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INFLUENCE OF OFF-DIAGONAL DISPERSION ON THE CONCENTRATION OF CONTAMINANT IN A TWO-DIMENSIONAL CONTAMINANT FLOW: A SEMI-ANALYTICAL APPROACH

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Abstract

The equation which describes the two-dimensional contaminant flow model is a partial differential equation characterized by advection, dispersion, adsorption, first order decay and zero-order source. In this paper, the off-diagonal dispersion parameter is introduced into the two dimensional contaminant flow model in order to study its effect on the concentration of the contaminant. It is assumed that the adsorption term is modeled by Freudlich isotherm. The parameter expanding method is applied on the equation to obtain a set of differential equations which are then solved successively using the Eigen functions expansion technique to obtain the analytical solution. The results obtained are plotted into graphs to show the effect of change in the parameters on the concentration of the contaminants. Findings from this research show that the contaminant concentration decreases with increase in distance as the off-diagonal dispersion coefficient, zero-order source coefficient and vertical dispersion coefficient increases.

***Keywords*:** Advection, dispersion, adsorption, contaminant, off-diagonal dispersion.

1.0 Introduction

The problem of dispersion of contaminant in soil, water channels, groundwater and surface water has been an evolving research in geology and hydro-geological centers for many years. This is not unconnected with the increased awareness of significant contamination of groundwater and surface water by industrial and human activities such as agricultural chemicals, accidental spills, landfills, toxic wastes and buried hazardous materials. While agricultural chemicals are generally useful in the surface of the soil, their penetration into the unsaturated zone and groundwater could contaminate groundwater.

Groundwater in its natural state is generally of excellent quality because the physical structure and mineral constituents of rock made it self-purifying. Before the coming of industries, the major threat to groundwater came from viruses and bacteria. The presence of these microbiological contaminants like bacteria, viruses and parasites in groundwater constitute some threat to community health.

The movement of contaminants through groundwater and surface water environments is modeled by transport equations[1]. In order to predict the contaminant migration in the geological formation more accurately, a tasking job emerges for scientists. The problem involves providing the solution of transport equation, defining the flow lines of groundwater of the aquifers, the travel time of water along the flow lines and to predict the chemical reaction and zero order source coefficient which affect the concentration during transport.

Most researchers posits that the flow in the solute transport or contaminant flow model is predominantly horizontal as found in [1], but nonetheless, appreciable vertical flow components may occur in the domain of vertically penetrating wells and streams [2]. In an effort to provide solutions to the contaminant flow problems, a lot of successes were achieved by some researchers but mostly on one-dimensional cases with various initial and boundary conditions. This includes the study of the influence of the retardation factor on the contaminant in a nonlinear contaminant flow [3]. On the dispersion of solute, the effect of longitudinal dispersion of miscible fluid flow in one dimension through porous media was explored [4].

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In line with desire to understand the behavior of contaminant in a flow, an analytical solution to temporally dependent dispersion through semi-infinite homogeneous porous media by Laplace transform technique (LTT) was provided [5]. Computational analyses on the effect of reactive and non-reactive contaminant on the flow were carried out on one-dimensional non-linear contaminant flow with an initial continuous point source and discovered that the concentration decreases with increase in time and distance from the origin for the non-reactive case by homotopy perturbation method [5, 7].

In this research, we provide asemi-analytical solution of the two-dimensional contaminant flow problem incorporating flow in both horizontal directions, off-diagonal dispersion parameters in addition to first-order decay and zero order sources using the parameter expanding technique and Eigen functions expansion method.

2.0 Formulation of the Problem

We consider an incompressible fluid flow through a finite homogeneous porous media with non-zero initial concentration in the transport domain. It is assumed that the flow is two-dimensional and in the direction of x and y-axis. The source concentration is assumed at the origin (i.e. at time $t=0$). It is assumed that the contaminant invades the groundwater level from point source in a finite homogeneous porous media.

Following [1, 5, 8, 9, 10, 15], we introduce the off-diagonal dispersion parameter into the two dimensional parabolic partial differential equation describing hydrodynamic dispersion in adsorbing homogeneous, isotropic porous medium and obtained:

$$\frac{\partial C}{\partial t}+\frac{\partial S}{\partial t}=D_{xx}\frac{\partial^2 C}{\partial x^2}+D_{yy}\frac{\partial^2 C}{\partial y^2}+D_{xy}\frac{\partial^2 C}{\partial x\partial y}+D_{yx}\frac{\partial^2 C}{\partial y\partial x} -u(t)\frac{\partial C}{\partial x}-v(t)\frac{\partial C}{\partial y}-\gamma(t)C+\mu(t), \quad (1)$$

where C is the concentration of the contaminant in the flow, S is the concentration of the contaminant adsorbed to the porous media, D_{xx} is the dispersion in the horizontal direction, D_{yy} is the dispersion in the vertical direction, D_{xy} and D_{yx} are off-diagonal dispersion coefficients. $\gamma(t)$ is the decay parameter, $\mu(t)$ is the source term, $u(t)$ and $v(t)$ are the velocities in the horizontal and vertical directions respectively,

$$\left.\begin{aligned} D_{xx}&=\frac{\alpha_L u^2}{\sqrt{u^2+v^2}}+\frac{\alpha_T v^2}{\sqrt{u^2+v^2}}\\ D_{yy}&=\frac{\alpha_L v^2}{\sqrt{u^2+v^2}}+\frac{\alpha_T u^2}{\sqrt{u^2+v^2}}\end{aligned}\right\} \quad (2)$$

$$D_{xy}=D_{yx}=\frac{(\alpha_L-\alpha_T)uv}{\sqrt{u^2+v^2}}, \quad (3)$$

as in [14].

The adsorbed contaminant is assumed to be directly proportional to the contaminant concentration. i.e.,

$$S=K_d C \quad (4)$$

$$\frac{\partial S}{\partial t}=K_d\frac{\partial C}{\partial t}, \quad (5)$$

as found in [16].

Following the relationship (4) and (5), equation (1) can be rewritten as

$$R\frac{\partial C}{\partial t}=D_{xx}\frac{\partial^2 C}{\partial x^2}+D_{yy}\frac{\partial^2 C}{\partial y^2}+2D_{xy}\frac{\partial^2 C}{\partial x\partial y}-u(t)\frac{\partial C}{\partial x}-v(t)\frac{\partial C}{\partial y}-\gamma(t)C+\mu(t) \quad (6)$$

D_{xy} is the initial off-diagonal dispersion component. The initial and boundary conditions are chosen as

$$\left.\begin{aligned} C(x,y,t)&=c_i;x\ge 0,y\ge 0,t=0\\ C(x,y,t)&=C_0(1+e^{-qt});x=0,y=0,t<0\\ C(x,y,t)&=c_p;x\to L,y\to L,t\ge 0\end{aligned}\right\} \quad (7)$$

We let

$$\left.\begin{aligned} D_x&=D_{x0}f(t)\\ u(t)&=u_0f(t)\\ v(t)&=v_0f(t)\\ D_y&=D_{y0}f(t)\\ \mu(t)&=\mu_0f(t)\\ \gamma(t)&=\gamma_0f(t)\\ D_{xy}&=D_{xy0}f(t)\\ D_{xy0}f(t)&=\frac{(\alpha_L-\alpha_T)u_0v_0}{\sqrt{u_0^2+v_0^2}}.\end{aligned}\right\} \quad (8)$$

where $f(t)$ is arbitrary function of time as used in [5]. D_{x0} is initial horizontal dispersion coefficient, D_{y0} is the initial vertical dispersion coefficient, D_{xy0} is the initial off-diagonal dispersion coefficient, v_0 is the initial vertical velocity, u_0 is the initial horizontal velocity, μ_0 is the initial zero-order source coefficient and γ_0 is the initial first order decay coefficient. We also introduced a new time variable as in [11]:

$$\left.\begin{array}{l}\tau=\int_0^t f(t)dt\\ f(t)=Re^{-qt}\end{array}\right\} \quad (9)$$

By substituting the components of equations (8) and (9) in (6), the following equation is obtained:

$$\frac{\partial C}{\partial \tau}=D_{x0}\frac{\partial^2 C}{\partial x^2}+D_{y0}\frac{\partial^2 C}{\partial y^2}+2D_{xy0}\frac{\partial^2 C}{\partial x\partial y}-u_0\frac{\partial C}{\partial x}-v_0\frac{\partial C}{\partial y}-\gamma_0 C+\mu_0 \quad (10)$$

$$\left.\begin{array}{l}C(x,y,\tau)=c_i;x\geq 0,y\geq 0,\tau=0\\ C(x,y,\tau)=C_0(2-q\tau);x=0,y=0,\tau\geq 0\\ C(x,y,\tau)=c_p;x\to L,y\to L,\tau\geq 0\end{array}\right\} \quad (11)$$

where all the parameters as defined previously.
A new special variable is introduced below as used in [14]:

$$\eta=x+y\sqrt{\frac{D_{y0}}{D_{x0}}} \quad (12)$$

Then, substituting equation (12) in equation (10), we have

$$\frac{\partial C}{\partial \tau}=\left(D_{x0}+\frac{D_{y0}^{\ 2}}{D_{x0}}+2D_{xy0}\sqrt{\frac{D_{y0}}{D_{x0}}}\right)\frac{\partial^2 C}{\partial \eta^2}-\left(u_0+v_0\sqrt{\frac{D_{y0}}{D_{x0}}}\right)\frac{\partial C}{\partial \eta}-\gamma_0 C+\mu_0 \quad (13)$$

i.e.
$$\frac{\partial C}{\partial \tau}=D\frac{\partial^2 C}{\partial \eta^2}-U\frac{\partial C}{\partial \eta}-\gamma_0 C+\mu_0 \quad (14)$$

$$\left.\begin{array}{l}C(\eta,\tau)=C_i;\ \eta\geq 0,\tau=0\\ C(\eta,\tau)=C_0(2-q\tau);\ \eta=0,\tau\geq 0\\ C(\eta,\tau)=c_p;\ \eta\to L,\tau\geq 0\end{array}\right\} \quad (15)$$

where

$$D=D_{x0}\left(1+\left(\frac{D_{y0}}{D_{x0}}\right)^2+2\frac{D_{xy0}}{D_{x0}}\sqrt{\frac{D_{y0}}{D_{x0}}}\right) \quad (16)$$

$$U=\left(u_0+v_0\sqrt{\frac{D_{y0}}{D_{x0}}}\right) \quad (17)$$

2.1 Non-Dimensionalization

Equation (14) is non-dimensionalized by using the following dimensionless variables:

$$\left.\begin{array}{l}\tau=\frac{L}{U}\tau'\\ \eta=\eta' L\\ C=C_0 C^*\\ \partial\tau=\frac{L}{U}\partial\tau'\\ \partial\eta=L\partial\eta'\end{array}\right\} \quad (18)$$

On substituting the above dimensionless variables in equation (14), the following equation is obtained.

$$\frac{\partial C^*}{\partial \tau'}=\frac{D}{LU}\frac{\partial^2 C^*}{\partial \eta'^2}-\frac{\partial C^*}{\partial \eta'}-\frac{\gamma_0 LC^*}{U}+\frac{L}{C_0 U}\mu_1 \quad (19)$$

For convenience, the primes are dropped and obtained

$$\frac{\partial C^*}{\partial \tau}=D_2\frac{\partial^2 C^*}{\partial \eta^2}-\frac{\partial C^*}{\partial \eta}-\gamma_0 C^*+\mu_1 \quad (20)$$

where,

$$D_2=\frac{D}{LU} \quad (21)$$

The dimensionless equation together with the initial and boundary conditions is

$$\left.\begin{aligned}
&\frac{\partial C^*}{\partial \tau} = D_2 \frac{\partial^2 C^*}{\partial \eta^2} - \frac{\partial C^*}{\partial \eta} - \gamma C^* + \mu_1 \\
&C^*(\eta, 0) = \frac{c_i}{c_0} \\
&C^*(0, \tau) = 2 - q\tau, \ \tau \geq 0 \\
&C^*(1, \tau) = \frac{c_p}{c_0}, \ \tau \geq 0
\end{aligned}\right\} \quad (22)$$

2.2 Solution of the Model

The above problem (22) is solved by using parameter expanding method. The parameter expanding method breaks the equation (22) into simpler ones which can be easily solved successively. To achieve this, let

$$1 = a\gamma_0 \quad (23)$$

in the advection term of equation (22) and

$$C^*(\eta, \tau) = C_0(\eta, \tau) + \gamma_0 C_1(\eta, \tau) + \ldots \quad (24)$$

as in [12], [13] and [11]. When equation (24) is substituted in equation (22), the following was obtained.

$$\begin{aligned}
&\frac{\partial}{\partial \tau}(C_0(\eta,\tau) + \gamma_0 C_1(\eta,\tau) + \gamma_0^2 C_2(\eta,\tau) + \ldots) \\
&= D_2 \frac{\partial^2}{\partial \eta^2}(C_0(\eta,\tau) + \gamma_0 C_1(\eta,\tau) + \gamma_0^2 C_2(\eta,\tau) + \ldots) \\
&- a\gamma_0 \frac{\partial}{\partial \eta}(C_0(\eta,\tau) + \gamma_0 C_1(\eta,\tau) + \gamma_0^2 C_2(\eta,\tau) + \ldots) \\
&- \gamma_0(C_0(\eta,\tau) + \gamma_0 C_1(\eta,\tau) + \gamma_0^2 C_2(\eta,\tau) + \ldots) + \mu_1
\end{aligned} \quad (25)$$

Equating corresponding coefficients on both sides of equation (25), the resulting equations together with the initial and boundary conditions are given below:

$$\left.\begin{aligned}
&\frac{\partial C_0(\eta,\tau)}{\partial \tau} = D_2 \frac{\partial^2 C_0}{\partial \eta^2} + \mu_1 \\
&C_0(\eta, 0) = \frac{c_i}{c_0} \\
&C_0(0, \tau) = 2 - q\tau \\
&C_0(1, \tau) = \frac{c_p}{c_0}
\end{aligned}\right\} \quad (26)$$

$$\left.\begin{aligned}
&\frac{\partial C_1(\eta,\tau)}{\partial \tau} = D_2 \frac{\partial^2 C_1}{\partial \eta^2} - a\frac{\partial C_0}{\partial \eta} - C_0 \\
&C_1(\eta, 0) = 0 \\
&C_1(0, \tau) = 0 \\
&C_1(1, \tau) = 0
\end{aligned}\right\} \quad (27)$$

$$\left.\begin{aligned}
&\frac{\partial C_2(\eta,\tau)}{\partial \tau} = D_2 \frac{\partial^2 C_2}{\partial \eta^2} - a\frac{\partial C_1}{\partial \eta} - C_1 \\
&C_2(\eta, 0) = 0 \\
&C_2(0, \tau) = 0 \\
&C_2(1, \tau) = 0
\end{aligned}\right\} \quad (28)$$

Equations (26), (27) and (28) are transformed to satisfy the homogeneous boundary conditions and solved successively using the Eigen functions expansion method. To accomplish this, a function is chosen which satisfies the given boundary conditions. i.e.,

$$w(\eta, \tau) = \alpha(\tau) + \eta(\beta(\tau) - \alpha(\tau)) \quad (29)$$

where

$$\left.\begin{aligned}
&\alpha(\tau) = 2 - q\tau \\
&\beta(\tau) = \frac{c_p}{c_0}
\end{aligned}\right\} \quad (30)$$

such that

$$C_0(\eta, \tau) = v_0(\eta, \tau) + w(\eta, \tau) \quad (31)$$

That is

$$C_0(\eta, \tau) = v_0(\eta, \tau) + (2 - q\tau) + \eta(c_p - (2 - q\tau)) \quad (32)$$

By application of change of variables,

$$\frac{\partial C_0}{\partial \tau} = \frac{\partial C_0}{\partial v_0} \times \frac{\partial v_0}{\partial \tau} + \frac{\partial C_0}{\partial w_0} \times \frac{\partial w_0}{\partial \tau} \quad (33)$$

$$\frac{\partial C_0}{\partial \tau} = \frac{\partial v_0}{\partial \tau} + (\eta - 1)q \quad (34)$$

i.e.

$$\frac{\partial C_0}{\partial \eta} = \frac{\partial v_0}{\partial \eta} + \left(\frac{c_p}{c_0} - (2 - q\tau)\right) \quad (35)$$

$$\frac{\partial^2 C_0}{\partial \eta^2} = \frac{\partial}{\partial \eta}\left(\frac{\partial C_0}{\partial \eta}\right) \quad (36)$$

Therefore,

$$\frac{\partial^2 C_0}{\partial \eta^2} = \frac{\partial^2 v_0}{\partial \eta^2} \quad (37)$$

We substitute (34) and (37) in equation (26) to obtain

$$\frac{\partial v_0}{\partial \tau} = D_2 \frac{\partial^2 C_0}{\partial \eta^2} + \mu_0 - (\eta - 1)q \quad (38)$$

Then for the initial and boundary conditions,

$$C_0(0,\tau) = v_0(0,\tau) + w(0,\tau) \quad (39)$$

$$\Rightarrow v_0(0,\tau) = 0 \quad (40)$$

Similarly,

$$C_0(1,\tau) = v_0(1,\tau) + w_0(1,\tau) \quad (41)$$

$$v_0(1,\tau) = 0 \quad (42)$$

For the initial condition, $C_0(\eta,0) = 0$

$$C_0(\eta,0) = v_0(\eta,0) + 2 + \eta\left(\frac{c_p}{c_0} - 2\right) = 0 \quad (43)$$

$$\Rightarrow v_0(\eta,0) = \frac{c_i}{c_0} + 2(\eta - 1) - \eta\frac{c_p}{c_0} \quad (44)$$

The partial differential equation (26) with the homogeneous boundary conditions (40) and (42) and the initial condition (44) is solved using the Eigen Function expansion method and obtain the following result:

$$v_0(\eta,\tau) = \sum_{n=1}^{\infty} C_n(\tau)\sin(n\pi\eta) \quad (45)$$

$$C_0(\eta,\tau) = v_0(\eta,\tau) + w(\tau) \quad (46)$$

i.e.,

$$C_0(\eta,\tau) = w(\tau) + \sum_{n=1}^{\infty} C_n(\tau)\sin(n\pi\eta) \quad (47)$$

$$C_0(\eta,\tau) = (2 - q\tau)(1 - \eta) + \eta c_p + \sum_{n=1}^{\infty}\left(\begin{array}{l}\frac{2}{D_2(n\pi)^3}\left(q - \mu_1(\cos(n\pi) - 1)\right)\left(1 - e^{\left(-D_2(n\pi)^2\tau\right)}\right) \\ + \left(\begin{array}{l}-\frac{2c_i}{n\pi c_0}(\cos(n\pi) - 1) \\ -\frac{4}{n\pi} + \frac{2c_p}{n\pi c_0}\cos n\pi\end{array}\right)e^{\left(-D_2(n\pi)^2\tau\right)}\end{array}\right)\sin(n\pi\eta) \quad (48)$$

Similarly, when equation (27) and (28) are solved by Eigen Functions expansion method, the following results were obtained:

$$C_1(\eta,\tau)=-\sum_{n=1}^{\infty}\left(\begin{array}{l}\sum_{n=1}^{\infty}\frac{2}{D_2(n\pi)^3}\left(q-\mu_1(\cos(n\pi)-1)\right)\left(\begin{array}{l}\frac{1}{D_2(n\pi)^2}\\-\tau e^{\left(-D_2(n\pi)^2\tau\right)}\\-\frac{e^{\left(-D_2(n\pi)^2\tau\right)}}{D_2(n\pi)^2}\end{array}\right)\sin(n\pi\eta)\\-\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\left(\begin{array}{l}-\frac{2c_i}{n\pi c_0}(\cos(n\pi)-1)\\-\frac{4}{n\pi}+\frac{2c_p}{n\pi c_0}\cos n\pi\end{array}\right)e^{\left(-D_2(n\pi)^2\tau\right)}\tau\sin(n\pi\eta)\right)\end{array}\right) \quad (49)$$

$$C_{n2}(\tau)=\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\left(\frac{1}{D_2(n\pi)^3}\right)\left(q-\mu_1(\cos(n\pi)-1)\right)\left(\begin{array}{l}\frac{1}{D_2^{\,2}(n\pi)^4}-\frac{\tau^2}{2}e^{\left(-D_2(n\pi)^2\tau\right)}\\-\frac{\tau e^{\left(-D_2(n\pi)^2\tau\right)}}{D_2(n\pi)^2}\\-\frac{1}{D_2^{\,2}(n\pi)^4}e^{\left(-D_2(n\pi)^2\tau\right)}\end{array}\right)\right) \quad (50)$$

$$+\sum_{n=1}^{\infty}\left(\sum_{n=1}^{\infty}\left(\begin{array}{l}-\frac{2c_i}{n\pi c_0}(\cos(n\pi)-1)\\-\frac{2}{n\pi}+\frac{c_p}{n\pi c_0}\cos n\pi\end{array}\right)\frac{\tau^2}{2}e^{\left(-D_2(n\pi)^2\tau\right)}\right)$$

$$C_2(\eta,\tau)=\sum_{n=1}^{\infty}C_{n2}(\tau)\sin(n\pi\eta) \quad (51)$$

The solution of the contaminant flow equation (22) where $c_i \neq 0$ is therefore,

$$C^*(\eta,\tau)=C_0(\eta,\tau)+\gamma C_1(\eta,\tau)+\gamma^2 C_2(\eta,\tau)+\ldots \quad (52)$$

where $C_0(\eta,\tau)$, $C_1(\eta,\tau)$ and $C_2(\eta,\tau)$ are as given in (48), (49) and (51) respectively.

3.0 Results and Discussion

The analytical solution obtained in equation (52) is plotted into graphs with the help of input data and Maple software (Maple 16) package as presented in the following figures.

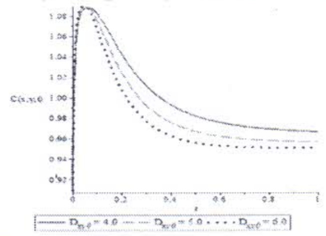

Figure 1: Contaminant Concentration profile for $D_{xy0}=4, D_{xy0}=5,\ D_{xy0}=6$ when $c_0=1, \mu_1=0.3, c_p=1,\ \gamma_0=0.1,$ $q=3,\ D_{xo}=1,\ D_{y0}=1.5,\ u_0=0.1, v_0=0.1$ with y and x fixed as 0.5.

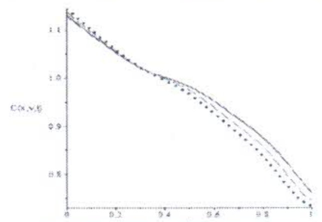

Figure 2: Contaminant Concentration profile for $D_{y0}=1.5, D_{y0}=1.6,\ D_{y0}=1.7$ when $c_0=1, \mu_1=0.3, c_p=1,\ \gamma_0=0.1,$ $q=3,\ D_{xo}=1,\ D_{xy0}=4,\ u_0=0.1, v_0=0.1$ with x and t fixed as 0.5.

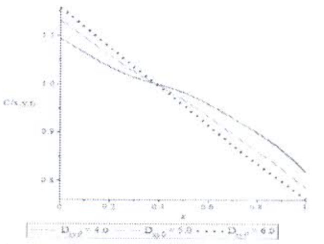

Figure 3: Contaminant Concentration profile for $D_{xy0}=4, D_{xy0}=5, D_{xy0}=6$ when $c_0=1, \mu_1=0.3, c_p=1, \gamma_0=0.1, q=3,$ $D_{xo}=1, D_{y0}=1.5, u_0=0.1, v_0=0.1$ with y and t fixed as 0.5.

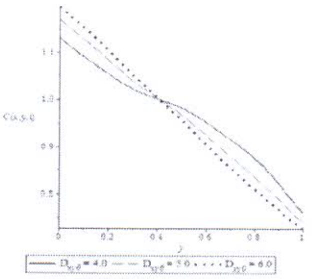

Figure 4: Contaminant Concentration profile for $D_{xy0}=4, D_{xy0}=5, D_{xy0}=6$ when $c_0=1, \mu_1=0.3, c_p=1, \gamma_0=0.1, q=3,$ $D_{xo}=1, D_{y0}=1.5, u_0=0.1, v_0=0.1$ with x and t fixed as 0.5.

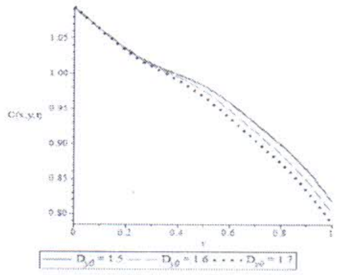

Figure 5: Contaminant Concentration profile for $D_{y0}=1.5, D_{y0}=1.6, D_{y0}=1.7$ when $c_0=1, \mu_1=0.3, c_p=1,$ $\gamma_0=0.1, q=3, D_{yo}=1.5, D_{xy0}=4, u_0=0.1, v_0=0.1$ with y and t fixed as 0.5.

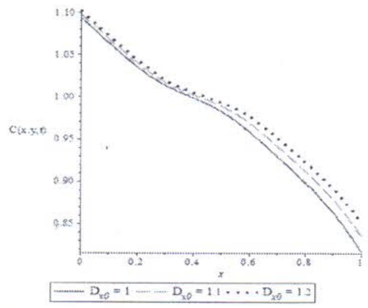

Figure 6: Contaminant Concentration profile for $D_{x0}=1, D_{x0}=1.1, D_{x0}=1.2$ when $c_0=1, \mu_1=0.3, c_p=1, \gamma_0=0.1,$ $q=3, D_{yo}=1.5, D_{xy0}=4, u_0=0.1, v_0=0.1$ with y and t fixed as 0.5.

Figure1 is the graph of contaminant concentration with respect to time for varying off-diagonal dispersion coefficient from 4.0 to 6.0. The graph shows that as the off-diagonal dispersion coefficient increases, the concentration decline with time. Figures 2 and 5 are the concentration profile of contaminant for varying vertical dispersion coefficient. From the graphs, as the vertical dispersion coefficient increases, the contaminant concentration decreases with distances x and y.

The impact of off-diagonal dispersion on the contaminant concentration is illustrated by figures 3 and 4. These figures show that as the off-diagonal dispersion coefficient increases, the contaminant concentration decreases with increase distances in both x and y directions. Lastly, in figure 6, as the horizontal dispersion coefficient increases, the contaminant concentration decreases with increase in the horizontal distance x. The results obtained in this study may assist the geologist in knowing the distance from the source of contaminant that is good for location of wells. These are the distances at which the contaminant concentration is zero.

4.0 Conclusion

In this work, the effect of off-diagonal dispersion of contaminant on the concentration is studied. The semi-analytical solution of the two-dimensional contaminant flow problem incorporating the off-diagonal dispersion coefficient has been solved by the method of Eigen functions expansion technique and the study revealed that the contaminant concentration decreases with increase in time and distances as the off-diagonal dispersion, horizontal dispersion and vertical dispersion coefficients increase.

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