ANALYTICAL STUDY OF HEAT TRANSFER ON FLOW OF A NANOFLUID IN A POROUS MEDIUM WITH HEAT GENERATION

SULEIMAN, A., & YUSUF, A.
Mathematics Department,
Federal University of Technology, PMB 65, Minna, Nigeria, Niger State, Nigeria
E-mail: lebesque500@yahoo.com

Abstract
Analytical study of heat transfer on flow of a nanofluid in a porous medium with heat generation is presented. The partial differential equation representing the problem was reduced to ordinary differential equation using some similarity transformation variables. The transformed equations were solved using the Adomian decomposition method which results were compared with existing results in the literatures. A good agreement was established between the new method and the existing ones, which shows the reliability of the present method. The physical parameters that occurred in the solutions such as magnetic parameter, Darcy number, Ekcert number, Prandtl number, Schmdt number were varied to determine their respective effects on the flow. It was observed that the Magnetic parameter and inverse Darcy number are all reduction agents of the fluid velocity.

Keywords: Adomian decomposition method, Brownian motion, Darcy number, Eckert number, Magnetic parameter, Nanofluid

Introduction

Nanofluid is a new kind of heat transfer medium, containing nanoparticles (1–100 nm) which are uniformly and stably distributed in a base fluid. These distributed nanoparticles, generally a metal or metal oxide greatly enhance the thermal conductivity of the nanofluid, increases conduction and convection coefficients, allowing for more heat transfer (Yusuf et al., 2018).

Vasu and Manish (2015) studied the problem of two-dimensional transient hydrodynamic boundary-layer flow of an incompressible Newtonian nanofluid past a cone and plate with constant boundary conditions. Gireesha et al. (2015) introduced a numerical solution for hydromagnetic boundary-layer flow and heat transfer past a stretching surface embedded in a non-Darcy porous medium with fluid-particle suspension. The unsteady forced convective boundary-layer flow of an incompressible non-Newtonian nanofluid over a stretching sheet when the sheet is stretched in its own plane is investigated by Gorla and Vasu (2016). Gorla et al. (2016) investigated the transient mixed convective boundary-layer flow of an incompressible non-Newtonian quiescent nanofluid adjacent to a vertical stretching surface. The unsteady flow and heat transfer of a nanofluid over a contracting cylinder was studied by Zaimi et al. (2014). Srinivasacharya and Surender (2014) studied the effects of thermal and mass stratification on natural convection boundary-layer flow over a vertical plate embedded in a porous medium saturated by a nanofluid.

An analytical study of heat transfer on flow of a nanofluid in a porous medium with heat generation using the adomian decomposition method is presented, which is new in the literature.

Problem Formulation

Considering two-dimensional, incompressible viscous flow of a water-based nanofluid past over a stretching sheet. The sheet stretches with a velocity ax, where a is a constant and x is the coordinate measured along the stretching surface. The fluid flow at y=0, where y is the coordinate normal to the surface. The surface temperature is taken as T_w and at larger values, it is taken as T_∞. The nanoparticle concentration C_w is assumed constant on the stretching surface and C_∞ at larger values of y. Following the formulation in Mabood and Mastroberardino (2015) in a porous medium with heat generation, the governing equations of continuity, momentum, temperature, and nanoparticle concentration are written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\sigma B_0^2}{\rho_f}u-\frac{\upsilon\varphi}{k_0}u \tag{2}$$

$$\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\tau\left[D_B\left(\frac{\partial\varphi\partial T}{\partial x\partial x}+\frac{\partial\varphi\partial T}{\partial y\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial x}+\frac{\partial T}{\partial y}\right)^2\right]+ \frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{Q}{\rho c_p}(T-T_w) \tag{3}$$

$$\left(u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}\right)=D_B\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

Subject to the boundary condition:

$$\left.\begin{array}{l} y=0: u=ax,\ T=T_w, C=C_w \\ y\to\infty: u=0,\ T=T_\infty, C=C_\infty \end{array}\right\} \tag{5}$$

where velocity along x and y axes are u and v respectively, ρ_f is the density of the base fluid, υ is the kinematic viscosity, σ is the electrical conductivity, α is the heat diffusivity, B_0 external magnetic field, C_p is the specific heat capacity at constant pressure, D_B is the Brownian diffusion coefficient, D_T is the thermopheric diffusion coefficient and $\tau=\frac{(\rho c)_p}{(\rho c)_f}$ is the ratio between the effective heat capacity of the fluid with k_0 as permeability, Q is heat generation, φ is porousity.

In other to reduce the PDEs into ODEs, the following similarity transformational variables are defined as follows:

$$\eta=\sqrt{\frac{a}{\upsilon}}y,\ u=axf'(\eta),\ v=-\sqrt{a\upsilon}f(\eta),\theta=\frac{T-T_\infty}{T_w-T_\infty},\phi=\frac{C-C_\infty}{C_w-C_\infty} \tag{6}$$

where η, $f(\eta)$, $\theta(\eta)$, $\phi(\eta)$ are the dimensionless fluid distance, velocity, temperature, and nanoparticle concentrations.

$$\left.\begin{aligned}
&\eta = \sqrt{\frac{a}{\upsilon}}y, u = axf', v = -\sqrt{a\upsilon}f(\eta)\\
&\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y} = axf''\sqrt{\frac{a}{\upsilon}},\ \frac{\partial u}{\partial x} = af'\\
&\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(axf''\sqrt{\frac{a}{\upsilon}}\right) = \frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\left(axf''\sqrt{\frac{a}{\upsilon}}\right) = \frac{a^2 x}{\upsilon}f'''\\
&u\frac{\partial u}{\partial x} = axf'af' = a^2xf'^2\\
&v\frac{\partial u}{\partial y} = -\sqrt{a\upsilon}f(\eta)a\sqrt{\frac{a}{\upsilon}}xf'' = -a^2xff''\\
&T = T_\infty + (T_w - T_\infty)\theta(\eta), C = C_\infty + (C_W - C_\infty)\phi(\eta)\\
&\frac{\partial T}{\partial x} = 0,\ \frac{\partial T}{\partial y} = (T_w - T_\infty)\frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\theta(\eta) = (T_w - T_\infty)\sqrt{\frac{a}{\upsilon}}\theta'\\
&\frac{\partial C}{\partial x} = 0,\ \frac{\partial C}{\partial y} = (C_W - C_\infty)\frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\phi(\eta) = (C_W - C_\infty)\sqrt{\frac{a}{\upsilon}}\phi'\\
&\frac{\partial^2 T}{\partial y^2} = \frac{\partial}{\partial y}\left((T_w - T_\infty)\sqrt{\frac{a}{\upsilon}}\theta'\right) = \frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\left((T_w - T_\infty)\sqrt{\frac{a}{\upsilon}}\theta'\right) = (T_w - T_\infty)\frac{a}{\upsilon}\theta''\\
&\frac{\partial^2 C}{\partial y^2} = \frac{\partial}{\partial y}\left((C_W - C_\infty)\sqrt{\frac{a}{\upsilon}}\phi'\right) = \frac{\partial \eta}{\partial y}\frac{\partial}{\partial \eta}\left((C_W - C_\infty)\sqrt{\frac{a}{\upsilon}}\phi'\right) = (C_W - C_\infty)\frac{a}{\upsilon}\phi''\\
&v\frac{\partial T}{\partial y} = -\sqrt{a\upsilon}f(\eta)\sqrt{\frac{a}{\upsilon}}(T_w - T_\infty)\theta' = a(T_w - T_\infty)f\theta'\\
&v\frac{\partial C}{\partial y} = -\sqrt{a\upsilon}f(\eta)\sqrt{\frac{a}{\upsilon}}(C_w - C_\infty)\phi' = a(C_w - C_\infty)f\phi'
\end{aligned}\right\} \quad (7)$$

Introducing equation (7) into equations (1) to (5), the PDEs reduces to

$$\left.\begin{aligned}
&f''' + ff'' - f'^2 - \left(M + Da^{-1}\right)f' = 0\\
&\theta'' + P_r f\theta' + P_r N_b\theta'\phi' + P_r N_t\theta'^2 + P_r Ecf''^2 + P_r Q_0\theta = 0\\
&\phi'' + S_C f\phi' + \frac{N_t}{N_b}\theta'' = 0\\
&f(0) = 0, f'(0) = 1, \theta(0) = 1, \phi(0) = 1\\
&f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0
\end{aligned}\right\} \quad (8)$$

where

$$M = \frac{\sigma B_0^2}{a\rho_f},\ Da^{-1} = \frac{\upsilon\varphi}{ak_0}, P_r = \frac{\upsilon}{\alpha}, \mathrm{N}_b = \frac{\tau D_B(C_w - C_\infty)}{\upsilon}, \mathrm{N}_t = \frac{\tau D_T}{\upsilon T_\infty}(T_w - T_\infty), \mathrm{Ec} = \frac{a^2x^2\upsilon}{c_p(T_w - T_\infty)}$$

$$Q_0 = \frac{Q}{a\rho c_p}, \mathrm{S}_C = \frac{\upsilon}{D_B}.$$

Are Magnetic parameter, inverse Darcy number, Prandtl number, Brownian motion, thermophoresis parameter, Eckert number, Heat generation and Schmidt number.

Method of Solution

The method of Adomian (1994) is employ to obtain the solution of problem (8) by letting $\frac{d^3}{d\eta^3} = L_1$ and $\frac{d^2}{d\eta^2} = L_2$ and from problem (8), we have

$$\left.\begin{aligned} f''' &= -ff'' + f'^2 + \left(M + Da^{-1}\right) f' \\ \theta'' &= -P_r f\theta' - P_r N_b \theta' \phi' - P_r N_t \theta'^2 - P_r E_C f'' - P_r Q_0 \theta \\ \phi'' &= -S_C f \phi' - \frac{N_t}{N_b} \theta'' \end{aligned}\right\} \quad (9)$$

Introducing the operators into equations (9), we have

$$\left.\begin{aligned} L_1^{-1} L_1 \left[f(\eta) \right] &= L_1^{-1} \left[-ff'' + f'^2 + \left(M + Da^{-1}\right) f' \right] \\ L_2^{-1} L_2 \left[\theta(\eta) \right] &= L_2^{-1} \left[-P_r f\theta' - P_r N_b \theta' \phi' - P_r N_t \theta'^2 - P_r E_C f''^2 - P_r Q_0 \theta \right] \\ L_2^{-1} L_2 \left[\phi(\eta) \right] &= L_2^{-1} \left[-S_C f \phi' - \frac{N_t}{N_b} \theta'' \right] \end{aligned}\right\} \quad (10)$$

Where $L_1^{-1} = \int\int\int(\bullet)\, d\eta d\eta d\eta$ and $L_2^{-1} = \int\int(\bullet)\, d\eta d\eta$ (11)

Introducing the Adomian polynomials into (10) we have

$$\left.\begin{aligned} &\sum_{n=0}^{\infty} f_n = -L_1^{-1} \sum_{n=0}^{\infty} A_n + L_1^{-1} \sum_{n=0}^{\infty} B_n + \left(M + Da^{-1}\right) L_1^{-1} \sum_{n=0}^{\infty} f_n' \\ &\sum_{n=0}^{\infty} \theta_n = -P_r L_2^{-1} \sum_{n=0}^{\infty} C_n - P_r N_b L_2^{-1} \sum_{n=0}^{\infty} D_n - P_r N_t L_2^{-1} \sum_{n=0}^{\infty} E_n - P_r E_C L_2^{-1} \sum_{n=0}^{\infty} F_n - P_r Q_0 L_2^{-1} \sum_{n=0}^{\infty} \theta_n \\ &\sum_{n=0}^{\infty} \phi_n = -S_C L_2^{-1} \sum_{n=0}^{\infty} G_n - \frac{N_t}{N_b} L_2^{-1} \sum_{n=0}^{\infty} \theta_n'' \\ &\text{where } A_n = f_n f_{n-k}'', B_n = f_n' f_{n-k}', C_n = f_n \theta_{n-k}', D_n = \theta_n' \phi_{n-k}', E_n = \theta_n' \theta_{n-k}', F_n = f_n'' f_{n-k}'' \\ &G_n = f_n \phi_{n-k}', \end{aligned}\right\} \quad (12)$$

$$
\left.\begin{aligned}
f_{n+1} &= -L_1^{-1}\sum_{k=0}^{n} f_n f_{n-k}^{//} + L_1^{-1}\sum_{k=0}^{n} f_n^{/} f_{n-k}^{/} + \left(M + Da^{-1}\right)L_1^{-1} f_n^{/} \\
\theta_{n+1} &= -P_r L_2^{-1}\sum_{k=0}^{n} f_n \theta_{n-k}^{/} - P_r N_b L_2^{-1}\sum_{k=0}^{n} \theta_n^{/}\phi_{n-k}^{/} - P_r N_t L_2^{-1}\sum_{k=0}^{n} \theta_n^{/}\theta_{n-k}^{/} \\
&\quad - P_r E_C L_2^{-1}\sum_{k=0}^{n} f_n^{//} f_{n-k}^{//} - P_r Q_0 L_2^{-1}\theta_n^{/} \\
\phi_{n+1} &= -S_C L_2^{-1}\sum_{k=0}^{n} f_n \phi_{n-k}^{/} - \frac{N_t}{N_b} L_2^{-1}\theta_n^{//}
\end{aligned}\right\} \qquad (13)
$$

In order to obtain the solution to problem (8), the initial guess for (13) which satisfied the initial condition, are taking as:

$$
\left.\begin{aligned}
f_0(\eta) &= \eta - \frac{\alpha_1\left(\sqrt{M + Da^{-1}}\right)\eta^2}{2} \\
\theta_0(\eta) &= 1 + \eta\alpha_2 \\
\phi_0(\eta) &= 1 + \eta\alpha_3
\end{aligned}\right\} \qquad (14)
$$

Using maple18 to evaluate the integrals we have the final solutions as:

$$
\left.\begin{aligned}
f(\eta) &= \sum_{n=0}^{3} f_n(\eta) \\
\theta(\eta) &= \sum_{n=0}^{3} \theta_n(\eta) \\
\phi(\eta) &= \sum_{n=0}^{3} \phi_n(\eta)
\end{aligned}\right\} \qquad (15)
$$

Note: The infinity was observed at $\eta = 3$

Results and Discussion

The results obtained from the above section are presented and discussed in this section. Table 1 show the comparison of the present method with the existing method in the literature and a good agreement is observed among the methods.

Table 4.1: Comparison of values of $-f''(0)$ with existing solutions

M	Present Results	Mabood and Mastroberardino (2015)	Xu and Lee (2013)
0	-	-1.000008	-
1	1.3305610	1.4142135	1.41421
5	2.384890	2.4494897	2.4494
10	3.267599	3.3166247	3.3166
50	7.118200	7.1414284	7.1414
100	10.03333	10.049875	10.0498
500	22.375586	22.383029	22.38302
1000	31.633317	31.638584	

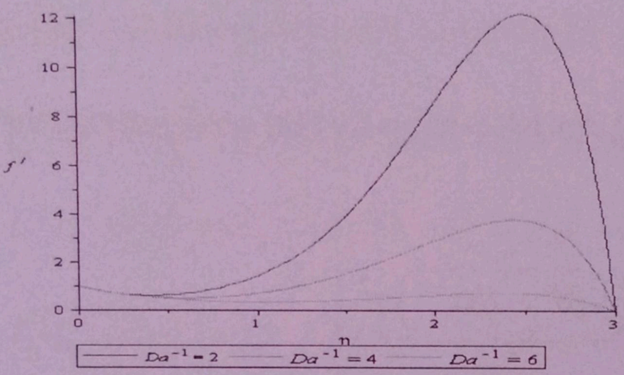

Figure 1: Variation of inverse Darcy number on velocity

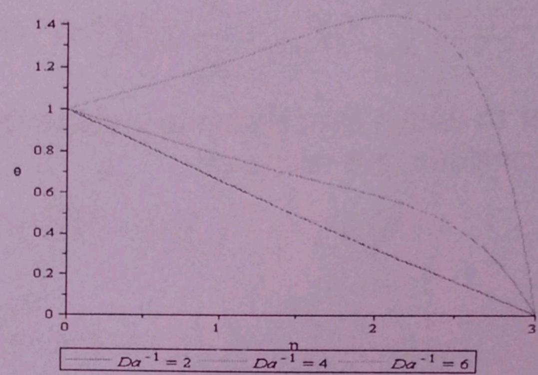

Figure 2: Variation of inverse Darcy number on temperature

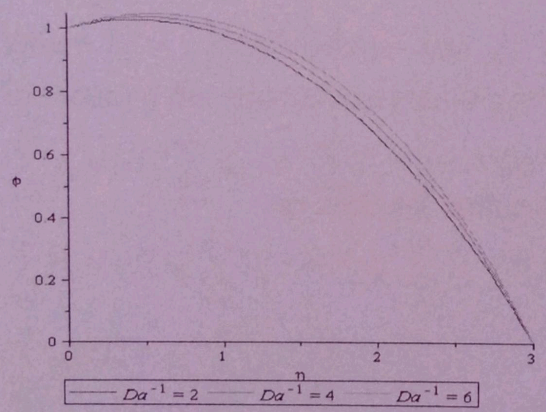

Figure 3: Variation of inverse Darcy number on concentration

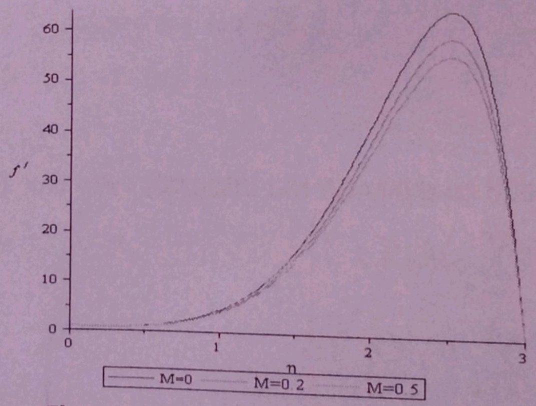

Figure 4: Variation of Magnetic number on velocity

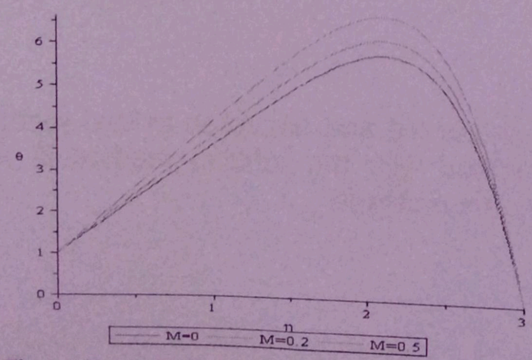

Figure 5: Variation of magnetic number on temperature

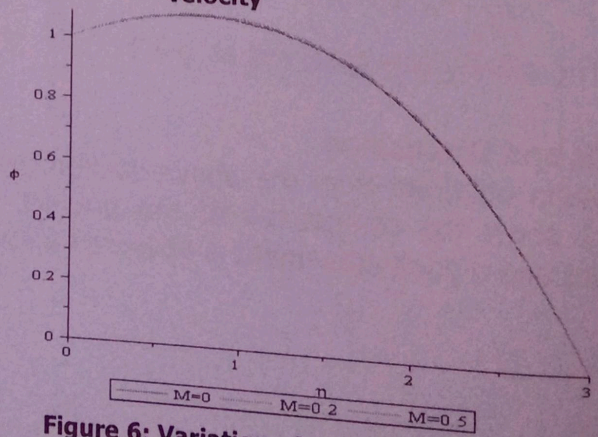

Figure 6: Variation of magnetic number on concentration

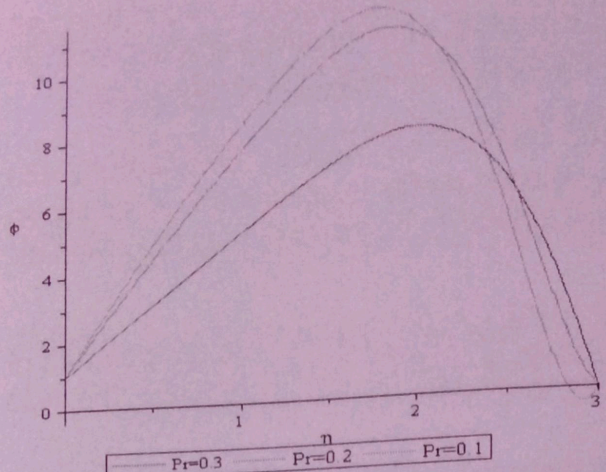

Figure 7: Variation of Prandtl number on temperature

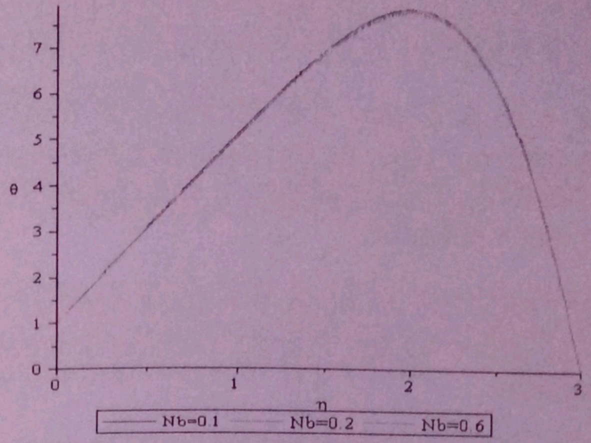

Figure 8: Variation of Brownian motion on temperature

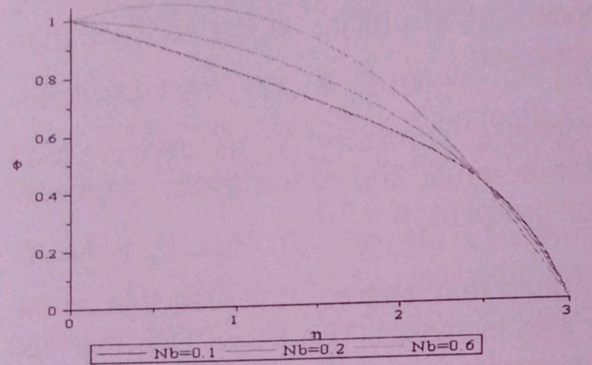

Figure 9: Variation of Brownian motion on concentration

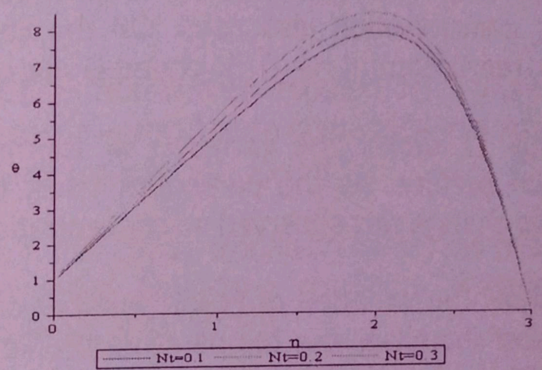

Figure 10: Variation of thermopheric parameter on temperature

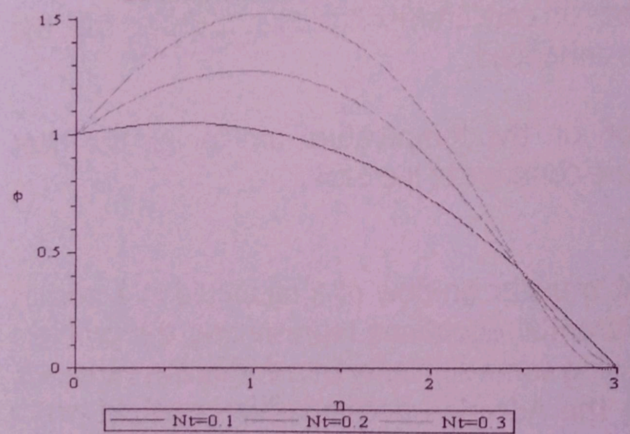

Figure 11: Variation of thermopheric parameter on concentration

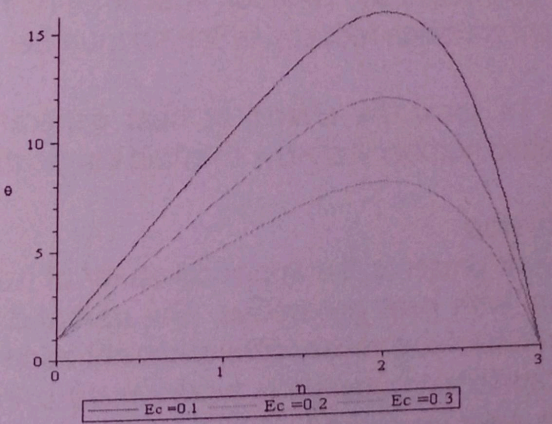

Figure 12: Variation of thermopheric parameter on concentration

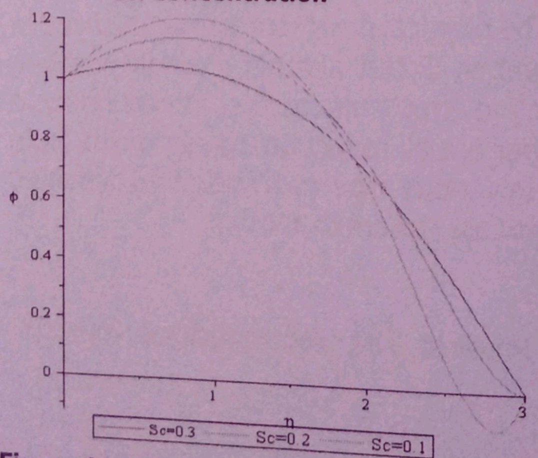

Figure 13: Variation of Schmdt number on concentration

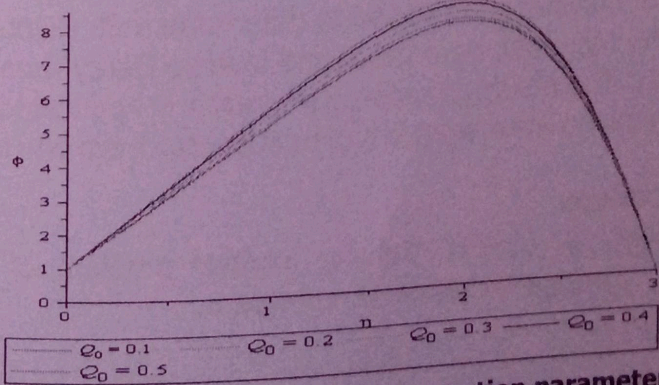

Figure 14: Variation of heat generation parameter on temperature

Figure 1 to 3 present the variation of inverse Darcy number on velocity, temperature and concentration profile. As the inverse Darcy number increases, velocity profile is observed to be a reduction agent while the temperature and concentration appear as increasing agents.

Figure 4 to 6 display the variation of magnetic number on velocity, temperature and concentration profile. As the magnetic increases, velocity profile is observed to drop due to drag like force. The temperature and concentration appear to be increasing as the magnetic parameter is enhance.

Figure 7 show the variation of Prandtl number on the fluid temperature. The temperature of the fluid drops as the Prandtl number increases which can be use to regulate the fluid temperature.

Figures 8 to 9 are the graphs showing the variation of Brownian motion on temperature and concentration respectively. As the Brownian motion increases, the fluid temperature rises slightly and concentration also rises. On the concentration profile, as the concentration approaches free stream $(\eta = 2.5)$ no changes was observed.

Figure 10 to 11 depict the variation of thermopheric parameter on temperature and concentration profiles. As the parameter rises, temperature and concentration profile all increases. No change was observed on concentration profile at $\eta = 2.5$

Figure 12 show the variation of Eckert number on fluid temperature. It is seen that as the Eckert number increases the temperature profile also increases. This shows that the fluid temperature boundary thickness thickens as the fluid becomes more viscous.

Figure 13 present the variation of Schmdt number on concentration profile. It shows that the fluid concentration reduces as Schmdt number is enhanced.

Figure 14 show the effects of heat generation on the temperature profile. As the heat generation number increase, the fluid temperature continue to increase.

Conclusion

This work presents the analytical study of heat transfer on flow of a nanofluid in a porous medium with heat generation. The partial differential equations representing the problem were reduced to ordinary differential equation using some similarity transformation variables. The transformed equations were solved using the Adomian decomposition method which results were compared with existing results in the literatures. A good agreement was established between the new method and the existing ones, which depicts the efficiency of the present method. The infinity was observed at $\eta = 3$ The physical parameters that occurred in the solutions such as magnetic parameter, Darcy number, Ekcert number, Prandtl number, Schmdt number were varied to determine their respective effects on the flow. It was observe that the Magnetic parameter and inverse Darcy number are all reduction agents of the fluid velocity. The results presented are real and applicable in various aspects of industrial activities that involved the movement of heat energy from one process stream to another.

References

Adomian, G. (1994). Solving frontier problem of Physics: The decomposition method. *Springer-Science +Business*, 6.

Gireesha, B. J., Mahanthesh, B., Manjunatha, P. T., & Gorla, R .S. R (2015). Numerical solution for hydromagnetic boundary layer flow and heat transfer past a stretching surface embedded in non-darcy porous medium with fluid-particle suspension. *Journal of Nigeria Mathematical Society*, 34, 267–285, 2015.

Gorla, R. S. R., & Vasu, B. (2016). Unsteady convective heat transfer to a stretching surface in a non-newtonian nanofluid. *Journal of Nanofluids*, 5, 581–594.

Mabood, F., & Mastroberardino, A. (2015). Melting heat transfer on MHD convective flow of a nanofluid over a stretching sheet with viscous dissipation and second order slip. *Journal of the Taiwan institute of Chemical engineers*, 17 (7), 1-7.

Vasu, B., & Manish, K. (2015). Transient boundary layer laminar free convective flow of a nanofluid over a vertical cone/plate. *International Journal of Applied Computational Mathematics*, 1, 427–448.

Srinivasacharya, D., & Surender, O. (2014). Non-similar solution for natural convective boundary layer flow of a nanofluid past a vertical plate embedded in a doubly stratified porous medium. *Journal of Heat Mass Transfer*, 71, 431–438.

Yusuf, A., Bolarin, G., Jiya, M., & Aiyisimi, Y. M. (2018). Boundary layer flow of a nanofluid in an inclined wavy wall with convective boundary condition. *Communication in Mathematical Modeling and Applications*, 3(4), 48-56.

Zaimi, K., Ishak, A., & Pop, I. (2014). Unsteady flow due to a contracting cylinder in a nanofluid using Buongiorno's model. *International Journal of Heat and Mass Transfer*, 68, 509–513.