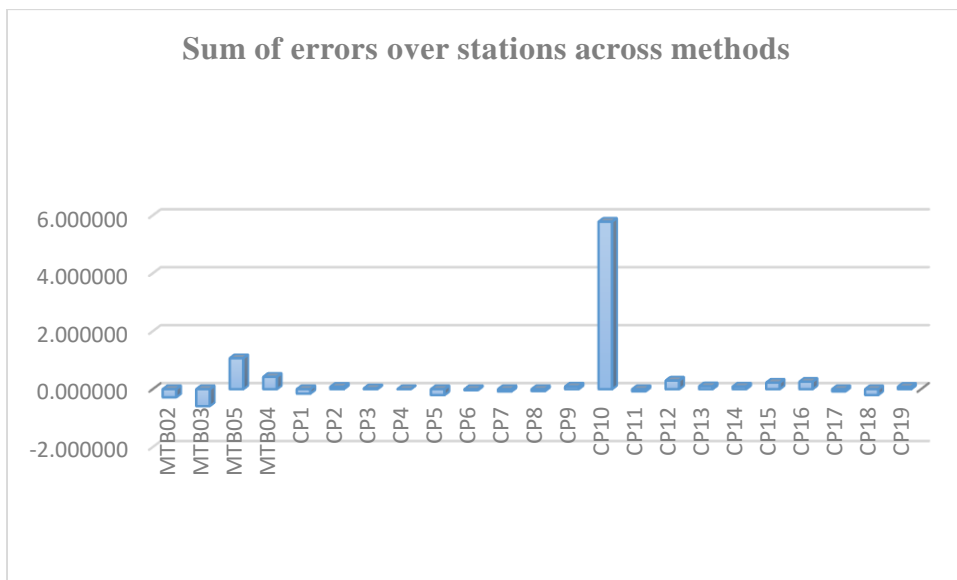
Table 3. Heights comparison

<b>POINT ID</b>	<b>CONTROL HT</b>	<b>IDW</b>	<b>KRIGING</b>	<b>NN</b>	<b>SPLINE</b>
MTB02	190.630	190.635	190.699	190.700	190.765
MTB03	188.887	188.951	189.064	189.154	188.964
MTB05	207.074	206.660	206.765	206.772	207.030
MTB04	199.700	199.574	199.609	199.617	199.574
CP1	189.78	189.838	189.876	189.770	189.789
CP2	190.834	190.83	190.807	190.806	190.810
CP3	192.786	192.753	192.776	192.772	192.814
CP4	193.429	193.428	193.429	193.427	193.432
CP5	192.259	192.390	192.419	192.411	192.020
CP6	198.865	198.969	198.747	199.008	198.770
CP7	196.399	196.402	196.426	196.424	196.423
CP8	199.957	199.979	199.961	199.961	199.994
CP9	201.763	201.750	201.770	201.675	201.766
CP10	205.927	204.957	204.435	204.340	204.193
CP11	208.833	208.841	208.845	208.849	208.870
CP12	220.945	220.753	220.779	220.781	221.170
CP13	213.494	213.427	213.467	213.455	213.528
CP14	219.748	219.746	219.723	219.719	219.713
CP15	198.384	198.306	198.328	198.317	198.360
CP16	200.109	200.057	200.030	200.033	200.059
CP17	190.099	190.104	190.119	190.124	190.126
CP18	193.144	193.322	193.105	193.263	193.087
CP19	193.693	193.701	193.758	193.473	193.757

	Delta 1	Delta 2	Delta 3	Delta 4	ABSV IDW	ABSVKG	ABSVNN	ABSVSP	SqDelta 1	SqDelta 2	SqDelta 3	SqDelta 4	SUM errors across methods
	-0.005	-0.069	-0.070	-0.135	0.005	0.069	0.07	0.135	0.000025	0.004761	0.004900	0.018225	-0.279000
	-0.064	-0.177	-0.267	-0.077	0.064	0.177	0.267	0.077	0.004096	0.031329	0.071289	0.005929	-0.585000
	0.414	0.309	0.302	0.044	0.414	0.309	0.302	0.044	0.171396	0.095481	0.091204	0.001936	1.069000
	0.126	0.091	0.083	0.126	0.126	0.091	0.083	0.126	0.015876	0.008281	0.006889	0.015876	0.426000
	-0.058	-0.096	0.010	-0.009	0.058	0.096	0.01	0.009	0.003364	0.009216	0.000100	0.000081	-0.153000
	0.004	0.027	0.028	0.024	0.004	0.027	0.028	0.024	0.000016	0.000729	0.000784	0.000576	0.083000
	0.033	0.010	0.014	-0.028	0.033	0.01	0.014	0.028	0.001089	0.000100	0.000196	0.000784	0.029000
	0.001	0.000	0.002	-0.003	0.001	0	0.002	0.003	0.000001	0.000000	0.000004	0.000009	0.000000
	-0.131	-0.160	-0.152	0.239	0.131	0.16	0.152	0.239	0.017161	0.025600	0.023104	0.057121	-0.204000
	-0.104	0.118	-0.143	0.095	0.104	0.118	0.143	0.095	0.010816	0.013924	0.020449	0.009025	-0.034000
	-0.003	-0.027	-0.025	-0.024	0.003	0.027	0.025	0.024	0.000009	0.000729	0.000625	0.000576	-0.079000
	-0.022	-0.004	-0.004	-0.037	0.022	0.004	0.004	0.037	0.000484	0.000016	0.000016	0.001369	-0.067000
	0.013	-0.007	0.088	-0.003	0.013	0.007	0.088	0.003	0.000169	0.000049	0.007744	0.000009	0.091000
	0.970	1.492	1.587	1.734	0.97	1.492	1.587	1.734	0.940900	2.226064	2.518569	3.006756	5.783000
	-0.008	-0.012	-0.016	-0.037	0.008	0.012	0.016	0.037	0.000064	0.000144	0.000256	0.001369	-0.073000
	0.192	0.166	0.164	-0.225	0.192	0.166	0.164	0.225	0.036864	0.027556	0.026896	0.050625	0.297000
	0.067	0.027	0.039	-0.034	0.067	0.027	0.039	0.034	0.004489	0.000729	0.001521	0.001156	0.099000
	0.002	0.025	0.029	0.035	0.002	0.025	0.029	0.035	0.000004	0.000625	0.000841	0.001225	0.091000
	0.078	0.056	0.067	0.024	0.078	0.056	0.067	0.024	0.006084	0.003136	0.004489	0.000576	0.225000
	0.052	0.079	0.076	0.050	0.052	0.079	0.076	0.05	0.002704	0.006241	0.005776	0.002500	0.257000
	-0.005	-0.020	-0.025	-0.027	0.005	0.02	0.025	0.027	0.000025	0.000400	0.000625	0.000729	-0.077000
	-0.178	0.039	-0.119	0.057	0.178	0.039	0.119	0.057	0.031684	0.001521	0.014161	0.003249	-0.201000
	-0.008	-0.065	0.220	-0.064	0.008	0.065	0.22	0.064	0.000064	0.004225	0.048400	0.004096	0.083000
<b>COUNT</b>	<b>23</b>	<b>23</b>	<b>23</b>	<b>23</b>	<b>23</b>	<b>23</b>	<b>23</b>	<b>23</b>	<b>23</b>	<b>23</b>	<b>23</b>	<b>23</b>	
<b>SUM</b>	<b>1.366000</b>	<b>1.802000</b>	<b>1.888000</b>	<b>1.725000</b>	<b>2.538000</b>	<b>3.076000</b>	<b>3.530000</b>	<b>3.131000</b>	<b>1.247384</b>	<b>2.460856</b>	<b>2.848838</b>	<b>3.183797</b>	
<b>AVG</b>	<b>0.059</b>	<b>0.078</b>	<b>0.082</b>	<b>0.075</b>									
<b>SD</b>	<b>0.230</b>	<b>0.325</b>	<b>0.350</b>	<b>0.373</b>									
<b>SE</b>	<b>0.048009</b>	<b>0.067708</b>	<b>0.072965</b>	<b>0.077694</b>									
<b>MSE</b>	<b>0.054234</b>	<b>0.106994</b>	<b>0.123863</b>	<b>0.138426</b>									
<b>RMSE</b>	<b>0.232882</b>	<b>0.327099</b>	<b>0.351941</b>	<b>0.372056</b>									
<b>MAE</b>	<b>0.110</b>	<b>0.134</b>	<b>0.153</b>	<b>0.136</b>									



**Figure 5a and b. Visualization of Statistical charts Performance measure**



**Figure 6. Summation of errors over station across the various methods of interpolation**

### 3.2 Discussion

Heights interpolation was performed by four different interpolation techniques viz: IDW, KG, NN and SP on the basis of point distributions within the study area. The techniques were statistically compared by several tables and graphics. The accuracy of the methods was calculated by RMSE comparing to the original control points obtained by DGPS observation since RMSE has gain popularity as the most common way to quantify the difference between predicted values (forecasted) and ground truth values. The research also exploited MSE and MAE as other statistical measures to assess the best fit model. Additionally other statistical

parameters such as arithmetic mean of height differences, standard deviation and standard error as well as enhanced visual techniques can be utilized for quality assessment.

The statistics (Average, standard deviation, standard error, mean standard error, root mean square error and mean absolute error) of errors are given in Table 4 and the statistical chart of performance measure is shown in figure 5a and b. Figure 6 clearly shows the summation of errors on each station (control point) over the four techniques of interpolation used. The IDW interpolation method has a SD of 0.230, SE as 0.0048, MSE of 0.0542, RMSE as 0.232882 and MAE of 0.110. This was closely followed by the KG interpolation method the the NN model and finally the SP model respectively. By comparing the fits of these different interpolation models, based on smaller values indicates a better fit. In this research the IDW interpolation model has proven to be the best interpolation model while the SP model did not perform very well. The summation of errors across the different interpolation methods over the control points ranges between -0.5 to +5.7m as shown in table 4. The highest sum of errors was on station CP10 with +5.783 and the least was on station MTB03 as -0.585 while station CP4 has negligible or no error as 0.000000.

#### **4.0 Conclusion**

The research did establish that extracted spot heights from interpolated surfaces may not exactly match actual leveled heights on the ground but it is possible to compute the statistics of residuals using different statistical measures. Thereby establishing the best fit model base on the fact that smaller values presumes better fit. Based on the results and analysis, The IDW interpolation method has a SD of 0.230, SE as 0.0048, MSE of 0.0542, RMSE as 0.232882 and MAE of 0.110 while the KG interpolation method, NN model and SP model followed respectively the range of error across the different interpolation methods over the control points ranges between -0.5 to +5.7m. Station CP10 has +0.5783 while MB03 was -0.585 and CP4 has no or negligible error of 0.000000m. The Interpolation method with the least error was IDW using point map data against GPS points, while the highest error was obtained from SP method. This study concluded that the best interpolation method was IDW and closely followed by KG while the SP model performed poorly.

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