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ORIGINAL ARTICLE

Multi-step Homotopy Analysis Method for Solving Malaria Model

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Abstract

In this paper, we consider the modified epidemiological malaria model proposed by Abadi and Harald. The multi-step homotopy analysis method (MHAM) is employed to compute an approximation to the solution of the model of fractional order. The fractional derivatives are described in the Caputo sense. We illustrated the profiles of the solutions of each of the compartments. Figurative comparisons between the MHAM and the classical fourth-order reveal that this method is very effective

Keywords: Multi-step homotopy analysis method; Malaria; Runge-Kutta Method

Introduction

Malaria is one of the oldest diseases studied for a long-time from all angles, and vast literature exists describing a host of modelling approaches. Different approaches are helpful in guiding different stages of the disease and its transmission through synthesizing available information and extrapolating it. It is felt that a combination of different approaches, rather than a single type of modelling, may have long term usefulness in prevention and control. Malaria is a parasitic vector borne disease, endemic in many parts of the world. At present, at least 300 million people are affected worldwide and there are between 1-1.6 million malaria related deaths annually (Nedelman, 1983).

There have been a number of control strategies against the transmission of malaria and most importantly antimalarial drugs that have played a mainstream role through the treatment of clinical cases, prophylaxis of the high risk groups (infants, nonimmune travellers, and pregnant women). Early treatment of suspected cases with adequate drugs continue to be the main control strategy of malaria in the sub-Saharan Africa. However, despite intensive control efforts, global incidence of malaria case is increasing especially in the sub-Saharan Africa. This has been attributed to poverty, war and the collapse of health care systems following abandonment of WHO's Eradicate Malaria Campaign(Ngwa,2004). In the 1960s, another important factor has been largely due to emergence of drug resistant strains of Plasmodium to cheap and affordable antimalarial drugs such as chloroquine (CQ) and sulfadoxine-pyrimethamine (SP, trade name: Fansidar) that have been the mainstay of malaria treatment in disease endemic regions (Ridley, 2002; Baird, 2000; Peters, 1998).

Several researchers work on the fractional order differential equations because of best presentation of many phenomena. Fractional calculus has been used to model physical and engineering processes, which are found to be best described by fractional differential equations. It is worth noting that the standard mathematical models of integer-order derivatives, including nonlinear models, do not work adequately in many cases. In the recent years, fractional calculus has played a vital role in various fields such as mechanics, electricity, mathematics, biology, economics, notably control theory, and signal and image processing see for example. (Ertürk, and Momani, 2011; Lin, 2007; Miller, 1993).

In this study, we employ the Multi-step Homotopy Analysis Method (MHAM) to the system of differential equations proposed by Gebremeskel and Krogstad (2015), which describe our model and approximating the solutions in a sequence of time intervals. In other to illustrate the accuracy of the MHAM, the obtained results are compared with fourth-order Runge-Kutta Method. Other semi-analytical methods to determine the solutions of nonlinear system includes: Homotopy Pertubation Method (HPM), Reduced Differential Transform Method (RDTM), Viarational Iterational Method (VIM), Differential Transform Method. The methods mentioned above have been used as a tools to approximate linear and non-linear problems in Physics and Engineering respectively (Abbasbandy and Shivanian, 2009; Abdallah, 2009; Peter and Akinduko, 2018; Peter and Ibrahim, 2017; Peter et al., 2018).

Materials and Methods

The endemic malaria model transmission considered in this study is *SIR* and *SI* in human a mosquito population respectively. The model is formulated for the spread of malaria in the human and mosquito population with the total population size at time *t* denoted by $N_h(t)$ and $N_v(t)$, respectively. These are further compartmentalized into epidemiological subclasses as susceptible $S_h(t)$, infected $I_h(t)$, and recovered $R_h(t)$ human population, and susceptible $S_v(t)$ and infected $I_v(t)$ vector population. The vector component of the model does not include an immune class as mosquitoes does not recover from the infection, that is, their infective period ends with their death due to their relatively short lifecycle. Thus, the immune class in the mosquito population is negligible and death occurs equally in all class. The model can be used for diseases that persist in a population for a long period of time with vital dynamics. The basic model incorporates a set of assumptions. Both the human and vector total population sizes are assumed to be constant.

The recovered individuals in human population develop permanent immunity. The populations in compartments of both humans and vectors are non-negative. All newborns are susceptible to infection, and the development of malaria starts when the All human individuals, whatever their status, are subject to an infectious female mosquito bites the human host. The vectors do not die from the infection or are otherwise harmed. Individuals move from one class to the other as their status with respect to the disease evolves. Humans enter the susceptible class through birth rate μ_h and leave from the susceptible class through death rate α_h, and infective rate $\beta_h I_h$. natural death, which occurs at a rate α_h, and disease induced death rate ρ_h. In this model, $\mu_h N_h$ and $\mu_v N_v$ are denoted the total birth rates of human and mosquito, respectively. The terms $\alpha_h S_h$,, $\alpha_h I_h$ and $\alpha_h R_h$ refer to the total number of removed susceptible, infected and recovered humans per unit of time. The terms $\alpha_v S_v$ and $\alpha_v I_v$ are the number of removed susceptible and infected mosquito populations per unit of time. The term $\rho_h I_h$ is the number of removed human population because of the disease per unit of time, whereas $\gamma_h I_h$ is the total recovered human population per unit of time. The term $\beta_h S_h I_v$ denotes the rate at which the human hosts S_h get infected by the mosquito vector. I_v, and $\beta_v S_v I_h$ refers to the rate at which the susceptible mosquitoes S_v are infected by the human hosts I_h at a time, *t*. Thus, both these terms are important parts of the model describing the interaction between the two populations. The aim of this work is to extend HAM to solve the system of the model equations (1). This modification is called Multi-Step Homotopy Analysis Method (MHAM). The ordinary differential equations which describe the dynamics of malaria in the human and mosquito populations become:

$$\frac{dS_h}{dt} = \mu_h N_h - \beta_h S_h I_v - \alpha_h S_h$$
$$\frac{dI_h}{dt} = \beta_h S_h I_v - \alpha_h S_h - \delta_h I_h - \gamma_h I_h - \alpha_h I_h$$
$$\frac{dR_h}{dt} = \gamma_h I_h - \alpha_n R_n$$
$$\frac{dSv}{dt} = \mu_v N_v - \beta_v S_v I_h - \alpha_v S_v \qquad (1)$$
$$\frac{dI_v}{dt} = \beta_v S_v I_h - \alpha_v I_v$$

Basic Definitions and Notation

Lemma 1 [Generalized Mean Value Theorem (Lin, 2007)]
Let $\mathrm{p(x)} \in \mathrm{C[e,f]}$ and $\mathrm{D^{\alpha}\,p(x)} \in \mathrm{C[e,f]}$ for $0 < \alpha \leq 1$, then we have

$$\mathrm{p(x)} = \mathrm{p(e)} + \frac{1}{\Gamma(\alpha)} \mathrm{D^{\alpha} p(\varepsilon)(x-e)^{\alpha}}$$
$$0 \leq \varepsilon < x \; \forall \, \mathrm{x} \in \mathrm{[e,f]}$$

Remark
Suppose $\mathrm{p(x)} \in \mathrm{C[e,f]}$ and $\mathrm{D^{\alpha}p(x)} \in \mathrm{C[e,f]}$ for $0 < \alpha \leq 1$. It is obvious from Lemma 1 that if $\mathrm{D^{\alpha}\,p(x)} \geq 0, \forall \mathrm{x} \in (0,\mathrm{f})$ then the function p is non-deceasing and if $\mathrm{D^{\alpha}\,p(x)} \leq 0, \forall \mathrm{x} \in (0,\mathrm{f})$ then the functionp is non-increasing

Definition 1
A function $g(x)$ having a position value of x is defined in the space D_{α} ($\alpha \in \mathbb{R}$) if it is expressed in the form $g(x) = x^{a} g(x)$ and for some $a > \alpha$ where $g(x)$ is continuous in $[0,\infty]$ and it is identified to be in the space D_{α}^{n} if $g^{(n)} \in Dn \in \mathbb{N}$

Definition 2
The Riemann Liouvoille integral Operator of a given order$\alpha > 0$ with $b \geq 0$ is expressed as:

$$(J_a^{\alpha} g)(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{n-1}\, g(t)dt, x > \alpha$$
$$(J_b^0 g)(x) = g(x).$$

We require only the following for:

$g \in \beta_n, \alpha > 0, \beta > 0, c \in \mathbb{R}$ and $\gamma > -1$, we get

$$J_b^{\alpha} x^{\gamma} = \frac{x^{\gamma+\alpha}}{\Gamma(b)} \beta(x-b)(\alpha,\gamma+1)$$

Where $\beta_{\omega}\,(\alpha,\gamma+1)$ characterized the incomplete beta function stated as:

$$D_{\omega}(\alpha,\gamma+1\,) = \int_0^{\omega} t^{\alpha_1}(1-t)^{\gamma} dt,$$
$$J_b^{\alpha} f^{(x)} = f^{(x)}(x-b)^{\alpha} \sum_{k=0}^{\infty} \frac{[c(c-b)]^k}{\Gamma(\alpha+k+1)}$$

The Riemann Liouville derivative possesses some setbacks when applying to a real life situation with fractional differential equations. Thus we employ a modified version of fractional differential operator D_b^{α} which has been employed in the Caputo work on the theory of viscoelasticity.

Definition 3

The caputo fractional derivative of $p(x)$ order $\alpha > 0$ with $a \geq 0$ is given as

$$(D_b^{\alpha} g)(x) = (J_b^{m-\alpha})(x) = \frac{1}{\Gamma(m-\alpha)} \int_b^x \frac{g^{(m)}(t)}{(x-t)^{\alpha+1-m}} dt$$

For $m-1 < \alpha \leq m, m \in \mathbb{N}, x \geq bf(x) \in D_{-1}^{m}$

Multistep Homotopy Analysis Method

The principle of homotopy analysis method are given in (Ibrahim, et al., 2017; Liao, 1992). HAM is used to provide approximate solutions for all wide range of non-linear problem in terms of convergent series. The HAM has been extended by many researchers to solve linear and nonlinear problems in terms of convergent series with easily computable components, however it does have some drawbacks: the series solution always converges in a very small region and it has slow convergent rate or is completely divergent in the wider region (Alomari, et al., 2009; Zurigat, et al., 2010; Cang, et al., 2009; Zurigat, et al., 2010).

It is only a simple modification of HAM and this can ensure the validity of the approximant solution for a large interval. To overcome the shortcoming of HAM we present (MHAM) that we have developed for the numerical solution of the system of fractional differential equations.

$$\begin{aligned}
&D^{\alpha_1} S_h(t) = \mathcal{U}_n N_h - \beta_h S_h(t) I_v(t) - \alpha_h S_h(t) \\
&D^{\alpha_2} I_h(t) = \beta_h S_h(t) I_v(t) - \delta_h I_h(t) - \gamma_h I_h(t) - \alpha_h I_h(t) \\
&D^{\alpha_3} R_h(t) = \gamma_h I_h(t) - \alpha_n R_h(t) \\
&D^{\alpha_4} S_v(t) = \mathcal{U}_v N_v - \beta_v S_v(t) I_h(t) - \alpha_v S_v(t) \\
&D^{\alpha_5} I_v(t) = \beta_v S_v(t) I_h(t) - \alpha_v I(t) \qquad (2)
\end{aligned}$$

To expand the solution over the interval $[0, t]$, we subdivide the interval $[0, t]$ into n subintervals of equal length:

$$\Delta t, [t_0, t_1), [t_1, t_2], [t_2, t_3), [t_3, t_4) \dots\dots\dots\dots\dots\dots [t_{n-1}, t_n)$$

with $t_0 = 0$, and $t_n = t$

We let t^* be the initial value for each subinterval $[t_{j-1}, t_j]$; $j = 1,2, \cdots n$ with initial guesses:

$$\begin{aligned}
S_{h_1}(t) = 3, S_{(h,j)(t^*)} = S_{h,j}(t_{j-1}) = S_{hj-1}(t_{j-1}) \\
I_{h_2}(t^*) = 1, I_{h,j}(t^*) = i_{h,j}(t_{j-1}) = i_{hj-1}(t_j - 1) \\
R_{3(t^*)} = 1, R_{h,j}(t^*) = r, j(t_{j-1}) = r_{j-1}(t_{j-1}) \\
S_{v4}(t^*) = 1\ S_{(v,j)(t^*)} = S_{v,j}(t_{j-1}) = S_{vj-1}(t_{j-1}) \\
I_{v5}(t^*) = 1\ I_{(v,j)(t^*)} = i_{v,j}(t_{j-1}) = i_{vj-1}(t_{j-1})
\end{aligned} \qquad (3)$$

Now, we constructed the Zeroth order transformation of the model equation (1):

$$(1-p)[\phi_{1,j}(t;p) - S_{hj}(t^*)] = ph\,[D^{\alpha_1} \phi_{1_j}(t;p) - \mathcal{U}_n N_n + \beta_h \phi_{1,j}(t;p) \phi_{5,j}(t,p) + \alpha_h \phi_{1,j}(t;p)]$$

$$(1-p)[\phi_{2,j}(t;p) - I_{hj}(t^*)] = \rho h\,[D^{\alpha_2} \phi_{2,j})(t;p) - \beta_h \phi_{1,j}(t;p)(t;p) \phi_{5,j}(t,p) + \delta_h \phi_{2,j}(t;p) + \gamma_h \phi_{2,j}(t,p) + \alpha_h \phi_{2,j}(t,p)]$$

$$(1-p)L[\phi_{3,j}(t;p) - R_h(t^*)] = ph[D^{\alpha_3} \phi_{3,j}(t;p) - \gamma_h \phi_{2,j}(t,p) + \alpha_h \phi_{3,j}(t,p)]$$

$$(1-p)L\big(\phi_{4,j}(t;p)-S_v(t^*)\big] \\ = ph\big[D^{\alpha_4}\phi_{4,j}\,(t,p)\big]-\mathcal{U}_vN_v+\beta_v\phi_{4,j}(t,p)\phi_{2,j}(t,p)-\alpha_v\big(\phi_{4,j}(t;p)\big]$$

$$(1-p)L\big[\phi_{5,j}(t;p)-I_v(t^*)\big]= ph\big[D^{\alpha_5}\phi_{5,j}(t,p)-\beta_v\phi_{4,j}(t;p)\phi_{2,j}(t,p)+\alpha_v\phi_{5,j}(t,p)\big]$$

Where $p\in[0,1]$ is an embedding parameter, L is an auxiliary linear operator, $h\neq 0$, is an auxiliary parameter and $\phi_{i,j}(t,p), i=1,2,3,4,5, j=1,2,3,\cdots n$ are unidentified function. When $p=0$, we have:

$$\begin{gathered}\phi_{1,1}(t;0)=3,\phi_{1,j}(t;0)=S_{h_{j-1}}\big(t_{j-1}\big),\\ \phi_{2,1}(t;0)=1,\phi_{2,j}(t;0)=I_{h_{j-1}}\big(t_{j-1}\big),\\ \phi_{3,1}(t;0)=1,\phi_{3,j}(t;0)=R_{h_{j-1}}\big(t_{j-1}\big),\qquad j=1,2,\cdots n\\ \phi_{4,1}(t;0)=1,\phi_{4,j}(t;0)=S_{v_{j-1}}\big(t_{j-1}\big),\\ \phi_{5,1}(t;0)=1,\phi_{5,j}(t;0)=I_{v_{j-1}}\big(t_{j-1}\big),\end{gathered}$$

And when $p=1$, we obtain:

(4)

$$\begin{gathered}\phi_{1,j}(t;0)=S_{h_j}(t_j),\\ \phi_{2,j}(t;0)=I_{h_j}(t_j),\\ \phi_{3,j}(t;0)=R_{h_j}(t_j),\\ \phi_{4,j}(t;0)=S_{v_j}(t_j),\qquad j=1,2,\cdots n.\\ \phi_{5,j}(t;0)=I_{v_j}(t_j)\end{gathered}$$

Expanding $\phi_{i,j}(t;p)$, $i=1,2,3,4,5$ and $j=1,2,3,\cdots n$ using Taylor's series expansion with respect to p, we obtain:

$$\begin{gathered}\phi_{1,j}(t;p)=S_{h_j}(t^*)+\sum_{m=1}^{\infty}S_{h_{j,m}}(t)P^m\\ \phi_{2,j}(t;p)=I_{h_j}(t^*)+\sum_{m=1}^{\infty}I_{h_{j,m}}(t)P^m\\ \phi_{3,j}(t;p)=R_{h_j}(t^*)+\sum_{m=1}^{\infty}R_{h_{j,m}}(t)P^m\\ \phi_{4,j}(t;p)=S_{v_j}(t^*)+\sum_{m=1}^{\infty}S_{v_{j,m}}(t)P^m j=1,2,\cdots n.\\ \phi_{5,j}(t;p)=I_{v_j}(t^*)+\textstyle\sum_{m=1}^{\infty}I_{v_{j,m}}(t)P^m\end{gathered}$$

Where (5)

$$\begin{gathered}S_{h_{j,m}(t)}=\frac{1}{m!}\frac{\partial^m\phi_{1,j}(t,p)}{\partial p^m}|p=0\quad j=1,2,\cdots n\\ I_{h_{j,m}(t)}=\frac{1}{m!}\frac{\partial^m\phi_{2,j}(t,p)}{\partial p^m}|p=0\\ R_{h_{j,m}(t)}=\frac{1}{m!}\frac{\partial^m\phi_{3,j}(t,p)}{\partial p^m}|p=0\\ S_{v_{j,m}(t)}=\frac{1}{m!}\frac{\partial^m\phi_{4,j}(t,p)}{\partial p^m}|p=0\\ I_{v_{j,m}(t)}=\frac{1}{m!}\frac{\partial^m\phi_{5,j}(t,p)}{\partial p^m}|p=0\end{gathered}\qquad(6)$$

If the auxiliary linear operator L together with the initial guesses $S_{h_j}(t^*), I_{h_j}(t^*), R_{h_j}(t^*), S_{v_h}, I_{v_j}(t^*)$ and the nonzero auxiliary parameter h are power series selected in order that the power series (1) converges at $p = 1$, we obtain:

$$S_{h_j}(t) = \phi_{1,j}(t;1) = S_{h_j}(t^*) + \sum_{m=1}^{\infty} S_{h_{j,m}}(t)$$
$$I_{h_j}(t) = \phi_{2,j}(t;1) = I_{h_j}(t^*) + \sum_{m=1}^{\infty} I_{h_{j,m}}(t)$$
$$R_{h_j}(t) = \phi_{3,j}(t;1) = R_{h_j}(t^*) + \sum_{m=1}^{\infty} R_{h_{j,m}}(t)$$
$$S_{v_j}(t) = \phi_{4,j}(t;1) = S_{v_j}(t^*) + \sum_{m=1}^{\infty} S_{v_{j,m}}(t)$$
$$I_{v_j}(t) = \phi_{5,j}(t;1) = I_{v_j}(t^*) + \sum_{m=1}^{\infty} I_{v_{j,m}}(t) \tag{7}$$

Define the vectors:

$$\vec{S}_{h_{j,m}}(t) = \left\{S_{h_{j,0}}(t), S_{h_{j,1}}(t) \cdots S_{h_{j,m}}(t)\right\}$$
$$\vec{I}_{h_{j,m}}(t) = \left\{I_{h_{j,0}}(t), I_{h_{j,1}}(t) \cdots I_{h_{j,m}}(t)\right\}$$
$$\vec{R}_{h_{j,m}}(t) = \left\{R_{h_{j,0}}(t), R_{h_{j,1}}(t) \cdots R_{h_{j,m}}(t)\right\}$$
$$\vec{S}_{v_{j,m}}(t) = \left\{S_{v_{j,0}}(t), S_{v_{j,1}}(t) \cdots S_{v_{j,m}}(t)\right\} \tag{8}$$
$$\vec{I}_{v_{j,m}}(t) = \left\{I_{v_{j,0}}(t), I_{v_{j,1}}(t) \cdots I_{h_{j,m}}(t)\right\}$$

Differentiating the zero-order definition in (3) m times with regard to p then putting $p = 0$ and partitioning them by $m!$. Finally using (6) we obtain the higher order definition equations:

$$L\left[S_{h_{j,m}}(t) - X_m S_{h_{j,m-1}}(t)\right] = h\,\mathbb{R}'_{j,m}(\vec{S}_{h_{j,m-1}}(t)),$$
$$L\left[I_{h_{j,m}}(t) - X_m I_{h_{j,m-1}}(t)\right] = h\,\mathbb{R}^2_{j,m}(\vec{I}_{h_{j,m-1}}(t)),$$
$$L\left[R_{h_{j,m}}(t) - X_m R_{h_{j,m-1}}(t)\right] = h\,\mathbb{R}^3_{j,m}(\vec{R}_{h_{j,m-1}}(t)),$$
$$L\left[S_{v_{j,m}}(t) - X_m S_{v_{j,m-1}}(t)\right] = h\,\mathbb{R}^4_{j,m}(\vec{S}_{v_{j,m-1}}(t)), \tag{9}$$

$$L\left[I_{v_{j,m}}(t) - X_m I_{v_{j,m-1}}(t)\right] = h\,\mathbb{R}^5_{j,m}(\vec{I}_{h_{j,m-1}}(t)), \tag{10}$$

Subject to the initial condition:

$$S_{h_{j,m}}(0) = I_{h_{j,m}}(0) = R_{h_{j,m}}(0) = S_{v_{j,m}}(0) = I_{v_{j,m}}(0) = 0 \tag{11}$$

$j = 1,2,\cdots n,\;\; m = 1,2,\cdots n$

Where

$$\mathbb{R}^1_{j,m} = \left(S_{h_{j,m-1}}(t)\right) = D^{\alpha_1} S_{h_{j,m-1}}(t) + \beta_h \sum_{i=0}^{m-1} S_{h_{j,i}}(t) I_{v_{j,m-i-1}} - \mathcal{U}_h N_h + \alpha_h S_{h_{j,m-1}}(t)$$

$$\mathbb{R}^2_{j,m} = \left(I_{h_{j,m-1}}(t)\right) = D^{\alpha_2} I_{h_{j,m-1}}(t) - \beta_h \sum_{i=0}^{m-1} S_{h_{j,i}}(t) I_{v_{j,m-i-1}}(t) + \delta_h I_{h_{j_{m-1}}}(t) + \gamma_h I_{h_{j,m-1}}(t) + \alpha_h I_{h_{j,m-1}}(t)$$

$$\mathbb{R}^3_{j,m} = \left(R_{h_{j,m-1}}(t)\right) = D^{\alpha_3} R_{h_{j,m-1}}(t) - \gamma_h I_{h_{j,m-1}}(t) + \alpha_h R_{h_{j,m-1}}(t)$$

$$\mathbb{R}^4_{j,m} = \left(S_{v_{j,m-1}}(t)\right) = D^{\alpha_4} S_{v_{j,m-1}}(t) - \mathcal{U}_v N_v + \beta_v \sum_{i=0}^{m-1} S_{v_{j,i}}(t) I_{h_{j,m-i-1}}(t) + \alpha_v S_{v_{j_{m-1}}}(t)$$

$$\mathbb{R}^5_{j,m} = \left(I_{v_{j,m-1}}(t)\right) = D^{\alpha_5} I_{v_{j,m-1}}(t) - \beta_v \sum_{i=0}^{m-1} S_{v_{j,i}}(t) I_{h_{j,m-i-1}}(t) + \alpha_v I_{v_{j,m-1}}(t) \quad (12)$$

Where

$$X_m = \begin{cases} 0 & m \le 1 \\ 1 & m > 1 \end{cases}$$

Selecting the linear operator $L = D^{\alpha_i}, i = 1,2,3,\cdots n$ then, the m^{th} - order deformation in (7) can be written in the form:

$$\begin{aligned}
S_{h_{j,m}}(t) &= X_m S_{h_{j,m-1}}(t) + hJ^{\alpha_1}\left[\mathbb{R}^1_{j,m}\left(\vec{S}_{h_{j,m-1}}\right)\right], \\
I_{h_{j,m}}(t) &= X_m I_{h_{j,m-1}}(It) + hJ^{\alpha_2}\left[\mathbb{R}^2_{j,m}\left(\vec{I}_{h_{j,m-1}}\right)\right], \\
R_{h_{j,m}}(t) &= X_m R_{h_{j,m-1}}(t) + hJ^{\alpha_3}\left[\mathbb{R}^3_{j,m}\left(\vec{R}_{h_{j,m-1}}\right)\right], \\
S_{v_{j,m}}(t) &= X_m S_{v_{j,m-1}}(t) + hJ^{\alpha_4}\left[\mathbb{R}^4_{j,m}\left(\vec{S}_{v_{j,m-1}}\right)\right], \\
I_{v_{j,m}}(t) &= X_m I_{v_{j,m-1}}(t) + hJ^{\alpha_5}\left[\mathbb{R}^5_{j,m}\left(\vec{I}_{v_{j,m-1}}\right)\right]
\end{aligned} \quad (13)$$

The solutions of system (2) in every subinterval $[t_{j-1}, t_j]$, $j = 1,2,3,\cdots n$ has the structure :

$$\begin{aligned}
s_{h_j}(t) &= \sum_{m=0}^{\infty} S_{h_{j,m}}(t - t_{j-1}) \\
i_{h_j}(t) &= \sum_{m=0}^{\infty} I_{h_{j,m}}(t - t_{j-1}) j = 1,2,\cdots n \\
r_{h_j}(t) &= \sum_{m=0}^{\infty} R_{h_{j,m}}(t - t_{j-1}) \\
s_{v_j}(t) &= \sum_{m=0}^{\infty} S_{v_{j,m}}(t - t_{j-1}) \\
I_{v_j}(t) &= \sum_{m=0}^{\infty} I_{v_{j,m}}(t - t_{j-1})
\end{aligned} \quad (14)$$

and the solution of the system (6) for $[0, T]$ as:

$$\begin{aligned}
S_{h_j}(t) &= \sum_{m=0}^{\infty} s_{h_{j,m}}(t - t_{j-1}) \\
I_{h_j}(t) &= \sum_{m=0}^{\infty} i_{h_{j,m}}(t - t_{j-1}) \quad j = 1,2,\cdots n
\end{aligned}$$

$$R_{h_j}(t) = \sum_{m=0}^{\infty} r_{h_{j,m}}(t - t_{j-1})$$
$$S_{v_j}(t) = \sum_{m=0}^{\infty} s_{v_{j,m}}(t - t_{j-1})$$
$$I_{v_j}(t) = \sum_{m=0}^{\infty} I_{v_{j,m}}(t - t_{j-1}) \qquad (15)$$

Where

$$X_p = \begin{cases} 0, & t \in [t_{j-1}, t_j] \\ 1, & t \notin [t_{j-1}, t_j] \end{cases}$$

Results and Discussion

Numerical Simulation and Graphical Illustration of the Model

We present the numerical simulation which demonstrate the analytical results for the model. This is achieved by using some set of parameter values. The MHAM provides approximate solutions to linear as well as nonlinear differential equations. We choose the auxiliary parameter h= -1 and partition the interval [0, 25] into subintervals with step size $\Delta t = 0.1$ and thereafter we obtain HAM series solutions of order k=5 at every subintervals. We also employ MHAM algorithm constructed on the interval [0, 30]. The MHAM is demonstrated against maple in-buit fourth order Runge-Kutta procedure for the solution of the model. We take into consideration the following set of parameters values μ_h=0.1, β_h=0.4, α_v=0.2 γ_h=40, ρ_h=1, α_h=1, β_v=0.4, μ_v=0.1

Figure 1 to Figure 5 show the combined plots of the solutions of S_h, I_h, R_h, S_v and I_v by MHAM and RK4.

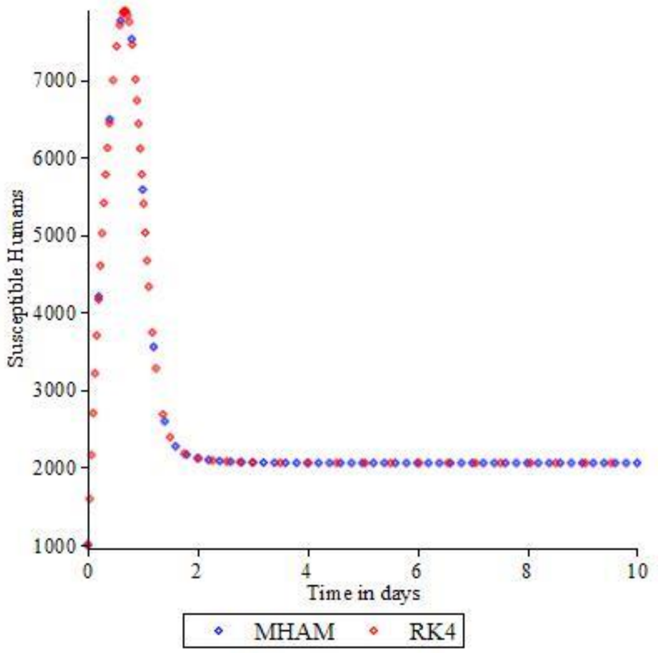

Figure 1. Solution of Susceptible Human Population by MHAM and RK4.

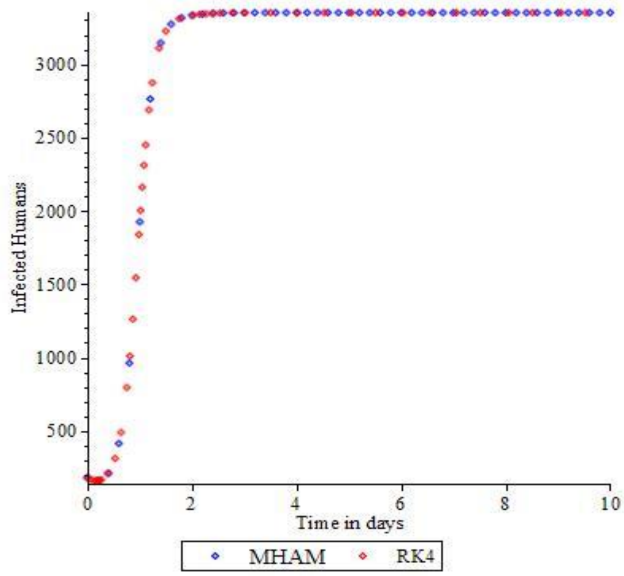

Figure 2. Solution of Infected Human Population by MHAM and RK4.

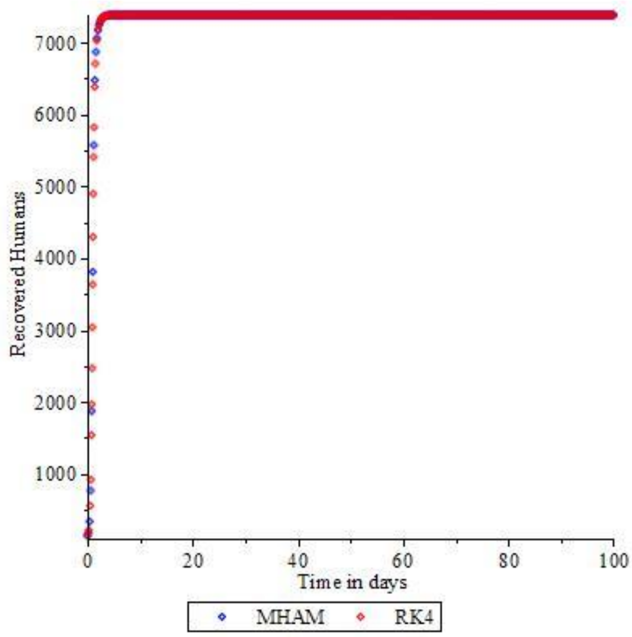

Figure 3. Solution of Recovered Human Population by MHAM and RK4.

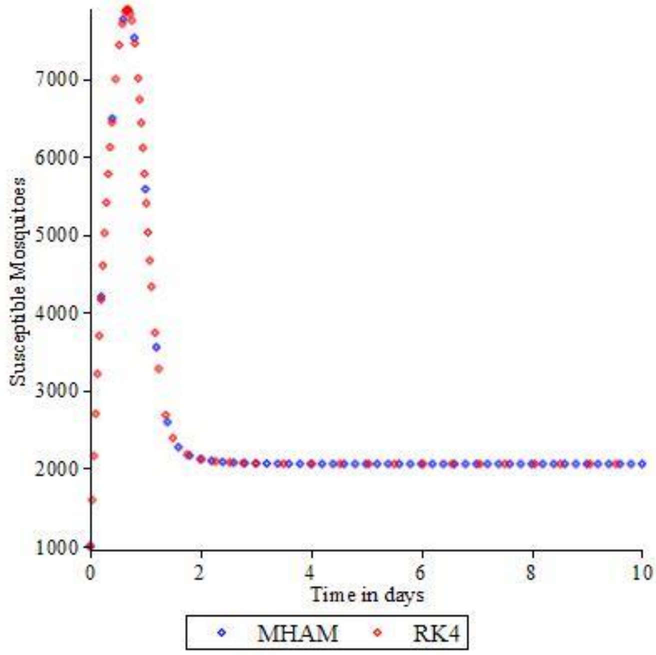

Figure 4. Solution of Susceptible Mosquitoes Population by MHAM and RK4.

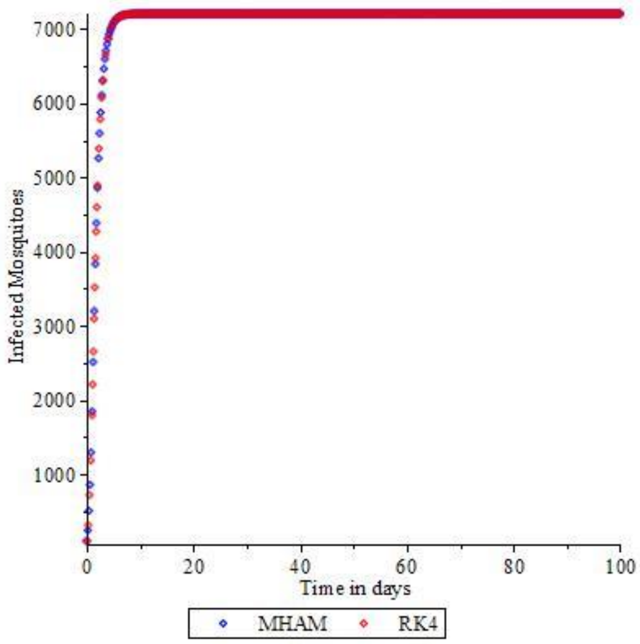

Figure 5. Solution of Infected Mosquitoes Population by MHAM and RK4.

Conclusion

In this paper, a fractional order differential S_h, I_h, R_h, S_v, I_v model is studied and its approximate solution is presented using a MHAM. The approximate solutions obtained by MHAM are highly accurate and valid for a long time. The reliability of the method and the reduction in the size of computational domain gives impetus of broad applicability. The comparison between MHAM and Runge-Kutta (RK4) were performed which were found to be efficient, accurate and rapidly convergence.

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