# Power Quality Disturbances Analysis using Two Forms of Wigner-Ville Distribution

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Abstract—Power quality (PO) is a major concern to both power utilities and consumers due to the widespread use of sensitive loads, power electronic devices and industrial equipments. This paper presents an analysis of PQ disturbances as a first step of their appropriate identification using time-frequency analysis. The research focused on the two most common forms of PQ disturbances in Nigeria: voltage sag and swell. The disturbance analysis was based on two basic forms of Wigner-Ville distribution (WVD); the windowed WVD and the filtered WVD used in conjunction with Hilbert transform. Results obtained showed that both forms of WVD are able to accurately capture the disturbances with concentration of energy along the main power supply frequency of 50Hz within the PQ disturbance duration of 0.1s. The proposed methodology showed that the WVDs would be a good signal processing method for subsequent classification of PQ disturbances in electrical power smart grid.

Index Terms—Power Quality (PQ), time-frequency analysis, voltage sag, voltage swell, Wigner-Ville distribution (WVD).

### I. INTRODUCTION

Power system is a multi-level connected system consisting of primarily generation, transmission and distribution units. These units consist of vast range of equipment such as power transformers, synchronous machines, instrument transformers, induction motors, power electronic devices, and so on for proper functioning. Given these constituents, signal processing is an important assessment tool that enables researchers to understand, plan, design and operate the complex and smart electronic grid of the future [1]. It is normally qualified by the analytical aspects of the electrical systems, and can help to expose and characterize the diversity, unity, meaning and intrinsic purpose of electrical parameters, system phenomena and events.

Power quality (PQ) is characterized by events which require analysis of voltage waveforms as part of power system monitoring. These events can be characterized by various forms of PQ disturbances such as voltage sags, swells, harmonics, transients, flicker and combination of these disturbances [2]. As such, these disturbances need to be detected, analysed, and classified reliably before appropriate action can be taken to counteract or mitigate them [3]. The

aim of this paper is to further promote the use of digital signal processing within power systems context through analysis of some of the main PQ disturbances as a key aspect of smart grids. A smart grid is a computerized electrical grid that uses the merged technologies of sensors, computers, and other digitized systems for fault detection and resolution [4].

Various signal processing methods have been developed and used for PQ disturbances characterization where good results were achieved within some certain scope but accompanied by new limitations. Most recent examples of these signal processing methods for PQ disturbances analysis includes empirical mode decomposition and multilayer perceptron [5], independent component analysis (ICA) and support vector machine (SVM) [6], fast Fourier transform (FFT) and adaptive filter[7], S-transform based on optimally concentrated window [8], tunable-Q wavelet transform and dual multiclass SVM [9] and curvelet transform (CT) and deep convolutional neural networks (CNN) [10]. A most recent significant and comprehensive review of researches associated with PQ events over the last three decades can be found in [11].

In line with the previous researches, the paper focused on analyzing voltage swell and sag aspects of PQ disturbances using two forms of Wigner-Ville distribution (WVD) in conjunction with Hilbert transform (HT). The selected PO disturbances are the most common disturbances in the home country of the researchers based on a survey carried out amongst 15 multinational companies [12]. WVD is used as the main signal processing tool due to its characteristics of being the key and the main origin of the quadratic class of timefrequency distributions (TFDs) [13]. However, the popular HT is used to convert the real signal to analytical format before the use of WVD for better signal processing. All illustrations and analysis was carried out using the MATLAB R2015a. The rest of the paper is as follows: model for the PQ disturbance events considered in this paper are presented in section II, proposed methodology involving Hilbert transform and WVD forms is presented in section III. Section IV presents the analysis of results and discussion while the paper is concluded in section V.

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### II. POWER QUALITY DISTURBANCES MODEL

The research in this paper focused on the two main PQ disturbances; voltage sag and swell. However, these two waveforms for the PQ disturbances were normalized with respect to a pure 50Hz sine wave. This pure sine wave is mathematically given in (1).

$$s(t) = A\sin(2\pi f t) \tag{1}$$

Where s(t) is the signal, as function of time, A is the amplitude with a normalized voltage value of 1 and f is the frequency pegged at 50Hz to denote the normal power supply voltage frequency. The values of these aforementioned defined parameters are also adopted for the remaining PQ disturbances model to be presented. The pure sine wave is graphically depicted in Fig. 1

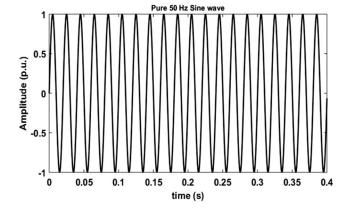


Figure 1. Pure sine waveform

It is clearly seen in Fig. 1 that sine wave is constant at 50Hz with no form of disturbance. The vertical axis is labeled amplitude to a normalized value of 1 per unit (p. u.) as given in (1). The voltage sag is the most common type of PQ disturbances that denotes a long-term reduction in voltage as opposed to short-term reduction in voltage dip [14]. It can be caused by unreliable grid system and non-suitability of power distributor tolerances for voltage sensitive equipment among others [12]. The voltage sag model is mathematically given in (2) with controlling parameters of (3) [3].

$$s(t) = A(1 - \alpha(u(t - t_1) - u(t - t_2)))sin(2\pi f t)$$
 (2)  

$$0.1 \le \alpha \le 0.9; \quad T \le t_2 - t_1 \le 9T$$
 (3)

Where  $\alpha$  is the main voltage control parameter, T is the period and all other terms defined after presentation of (1). The parametric equation of (2) gives the primary advantage of wide and controlled signal parameters variation. Voltage sag is graphically depicted in Fig. 2.

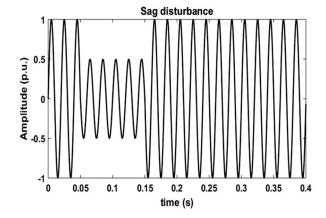


Figure 2. Votage sag PQ disturbance

It is noted from Fig. 2 that a PQ disturbance of voltage sag occurs between 0.05 to 0.15 seconds which was modeled by (2). Voltage swell denotes an increase in voltage outside the normal rated tolerance of equipment. It is long-term in nature when compared to voltage spikes, impulses or surges. It may be caused by unbalanced load on a three phase system in the non-fault phases due to voltage sag disturbance on the fault phase in a single line to ground fault among others [12]. It is mathematically given in (4) based on controlling parameters of (5) [3].

$$s(t) = A(1 + \alpha(u(t - t_1) - u(t - t_2)))sin(2\pi ft)$$
 (4)

$$0.1 \le \alpha \le 0.8; \quad T \le t_2 - t_1 \le 9T$$
 (5)

Equation (4) is very similar to (2) with the only difference of the swapping of first subtraction sign with addition sign. This is because they are highly related in the real world and as such their model also follows the same line. A good example is the earlier mentioned single line to ground fault. Their controlling parameters are basically the same, and hence similarities of (3) and (5). The voltage swell is graphically depicted in Fig. 3.

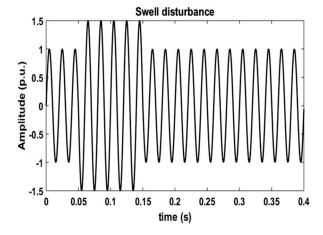


Figure 3. Votage swell PQ disturbance

Similar to Fig. 2, a voltage increase in noted between 0.05 and 0.15 seconds in Fig. 3 to depict the voltage swell type of PQ disturbance. The similarity in time of PQ disturbance is due to using the same main voltage control parameter  $(\alpha)$  of

0.5 based on literature presented in [3]. In order to analyze each PQ disturbance among many other types, time-frequency analysis was adopted in this research and is presented in the next section.

### III. TIME-FREQUENCY ANALYSIS

Time-frequency analysis is the main methodology adopted and developed in this paper. Time-frequency analysis arises from the need to represent signal in the time-frequency domain due to inadequacies of information of the signal to be analysed when accessed in time or frequency domain. It is an advancement of mathematical postulations and ideas used in the analysis of time-varying spectra of signals in order to cater for various problems in different fields [15]. One of such fields of interest considered in this paper is power system related problems. The proposed methodology is divided into two parts; HT and WVD.

## A. Hilbert Transform (HT)

Simply put, HT is used to introduce a phase lag of  $90^{\circ}$ . For a signal s(t), its Hilbert transform is given by (6).

$$H\{s(t)\} = F^{-1}\{(-j \, sgn \, f)F\{s(t)\}\} \tag{6}$$

Where  $F\{\}$  is the Fourier transform of the expression in bracket from time to frequency,  $F^{-1}\{\}$  is the inverse Fourier transform of the expression in bracket from frequency to time, f is frequency of the signal and , sgn is the sign to determine multiplication factor for the frequency. Equation (6) indicates that for positive frequency, sinusoidal signal become cosinusoidal signal and vice versa. This transformation is expressed in (7) and (8).

$$H\{\cos(2\pi f t)\} = \sin(2\pi f t) \tag{7}$$

$$H\{\sin(2\pi f t)\} = -\cos(2\pi f t) \tag{8}$$

However, the main point of interest of HT to this paper is the generation of the signal's complex form to eliminate the non-required negative frequencies generated by Fourier transform. It also provides two main advantages through halving the total bandwidth, hence allowing sampling at half the Nyquist rate without aliasing and avoiding of interference terms generated by quadratic time–frequency distributions (TFD)'s interaction of positive and negative components [15]. The mathematical expression for the complex form based on Hilbert transform is given in (9).

$$z(t) = s(t) + i H\{s(t)\}$$
 (9)

Where z(t) is the complex (or analytic) form of signal s(t). Substitution of either (7) or (8) into (9) would indicate that the analytic form would be an exponential function of single frequency after Fourier transform instead of its original sinusoidal (or cosinusoidal) functions of double frequencies.

### B. Wigner-Ville Distribtuion (WVD)

Quadratic TFDs were developed to mitigate the limitation of linear TFDs through the use of quadratic functions. The Wigner-Ville distribution (WVD) is the baseline quadratic TFD that uses a quadratic function to concentrate the signal energy along the signal's instantaneous frequencies. The WVD for a complex signal is given (10) - (12).

$$W_{z}(t,f) = \underset{\tau \to f}{F} \{ K_{z}(t,\tau) \}$$
 (10)

$$W_z(t,f) = F_{\tau \to f} \left\{ z \left( t + \frac{\tau}{2} \right) z^* \left( t - \frac{\tau}{2} \right) \right\}$$
 (11)

$$W_z(t,f) = \int_{-\infty}^{\infty} z \left(t + \frac{\tau}{2}\right) z^* \left(t - \frac{\tau}{2}\right) e^{-j2\pi f \tau} d\tau \qquad (12)$$

Where,  $K_z(t,\tau)$  is the instantaneous autocorrelation function (IAF) (i.e. quadratic function),  $F_{\tau \to f}$  denotes taking a Fourier transform with respect to  $\tau$ , z(t) is the complex form associate of the signal s(t),  $\tau$  is the time-lag and \* denotes the complex conjugate of the signal of interest. Unfortunately, the WVD comes with the undesired cross and inner terms as a result of this quadratic nature which often lead to mistranslation and lack of clarity of the information presented [15]. As such, these led to development of other TFDs through its modification.

This paper considered two basic forms of modifying WVD distribution through its IAF in order to verify their capabilities and reduce computational complexity by avoiding adaptive windows. The first way is through windowing the IAF in the lag direction with new WVD referred to as windowed WVD (WWVD). The lag domain ( $\tau$ ) is the resulting transformed time domain as a result of the IAF. WWVD is mathematically given in (13).

$$W_{z,w}(t,f) = \underset{\tau \to f}{F} \{ g(\tau) K_z(t,\tau) \}$$
 (13)

Where  $g(\tau)$  is the window function. For this research, the hamming window function is selected due to its minimum sidelobe [16] among other windows. This function is mathematically given in (14).

$$g(\tau) = 0.54 - 0.46 \cos\left(\frac{2\pi\tau}{T}\right), \quad 0 \le \tau \le T$$
 (14)

The hamming window can be considered as an optimized version of von-Hann window. The second way in which WVD considered in this paper was modified was through filtering (or smoothing) the IAF in the original time direction referred to as filter WVD (FWVD). The FWVD is mathematically given in (15).

$$W_{z,f}(t,f) = \underset{\tau \to f}{F} \{ h(t) * K_z(t,\tau) \}$$
 (15)

Where h(t) is the filter function. For this research, the Kaiser filter function was selected due to its better ripple factor control [17]. It is mathematically given in (16).

$$g_{1}(t) = \frac{I_{0}\left\{\beta\sqrt{1-\left(\frac{t}{T}\right)^{2}}\right\}}{I_{0}\left\{\beta\right\}}, \ 0 \le t \le T$$
 (16)

Where a Bessel constant default value of  $\beta = \frac{1}{2}$  is used and  $I_0\{.\}$  is the modified zero-order Bessel function. It is a rapidly converging power series of the form of (17).

$$I_0\{x\} = 1 + \sum_{k=1}^{\infty} \left[ \frac{\left(\frac{x}{2}\right)^k}{k!} \right]^2$$
 (17)

Thereafter, a set of algorithms were then designed based all the equations associated with the methodology presented in this section and applied to the PQ disturbances presented in the previous section to obtain the required results.

### IV. RESULTS AND DISCUSSION

In this paper, the two PQ disturbances considered were analyzed using the WWVD and FWVD. The selected length for the window and filter function durations were the same at 0.004s (1% of the total observation duration) to avoid unnecessary expansion. Also the duration for the observation was based on [3]. Finally, the output of the WVDs considered just like all other TFDs is a function of power, time and frequency. The results presented in this section are functions of time and frequency with internal lines denoting power. The closer the internal line to the center, the higher the magnitude power. Two main reasons are identified as reason for the presenting these results in this format. Firstly, the main tool of analysis is not always capable of showing a very clear pictorial 3D for the TFDs' output. Secondly, the focus of this paper is on the time-frequency relationship analysis for subsequent classification in future research. The results for the two WVDs analysis of the voltage sag PQ disturbance is shown in Fig. 4.

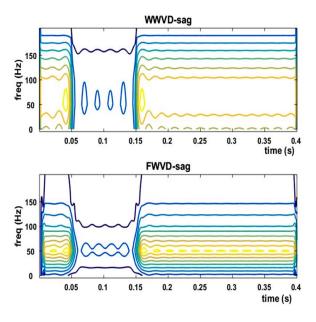


Figure 4. WWVD and FWVD for voltage sag PQ disturbance.

It is seen from Fig. 4 that the 0.1s time of occurrence of the PQ disturbance between 0.05s and 0.15s is accurately captured by the two WVDs in line with Fig. 2 and Fig. 3. Furthermore, the power supply frequency of 50Hz is also captured accurately in Fig. 4 as the center of energy concentration is at this frequency. However, it is seen that during the voltage sag, there is less concentration of energy along the frequency to indicate the disturbance. This showed that two WVDs can accurately capture this disturbance. If Fig. 4 is individually examined, it will be observed that FWVD provides a better PQ disturbance capture than WWVD as energy concentration are more closely knitted together. This is because the

windowing of IAF leads to expansion of signal pulse in the frequency domain while the filtering of IAF tries to smooth the signal pulse in frequency domain without necessarily expanding it. The result obtained for the voltage swell PQ disturbance is given in Fig. 5.

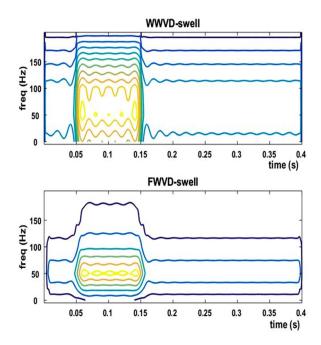


Figure 5. WWVD and FWVD for voltage swell PQ disturbance.

Similar to the result obtained in Fig. 4, the disturbance occurrence duration of 0.1s between 0.05s and 0.15s and the supply frequency of 50Hz is accurately captured in Fig. 5. Also, the individual examination of each subplot of Fig. 5 also demonstrates the superiority of FWVD over the WWVD based on the aforementioned discussion. However, the major difference is that this instance in Fig. 5, the energy concentration is more along the disturbance than main supply voltage signal. Results of Fig. 4 and Fig. 5 therefore show that the WVDs concentrate more energy along the lesser frequency or higher voltage amplitude irrespective of the disturbance.

Further research will involve further signal processing of converting 3D output to actual 2D output based on time-frequency representations of Figs. 4 and 5. These further signal processing may be signal-based or image-based with feature extraction for sophisticated classifiers such as SVM and CNN.

### V. CONCLUSION

The research involving the analysis of voltage sag and swell PQ disturbances using WWVD and FWVD was presented in this paper. The two WVDs were able to accurately capture both the time duration of the PQ disturbance of 0.1s and supply frequency voltage of 50Hz. However, the FWVD analyzed the disturbances better than the WWVD as energy concentration are more closely knitted together along the frequency axis. Finally, it was observed that

the higher the voltage amplitude (or lesser the voltage frequency) the more the energy concentration. Therefore, further signal processing may be applied using these WVDs involving feature extraction and classification for subsequent identification of each PQ disturbance.

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