Control of a Two Layered Coupled Tank: Application of IMC, IMC-PI and Pole-Placement PI Controllers

[1,*]Folorunso Taliha Abiodun, [2]Bello Salau Habeeb, [3]Olaniyi Olayemi Mikail and [4]Norhaliza Abdul Wahab

[1,3]Department of Computer Engineering, Federal University of Technology Minna, Nigeria
[2]Department of Telecommunication Engineering, Federal university of Technology Minna, Nigeria
[4]Department of Control and Mechatronics Engineering, Universiti Teknologi Malaysia
[1,]*funso.taliha@futminna.edu.ng

Abstract– **Proportional Integral Derivative (PID) controller remains the most widely and applicable type of controller used for process control applications in industries till date. However the efficacy of this controller lies in the tuning technique used in the determining its parameters. In this paper, we adopt a two layer coupled tank system whose transfer function was obtained via mathematical modeling. Thereafter the comparison of the performance of the PID controller based on two different tuning techniques – pole placement and internal model control (IMC) technique and that of another controller – the IMC controller was made. This is aimed at determining which tuning technique and control strategy produces a better performance on the basis of the transient response and Integral Absolute Error (IAE) for the system. The design and simulation is being carried out using the MatLab/ Simulink Toolbox and the simulation results shows that the IMC and IMC-tuned PI controller has an improved performance over the Pole-Placement-PI controller.**

Keywords— **PID/PI controller, Internal Model Control (IMC), Coupled-Tank System and Integral Absolute Error (IAE)**

I. INTRODUCTION

The application of coupled tank systems in most industrial manufacturing processes cannot be over emphasized, as the tanks are used to store various liquid substances and likewise also used for mixing purposes at various production stages in production operations [1]. To ensure the adequate and correct control of these tanks in the production process, there is a need for adequate representation and modeling of the tank system. This will not only ensure efficient control of the process but also enhance the safe operation of the system thus ensuring an optimized production process.

Conventionally, the Proportional Integral Derivative (PID) controller remains one of the most widely used control algorithm for the control of the liquid level in the coupled tank systems due to its robustness, reliability, simple structure and ease of tuning [1]-[4]. It is required and necessary to tune the PID controllers in order to obtain its optimal parameters namely the proportional gain Kp, the integral gain Ki and the derivative gain Kd. In tuning for these parameters there exist a number of applicable techniques such as the Ziegler-Nichols, cohen-coon, pole placement, Internal Model Control (IMC) methods and lots more [2], [4]-[7].

The efficiency and performance of the PID controller lies on the reliability of the tuning technique adopted as each of the techniques has its own pros and cons. In this paper, we investigate the performance of the PI controller on the coupled tank system using two of the aforementioned techniques, the pole placement method and the IMC method owing to their simplicity, robustness and ease of computation.

The rest part of this work is organized into 5 sections. Section II gives a detailed description of the system under review. In section III the mathematical model of the system is derived, while section IV focuses on the controller design for the system. Section V consists the detailed simulation and the discussion of the results obtained and the conclusion is presented in section VI.

II. THE SYSTEM DESCRIPTION

The system under-review is a double coupled tank system, whose schematic diagram is shown in Figure 1. The system consists of two tanks namely, Tank 1(primary tank) and Tank 2 (secondary). The outlet of primary tank serves as the inlet to the secondary. The primary tank is fed by the aid of a pipe connected via a pump whose rate of pumping is proportional to the applied voltage Vp. The pump ensures the liquid is pumped from the bottom basin after exit from the secondary back into the primary. The system requirement is to ensure the water level in the system is maintained at a particular level L1 and L2 as shown. For the purpose of this paper, it is required that the liquid level in the primary tank is controlled at an operating point of 15cm. Hence the liquid level in the primary tank must not exceed the operating point specified. Table 1, shows other parameters of the system.

III. SYSTEM MATHEMATICAL MODELING

Considering the primary tank in Figure 1, the flow rate is derived based on the conservation of mass and rate of change in volume [8]-[9] given as;

$$\frac{d(\rho V_1)}{dt} = \rho q_i - \rho q_o \qquad (1)$$

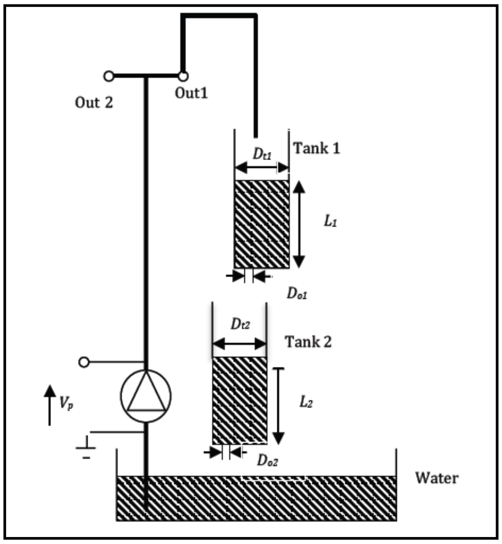

Figure 1: schematic diagram of the coupled tank system

TABLE I. PARAMTERS VALUES

Parameter	*Description*	*Value*
K_p	Constant related to the flow rate into the tank	3.3 $cm^3/(s.V)$
a_1	The cross sectional area of tank one outlet hole	0.1781 cm^2
a_2	The cross sectional area of tank two outlet hole	0.1781 cm^2
g	Gravity constant	981 cm/s^2
A_1	The cross sectional area of tank one	15.5179 cm^2
A_2	The cross sectional area of tank two	15.5179 cm^2
L_{10} , L_{20}	The operating points for the water levels in tank one & tank two	15 cm

Where V_1 the volume of tank one, q_i is the inflow, q_o is the outflow and ρ is the density of the liquid. Canceling the common variables and replacing the volume with

$$V_1 = A_1 L_1 \tag{2}$$

The relationship in (1) becomes:

$$A_1 \frac{d(L_1)}{dt} = q_i - q_o \tag{3}$$

The inflow rate relative to the voltage from controller and the constant K_p is given as:

$$q_i = K_p V_p \tag{4}$$

The outlet flow rate can be calculated as:

$$Outlet\ Flow\ rate = (Area\ Of\ the\ exist\ pipe * speed\ of\ flow)$$

$$q_o = a_1 v_1 \tag{5}$$

The exit velocity or speed is calculated using the relationship of the outlet opening and the height of the liquid in the primary tank as defined by Torricelli's theorem and it is given as:

$$q_o = a_1 \sqrt{2gL_1} \tag{6}$$

Substituting equation (4) and equation (6) into equation (3), the non-linear equation representing the system is obtained and given by equation (7):

$$\frac{d(L_1)}{dt} = \frac{K_p}{A_1} V_p - \frac{a_1}{A_1} \sqrt{2gL_1} \tag{7}$$

The linearized model is obtained as follows using the Taylors series expansion method. The expansion of the non-linear term in the model gives;

$$\sqrt{2gL_1} = \sqrt{2gL_{10}} + g\left(\frac{1}{\sqrt{2gL_{10}}}\right) * (L_1 - L_{10}) \tag{8}$$

Substituting (8) into (7):

$$\frac{d(L_1)}{dt} = \frac{K_p}{A_1} V_p - \frac{a_1}{A_1}\left(\sqrt{2gL_{10}} + g\left(\frac{1}{\sqrt{2gL_{10}}}\right) * (L_1 - L_{10})\right) \tag{9}$$

At $L_1 = L_{10}$ and $V_p = V_{po}$, the equation becomes:

$$\frac{d(L_{10})}{dt} = \frac{K_p}{A_1} V_{po} - \frac{a_1}{A_1}\left(\sqrt{2gL_{10}}\right) \tag{10}$$

Subtracting (10) from (9) and defining the following:

$$L_{11} = L_1 - L_{10}$$
$$V_{ps} = V_p - V_{po}$$

The linearized model is given as:

$$\frac{d(L_{11})}{dt} = \frac{K_p}{A_1} V_{ps} - \frac{a_1}{A_1} \frac{g}{\sqrt{2gL_{10}}} * (L_{11}) \tag{11}$$

Applying Laplace transform to (11) and rearranging to obtain the transfer function as:

$$\frac{L_{11}(S)}{V_{ps}(S)} = \frac{\frac{K_p\sqrt{2gL_{10}}}{ga_1}}{\frac{A_1\sqrt{2gL_{10}}}{ga_1}s + 1} = \frac{K_{dc1}}{\tau_1 s + 1} \tag{12}$$

The mathematical model is approximated to a first order system without dead time as shown in equation (12) and upon substituting all the variables in Table 1 therein the model equation is given as:

$$\frac{L_{11}(S)}{V_{ps}(S)} = \frac{3.240}{15.237s + 1} \tag{13}$$

IV. CONTROLLER DESIGN

In this paper, the performance of two main controllers is investigated based on the above developed model namely the PI controller and the IMC controller. As a requirement for design, the controllers should be able to satisfy the following system requirements:

1) The operating point L_1 = 15 cm
2) The percent overshoot less than 10%, thus; $PO_1 \leq 10.0$ ["%"]
3) The settling time less than 20 sec, thus; $T_{s1} \leq 20.0$ [s]
4) The response has no steady state error, thus $e_{ss} = 0$

A. Internal Model Controller (IMC)

The internal model controller (IMC) is an effective process model for feedback control with limited computational requirement [1], [4], [7]. The IMC uses a process model and inverts parts of the model for use as a controller for the process. However some parts of some models are not invertible such as the delay and right half plane [1]. In such situations linear filters are added to make the model invertible and the tuning of these filter parameters determines the performance of the IMC controller.

The structure of a feedback control system with an IMC controller is shown in Figure 2. Where $G_p(s)$ is the system model, $G_m(s)$ is the process model which is used in the controller design, $G_{IMC}(s)$is the IMC controller.

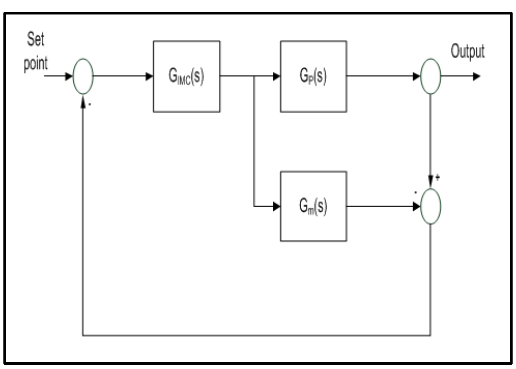

Figure 2: IMC structure for feedback system

The controller $G_{IMC}(s)$ is obtained by factorizing the model process $G_m(s)$into invertible and non-invertible parts as follows:

$$G_m(s) = G_m^+(s) * G_m^-(s) \tag{14}$$

Since the system is a first order system without delay/dead time the non-invertible part $G_m^+(s)$ can be eliminated as it thus leads to instability and as realization issues if inverted, and taking into consideration only the invertible part $G_m^-(s)$ which is stable and causal [10]. The controller $G_C(s)$is set to be equal to the inverse of the invertible part as:

$$G_C(s) = [G_m^-(s)]^{-1} \tag{15}$$

In quest to ensure the reliability, robustness and increase in system performance [4], a filter $G_f(s)$ with tunable parameter λ_f is added to the $G_C(s)$. The filter also ensures that the system is stable at all times.

$$G_{IMC}(s) = G_C(s) * G_f(s) \tag{16}$$

$$G_f(s) = \frac{1}{\left[\lambda_f s + 1\right]^n} \tag{17}$$

Where λ_f the filter parameter and n is is the order of the filter.

As a rule of thumb, the filter parameter λ_f is selected to be at least twice as fast as the open loop response of the process model $G_P(s)$. Using this as an initial value for the filter parameter and continuously tuning it till when a desirable optimal response is obtained at $\lambda_f = 2$. With that, the overall IMC controller is given as:

$$G_{IMC}(s) = \frac{15.237s + 1}{6.48s + 3.24} \tag{18}$$

Having designed the IMC controller, the PI controller can be tuned using its optimized parameters, based on tuning rules developed [7] and presented in Table 2.

TABLE II. IMC TUNING RULES

Parameter	*IMC-PI*
Controller Gain K_C	$\frac{\tau_d}{K_{dc}\lambda_f}$
Integral Time T_i	τ_d

Substituting the values of the parameters into the rules the values of the KC= 3.81 and the integral time T_i=0.25.

Thus the IMC-PI for the system is given by

$$G_{PI}(s) = K_C\left[1 + \frac{1}{T_i s}\right] = 1.57\left[1 + \frac{1}{15.237s}\right] \tag{19}$$

B. Pole Placement Method

This method of controller design entails the placement of poles of the closed loop system at some specific point to ensure the stability of the system. The design is achieved by determining some desired loop poles via the frequency or transient response of the system [5], [9], [11]. In this work the frequency responses namely the damping ratio and natural frequency are being used to determine the control parameters from the design parameters specified earlier.

The transfer function of PI controller is given as [5], [8]-[11]:

$$G_c(S) = K_c + \frac{K_I}{s} \quad (20)$$

The characteristic equation upon finding the closed loop transfer function of the controller equation (20) and the plant equation (12):

$$s^2 + \frac{(1 + K_{dc1}K_c)s}{\tau_1} + \frac{(K_{dc1}K_I)}{\tau_1} = 0 \quad (21)$$

The PI tuning for Tank 1 using can be found using Pole placement method by comparing equation (21) to equation (22):

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad (22)$$

Using the following two relations to calculate ω_n *and* ξ:

$$\xi = \frac{\left| In\left(\frac{PO_1}{100}\right) \right|}{\sqrt{In(\frac{PO_1}{100})^2 + \pi^2}}; \ \omega_n = \frac{4}{\xi T_s}$$

That gives: $\xi = 0.6$ $\qquad \omega_n = \frac{1}{3}$

Solving for K_c *and* K_I , and substituting to obtain the PI transfer function; $K_c = 1.554$ $\quad K_I = 0.523$

$$G_c(S) = 1.554 + \frac{0.523}{s} \quad (23)$$

V. SIMULATIONS AND DISCUSSION

The simulation was carried out using Matlab-simulink using a periodic step input as input to the system. An open loop analysis on the system model in (13), shows the first order model with a time constant of $\tau_1 = 15.237$ and gain of $K_{dc1} = 3.240$ has one pole at $s = \frac{-1}{15.237}$. Hence a stable system and the system responses reach steady state at about 80secs as shown in Figure 3. Thus requires a controller to suit the requirements specified earlier.

From Fig. 4, Fig. 5 and Fig. 6 show the system response of the IMC controller, the IMC tuned PI controller and that of the Pole Placement tuned PI controller respectively. As seen from the response, it takes the open loop with no controller more than 80 seconds to reach the various set points. Upon the introduction of PI controller using pole assignment method of tuning, far more improvement was achieved in terms of its rise time and settling time over the open loop. However, the performance of that controller is not satisfactory because of the overshoot which was estimated to be around 24% fails to meet the maximum system requirement of 10%. This can be considered as undesirable as this could lead to unwanted performance

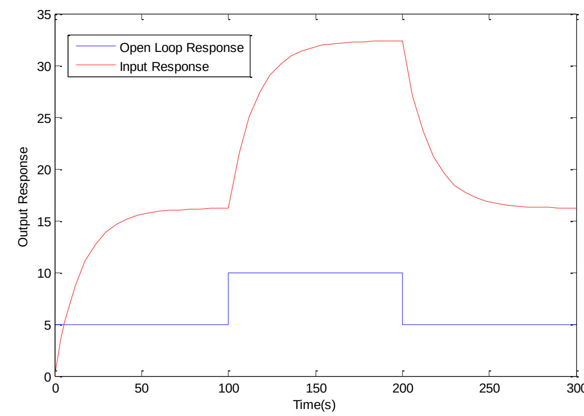

Figure 3: Open Loop Response

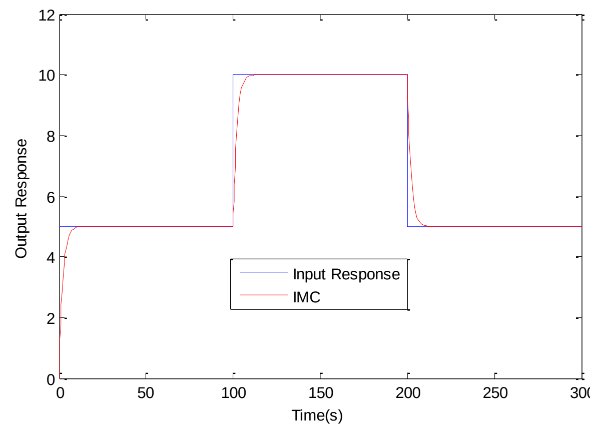

Figure 4: Output response of the IMC Controller

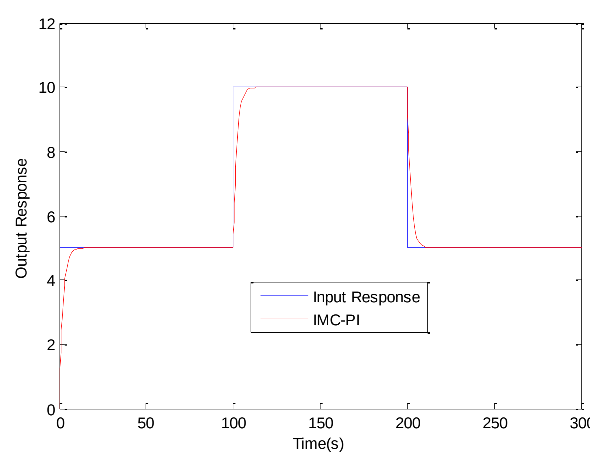

Figure 5: Output response of the IMC tuned PI controller

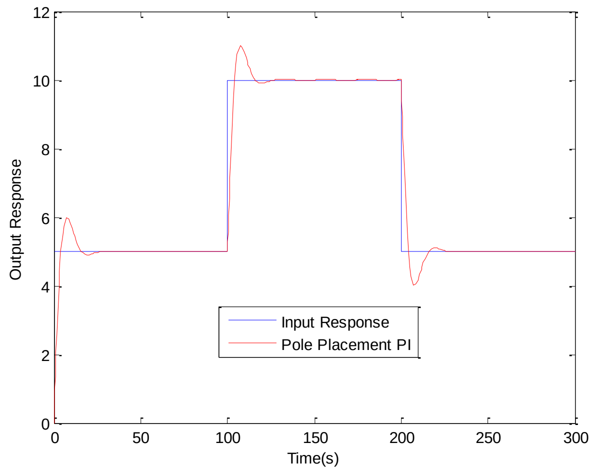

Figure 6: Output response of the Pole Placement tuned PI controller.

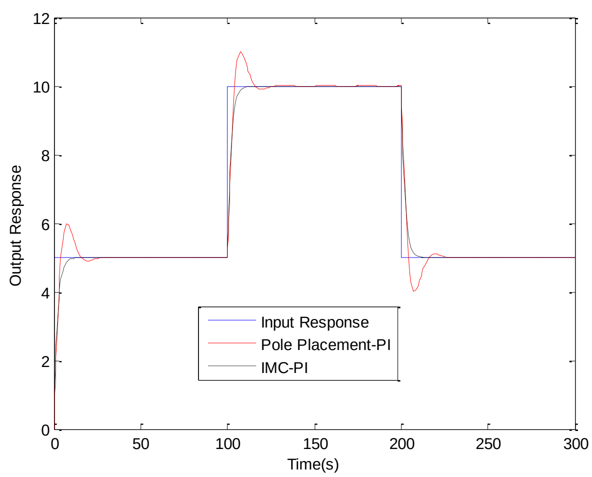

Figure 7: Comparison of the output response of the IMC-PI and the Pole Placement PI

in real time applications. Furthermore, great deal of improvement is achieved using the IMC and IMC-PI over the PPM-PI. The response reaches the setting points with no noticeable overshoot and acceptable settling time. A comparison of the pole placement PI and the IMC-PI presented in Figure 7 shows both controllers has a fairly close rise time, with the IMC –PI exhibiting no overshoot and a faster settling time as compared to the pole placement PI. The transient response characteristics of the controllers are as shown in Table III.

In a like manner, for further analysis and comparison the Integral of the absolute error (IAE) is used. It is one of the methods which have been used intensively for early performance degradation detection, depending on desired output. Table IV shows the IAE results obtained from the open loop to the use of the both PI- controllers and that of the IMC. From these results it can be deduced that the IMC controller has a significantly high performance as been the controller with the least value of 30 next to the IMC-PI with a value of 31.10 which is slightly higher than the IMC controller and the pole placement PI (PPM-PI) with a value of 48.24 with the least performance.

TABLE III. SUMMARY OF THE CHARACTERISTIC PERFORMANCE OF THE CONTROLLERS

	IMC	**IMC-PI**	**PPM-PI**
Rise Time(sec) Tr	1.8	1.85	2.1
Settling Time(sec) Ts	10.25	11	17
Over Shoot (%)	0	0	24
Steady Error ess	0	0	0

TABLE IV. INTEGRAL OF THE ABSOLUTE ERROR COMPARISON

System	***Value***
Open loop	4259
IMC	30
IMC-PI	31.10
PPM-PI	48.24

VI. CONCLUSION

In this work, we have carried out the mathematical modeling of the two layered couple tank system with a view of obtaining its system transfer function. Also carried out is the design of a PI controller for the system using two different tuning technique (the IMC tuning and the pole placement technique). In addition to the PI controller an IMC controller was also designed for application on the system. This is aimed at investigating the applicability and performance evaluation of the PI controller compared with that of the IMC controller. The result obtained shows, the IMC controller a model based controller enjoys a zero overshoot (no overshoot), faster response in terms of its rise time and settling time, thus providing an improved performance as compared with that of the PI controller. However for the case of the PI tuned using the pole placement method (PI-PPM) the responses suffers significant amount of overshoot as compared with that of the IMC-tuned PI and the IMC as could be seen in Figure 6. The PI controller produced a response similar to that of IMC controller when tuned using the IMC- technique to obtain the PI parameters as seen in Figure 4 and Figure 5, also outweighing the performances of the PI-PPM. In a further investigation of the performance evaluation using the IAE as a yard stick shows the IMC controller having the best of performance next to the IMC tuned PI and the Pole Placement PI controller with the least performance.

REFERENCES

[1] E. P. Gatzke, E. S. Meadows, C. Wang, and F. Doyle, "Model based control of a four-tank system," *Computers & Chemical Engineering,* vol. 24, pp. 1503-1509, 2000.

[2] M. O. Ali, "Genetic algorithm tuning based PID Controller for liquid-level tank system," in *international conference on Man-Machine systems (ICoMMS)*, 2009.

[3] M. I. Menhas, L. Wang, H. Pan, and M. Fei, "CFBB PID Controller Tuning with Probability based Binary Particle Swarm Optimization Algorithm," in *LSMS/ICSEE 2010*, 2010, pp. 44-51.

[4] D. E. Rivera, M. Morari, and S. Skogestad, "Internal model control: PID controller design," *Industrial & engineering chemistry process design and development,* vol. 25, pp. 252-265, 1986.

[5] K. J. Astrom and T. Hagglund, "PID Controllers: Theory,Design and Tuning," second ed: Instrument society of America, 1995.

[6] S. Bdran, M. Shuyuan, S. Saifullah, and H. Jie, "Comparison of PID, Pole Placement and LQR Controllers for speed ration control of continuously Variable Transmission (CVT)," in *ICAMECE* 2013, pp. 149-154.

[7] D. E. Rivera, "Internal Model Control, A Comprehensive View," *Arizona State University, Tempe, Arizona 85287-6006,* October 27, 1999.

[8] T. Marlin, *Process Control, Designing Processes and Control Systems for Dynamic Performance*, second ed. New York: McGraw Hill, 2000.

[9] D. E. Seborg, T. F. Edgar, and D. A. Mellichamp, *Process Control Dynamics*, second ed.: John Wiley and Sons Inc., 2004.

[10] W. S. Levine, *The control Handbook* CRC Press and IEEE Press, 1996.

[11] K. Ogata, *Discrete-Time control systems*, second edition ed. Upper Saddle River, New jersey: Prentice Hall, 1994.