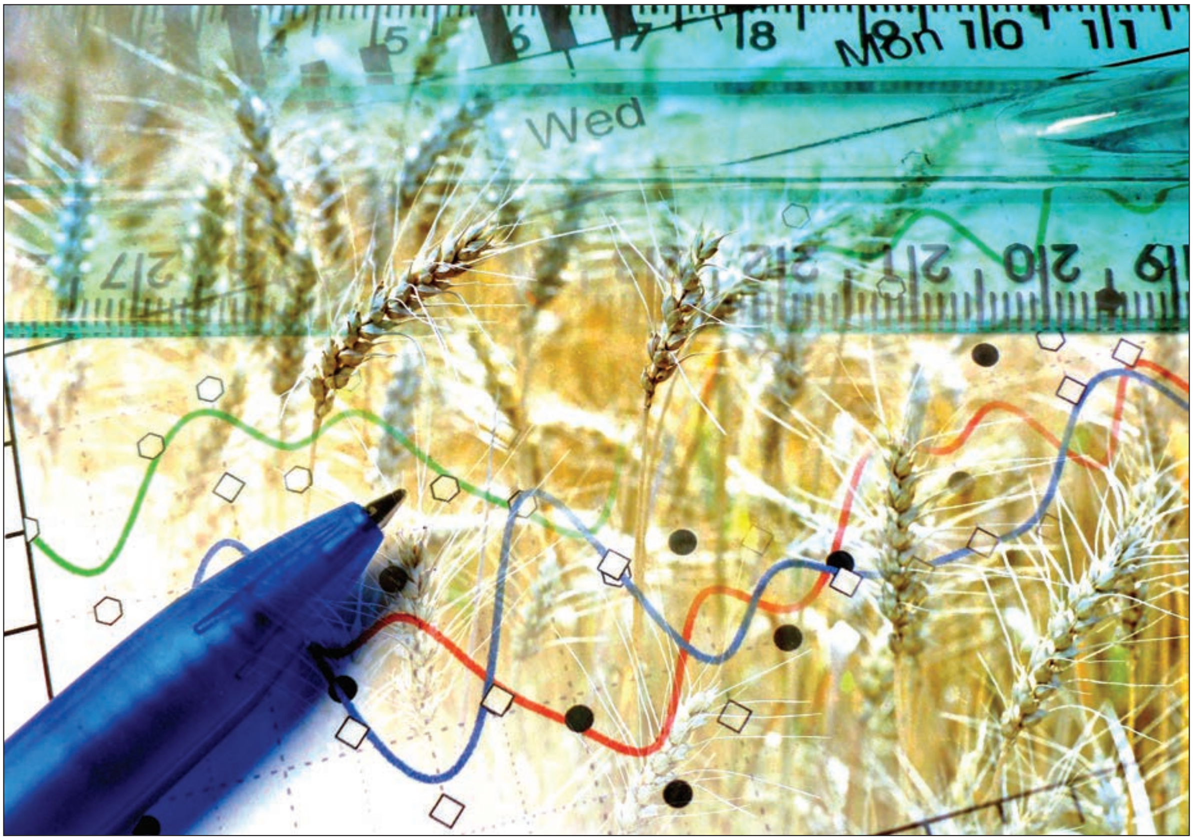

Trends in Agricultural Economics

ISSN 1994-7933

science alert

ANSInet
an open access publisher
http://ansinet.com

Trends in Agriculture Economics, 2 (1): 17-27, 2009
ISSN 1994-7933

Spline Functions: Assessing their Forecasting Consistency with Changes in the Type of Model and Choice of Joint Points

J.N. Nmadu, E.S. Yisa and U.S. Mohammed
Department of Agricultural Economics and Extension Technology,
Federal University of Technology, Minna, Nigeria

Abstract: The possibility of the type of spline function and joint points selected affecting the consistency of the ex-post and ex-ante forecasts were tested using cereal production (1961-2006) and percent contribution of agriculture to GDP (1961-2004) in Nigeria. Three types of model, that is, Linear-Quadratic-Linear, Quadratic-Quadratic-Linear and Linear-Quadratic-Quadratic, were used. The result indicated that there is no universality as to which model is appropriate, rather all possible models should be tried and the one that gives most consistent result when compared to observed data and other factors should be used.

Key words: Grafted polynomial, sub-periods, joint points, ex-post, ex-ante

INTRODUCTION

Since, Fuller (1969) introduced the concept of spline or grafted polynomial, many researchers have utilized it to make ex-post and ex-ante forecast of economic time series data beyond the estimation period. Such studies include those by Phillip (1990), Rahman and Damisa (1999), Nmadu and Amos (2002), Nmadu and Phillp (2001) and Nmadu *et al*. (2004). For example Bormann *et al*. (2002) estimated lactation stage, age at milking, previous days open and days pregnant using quadratic polynomials by fitting joint points. Meyer (2005) successfully modeled growth of Australian Angus cattle using the spline function. Some other researchers have made innovations to the original model. Those include Fox and Grafton (2000), Parsons and Hunt (1981) and Marsh (1986). Fox and Grafton (2000) used capital and model selection criteria rather than trend to determine appropriate break points.

The concept is based on the visual examination of the scatter diagram of the available data series against trend in order to divide the data into sub-periods and to suggest suitable joint points to capture all the sub-periods into a single model (Fuller, 1969; Phillip, 1990; Meyer, 2005; Pierre *et al*., 1987). Since, the eventual model estimated is subject to the visual examination of the base data by the researcher, it therefore means that the appropriateness of the eventual estimated function and the forecast based on it is accurate to the extent of the accuracy of the researcher's visualization. In this circumstance, the same data can be modeled along different lines depending on the researcher. Hence, there is need to find out if the quality of the model resulting from this modeling is affected by the type of function and other factors. The main objective of this study is to investigate whether the choice of joint points in a spline function and the type of model selected affects the forecasting ability of the resulting estimated coefficients.

MATERIALS AND METHODS

The data used in this research were mainly secondary data sourced from Earth Trend (2006). The data included cereal grains production in Nigeria in metric tones between 1961 and 2005, percent

Corresponding Author: J.N. Nmadu, Department of Agricultural Economics and Extension Technology, Federal University of Technology, Minna, Nigeria Tel/Fax: 234-803-5861170

agriculture contribution to Nigerian GDP between 1965 and 2004 and aggregate fertilizer consumption by Nigerian farmers in metric tones between 1961 and 2001.

Normally, the available data is plotted against trend in order to divide the series into segments based on visual examination. Traditionally, the data is usually divided into three sub-periods and no attempt was made in this study to go beyond that.

There are two commonly used models, that is, Linear-Quadratic-Linear and Quadratic-Quadratic-Linear models. These two are preferred because it is normal to have linear portion as the terminal (Fuller, 1969; Phillip, 1990) as that enhances forecasting which is the main objective of using the system. However, an attempt was made to explore all possible models in order to show if the eventual model is acceptable for forecasting. Therefore, Linear-Quadratic-Quadratic, Linear-Linear-Quadratic, Linear-Linear-Linear and Quadratic-Quadratic-Quadratic models were tried. Linear-Linear-Quadratic was dropped because some of the coefficients were over-identified while Linear-Linear-Linear was dropped because the variables were over-identified and any of the linear regression models can be applied to a data series that is linear over the entire trend and there will be no need to divide it into sub-periods. In the case of Quadratic-Quadratic-Quadratic, the model was dropped because the variables were over-identified and the data series with this type of behavior is better estimated with higher polynomial instead of dividing into sub-periods.

The details of the models and the mean equation are shown below for the three models left. The detail of how the mean equations were obtained is shown for one of the models in Appendix.

Linear Quadratic Linear

A graphical examination of the data may show that it can be divided into three segments; hence the following trend function was suggested:

$$Y_t = \alpha_0 + \beta_0 t \qquad t \le JP_1 \qquad (1)$$

$$Y_t = \alpha_1 + \beta_1 t + \phi_1 t^2 \qquad JP_1 < t \le JP_2 \qquad (2)$$

$$Y_t = \alpha_2 + \beta_2 t \qquad t > JP_2 \qquad (3)$$

Where:

Y_t = Data series in year t

t = Trend

α's, β's and ϕ = Structural parameters to be estimated

JP_1 and JP_2 = Joint point 1 and 2, respectively

Equation 1-3 are then reworked as shown below:

$$Y_t = \alpha_2 + \beta_2 t + \phi_1(JP_2^2 - JP_1^2 + - 2JP_2 t + 2JP_1 t) \qquad t \le JP_1 \qquad (4)$$

$$Y_t = \alpha_2 + \beta_2 t + \phi_1(JP_2^2 - 2JP_2 t + t^2) \qquad JP_1 < t \le JP_2 \qquad (5)$$

$$Y_t = \alpha_2 + \beta_2 t \qquad t > JP_2 \qquad (6)$$

Equation 4-6, are then formed into a single equation for estimation as follows:

$$Y_t = \mu_0 Z_0 + \mu_1 Z_1 + \mu_1 Z_2 + U_t \qquad (7)$$

Where:

Z_o	$= 1$	$\forall t$, $\forall$ = for all
Z_1	$= t$	$\forall t$
Z_2	$= JP_2^2 - JP_1^2 - 2t\,(JP_2 - JP_1)$	$t \leq JP_1$
	$= (t - JP_2)^2$	$JP_1 < t \leq JP_2$
	$= 0$	$t > JP_2$
U_t	= Error term assumed to be well behaved	

Quadratic Quadratic Linear

A graphical examination of a data series may reveal that it can be divided into different segments as the trend equation below:

$$Q_t = \alpha_o + \beta_o t + \phi_o t^2 \qquad t \leq JP_1 \tag{8}$$

$$Q_t = \alpha_1 + \beta_1 t + \phi_1 t^2 \qquad JP_1 < t \leq JP_2 \tag{9}$$

$$Q_t = \alpha_2 + \beta_2 t \qquad t > JP_2 \tag{10}$$

Where:

Q_t = Data series in year t

t = Trend

α's, β's and ϕ = Structural parameters to be estimated

JP_1 and JP_2 = Joint point 1 and 2, respectively

Equation 8-10 are then reworked as shown below:

$$Q_t = \alpha_2 + \beta_2 t + \phi_1(JP_2-t)^2 + (\phi_1-\phi_0)(JP_1-t)^2 \qquad t \leq JP_1 \tag{11}$$

$$Q_t = \alpha_2 + \beta_2 t + \phi_1(JP_2-t)^2 \qquad JP_1 < t \leq JP_2 \tag{12}$$

$$Q_t = \alpha_2 + \beta_2 t \qquad t > JP_2 \tag{13}$$

Equation 11-13, are then formed into a single equation for estimation as follows:

$$Q_t = \mu_o Z_o + \mu_1 Z_1 + \mu_2 Z_2 + \mu_3 Z_3 + U_t \tag{14}$$

Where:

Z_o	$= 1$	$\forall t$
Z_1	$= t$	$\forall t$
Z_2	$= (t-JP_2)^2$	$t \leq JP_1$
	$= (t-JP_2)^2$	$JP_1 < t \leq JP_2$
	$= 0$	$t > JP_2$
Z_3	$= (t-JP_1)^2$	$t \leq JP_1$
	$= 0$	$JP_1 < t \leq JP_2$
	$= 0$	$t > JP_2$
μ's	= Structural parameters to be estimated	
U_t	= Error term assumed to be well behaved	

Linear Quadratic Quadratic

A graphical examination of a data series may reveal that it can be divided into different segments as the trend equation below:

$$GD_t = \alpha_o + \beta_o t \qquad t \le JP_1 \qquad (15)$$

$$GD_t = \alpha_1 + \beta_1 t + \phi_1 t^2 \qquad JP_1 < t \le JP_2 \qquad (16)$$

$$GD_t = \alpha_2 + \beta_2 t + \phi_2 t^2 \qquad t > JP_2 \qquad (17)$$

Where:

GD_t = Data series in year t

t = Trend

α's, β's and φ = Structural parameters to be estimated

JP_1 and JP_2 = Joint point 1 and 2, respectively

Equation 15-17, are then reworked as shown below:

$$GD_t = \alpha_2 + \beta_2 t + (2JP_2 t - JP_2^2)(\phi_2 - \phi_1) + (2JP_1 t - JP_1^2)\phi_1 \qquad t \le JP_1 \qquad (18)$$

$$GD_t = \alpha_2 + \beta_2 t + (2JP_2 t - JP_2^2)(\phi_2 - \phi_1) + \phi_2 t^2 \qquad JP_1 < t \le JP_2 \qquad (19)$$

$$GD_t = \alpha_2 + \beta_2 t + \phi_2 t^2 \qquad t > JP_2 \qquad (20)$$

Equation 18-20 are then formed into a single equation for estimation as follows:

$$GD_t = \mu_o Z_o + \mu_1 Z_1 + \mu_2 Z_2 + \mu_3 Z_3 + \mu_3 Z_3 + \mu_4 Z_4 + U_t \qquad (21)$$

Where:

Z_o	$= 1,$	$\forall t$
Z_1	$= t,$	$\forall t$
Z_2	$= 2JP_2 t - JP_2^2$	$t \le JP_1$
	$= 2JP_2 t - JP_2^2$	$JP_1 < t \le JP_2$
	$= 0,$	$t > JP_2$
Z_3	$= 2JP_1 t - JP_1^2$	$t \le JP_1$
	$= 0$	$JP_1 < t \le JP_2$
	$= 0$	$t > JP_2$
Z_4	$= 0$	$t \le JP_1$
	$= t^2$	$JP_1 < t \le JP_2$
	$= t^2,$	$t > JP_2$
μ's	= Structural parameters to be estimated	
U_t	= Error term assumed to be well behaved	

Equations 7, 14 and 21 are the mean equations; they are continuous with the various restrictions relating to each model.

The three final equations were applied to cereal grains production in Nigeria in metric tones between 1961 and 2005 and percent agriculture contribution to Nigerian GDP between 1965 and 2004.

In addition, the models were applied to cereal grains production when either GDP or aggregate fertilizer consumption by Nigerian farmers in metric tones between 1961 and 2001 or both are added as explanatory variables. Ex-post forecast of the trend was then made for estimation period while ex-ante forecast was made to year 2020 and the forecasts compared with the observed data. The data were obtained from EarthTrend (2006). After the estimation of the models, the forecasting ability of each of them was assessed using Mean Square Error (MSE). MSE is given as:

$$\text{MSE} = \sqrt{\frac{1}{n}(Y_t - y_t)^2}$$

Where:

MSE = Mean Square Error
Y_t = Observed value
y_t = Estimated value
n = Sample size

The model with the least MSE is adjudged better than the other.

RESULTS AND DISCUSSION

The estimates of the various explanatory variables using the different models are presented in Table 1-3 while the ex-post and ex-ante estimates of the data series are shows in Fig. 1-4. Table 4 gives the MSE for all the models.

The result in Table 1 (Linear-Quadratic-Linear) shows that all variables are significant in the trend of cereal grains production during the period under study and the estimates of the coefficients of GDP or fertilizer or both were not significant as explanatory variables in the trend of cereal production. Table 1 also shows that the trend of GDP was not well explained by the variables included in the model. The result in Table 2 (Quadratic-Quadratic-Linear) shows that all the variables were significant in the trend of cereal production and GDP but the estimates of the coefficients of GDP was not a significant explanatory variable in cereal production.

Table 1: Estimates of the coefficient for Linear Quadratic Linear model

Variables	Cereal grains[1]	GDP[2]	Cereal grains/GDP[1]	Cereal grains/Fertilizer[1]	All[1]
Z_0	-1178952 (93927.25)***	-396.902 (640.99)[ns]	-1337201 (105564.6)***	-1220420 (119544.6)***	-1371891 (120028.3)***
Z_1	599.3687 (47.181)***	0.213245 (0.32)[ns]	678.6356 (53.09039)***	620.2491 (60.20337)***	696.2218 (62.18205)***
Z_2	25.6932 (3.12857)***	0.029212 (0.013054)**	36.31791 (5.659257)***	26.63412 (3.725489)***	41.35532 (6.950785)***
Z_3					
Z_4					
Z_5			5.499868 (59.068)[ns]		-35.874 (68.72507)[ns]
Z_6				0.019836 (3.839629)[ns]	4.512133 (3.788607)[ns]
F	93.540***	15.695***	63.929***	42.272***	39.326***
$\bar{R}^2$	0.81	0.43	0.83	0.75	0.81
N	45	40	39	42	38

(1) JP_1 = 1965, JP_2 = 1987; (2) JP_1 = 1980, JP_2 = 1988; Values in parenthesis are SE; *p<0.10; **p<0.05; ***p<0.01, ns: Not significant

Table 2: Estimates of the coefficient for the Quadratic Quadratic Linear model

Variables	Cereal grains[1]	GDP[2]	Cereal grains/GDP[1]	Cereal grains/Fertilizer[1]	All[1]
Z_0	-1414596 (100326.8)***	2113.035 (782.5428)***	-1503434 (115600.2)***	-1542672 (100839.1)***	-1580119 (108501.5)***
Z_1	717.3874 (50.34821)***	-1.0419 (0.39169)***	760.6251 (57.89634)***	780.553 (50.61249)***	798.4331 (54.55221)***
Z_2	41.15147 (5.076118)***	-0.09947 (0.03193)***	44.26467 (6.506164)***	61.84684 (6.35926)***	67.40492 (8.03633)***
Z_3	-83.164 (21.200334)***	0.187741 (0.045236)***	-164.346 (62.91912)***	-141.443 (22.83797)***	-212.082 (47.4194)***
Z_4					
Z_5			89.29117 (61.08482)[ns]		42.67508 (57.49479)[ns]
Z_6				10.28348 (3.229767)***	11.93835 (3.444629)***
F	86.967***	26.217***	55.838***	70.800***	53.578***
$\bar{R}^2$	0.85	0.66	0.85	0.87	0.87
N	45	40	39	42	38

(1) JP_1 = 1973, JP_2 = 1987; (2) JP_1 = 1988, JP_2 = 1998; Values in parenthesis are SE; *p<0.10; **p<0.05; ***p<0.01; ns: Not significant

Table 3: Estimates of the coefficient for the Linear Quadratic Quadratic model

Variables	Cereal grains[1]	GDP[2]	Cereal grains/GDP[1]	Cereal grains/Fertilizer[1]	All[1]
Z_0	57183180.8	169243.2967	57547194	59818881	57318927
	(9927363.681)***	(57915.753)***	(12053735)***	(19722368)***	(24781979)***
Z_1	-57704.981	-169.0473288	-58114.3	-60382.1	-57884.4
	(10010.67848)***	(58.335167)***	(12145.29)***	(19892.91)***	(24975.54)***
Z_2	-0.0015348	-0.0000015	-0.00144	-0.00149	-0.00144
	(0.000304639)***	(0.00000106)ns	(0.000326)***	(0.000406)***	(0.000422)***
Z_3	14.5609663	0.042215403	14.67467	15.2407	14.61679
	(2.523397653)	(0.0146886)***	(3.059364)***	(5.015514)***	(6.292296)***
Z_4	14.5615741	0.042220084	14.67488	15.24132	14.617
	(2.523608985)***	(0.014689)***	(3.0595)***	(5.015824)***	(6.292493)***
Z_5			55.71157		55.92123
			(51.25168)ns		(62.54331)ns
Z_6				-0.87662	-0.0651
				(4.768094)ns	(5.154615)ns
F	64.6386***	16.96332***	44.75053***	35.59105***	27.98958***
$\bar{R}^2$	0.85	0.62	0.85	0.81	0.81
N	45	40	39	42	38

(1) JP_1 = 1965, JP_2 = 1987; 2: JP_1 = 1980, JP_2 = 1992; Values in parenthesis are SE; *p<0.10; **p<0.05; ***p<0.01; ns: Not significant

Table 4: Estimates of MSE for all the models

Variables	Cereal grains	GDP	Cereal grains/GDP	Cereal grains/Fertilizer	All
LQL	2065.70	10.30	2479.20	2072.12	3074.76
QQL	1988.06	4.950	4448.31	1459.61	2607.15
LQQ	1765.86	5.150	1694.36	1778.36	1694.86

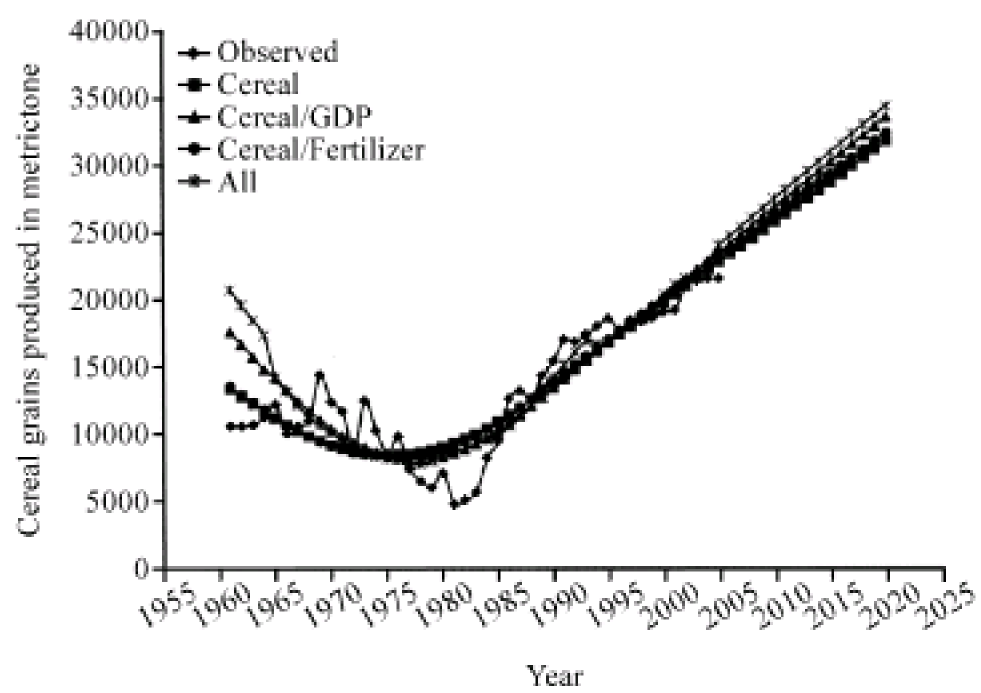

Fig. 1: Ex-post and ex-ante forecast of cereal grains using the Linear Quadratic Linear model

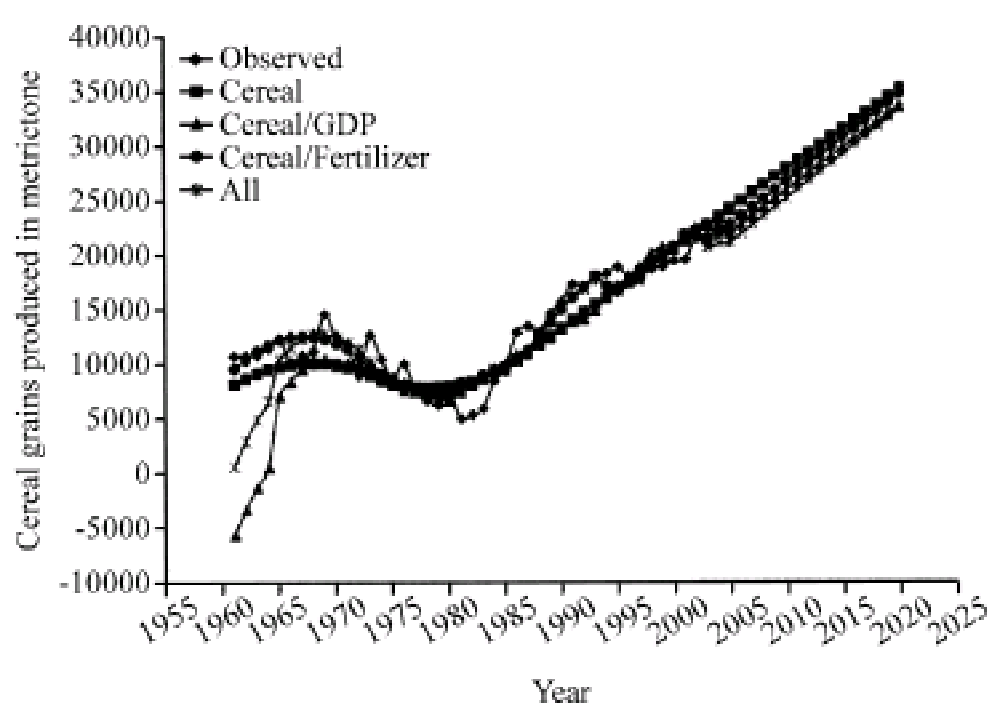

Fig. 2: Ex-post and ex-ante forecast of cereal grains using the Quadratic Quadratic Linear model

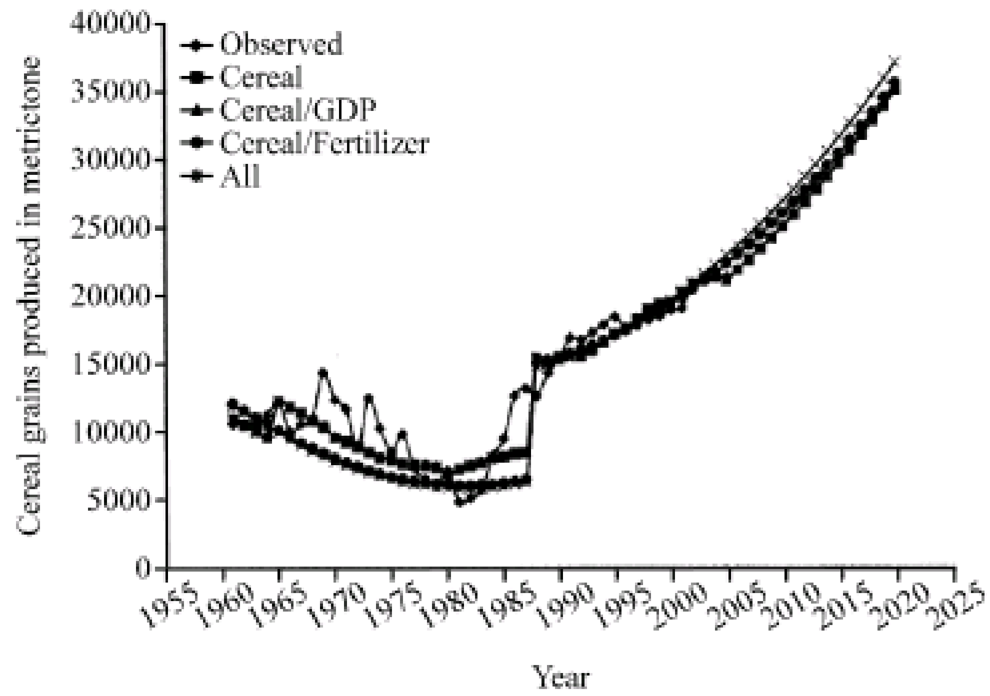

Fig. 3: Ex-post and ex-ante forecast of cereal grains using the Linear Quadratic Quadratic model

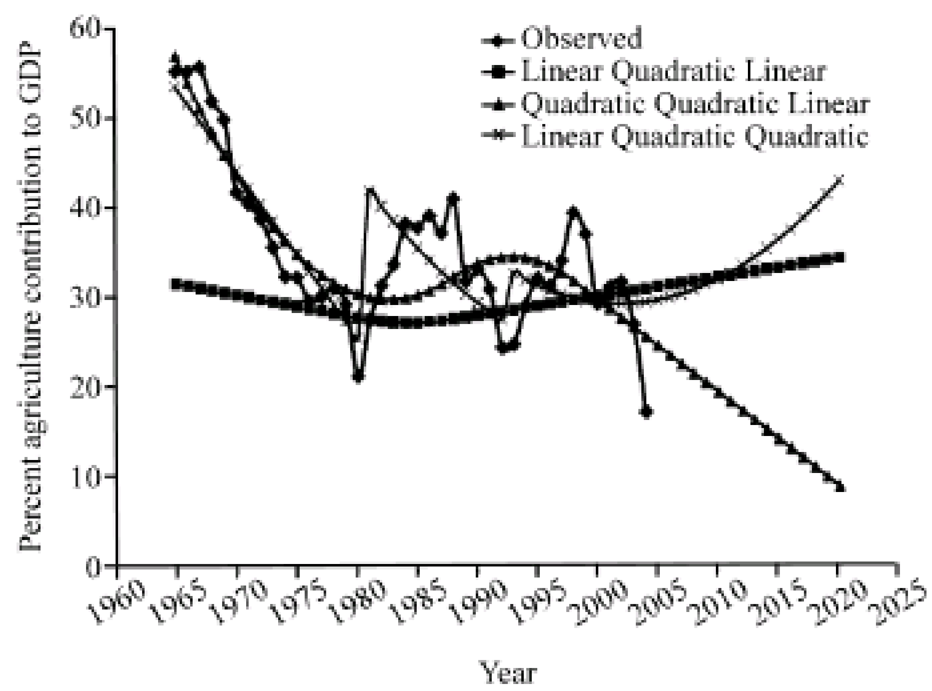

Fig. 4: Ex-post and ex-ante forecast of per cent contribution of agriculture to GDP using various models

The results shown in Table 3 indicate that the variables of the Linear-Quadratic-Quadratic model captured the trend in cereal production and GDP significantly. However, the estimates of the coefficient of GDP and fertilizer when added as explanatory variables are not significant in explaining the trend in cereal grains production. The non-significance of added explanatory variables in the trend equation is quite contrary to what Nmadu and Phillip (2001) and Nmadu *et al.* (2004) found in the case of sorghum. The result would seem to indicate that there is universality as to the appropriateness of the grafted model used. But that is sharply contrasted with the result of the ex-post and ex-ante forecast shown in Fig. 1-4, respectively. It would be noticed that similar results were obtained with models that have linear terminal but the result with Quadratic terminal is kinked at the joint points which is against one of the major requirement of the spline system (Fuller, 1969; Philip, 1990) even though the ex-ante forecast from the model compares favourably with the other models for cereal grains production and also compares favourably with results obtained from other series (Rahman and Damisa, 1999; Nmadu and Amos, 2002; Nmadu and Philip, 2001; Nmadu *et al.*, 2004). The ex-post and ex-ante forecast for the GDP from the three models show some interesting results. While the Linear-Quadratic-Linear model show a slow upward trend, the Quadratic-Quadratic-Linear show a sliding trend and Linear-Quadratic-Quadratic show a rapid upward trend. However, the forecast from the

Quadratic-Quadratic-Linear is most consistent with the observed trend. Given such scenario, it would seem that the choice of the spline model to use is not based solely on visual examination, but it will also depend on the nature of data series involved and the use to which the forecast would be put. While any of the models could be cautiously used for cereal grains production forecasting, other factors would have to be considered in choosing a model for forecasting GDP. In that regard, it is advised that all possible spline models should be tried and the one that gives best result should be utilized for further studies. For example, the result of the MSE in Table 4 shows that the best model is not uniform across. Different models may be recommended if type of spline system or number of explanatory variables in the various systems is considered. While QQL seams to be a better model with GDP and Cereal based splines; with regard to number of number of variables, the choice is a mixed bag. Changing of joint points has not shown any significant effect of the output of the models.

CONCLUSION

The effect of changing joint points and the type of spline function was investigated in this research. The result obtained show that the there was no universality as to the effect of the model and joint points chosen. Therefore, attempts should be made to model the data series with as many models as possible. The choice of the most acceptable should be based on the conformity of the ex-post and ex-ante forecasts to the observed data and economic sense.

APPENDIX

Full Details of the Grafting of Linear Quadratic Quadratic Model

A graphical examination of a data series may reveal that it can be divided into different segments as the trend equation below:

$$GD_t = \alpha_o + \beta_o t \qquad t \le JP_1 \qquad (1)$$

$$GD_t = \alpha_1 + \beta_1 t + \phi_1 t^2 \qquad JP_1 < t \le JP_2 \qquad (2)$$

$$GD_t = \alpha_2 + \beta_2 t + \phi_2 t^2 \qquad t > JP_2 \qquad (3)$$

Where:
Gd_t = Data series in year t
t = Trend
α's, β's and ϕ = Structural parameters to be estimated
JP_1 and JP_2 = Joint point 1 and 2, respectively

The restrictions (Fuller, 1969) on the Eq. 1-3 are:

$$\alpha_o + \beta_o K_1 = \alpha_1 + \beta_1 K_1 + \phi_1 K_1^2 \qquad (4)$$

$$\alpha_1 + \beta_1 K_2 + \phi_1 K_2^2 = \alpha_2 + \beta_2 K_2 + \phi_2 K_2^2 \qquad (5)$$

$$\beta_o = \beta_1 + 2\phi_1 K_1 \qquad (6)$$

$$\beta_1 + 2\phi_1 k_2 = \beta_2 + 2\phi_2 k_2 \qquad (7)$$

From Eq. 1-3, there are eight parameters with four restrictions as shown in Eq. 4-7, therefore, four parameters were estimated. We retain the terminal parameters being the most recent, hence α_2, β_2, ϕ_2 and ϕ_2-ϕ_1 were estimated while α_1 , β_1 , α_o and β_o were dropped. ϕ_2-ϕ_1 was estimated in order to study the transition from one phase to another in the data series. Equation 4-7 are now redefined in favour of the dropped parameters *viz*.:

By inspecting Eq. 4-7, it is obvious that it is better to start from Eq. 7, respectively because they have only one term, which we intend to drop i.e.,

$$(D_1) \quad \beta_1 = \beta_2 + 2\phi_2 k_2 - 2\phi_1 k_2 = \beta_2 + 2k_2(\phi_2 - \phi_1)$$

From Eq. 6, substituting (D_1), we obtain

$$(D_2) \quad \beta_o = \beta_2 + 2K_2(\phi_2 - \phi_1) + 2\phi_1 K_2$$

We now estimate α_1 from Eq. 5 substituting (D_1) i.e.,

$$\alpha_1 = \alpha_2 + \beta_2 K_2 + \phi_2 K_2^2 - K_2 \{\beta_2 + 2\phi_2 k_2 - 2\phi_1 k_2 = \beta_2 + 2k_2(\phi_2 - \phi_1)\} - \phi_1 K_2^2$$

$$(D_3) \quad \alpha_1 = \alpha_2 - K_2^2 (\phi_2 - \phi_1)$$

Finally we estimate α_o by making use of (D_1), (D_2) and (D_3)

$$\alpha_o = \alpha_1 + K_1 \{\beta_2 + 2\phi_2 k_2 - 2\phi_1 k_2 = \beta_2 + 2k_2(\phi_2 - \phi_1)\} + \phi_1 K_1^2 - K_1 \{ \beta_2 + 2K_2(\phi_2 - \phi_1) + 2\phi_1 K_2\}$$

$$(D_4) \qquad \alpha_o = \alpha_2 - K_2^2 (\phi_2 - \phi_1) - K_1^2 \phi_1$$

$$\alpha_o = \alpha_2 - K_2^2 (\phi_2 - \phi_1) - K_1^2 \phi_1 \tag{8}$$

$$\beta_o = \beta_2 + 2K_2(\phi_2 - \phi_1) + 2\phi_1 K_2 \tag{9}$$

$$\alpha_1 = \alpha_2 - K_2^2 (\phi_2 - \phi_1) \tag{10}$$

$$\beta_1 = \beta_2 + 2\phi_2 k_2 - 2\phi_1 k_2 = \beta_2 + 2k_2(\phi_2 - \phi_1) \tag{11}$$

The mean equation can now be obtained by substituting α_1 , β_1 , α_o and β_o in Eq. 1-3. From Eq. 1, substituting for α_o and β_o

$$Gd_t = \alpha_2 - K_2^2 (\phi_2 - \phi_1) - K_1^2 \phi_1 + t\{ \beta_2 + 2K_2(\phi_2 - \phi_1) + 2\phi_1 K_2\}$$

$$(E_1) \quad GD_t = \alpha_2 + \beta_2 t + (2K_2 t - K_2^2)(\phi_2 - \phi_1)(2K_1 t - K_1^2)\phi_1, \; t <= K_1$$

From Eq. 2, substituting for α_1 and β_1,

$$GD_t = \alpha_2 - K_2^2 (\phi_2 - \phi_1 + t\{\beta_2 + 2k_2(\phi_2 - \phi_1)\} + \phi_2 t^2$$

$$(E_2) \quad GD_t = \alpha_2 + \beta_2 t + (2K_2 t - K_2^2)(\phi_2 - \phi_1) + \phi_2 t^2, \; K_1 < t \le K_2$$

From Eq. 3, all coefficients were retained for forecasting purposes.

$$(E_3) \quad GD_t = \alpha_2 + \beta_2 t + \phi_2 t^2, \; t > K_2$$

The grafted Eq. 12-14, are then formed by inspection of (E_1), (E_2) and (E_3) above. The mean equation is continuous on the data set:

$$GD_t = \alpha_2 + \beta_2 t + (2JP_2t - JP_2^2)(\phi_2 - \phi_1) + (2JP_1t - JP_1^2)\phi_1 \qquad t \le JP_1 \qquad (12)$$

$$GD_t = \alpha_2 + \beta_2 t + (2JP_2t - JP_2^2)(\phi_2 - \phi_1) + \phi_2 t^2 \qquad JP_1 < t \le JP_2 \qquad (13)$$

$$GD_t = \alpha_2 + \beta_2 t + \phi_2 t^2 \qquad t > JP_2 \qquad (14)$$

Equation 12-14, are then formed into a single equation for estimation as follows:

$$GD_t = \mu_o Z_o + \mu_1 Z_1 + \mu_2 Z_2 + \mu_3 Z_3 + \mu_3 Z_3 + \mu_4 Z_4 + U_t \qquad (15)$$

Where:

Z_o	=	1	$\forall$ t
Z_1	=	t	$\forall$ t
Z_2	=	$2JP_2t - JP_2^2$	$t \le JP_1$
	=	$2JP_2t - JP_2^2$	$JP_1 < t \le JP_2$
	=	0	$t > JP_2$
Z_3	=	$2JP_1t - JP_1^2$	$t \le JP_2$
	=	t^2	$JP_1 < t \le JP_2$
	=	0	$t > JP_2$
Z_4	=	0	$t \le JP_1$
	=	0	$JP_1 < t \le JP_2$
	=	t^2	$t > JP_2$
μ's	=	Structural parameters to be estimated	
U_t	=	Error term assumed to be well behaved	

REFERENCES

Bormann, J., G.R. Wiggans, T. Druet and N. Gengler, 2002. Estimating effects of permanent environment, lactation stage, age and pregnancy on test-day yield. J. Dairy Sci., 85: 263-263.

EarthTrend, 2006. Agriculture and Food Agricultural Inputs: Fertilizer Consumption Retrieved on December 13, 2006. http//earthtrend.wri.org.

Fox, K.J. and R.Q. Grafton, 2000. Nonparametric estimation of returns to scale: Method and application. Can. J. Agric. Econ., 48: 341-354.

Fuller, W.A., 1969. Grafted polynomials as approximating functions. Aust. J. Agric. Econ., 13: 35-46.

Marsh, L.Ñ., 1986. Estimating the number and location of knots in spline regression. J. Applied Business Res., 3: 60-70.

Meyer, K., 2005. Random regression analyses using β-splines to model growth of Australian angus cattle. Genet. Sel. Evol., 37: 473-500.

Nmadu, J.N. and D.O.A. Phillip, 2001. Forecasting the yield of sorghum in Nigeria using alternative forecasting models. Nig. Agric. J., 32: 52-64.

Nmadu, J.N. and T.T. Amos, 2002. Comparison of grafted ex-ante forecast estimates and computed values of sorghum yield; acreage and output in Nigeria. J. Sustainable Trop. Agric. Res., 5: 40-44.

Nmadu, J.N., J.O. Olukosi, T.T. Amos and R.S. Musa, 2004. Forecasting sorghum acreage in Nigeria using grafted response analysis. J. Arid Agric., 14: 15-21.

Parsons, I.T. and R. Hunt, 1981. Plant growth analysis: A program for the fitting of lengthy series of data by the method of β-splines. Ann. Bot., 48: 341-352.

Phillip, D.O.A., 1990. Grafted polynomials as forecasting functions: Application to cotton production in Nigeria. Savanna J., 11: 103-108.

Pierre, N.R., C.S. Thraen and W.R. Harvey, 1987. Minimum cost nutrient requirements from a growth response function. J. Anim. Sci., 64: 312-327.

Rahman, S.A. and M.A. Damisa, 1999. Forecasting performance of a grafted model on cowpea price in Zaria area of Kaduna State, Nigeria. Nig. J. Rural Econ. Soc., 1: 22-28.