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Mathematical Analysis of a Contaminant Flow in a Finite Medium using the Weighted Residual Method

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Abstract

In this paper, a Galerkin weighted Residual method is used in providing an analytical solution of two-dimensional contaminant flow problem with non-zero initial concentration. The equation is described by advection, dispersion, adsorption, first order decay and zero-order source. It is assumed that the adsorption term is modeled by Freudlich isotherm. Using Bubnov-Galerkin method, the governing equation was converted to a discrete problem. Thereafter, the approximate solution of the resulting system of initial value problem was obtained. The results obtained are expressed in graphical form to show the effect of change in the parameters on the concentration of the contaminants. From the analysis of the results, it was discovered that the contaminant concentration decreases with increase in the distance from the origin while it increases with increase in the zero-order source coefficient.

Keywords: Advection, dispersion, adsorption, contaminant, Galerkin weighted residuals.

1. Introduction

The problem of contaminant transport in soil, groundwater and surface water has been in hydro-geological research history for many years. This is largely due to increased awareness of significant contamination of groundwater and surface water by industrial and human activities such as agricultural chemicals, accidental spills, landfills and buried hazardous materials. While agricultural chemicals are generally useful in the surface of the soil, their penetration into the vadose zone and groundwater could contaminate groundwater. Groundwater in its natural state is generally of excellent quality because the physical structure and mineral constituents of rock have facility for purifying water.

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Before the establishment of industries, the major threat to groundwater came from viruses and bacteria. The presence of these microbiological contaminants like bacteria, viruses and parasites in groundwater constitute some threat to community health.

The transport equation which models the movement of contaminants through groundwater and surface water environments was reported by (Bear, 1997). These equations describe advection, diffusion and interaction with the environment. They are often advection-dominated and require a lot of care when solved numerically. In order to predict the contaminant migration in the geological formation more accurately, a tasking job emerges for scientists. The problem involves defining the flow lines of groundwater of the aquifers, the travel time of water along the flow lines and to predict the chemical reaction and zero order source coefficient which alter the concentration during transport.

Most researchers are of the view that the flow in the solute transport or contaminant flow model is predominantly horizontal as found in (Bear, 1997). Further research by (Brainard and Gelhar, 1991) discovered that appreciable vertical flow components do occur in the domain of vertically penetrating wells and streams.

In an effort to provide solutions to the contaminant flow problems, a lot of successes were achieved by some researchers but mostly on one-dimensional cases with various initial and boundary conditions. (Okedayo and Aiyesimi, 2005) studied the influence of retardation factor on the nonlinear contaminant flow problem. Okedayo *et al*. (2011) worked on the 1-Dimensional nonlinear contaminant transport equation with an initial and instantaneous point source. Their investigation revealed that the contaminant concentration decreases with increase in the distance from the origin. On the dispersion of solute, Ramakanta and Mehta (2010) explored the effect of longitudinal dispersion of miscible fluid flow through porous media.

The analytical solution to temporally dependent dispersion through semi-infinite homogeneous porous media by Laplace transform technique (LTT) was provided by Yadav *et al.* (2011). On the effect of reactive and non-reactive contaminant on the flow, Aiyesimi and Jimoh (2012; 2013) explored the computational analysis of 1-dimensional non-linear contaminant flow problem with an initial continuous point source using homotopy perturbation method. They discovered that the concentration decreases with increase in time and distance from the origin for the non-reactive case.

In this research, we provide an analytical solution of the two-dimensional contaminant flow problem incorporating flow in both horizontal and vertical direction in addition to first-order decay and zero order sources using the weighted residual method (Bubnov-Galerkin method).

Bubnov-Galerkin Method

Bubnov-Galerkin method is a weighted residual method which is used in solving differential equations. When the problem at hand is an ordinary differential equation, we call the method Galerkin weighted residual method and it requires only one equation residual. If the problem is a partial differential equation, the method is a Bubnov-Galerkin method and requires more than one equation residual. The method of weighted residual requires two types of functions namely, the basis functions and weight functions. The former is used to construct the trial solution while the latter is used as criterion to minimize the residual. In applying Bubnov-Galerkin method, the trial solution is chosen to satisfy the boundary conditions while the basis functions must satisfy the homogeneous boundary conditions. In particular, the basis functions are chosen as the weight function.

2. Materials and Methods

Formulation of the Model

We consider an incompressible fluid flow through a semi-infinite homogeneous porous media with non-zero initial concentration in the transport domain. We assume that the flow is two-dimensional and in the direction of *x* and *y*-axis. The source concentration is assumed at the origin, that is, at $x = 0$ and $y = 0$.

Following (Bear, 1997; Yadav *et al.*, 2011; Freezer and Cherry, 1979), the two dimensional parabolic partial differential equation describing hydrodynamic dispersion in adsorbing homogeneous, isotropic porous medium can be written as

$$\frac{\partial C}{\partial t} + \frac{\partial S}{\partial t} = D_L \frac{\partial^2 C}{\partial x^2} + D_T \frac{\partial^2 C}{\partial y^2} - U \frac{\partial C}{\partial x} - V \frac{\partial C}{\partial y} - \lambda C + \mu \quad . \tag{1}$$

The adsorbed contaminant S is assumed to be a function of the concentration of the fluid. i.e.

$$S = K_d C, \tag{2}$$

$$\frac{\partial S}{\partial t} = K_d \frac{\partial C}{\partial t}. \tag{3}$$

Then, the two-Dimensional contaminant flow problem with the associated initial and boundary conditions is

$$\left.\begin{aligned}
&\frac{\partial C}{\partial t} = D_\alpha \frac{\partial^2 C}{\partial x^2} + D_\beta \frac{\partial^2 C}{\partial y^2} - U_\alpha \frac{\partial C}{\partial x} - V_\beta \frac{\partial C}{\partial y} - \gamma_s C + \mu_s \\
&C_0(x,y,t) = 1 - xy, \quad 0 < x < a, \quad 0 < y < b \\
&C(0,y,t) = 1 \\
&C(a,y,t) = 1 - ay \\
&C(x,0,t) = 1 \\
&C(x,b,t) = 1 - bx
\end{aligned}\right\}, \tag{4}$$

where, $R = 1 + K_d$, is the retardation coefficient, accounting for equilibrium linear sorption process, K_d is the distribution coefficient which is defined as the ratio of the adsorbed contaminant to the dissolved contaminants t is the time $[T]$,

x is the distance measured from the origin in the longitudinal direction $[L]$,

y is the distance measured from the origin in the transverse direction $[L]$,

S is the mass of adsorbed contaminant to the solid matrix per unit mass of the solid (dimensionless),

γ_s is a first order decay term $[T^{-1}]$,

μ is the zero order source term $[ML^{-3}T^{-1}]$,

D_α is the horizontal dispersion coefficient $[L^2T^{-1}]$,

D_β is the vertical dispersion coefficient $[L^2T^{-1}]$,

U_α is the flow velocity in the horizontal axis $[LT^{-1}]$,

V_β is the flow velocity in the vertical axis $[LT^{-1}]$,

μ_s is the source coefficient $[T^{-1}]$.

Method of Solution

In solving the above problem (4), we apply the Galerkin weighted residual method precisely the Bubnov-Galerkin method. We choose our basis functions to satisfy the homogeneous boundary conditions as in Ames (1972), Edward (1972) and Finlayson (1972). The initial approximation is chosen as:

$$\phi_0(x,y) = 1 - xy, \tag{5}$$

we use the basis functions:

$$\phi_1(x,y)=\left(\frac{x}{a}-\frac{x^2}{a^2}\right)\left(\frac{y}{b}-\frac{y^2}{b^2}\right), \tag{6}$$

$$\phi_2(x,y)=\left(\frac{x^2}{a}-\frac{x^3}{a^2}\right)\left(\frac{y^2}{b}-\frac{y^3}{b^2}\right). \tag{7}$$

We assume trial solution of the form:

$$C_T(x,y,t)=(1-xy)+A_1(t)\left(\frac{x}{a}-\frac{x^2}{a^2}\right)\left(\frac{y}{b}-\frac{y^2}{b^2}\right)+A_2(t)\left(\frac{x^2}{a}-\frac{x^3}{a^2}\right)\left(\frac{y^2}{b}-\frac{y^3}{b^2}\right), \tag{8}$$

we form the equation residual as follows:

$$R_E(x,y,A_1(t),A_2(t),t)=\frac{\partial C}{\partial t}-D_\alpha\frac{\partial^2 C}{\partial x^2}-D_\beta\frac{\partial^2 C}{\partial y^2}+U_\alpha\frac{\partial C}{\partial x}+V_\beta\frac{\partial C}{\partial y}+\gamma_s C-\mu_s. \tag{9}$$

By substituting equations (8) in the equation residual (9), we have

$$\begin{aligned}
R_E(x,y,A_1(t),A_2(t),t)&=A_1'(t)\left(\frac{x}{a}-\frac{x^2}{a^2}\right)\left(\frac{y}{b}-\frac{y^2}{b^2}\right)+A_2'(t)\left(\frac{x^2}{a}-\frac{x^3}{a^2}\right)\left(\frac{y^2}{b}-\frac{y^3}{b^2}\right)\\
&-D_\alpha\left\{A_1(t)\left(-\frac{2}{a^2}\right)\left(\frac{y}{b}-\frac{y^2}{b^2}\right)+A_2(t)\left(\frac{2}{a}-\frac{6x}{a^2}\right)\left(\frac{y^2}{b}-\frac{y^3}{b^2}\right)\right\}-D_\beta\left\{\begin{array}{l}A_1(t)\left(\frac{x}{a}-\frac{x^2}{a^2}\right)\left(-\frac{2}{b^2}\right)+\\ A_2(t)\left(\frac{x^2}{a}-\frac{x^3}{a^2}\right)\left(\frac{2}{b}-\frac{6y}{b^2}\right)\end{array}\right\}\\
&+U_\alpha\left\{-y+A_1(t)\left(\frac{1}{a}-\frac{2x}{a^2}\right)\left(\frac{y}{b}-\frac{y^2}{b^2}\right)+A_2(t)\left(\frac{2x}{a}-\frac{3x^2}{a^2}\right)\left(\frac{y^2}{b}-\frac{y^3}{b^2}\right)\right\}\\
&+V_\beta\left\{-x+A_1(t)\left(\frac{x}{a}-\frac{x^2}{a^2}\right)\left(\frac{1}{b}-\frac{2y}{b^2}\right)+A_2(t)\left(\frac{x^2}{a}-\frac{x^3}{a^2}\right)\left(\frac{2y}{b}-\frac{3y^2}{b^2}\right)\right\}\\
&+\gamma_s\left\{(1-xy)+A_1(t)\left(\frac{x}{a}-\frac{x^2}{a^2}\right)\left(\frac{y}{b}-\frac{y^2}{b^2}\right)+A_2(t)\left(\frac{x^2}{a}-\frac{x^3}{a^2}\right)\left(\frac{y^2}{b}-\frac{y^3}{b^2}\right)\right\}-\mu_s.
\end{aligned} \tag{10}$$

In order to minimize the equation residual, we have

$$\int_0^b\int_0^a R_E(x,y,A_1(t),A_2(t),t)\times\phi_1(x,y)\,dxdy=0, \tag{11}$$

$$\begin{aligned}
&\Rightarrow\frac{ab}{900}A_1'(t)+\frac{a^2b^2}{3600}A_2'(t)+\left(\frac{b}{90a}D_\alpha+\frac{a}{90b}D_\beta+\frac{ab}{900}\gamma_s\right)A_1(t)\\
&+\left(\frac{b^2}{360}D_\alpha+\frac{a^2}{360}D_\beta+\frac{ab^2}{3600}U_\alpha+\frac{a^2b}{3600}V_\beta+\frac{a^2b^2}{3600}\gamma_s\right)A_2(t)\\
&\left[\left(-\frac{ab^2}{72}U_\alpha-\frac{a^2b}{72}V_\beta+\left(\frac{ab}{36}-\frac{a^2b^2}{144}\right)\gamma_s-\frac{ab}{36}\mu_s\right)\right]=0.
\end{aligned} \tag{12}$$

Similarly, we apply the same procedure on the second weight function $\phi_2(x,y)$ to minimize the equation residuals as follows:

$$\int_0^b\int_0^a R_E(x,y,A_1(t),A_2(t)t)\times\phi_2(x,y)dxdy=0, \tag{13}$$

$$\Rightarrow \frac{a^2b^2}{3600}A_1'(t)+\frac{a^3b^3}{11025}A_2'(t)+\left(\frac{b^2}{360}D_\alpha+\frac{a^2}{360}D_\beta-\frac{ab^2}{3600}U_\alpha-\frac{a^2b}{3600}V_\beta+\frac{a^2b^2}{3600}\gamma_s\right)A_1(t)$$
$$+\left(\frac{2ab^3}{1575}D_\alpha+\frac{2a^3b}{1575}D_\beta+\frac{a^3b^3}{11025}\gamma_s\right)A_2(t)+\begin{pmatrix}-\frac{a^2b^3}{240}U_\alpha-\frac{a^3b^2}{240}V_\beta+\left(\frac{a^2b^2}{144}-\frac{a^3b^3}{400}\right)\gamma_s\\-\frac{a^2b^2}{144}\mu_s\end{pmatrix}=0. \tag{14}$$

From equations (12) and (14), we obtain the following equations:

$$A_1'(t)+\left(\frac{10}{a^2}D_\alpha+\frac{10}{b^2}D_\beta+\gamma_s+\frac{49}{15a}U_\alpha+\frac{49}{15b}V_\beta\right)A_1(t)+\begin{pmatrix}-\frac{64b}{15a}D_\alpha-\frac{64a}{15b}D_\beta\\+\frac{16b}{15}U_\alpha+\frac{16a}{15}V_\beta\end{pmatrix}A_2(t) \tag{15}$$
$$+\left(-\frac{13b}{3}U_\alpha-\frac{13a}{3}V_\beta+\left(25-\frac{41ab}{15}\right)\gamma_s-25\mu_s\right)=0,$$

$$A_2'(t)-\left(\frac{196}{15a^2b}U_\alpha+\frac{196}{15ab^2}V_\beta\right)A_1(t)+\left(\frac{406}{15a^2b}D_\alpha+\frac{406}{15b^2a}D_\beta+\gamma_s-\frac{49}{15a}U_\alpha-\frac{49}{15b}V_\beta\right)A_2(t)$$
$$+\left(-\frac{98}{3a}U_\alpha-\frac{98}{3b}V_\beta-\frac{539}{15}\gamma_s\right)=0. \tag{16}$$

Equation (14) and (16) can be rewritten as:

$$A_1'(t)+pA_1(t)+qA_2(t)+r=0, \tag{17}$$

$$A_2'(t)-sA_1(t)+wA_2(t)-z=0, \tag{18}$$

where

$$\left.\begin{aligned}
p &= \frac{10}{a^2}D_\alpha + \frac{10}{b^2}D_\beta + \gamma_s + \frac{49}{15a}U_\alpha + \frac{49}{15b}V_\beta \\
q &= -\frac{64b}{15a}D_\alpha - \frac{64a}{15b}D_\beta + \frac{16b}{15}U_\alpha + \frac{16a}{15}V_\beta \\
r &= -\frac{13b}{3}U_\alpha - \frac{13a}{3}V_\beta + \left(25 - \frac{41ab}{15}\right)\gamma_s - 25\mu_s \\
s &= \frac{196}{15a^2b}U_\alpha + \frac{196}{15ab^2}V_\beta \\
w &= \frac{406}{15a^2b}D_\alpha + \frac{406}{15b^2a}D_\beta + \gamma_s - \frac{49}{15a}U_\alpha - \frac{49}{15b}V_\beta \\
z &= \frac{98}{3a}U_\alpha + \frac{98}{3b}V_\beta + \frac{539}{15}\gamma_s
\end{aligned}\right\}. \tag{19}$$

To find the initial conditions of the equations (17) and (18), we use another Galerkin approximation which involves forming initial residual as in (Kythe *et al.*, 1997), that is, the initial residual is

$$R_I(x, y, A_1(0), A_2(0)) = \phi_0(x, y) - \phi_1(x, y)A_1(0) - \phi_2(x, y)A_2(0), \tag{20}$$

$$R_I(x, y, A_1(0), A_2(0)) = (1 - xy) - \left(\frac{x}{a} - \frac{x^2}{a^2}\right)\left(\frac{y}{b} - \frac{y^2}{b^2}\right)A_1(0) - \left(\frac{x^2}{a} - \frac{x^3}{a^2}\right)\left(\frac{y^2}{b} - \frac{y^3}{b^2}\right)A_2(0). \tag{21}$$

In order to minimize the initial residual:

$$\int_0^b\int_0^a R_I(x, y, A_1(0), A_2(0)) \times \phi_1(x, y)\,dxdy = 0, \tag{22}$$

$$\Rightarrow \frac{ab}{900}A_1(0) + \frac{a^2b^2}{3600}A_2(0) = \frac{ab}{36} - \frac{a^2b^2}{144}. \tag{23}$$

Similarly, to minimize the initial residual using the weight function, we have

$$\int_0^b\int_0^a R_I(x, y., A_1(0), A_2(0)) \times \phi_2(x, y)\,dxdy = 0, \tag{24}$$

$$\Rightarrow \frac{a^2b^2}{3600}A_1(0) + \frac{a^3b^3}{11025}A_2(0) = \frac{a^2b^2}{144} - \frac{a^3b^3}{400}. \tag{25}$$

We solve equations (23) and (25) simultaneously to obtain the values of $A_1(0)$ and $A_2(0)$ as follows:

$$A_2(0) = -\frac{539}{15}, \tag{26}$$

$$A_1(0) = \frac{41}{15}ab + 25. \tag{27}$$

Equations (26) and (27) serve as initial conditions for equations (17) and (18), respectively.

Rewriting equation (17) and (18) in matrix form, we have

$$\begin{pmatrix} A_1'(t) \\ A_2'(t) \end{pmatrix} = \begin{pmatrix} -p & -q \\ s & -w \end{pmatrix} \begin{pmatrix} A_1(t) \\ A_2(t) \end{pmatrix} + \begin{pmatrix} -r \\ z \end{pmatrix}, \tag{28}$$

with the initial conditions

$$\begin{pmatrix} A_1(0) \\ A_2(0) \end{pmatrix} = \begin{pmatrix} \frac{41}{15}ab + 25 \\ -\frac{539}{15} \end{pmatrix}. \tag{29}$$

The solution of the system (28) and (29) are obtained as:

$$\left.\begin{aligned} A_1(t) &= \frac{(C_0 - zq + rp)\left(-\frac{p}{2} + \frac{w}{2} + \frac{1}{2}\sqrt{p^2 - 2pw + w^2 - 4qs}\right) - q(C_1 - rs - wz)}{\sqrt{p^2 - 2pw + w^2 - 4qs}} \\ &\times \exp\left(-\frac{p}{2} - \frac{w}{2} + \frac{1}{2}\sqrt{p^2 - 2pw + w^2 - 4qs}\right)t \\ &+ \frac{(C_0 - zq + rp)\sqrt{p^2 - 2pw + w^2 - 4qs} - (C_0 - zq + rp)\left(-\frac{p}{2} + \frac{w}{2} + \frac{1}{2}\sqrt{p^2 - 2pw + w^2 - 4qs}\right) - q(C_1 - rs - wz)}{\sqrt{p^2 - 2pw + w^2 - 4qs}} \\ &\times \exp\left(-\frac{p}{2} - \frac{w}{2} - \frac{1}{2}\sqrt{p^2 - 2pw + w^2 - 4qs}\right)t + zq - pr \end{aligned}\right\} \tag{30}$$

and

$$\left.\begin{aligned} A_2(t) &= \left(\frac{(C_0 - zq + rp)\left(-\frac{p}{2} + \frac{w}{2} + \frac{1}{2}\sqrt{p^2 - 2pw + w^2 - 4qs}\right) - q(C_1 - rs - wz)}{q\sqrt{p^2 - 2pw + w^2 - 4qs}}\right) \\ &\times \left(-\frac{p}{2} + \frac{w}{2} - \frac{1}{2}\sqrt{p^2 - 2pw + w^2 - 4qs}\right)\exp\left(-\frac{p}{2} - \frac{w}{2} + \frac{1}{2}\sqrt{p^2 - 2pw + w^2 - 4qs}\right)t \\ &+ \left(\frac{\begin{array}{l}(C_0 - zq + rp)\sqrt{p^2 - 2pw + w^2 - 4qs} \\ -(C_0 - zq + rp)\left(-\frac{p}{2} + \frac{w}{2} + \frac{1}{2}\sqrt{p^2 - 2pw + w^2 - 4qs}\right) - q(C_1 - rs - wz)\end{array}}{q\sqrt{p^2 - 2pw + w^2 - 4qs}}\right) \\ &\times \left(-\frac{p}{2} + \frac{w}{2} + \frac{1}{2}\sqrt{p^2 - 2pw + w^2 - 4qs}\right)\exp\left(-\frac{p}{2} - \frac{w}{2} - \frac{1}{2}\sqrt{p^2 - 2pw + w^2 - 4qs}\right)t + rs + wz \end{aligned}\right\}. \tag{31}$$

Consequently, substituting equations (30) and (31) in equation (8), we have the solution of the contaminant flow problem (4) as:

$$C(x, y, t) = \phi_0(x, y) + A_1(t)\phi_1(x, y) + A_2(t)\phi_2(x, y), \tag{32}$$

where

$\phi_0(x,y)$, $\phi_1(x,y)$ and $\phi_2(x,y)$ are as defined in equations (5), (6) and (7) and $A_1(t)$, $A_2(t)$ are as given in equations (30) and (31), $C_o = \frac{41}{15}ab + 25$ and $C_1 = -\frac{539}{15}$.

3. Results and Discussion

The analytical solution obtained for equation (4) is displaced on the following graphs using hypothetical data and Maple software. The contaminant concentration is plotted against the distance with: $D_\alpha = 0.1,$ $D_\beta = 0.01,$ $U_\alpha = 0.1,$ $V_\beta = 0.01,$ $a = b = 1,$ $\mu_s = 0.1,$ $\gamma_s = 0.1,$ $R = 1,$ $y = 0.1$ and t ranges from 0.1 to 0.5 as shown in figure (4.1) below. By replacing values of the parameters with: $D_\alpha = 0.1,$ $D_\beta = 0.01, U_\alpha = 0.1, V_\beta = 0.01,$ $a = b = 1,$ $\gamma_s = 0.1,$ $R = 1,$ $y = 0.1, t = 0.1$ with μ_s ranging from 0 to 1, we have the Graph of Concentration against the zero order source coefficient as in figure (4.2) below.

Similarly, values of the parameters were replaced with the following:
$D_\alpha = 0.1, D_\beta = 0.01, U_\alpha = 0.1, V_\beta = 0.01, a = b = 1, \mu_s = 0.1, R = 1, y = 0.1, t = 0.1$ with γ_s ranging from 0 to 1 and concentration is plotted against the decay coefficient, we have figure (4.3) below. Lastly, figure 4.4 is obtained when μ_s is varied as 0.1, 0.2 and 0.3 and other parameters remain as in Figure 4.1. Figure 4.4 shows that as the source coefficient μ_s increases, the concentration increases and later decreases with increase in distance from source.

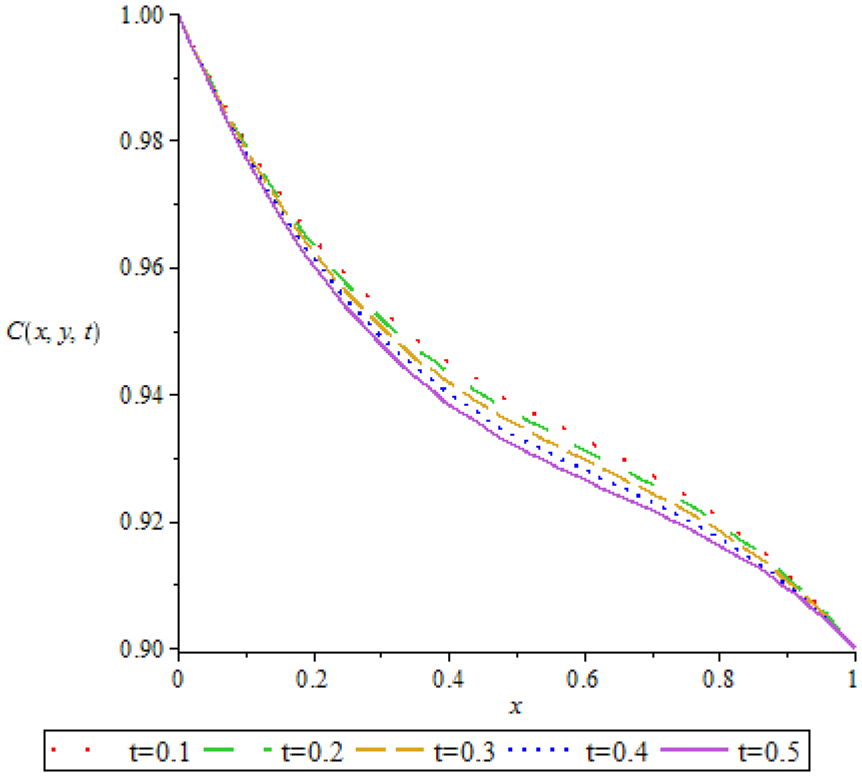

Fig. 4.1: Graph of concentration against Distance in the presence of first order decay and zero order source.

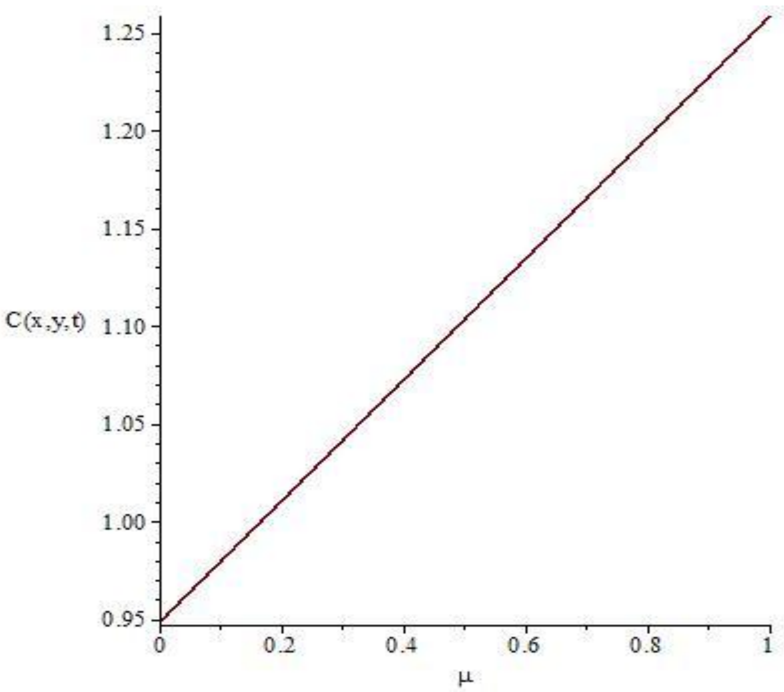

Fig. 4.2: Graph of Concentration against the zero order source coefficient μ

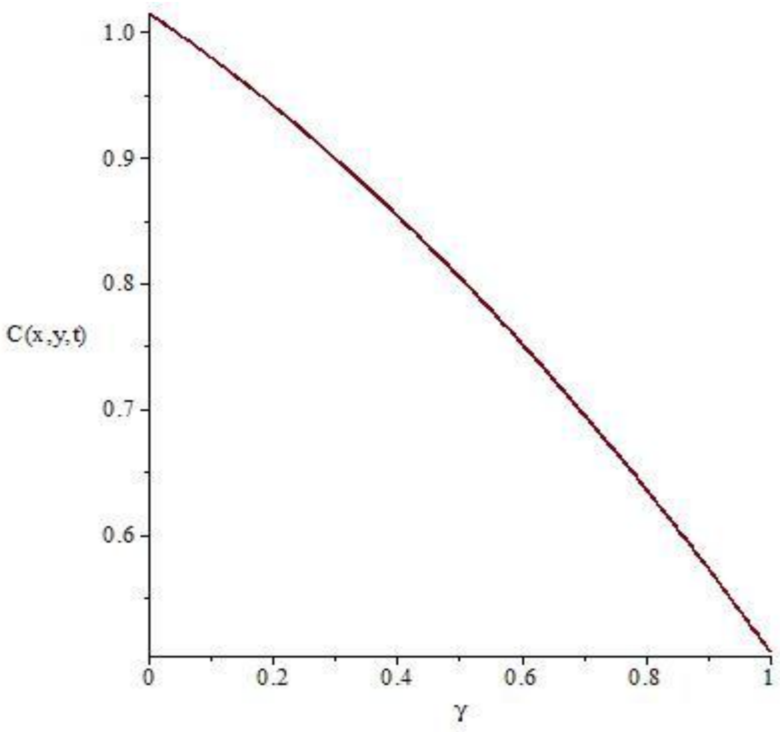

Fig. 4.3: Graph of Concentration against the decay coefficient

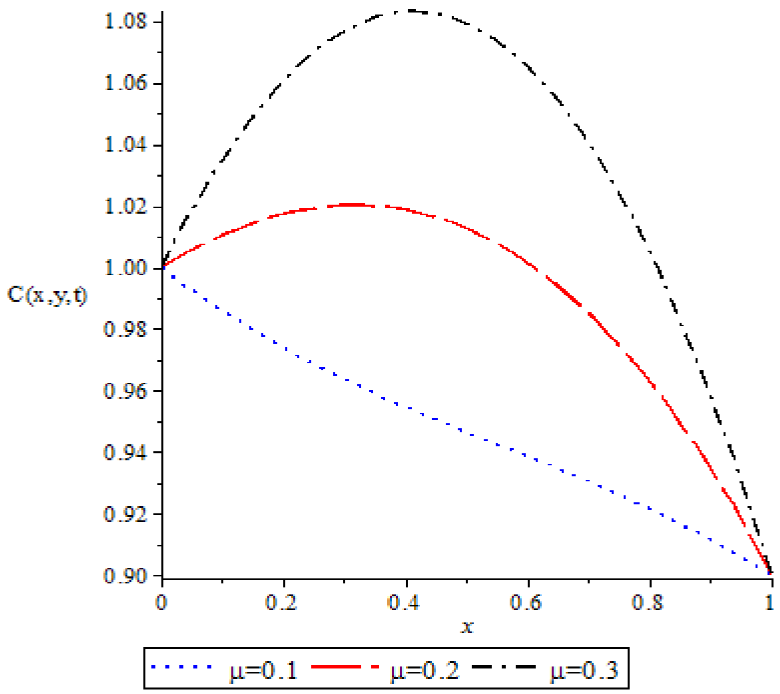

Fig. 4.4: Graph of Concentration of Contaminant against distance with varying values of source coefficient μ_s .

4. Conclusion

In this article, the Galerkin weighted residual method is used to solve a two-dimensional contaminant flow problem with non-zero initial concentration in a finite domain which has proven to be simple and effective. The study reveals that the concentration of the contaminant decreases with increase in the distance from the origin and increases with increase in the zero-order source coefficient. Similarly, as evident from the graphs, the contaminant concentration decreases with increase in the decay coefficient. As the source coefficient μ_s increases, the contaminant concentration increases and later decreases with increase in the distance from the source. This can be seen from figure 4.4.

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