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COMPUTER-AIDED ANALYSIS OF REINFORCED CONCRETE WAFFLE BRIDGE DECK USING METHOD OF GRILLAGES

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ABSTRACT

This paper aims to analyse a reinforced concrete waffle bridge deck using method of grillages where the topping and ribs are analysed as a monolithic unit in contrast to the conventional methods where the slab and beams are analyzed differently. In addition, the grillage approach accounts for the torsion that is usually lost in the conventional approach. The slab loading is in line with the HA loading of the BS 5400 part 2 for lightly loaded (accommodation) bridges. One of the strength of this approach is that it is amenable to computer application which has been demonstrated by using a code written in Matrix Laboratory (MATLAB) software and therefore easy for field use by practitioners. For the purpose of rendering this approach amenable to computer application, a program was used to determine the displacements, bending moments and torsional moments in the bridge deck. It is observed that the values of bending moments obtained from grillage analysis are lower than the moments from conventional beam-slab analysis carried out manually. Bending moments and other responses generated by conventional beam-slab approaches are usually exaggerated thereby reducing the anticipated benefits of waffle slab

Keywords: *Computer-aided, Grillage analysis, MATLAB, waffle bridge deck.*

1 INTRODUCTION

Waffle slabs are structural elements with a combination of top slab and a system of spaced longitudinal and transverse beams. They are efficient in resisting lateral loads than flat slabs, and are suitable for large spans. They can withstand heavier load and cover large span as they exhibit higher stiffness and smaller deflections. The waffle slab system is an evolution of the solid slab that results from the elimination of concrete below the neutral axis which allows an economic increase on the total thickness of the slab with the creation of voids in a rhythmic arrangement. As a result, the self-weight of the structure is reduced (Schwetz *et al,* 2009).

For quite some time in Nigeria, the use of conventional reinforced concrete system was so prevalent that no serious attention was paid to ribbed slab. However, in recent years, there has been a sudden increase in the use of waffle slabs. That however, makes it necessary to examine new ways in which it can be used in construction.

Principally, static analysis of waffle slabs aim to determine the amount and distribution of shear forces bending moment and torsional moments acting on the structure.

Over the years, researchers have analysed waffle slabs substantially based on conventional methods; both analytical and numerical but less research have been carried out on the use of grillage analysis for waffle bridge decks. Other methods available in literature include plate analogy by Timoshenko (Halkude and Mahamuni, 2014) Rankine Grashoff method (Mohammed *et al,* 2013). However the direct stiffness gives more accurate results as concluded by (Halkude and Mahamuni, 2014).

Up until now, waffle slabs are found more in number in building construction than in bridge construction. An argument against this is that loads are distributed in two orthogonal directions in waffle slabs as against the one-way loading system in bridges. As a result, engineers deem it incompatible with bridges as loads are transferred in one way only in bridges. However, technical reasoning has shown that when loads are transferred to bridges in one way only, large twisting moments are produced, the orthogonal rib system in a waffle slab provides an efficient means of resisting these twisting moments by incorporating large bending moments in the two orthogonal directions (Kennedy and Bahkt, 1983).

(Vaignan and Prashad, 2014) analysed voided and cellular deck slab using MIDAS civil and concluded that rectangular shaped cellular decks withstand more load than voided slabs.

For this purpose, serious attention needs to be given to the analysis of waffle slabs as bridge decks. Several methods have been used in the analysis of bridges, in each, the three dimensional structure is simplified based on assumptions on geometry, materials and relationship between components. The accuracy of analysis is dependent on the method used.
Bridge decks have been analyzed using several methods such as finite element grillage analogy, orthotropic plate theory as seen in past literatures (Schwetz *et al,* 2009).
Grillage method of analysis involves representing the bridge deck as a 2 by 2 system of interconnected beams intersecting each other. It is a numerical approach in analyzing bridge decks. It is easy to use and comprehend (Shreedar and Kharde, 2013).
According to (Shreedar and Kharde 2013) Lightfoot and Sawko made this method of analysis using grillages open to computer programming
As structures become complex and large, several methods of simplifying their analysis have been developed, among this is the computer aid. Computer aided analysis is a way of solving continuous system problems by dividing them into discrete elements thus simplifying analysis taking into consideration of compatibility and boundary conditions. A waffle slab is a three dimensional and complex structural element whose analysis requires very cumbersome calculations, hence the use of computer program for analysis. In this paper, the analysis of a waffle bridge deck using method of grillages was performed using direct stiffness method and MATLAB software as tools for writing the program as well as the analysis.
In the grillage analysis, the structure is represented by a plane grillage of discrete but interconnected beams. Almost any arrangement in plan is possible, so skew, curved, tapering or irregular decks can be analysed. But the usual layout is sets of parallel beams in two directions by assuming the plane of the grillage to be horizontal.
In a simple form of grillage analysis, each beam is assigned a torsional stiffness and flexural stiffness in the vertical plane. Vertical loads are applied only at the intersections of the beams. The matrix stiffness method of analysis is used by the existing software, to find the rotations about two horizontal axes and the vertical displacement at these nodes, and hence the bending and torsional moments and shear forces in the beams at each intersection. Warping stresses and shear lag are neglected in the analysis.

Location of grid lines

1. Grid lines should be adopted along line of strength.
2. The longitudinal gridlines run in parallel direction to the edge of the deck that is free. For longitudinal direction, it may be along the longitudinal webs, centre line of girders or edge beams etc.
3. Where isolated bearings are present, the grid line may be along the line joining center of bearing.
4. For transverse direction, it should be considered as one of each end connecting the center of bearing and along the center line of transverse beam (Surana And Agrawal, 1998).

Number and spacing of grid lines.

1. Where possible, odd numbers of gridlines should be chosen in both longitudinal and transverse directions.
2. The ratio of spacing of transverse grid line to those of longitudinal grids may be taken as 1 to 2.
3. As regards to the depth of slab, the minimum distance between longitudinal grid lines is limited to two to three times of the slab depth and the maximum separation of longitudinal members should not be more than one fourth of the effective span (Pandey and Maru, 2015).

A typical output gives the external reactions at each support. The bending and torsional moments will, in general, show a discontinuity at each joint. For an orthogonal grillage, each change in bending moment is equal to the change in torsional moment at that joint in the member at right angles to the one considered. Similarly, the change in torsional moment equals the change in bending moment in the perpendicular member.
Approximately one half of the local load can be distributed over the eight nodes of the vicinity to get correct results, even near the loaded point. An appropriate idealization for a continuous structure must be carefully selected. Each T-section of the longitudinal and transverse sides of a waffle slab is represented by a grillage beam. The transverse grillage members should extend to the edge of the real slab and their ends should be attached to longitudinal grillage beams, even if the real slab has no significant edge stiffening.

2. METHODOLOGY

Problem Formulation

TABLE 1: BRIDGE DATA

Bridge Deck Properties	
Width	7.3m
Width of Notional Lanes	3.65m
Thickness of Slab Topping	0.075m
Depth of Bridge Deck	0.37m
Width of Grid Beam	0.15mm
Depth of Asphalt Overlay	0.05m
Grade of Concrete	M30
Grade of Steel	E460
Rib spacing	1.2m

The loading considered is the self-weight, wearing course. The live load on the floor is HA loading as given in BS 5400-part 2 (1987) Clause 6.2.1. Load combination 1 of the BS 5400 part 2 is used. In this, eleven transverse members and five longitudinal members have been modeled. The grillage model has 77 nodes, 136 members and 231 degrees of freedom.

$$E = 3.61 \times 10^7 kN/m^2$$
$$G = 1.57 \times 10^7 kN/m^2$$
$$I = 1.93 \times 10^{-4} m^4$$
$$J = 1.063 \times 10^{-3} m^4$$

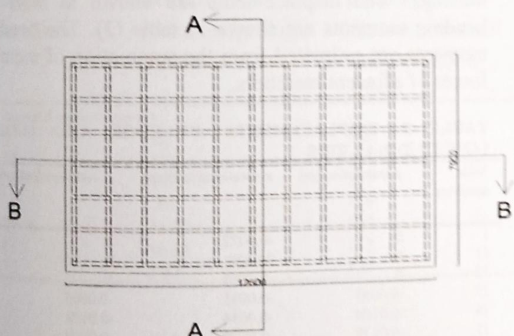


FIGURE1: PLAN OF WAFFLE BRIDGE DECK

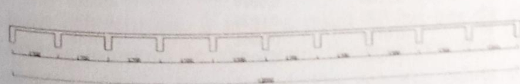


FIGURE 2: TRANSVERSE SECTION OF WAFFLE BRIDGE DECK.

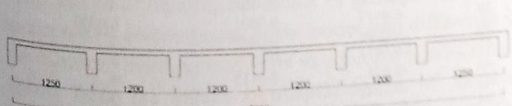


FIGURE 3: LONGITUDINAL SECTION OF THE WAFFLE BRIDGE DECK.

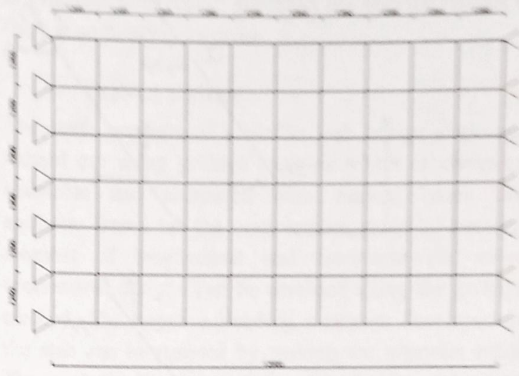
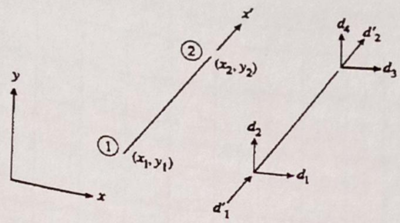


FIGURE 4: IDEALIZED GRILLAGE MODEL OF BRIDGE DECK

Analysis using MATLAB 2015a software (stiffness method)

1. Defining the nodal coordinates.
2. Numbering of numbers.
3. Defining the connectivity of elements
4. The length and angle of orientation
5. Material properties are modulus of elasticity and rigidities are defined.
6. For each element, the stiffness matrix computed the software.
7. The stiffness matrix for a grid member is a 6 by 6 matrix.
8. First the degrees of freedom at each node are identified and numbered; two perpendicular rotational displacement and one translational displacements $\Delta_1, \theta_2, \theta_3$.
9. The structures stiffness matrix for two nodes (one element) becomes;
   The global stiffness matrix is obtained by combining all the element stiffness matrices.
10. Assignments of boundary conditions.

Formulation of stiffness matrix



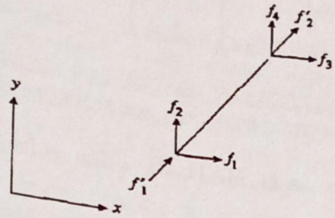


FIGURE 5: NODAL DEGREES OF FREEDOM FOR A GRID ELEMENT

Mathematical Model;

$$EI\frac{d^4v}{dx^4} = q$$

$$EI\frac{d^2v}{dx^2} = M \quad (1)$$

$$EI\frac{d^3v}{dx^3} = F \quad (2)$$

$$EI\frac{d^4v}{dx^4} = 0 \quad (3)$$

Integrating (4)

$$EIv = a_0 + a_1x + a_2x^2 + a_3x^3$$

The rotational degree of freedom (5)

$$\frac{dv}{dx} = 0;$$

(6)

Applying boundary conditions

Solving for coefficients,

$$x = 0:\ \frac{dv}{dx} = 0;\ v = 1; \Rightarrow a_0 = 0\ and\ a_1 = 1$$

(7)

$$x = L:\ \frac{dv}{dx} = 0;\ v = 0 \quad (8)$$

$$\frac{dv}{dx} = 0; \Rightarrow 2a_2 + 3a_3l \quad (9)$$

$$v = 1 + a_2l + a_3l^3 \quad (10)$$

$$\Rightarrow a_2 = \frac{3}{l^2}\ and\ a_3 = \frac{2}{l^3} \quad (11)$$

Equation (5) becomes

$$v = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \quad (12)$$

$$-EI\frac{d^3v}{dx^3} = F \Rightarrow -EI\left(\frac{12}{L^3}\right) = \frac{12}{L^3}EI \quad (13)$$

$$EI\frac{d^2v}{dx^2} = M \Rightarrow M_{x=0} = -\left(\frac{6}{L^2}\right) \quad (14)$$

$$k_{11} = -F_{x=0} = EI\left(\frac{12}{L^3}\right) \quad (15)$$

$$k_{21} = -M_{x=0} = EI\left(\frac{6}{L^2}\right) \quad (16)$$

By imposing a twisting moment at node 1, giving a rotation θ and applying boundary conditions the constant of integration found to be

$$T = \frac{GJ}{L}\theta \quad (17)$$

Therefore,

$$k_{33} = \frac{GJ}{L} \quad (18)$$

The remaining forces acting on the grid beam can be determined by applying unit displacement corresponding to translation and rotation at the two nodes of the beam.

$$\begin{bmatrix} k_{11}k_{12}k_{13}k_{14}k_{15}k_{16} \\ k_{21}k_{22}k_{23}k_{24}k_{25}k_{26} \\ k_{31}k_{32}k_{33}k_{34}k_{35}k_{36} \\ k_{41}k_{42}k_{43}k_{44}k_{45}k_{46} \\ k_{51}k_{52}k_{53}k_{54}k_{55}k_{56} \\ k_{61}k_{62}k_{63}k_{64}k_{65}k_{66} \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & \frac{-12EI}{L^3} & \frac{6EI}{L^2} & 0 \\ \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{2EI}{L} & 0 \\ 0 & 0 & \frac{GJ}{L} & 0 & 0 & \frac{-GJ}{L} & 0 \\ \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & 0 & \frac{12EI}{L^3} & \frac{-6EI}{L^2} & 0 \\ \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & \frac{-6EI}{L^2} & \frac{4EI}{L} & 0 \\ 0 & 0 & \frac{-GJ}{L} & 0 & 0 & \frac{GJ}{L} & 0 \end{bmatrix}$$

3. RESULTS AND DISCUSSION

Analysis of a waffle bridge deck have been carried by using grillage analogy method by simulating full the HA loading. The displacements are shown in table (2) bending moments are shown in table (3). The bending moments are estimated from the summation of member forces of adjacent members.

TABLE 2: DISPLACEMENTS AT EDGE AND MIDDLE LONGITUDINAL RIBS

Node numbers	y-translation(m)	x-rotations(radians)	Z-rotations(radians)
1	0	-0.0022	0
11	0	0.0020	0
12	0	-0.0057	0
13	-0.0059	-0.0045	-0.0066
14	-0.0104	-0.0034	-0.0123
15	-0.0139	-0.0029	-0.0169
16	-0.0170	-0.0025	-0.0204
17	-0.0190	-0.0001	-0.0255
18	-0.0200	0.0019	-0.0242
19	-0.0173	0.0028	-0.0209
20	-0.0135	0.0041	-0.0158
21	-0.0078	0.0059	-0.087
22	0	0.131	0
34	0	-0.0155	0
35	-0.0212	-0.0158	0
36	-0.0388	-0.0115	0
37	-0.0512	-0.007	0
38	--0.0593	-0.0047	0
39	-0.0627	0.0001	0.0000
40	-0.0592	0.0047	0
41	-0.0511	0.0078	0
42	-0.038	0.0115	0
43	-0.0211	0.015	0
44	0	-0.0000	0

The rotation in the vertical direction which causes torsion is higher towards the edge of the bridge deck. And then reduces towards mid-span of the deck. There is no rotation in the vertical direction at midspan in the longitudinal direction. This can be attributed to the symmetry in the grillage layout of the deck. This results in zero twisting moment in the longitudinal rib.

TABLE 3: COMPARISON BETWEEN VALUES OF BENDING MOMENTS (LONGITUDINAL RIB)

Node numbers	Bending moments(grillage analysis) kN-m	Bending moments(manual analysis) kN-m
Internal Longitudinal rib		
13	64.43	386.2
14	52.73	686.7
15	39.11	901
16	24.93	1030
17	1.46	1073
18	-23.93	1030
19	-39.47	901
20	-53.2	686.7
21	-45.19	386.2

TABLE 4: COMPARISON BETWEEN VALUES OF BENDING MOMENTS (TRANSVERSE RIB)

Node numbers	Bending moments(grillage analysis) kN-m	Bending moments(manual analysis) kN-m
Internal transverse rib		
6	3.63	132
17	43.97	113
28	27.07	177
39	1.48	198
50	-24.14	177
61	-46.36	113
72	-3.63	132

The values of bending moments obtained in the grillage analysis are lower than those obtained manually with beam line analysis, it is as a result of the neglect of twisting moments in the beam line analysis that is, twisting of beam is neglected. The negative values of bending moments is caused by the effect of support conditions adopted in the analysis. The stepped or saw-tooth values of bending moments in grillage analysis is as a result of discontinuity at the nodes and difference in the values of bending moments in adjacent beams which results from the values of displacements of member elements. The bending moments are higher at the supports to resist the large twisting moments developed at the edges and reduces gradually towards the middle at there is no twisting moment.

4. CONCLUSION

The static analysis of a waffle slab bridge deck was carried out using grillage analysis which is computer amenable and compared with manual beam line analysis. Based on the comparative study of bending moment of longitudinal and transverse rib, more economical designs can be obtained using the grillage method. The negative bending moments developed in the slab can be resisted by making the supports solid. The method also makes it possible for the slab to be analsed as an entity instead of the conventional slab-beam analysis. Large twisting moments developed at the support and edge of a bridge deck can be minimized using waffle slabs for bridge decks.

REFERENCES

British Standard Institution (1978). Code of practice for the design of steel, concrete and composite bridges. BS 5400 part 2 and 4

Halkude S. A and Mahamuni S.V (2014). Comparison of Various Methods of Analysis of Grid Floor Frame.*International Journal of Engineering Science Invention www.ijesi.org 3 (2) ISSN: 2319–6734*.pp 1-7

Kennedy J.B and Bakht B. (1983) Feasibility of waffle slabs for bridges. *Paper presented at The Annual Conference of The Canadian Society for Civil Engineering, Edmonton, Alberta.*

Pandey K.K and Maru S.(2015). Modelling Of Skew Bridge Deck by Grillage Method. *International journal of engineering research and general science, 3(4). ISSN: 2091-2730.12*. pp 1-7

Schwetz P. F. Gastal F. P. S. L. Silva F. L. C. P. (2009). Numerical and Experimental Study of a Real Scale Waffle Slab. *Ibracon structures and materials journal 2(4) ISSN:1983-4195*. 380-403.

Shreedhar, R. and Kharde, R (2014). Comparative study of grillage method and finite element method of RCC bridge deck. *International journal of scientific& engineering research volume 4(2). ISSN:2229-5518*. 1-10

Surana, C.S. and Agrawal, R. (1998) *Grillage Analogy in Bridge Deck Analysis*. India: Narosa Publishing House,6 Community Centre, Pansheel Park, New Delhi 110 017.

Vaignam, B and Prasad B.S.R (2014). Analysis of voided deck slab and cellular deck slab using Midas Civil. *International Journal Of Engineering Research and Technology* 3(9) *ISSN:2278-0181*.pp 1278-1290.