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APPROXIMATE SOLUTIONS FOR MATHEMATICAL MODELLING OF MONKEY POX VIRUS INCORPORATING QUARANTINE CLASS

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Abstract

In this paper we used Homotopy Perturbation Method (HPM) and Adomian Decomposition Method (ADM) to solve the mathematical modeling of Monkeypox virus. The solutions of HPM and (ADM) obtained were validated numerically with the Runge-Kutta-Fehlberg 4-5th order built-in in Maple software. The solutions were also presented graphically to give more insight into the dynamics of the monkeypox virus. It was observed that the two solutions were in agreement with each other and also with Runge-Kutta.

Keywords: Approximate solutions, mathematical modelling, monkeypox, numerical solutions

1. Introduction

Many a times the mathematical modeling of infectious diseases resulted into non-linear differential equations which are difficult to solve analytically. Therefore, semi-analytical methods are implored to solve these problems to get approximate or numerical solutions. The numerical and graphical solutions of these models help the mathematical biologist to explain the dynamics of the disease to a layman when sensitive parameters of the model are varied. Mathematical modeling of monkeypox virus is divided into two populations of human beings and rodents which are further subdivided into seven compartment. The seven compartments are; susceptible Humans S_h, Infected Humans I_h, Quarantine Infected Humans Q_h, Recovered Humans R_h, Susceptible Rodents S_r, Infected Rodents I_r and Recovered Rodents R_r which resulted into seven ordinary differential equation.

Monkeypox virus is another deadly disease that transmits through rodent to human beings. It occurs mostly in the rain forests of West and Central Africa [1]. The virus can spread from human to human by both respiratory (airborne) contact and contact with infected person's bodily fluids. Risk factors for transmission include sharing a bed, room or using the same utensils as an infected patient. The symptoms of Monkeypox are fever, headache, muscle aches, and exhaustion [2]. In 2017, 172 suspected and 61 confirmed cases of human monkeypox was reported by WHO from different parts Nigeria [3].

The first to developed HPM to solve non-linear problems was He [4]. The HPM provides an approximate analytical solution in a series form. Several scientists and engineers have overtime applied HPM to solved different kinds of physical problems that resulted into non-linear partial and ordinary differential equations. In [5] they solved mathematical modeling of measles using HPM. In their work, they considered three systems of equations and stopped the expansion at polynomial of degree two. The approximate solution of the mathematical modeling of Zika virus was obtained by [6] and they presented the solution graphically. HPM was used to solve the general SIR model of infectious diseases and obtained the numerical and graphical solutions [7]. In [8] Ebola epidemic model was solved by HPM and presented the obtained solution graphically. Atindiga *et al.* [9] applied HPM to solve mathematical modeling of infectious disease of seven compartments.

The Adomian Decomposition Method (ADM) was introduced by George Adomian in 1989 [10]. Deterministic model of malaria transmission was solved by Adomian decomposition method (ADM) and presented their solutions numerically and graphically [11]. Ibrahim *et al.* [12] solved the mathematical modeling for the control of Lassa fever using the revised ADM the solutions obtained are presented numerically. System of ordinary differential equations was solved using ADM [13].

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In this paper, HPM and ADM were used to solve the mathematical modeling of monkeypox virus. The details of the dynamics of the disease and other analysis can be found in [14]. The solutions obtained were presented numerically and graphically. Runge-Kutta solution obtained from Maple software was used to validate the numerical solutions of HPM and ADM. Both the numerical and graphical solutions show that the two methods used are in agreement with the Runge-Kutta

2. Materials and Methods

2.1 Formulation of Homotopy Perturbation Method

Basic Idea of Homotopy Perturbation Method (HPM)

He, [15] defined the non-linear differential equation as;

$$A(u) - f(r) = 0, \qquad r \in \Omega, \tag{1}$$

Subject to the boundary condition of:

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, \qquad r \in \Gamma \tag{2}$$

Where A is a general differential operator, B is a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω. .

Equation (1) can be written as

$$L(u) + N(u) - f(r) = 0, \qquad r \in \Omega, , \tag{3}$$

where L is the linear part and N is a non-linear
applying the Homotopy method to (3) with $v(r,p) : \Omega \times [0,1] \rightarrow R$ gives

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[L(v) + N(v) - f(r)] = 0 \tag{4}$$

$$p \in [0,1], \quad r \in \Omega \tag{5}$$

Simplifying (4) gives,

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0 \tag{6}$$

Using (4) in (2.6) gives

$$H(v,0) = L(v) - L(u_0) = 0 \tag{7}$$

and

$$H(v,1) = L(v) + N(v) - f(r) = 0 \tag{8}$$

Where u_0 is an initial approximation of (1)

Assuming the solution of (6) can be written as power series in p :

$$v = v_0 + pv_1 + p^2 v_2 + \dots \tag{9}$$

Setting $p = 1$ in (9) give the approximate solution of (1)

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \dots \tag{10}$$

The convergence rate of (10) depends on the non-linear operator $A(v)$ in most cases [16, 17].

Formulation of Adomian Decomposition Method

$$\left.\begin{aligned} \frac{dy_1}{dt} &= N_1(y_1, y_2, \dots, y_n) \\ \frac{dy_2}{dt} &= N_2(y_1, y_2, \dots, y_n) \\ &\vdots \\ \frac{dy_n}{dt} &= N_n(y_1, y_2, \dots, y_n) \end{aligned}\right\} \tag{11}$$

Equation (11) can be represented by using the ith equation as:

$$Ly_i = N_i(y_1, y_2, \dots, y_n), \quad i = 1, 2, \dots, n \tag{12}$$

Where L is the linear operator $\frac{d}{dt}$ with the inverse $L^{-1} = \int_0^t (\cdot)dt$ and N is the non-linear functions. Applying the inverse operator on (12) gives

$$y_i = y_i(0) + \int_0^t N_i(y_1, y_2, \cdots, y_n)dt \quad i = 1, 2, \cdots, n \tag{13}$$

In ADM the solution of (13) is consider to as the sum of a series

$$y_i = \sum_{j=0}^{\infty} y_{i,j} \tag{14}$$

And the integrand in (13), as the sum of the following series:

$$N_i(y_1, y_2, \cdots, y_n) = \sum_{j=0}^{\infty} A_{i,j}(y_{i,0}, y_{i,1}, \cdots, y_{i,j}) \tag{15}$$

Where $A_{i,j}(y_{i,0}, y_{i,1}, \cdots, y_{i,j})$ are called Adomian polynomials.

Substituting (15) into (13) gives

$$\begin{aligned}\sum_{j=0}^{\infty} y_{i,j} &= y_i(0) + \int_0^t \sum_{j=0}^{\infty} A_{i,j}(y_{i,0}, y_{i,1}, \cdots, y_{i,j})dt \\ &= y_i(0) + \sum_{j=0}^{\infty} \int_0^t A_{i,j}(y_{i,0}, y_{i,1}, \cdots, y_{i,j})dt\end{aligned} \tag{16}$$

From (16) we define

$$\left.\begin{aligned} y_{i,0} &= y_i(0) \\ y_{i,k+1} &= \int_0^t A_{i,k}(y_{i,0}, y_{i,1}, \cdots, y_{i,k})dt \end{aligned}\right\} \tag{17}$$

2.2 Model Equations

2.3.1 Solution by Homotopy Perturbation Method (HPM)

$$\frac{dS_h}{dt} = \Lambda_h - \left(\frac{a_1 I_r}{N_r} + \frac{a_2 I_h}{N_h}\right)S_h - (\mu_h + \varepsilon)S_h \tag{18}$$

$$\frac{dI_h}{dt} = \left(\frac{a_1 I_r}{N_r} + \frac{a_2 I_h}{N_h}\right)S_h - (\mu_h + \delta_h + \tau)I_h \tag{19}$$

$$\frac{dQ_h}{dt} = \tau I_h - (\mu_h + \delta_h + \gamma_h)Q_h \tag{20}$$

$$\frac{dR_h}{dt} = \varepsilon S_h + \gamma_h Q_h - \mu_h R_h \tag{21}$$

$$\frac{dS_r}{dt} = \Lambda_r - \frac{a_3 I_r S_r}{N_r} - \mu_r S_r \tag{22}$$

$$\frac{dI_r}{dt} = \frac{a_3 I_r S_r}{N_r} - (\mu_r + \delta_r + \gamma_r)I_r \tag{23}$$

$$\frac{dR_r}{dt} = \gamma_r I_r - \mu_r R_r \tag{24}$$

$$N_h = S_h + I_h + Q_h + R_h \tag{25}$$

$$N_r = S_r + I_r + R_r \tag{26}$$

Equation (18) to (24) can be written as

$$\left.\begin{aligned} &\frac{dS_h}{dt} - \Lambda_h + \left(\frac{a_1 I_r}{N_r} + \frac{a_2 I_h}{N_h}\right)S_h + A_1 S_h = 0 \\ &\frac{dI_h}{dt} - \left(\frac{a_1 I_r}{N_r} + \frac{a_2 I_h}{N_h}\right)S_h + A_2 I_h = 0 \\ &\frac{dQ_h}{dt} - \tau I_h + A_3 Q_h = 0 \\ &\frac{dR_h}{dt} - \varepsilon S_h - \gamma_h Q_h + \mu_h R_h = 0 \\ &\frac{dS_r}{dt} - \Lambda_r + \frac{a_3 I_r S_r}{N_r} + \mu_r S_r = 0 \\ &\frac{dI_r}{dt} - \frac{a_3 I_r S_r}{N_r} + A_4 I_r = 0 \\ &\frac{dR_r}{dt} - \gamma_r I_r + \mu_r R_r = 0 \end{aligned}\right\} \tag{27}$$

Where,

$$A_1=(\mu_h+\varepsilon),\ A_2=(\mu_h+\delta_h+\tau),\ A_3=(\mu_h+\delta_h+\gamma_h),\ A_4=(\mu_v+\delta_v+\gamma_v)$$

$$S_h(0)=S_{h0},\ I_h(0)=I_{h0},\ Q_h(0)=Q_{h0},\ R_h(0)=R_{h0},\ S_v(0)=S_{v0},\ I_v(0)=I_{v0},\ R_v(0)=R_{v0} \tag{28}$$

Let,

$$\left.\begin{aligned}
S_h &= a_0+pa_1+p^2a_2+p^3a_3+p^4a_4+\cdots\\
I_h &= b_0+pb_1+p^2b_2+p^3b_3+p^4b_4+\cdots\\
Q_h &= c_0+pc_1+p^2c_2+p^3c_3+p^4c_4+\cdots\\
R_h &= d_0+pd_1+p^2d_2+p^3d_3+p^4d_4+\cdots\\
S_v &= x_0+px_1+p^2x_2+p^3x_3+p^4x_4+\cdots\\
I_v &= y_0+py_1+p^2y_2+p^3y_3+p^4y_4+\cdots\\
R_v &= z_0+pz_1+p^2z_2+p^3z_3+p^4z_4+\cdots
\end{aligned}\right\} \tag{29}$$

Applying HPM to (27) gives

$$(1-p)\frac{dS_h}{dt}+p\left[\frac{dS_h}{dt}-\Lambda_h+\left(\frac{\alpha_1 I_v}{N_v}+\frac{\alpha_2 I_h}{N_h}\right)S_h+A_1S_h\right]=0 \tag{30}$$

$$(1-p)\frac{dI_h}{dt}+p\left[\frac{dI_h}{dt}-\left(\frac{\alpha_1 I_v}{N_v}+\frac{\alpha_2 I_h}{N_h}\right)S_h+A_2I_h\right]=0 \tag{31}$$

$$(1-p)\frac{dQ_h}{dt}+p\left[\frac{dQ_h}{dt}-\tau I_h+A_3Q_h\right]=0 \tag{32}$$

$$(1-p)\frac{dR_h}{dt}+p\left[\frac{dR_h}{dt}-\varepsilon S_h-\gamma_hQ_h+\mu_hR_h\right]=0 \tag{33}$$

$$(1-p)\frac{dS_v}{dt}+p\left[\frac{dS_v}{dt}-\Lambda_v+\frac{\alpha_3 I_h S_v}{N_h}+\mu_vS_v\right]=0 \tag{34}$$

$$(1-p)\frac{dI_v}{dt}+p\left[\frac{dI_v}{dt}-\frac{\alpha_3 I_h S_v}{N_h}+A_4I_v\right]=0 \tag{35}$$

$$(1-p)\frac{dR_v}{dt}+p\left[\frac{dR_v}{dt}-\gamma_vI_v+\mu_vR_v\right]=0 \tag{36}$$

Substituting equation (29) into (30) to (36), simplifying and collecting the coefficient of powers of p gives

$$\left.\begin{aligned}
&p^0: a_0'=0\\
&p^1: a_1'+\frac{\alpha_1}{N_v}a_0y_0+\frac{\alpha_2}{N_h}a_0b_0+A_1a_0-\Lambda_h=0\\
&p^2: a_2'+\frac{\alpha_1}{N_v}(a_0y_1+a_1y_0)+\frac{\alpha_2}{N_h}(a_0b_1+a_1b_0)+A_1a_1=0\\
&p^3: a_3'+\frac{\alpha_1}{N_v}(a_0y_2+a_1y_1+a_2y_0)+\frac{\alpha_2}{N_h}(a_0b_2+a_1b_1+a_2b_0)+A_1a_2=0\\
&p^4: a_4'+\frac{\alpha_1}{N_v}(a_0y_3+a_1y_2+a_2y_1+a_3y_0)+\frac{\alpha_2}{N_h}(a_0y_3+a_1y_2+a_2y_1+a_3y_0)+A_1a_3=0
\end{aligned}\right| \tag{37}$$

$$\left.\begin{aligned}
&p^0: b_0'=0\\
&p^1: b_1'-\frac{\alpha_1}{N_v}a_0y_0-\frac{\alpha_2}{N_h}a_0b_0+A_2b_0=0\\
&p^2: b_2'-\frac{\alpha_1}{N_v}(a_0y_1+a_1y_0)-\frac{\alpha_2}{N_h}(a_0b_1+a_1b_0)+A_2b_1=0\\
&p^3: b_3'-\frac{\alpha_1}{N_v}(a_0y_2+a_1y_1+a_2y_0)-\frac{\alpha_2}{N_h}(a_0b_2+a_1b_1+a_2b_0)+A_2b_2=0\\
&p^4: b_4'-\frac{\alpha_1}{N_v}(a_0y_3+a_1y_2+a_2y_1+a_3y_0)-\frac{\alpha_2}{N_h}(a_0y_3+a_1y_2+a_2y_1+a_3y_0)+A_2b_3=0
\end{aligned}\right| \tag{38}$$

$$\left.\begin{aligned}
&p^0: c_0' = 0\\
&p^1: c_1' - rb_0 + f_1c_0 = 0\\
&p^2: c_2' - rb_1 + f_1c_1 = 0\\
&p^3: c_3' - rb_2 + f_1c_2 = 0\\
&p^4: c_4' - rb_3 + f_1c_3 = 0
\end{aligned}\right\} \quad (39)$$

$$\left.\begin{aligned}
&p^0: d_0' = 0\\
&p^1: d_1' - \omega a_0 - \gamma_s b_0 + \mu_s d_0 = 0\\
&p^2: d_2' - \omega a_1 - \gamma_s b_1 + \mu_s d_1 = 0\\
&p^3: d_3' - \omega a_2 - \gamma_s b_2 + \mu_s d_2 = 0\\
&p^4: d_4' - \omega a_3 - \gamma_s b_3 + \mu_s d_3 = 0
\end{aligned}\right\} \quad (40)$$

$$\left.\begin{aligned}
&p^0: x_0' = 0\\
&p^1: x_1' + \frac{\alpha_1}{N_r}x_0y_0 + \mu_r x_0 - \Lambda_r = 0\\
&p^2: x_2' + \frac{\alpha_1}{N_r}(x_0y_1 + x_1y_0) + \mu_r x_1 = 0\\
&p^3: x_3' + \frac{\alpha_1}{N_r}(x_0y_2 + x_1y_1 + x_2y_0) + \mu_r x_2 = 0\\
&p^4: x_4' + \frac{\alpha_1}{N_r}(x_0y_3 + x_1y_2 + x_2y_1 + x_3y_0) + \mu_r x_3 = 0
\end{aligned}\right\} \quad (41)$$

$$\left.\begin{aligned}
&p^0: y_0' = 0\\
&p^1: y_1' - \frac{\alpha_1}{N_r}x_0y_0 + A_4y_0 = 0\\
&p^2: y_2' - \frac{\alpha_1}{N_r}(x_0y_1 + x_1y_0) + A_4y_1 = 0\\
&p^3: y_3' - \frac{\alpha_1}{N_r}(x_0y_2 + x_1y_1 + x_2y_0) + A_4y_2 = 0\\
&p^4: y_4' - \frac{\alpha_1}{N_r}(x_0y_3 + x_1y_2 + x_2y_1 + x_3y_0) + A_4y_3 = 0
\end{aligned}\right\} \quad (42)$$

$$\left.\begin{aligned}
&p^0: z_0' = 0\\
&p^1: z_1' - \gamma_r y_0 + \mu_r z_0 = 0\\
&p^2: z_2' - \gamma_r y_1 + \mu_r z_1 = 0\\
&p^3: z_3' - \gamma_r y_2 + \mu_r z_2 = 0\\
&p^4: z_4' - \gamma_r y_3 + \mu_r z_3 = 0
\end{aligned}\right\} \quad (43)$$

Integrating the each equation of equations (37) to (43) with initial conditions gives

$$\left.\begin{aligned}
&a_0 = S_{h0}, a_1 = B_1t, a_2 = C_1\frac{t^2}{2}, a_3 = D_1\frac{t^3}{6}, a_4 = E_1\frac{t^4}{24}\\
&b_0 = I_{h0}, b_1 = B_2t, b_2 = C_2\frac{t^2}{2}, b_3 = D_2\frac{t^3}{6}, b_4 = E_2\frac{t^4}{24}\\
&c_0 = U_{h0}, c_1 = B_3t, c_2 = C_3\frac{t^2}{2}, c_3 = D_3\frac{t^3}{6}, c_4 = E_3\frac{t^4}{24}\\
&d_0 = R_{h0}, d_1 = B_4t, d_2 = C_4\frac{t^2}{2}, d_3 = D_4\frac{t^3}{6}, d_4 = E_4\frac{t^4}{24}\\
&x_0 = S_{v0}, x_1 = B_5t, x_2 = C_5\frac{t^2}{2}, x_3 = D_5\frac{t^3}{6}, x_4 = E_5\frac{t^4}{24}\\
&y_0 = I_{v0}, y_1 = B_6t, y_2 = C_6\frac{t^2}{2}, y_3 = D_6\frac{t^3}{6}, y_4 = E_6\frac{t^4}{24}\\
&z_0 = R_{v0}, z_1 = B_7t, z_2 = C_7\frac{t^2}{2}, z_3 = D_7\frac{t^3}{6}, z_4 = E_7\frac{t^4}{24}
\end{aligned}\right\} \quad (44)$$

Where,

$$B_{1} = \left[\Lambda - \frac{\alpha_1}{N} S I - \frac{\alpha_2}{N} S I - A S\right] B + \left[\frac{\alpha_1}{N} S I + \frac{\alpha_2}{N} S I - A I\right] \quad (45)$$

$$B_{2} = (\ldots - A Q) B_{1} - (\varepsilon S - \tau Q - \mu R) + B \left[\Lambda - \frac{\alpha_2}{N} S I - \mu S\right]$$

$$B_{3} = \left[\frac{\alpha}{N} S I - A I\right] B_{1} - (\gamma I - \mu R)$$

$$C_{1} = \left[\frac{\alpha_1}{N}(S B + I B_1) + \frac{\alpha_2}{N_1}(S B_2 + I B_1) - A B\right]$$

$$C = \left[\frac{\alpha_1}{N}(S B + I B_1) + \frac{\alpha_2}{N_1}(S B_2 + I B_1) - A B\right]$$

$$C_{3} = (\tau B - A B) C_{4} + (\varepsilon B + \gamma B - \mu_4 B_4) C + \left[\frac{\alpha_2}{N_1}(S B_4 + I B_3) + \mu B_3\right]$$

$$C_{4} = \left[\frac{\alpha_1}{N}(S B_4 + I B_3) - A B_4\right] C + (\gamma B - \mu B)$$

(46)

$$D_{1} = \left[\frac{\alpha_1}{N}(S C_4 - 2B_1 B_4 - I C_1) + \frac{\alpha_2}{N_1}(S B - 2B B - I C) - A C\right] \quad (47)$$

$$D = \left[\frac{\alpha_1}{N}(S C_4 + 2B_1 B_4 - I C_1) - \frac{\alpha_2}{N_1}(S B + 2B B - I C) - A C\right]$$

$$D_{3} = (\tau C - A C) D_{4} - (\varepsilon C - \varepsilon C - \mu_4 C_4) D + \left[\frac{\alpha}{N}(S C_4 + 2B B\ [illegible]\right]$$

$$D_{4} = \left[\frac{\alpha_1}{N}(S C_4 + 2B B_4 - I C) - A C_4\right] D\ [illegible]$$

$$E_{1} = \left[\frac{\alpha}{N}(S D_4 + 3B C_4 - 3C B_4 - I D_1) + \frac{\alpha}{N_1}(S D - 3B C\ [illegible]\right] \quad (48)$$

$$E = \left[\frac{\alpha}{N}(S D_4 + 3B C_4 - 3C B_4 - I D_1) + \frac{\alpha}{N}(S D + 3B C - 3C B - I D) - A D\right]$$

$$E\ [illegible]$$

$$E_{4} = \left[\frac{\alpha}{N}(S D_4 + 3B C_4 + 3C B_4 - I D_1) - A D_4\right] E\ [illegible]$$

Substituting (44) into (29) gives

[illegible] (49)

[illegible]

Where:

[illegible] (51)

Hence, (49) becomes

[illegible] (52)

2.3.2 Solution by Adomian Decomposition Method (ADM)

Let,

$$S_h = y_1, I_h = y_2, Q_h = y_3, R_h = y_4, S_v = y_5, I_v = y_6 \text{ and } R_v = y_7 \quad (53)$$

[illegible] (54)

Computing the Adomian Polynomial using the alternate Adomian Polynomian of special case by [18].

Equation (54) will lead to the following scheme:

[illegible] (55)

and

[illegible] (56)

Where

Solving (56) for $n = 0$ gives

$$y_{1,1} = \left(B_{011}t + B_{012}\frac{t^2}{2}\right),\ y_{2,1} = \left(B_{021}t + B_{022}\frac{t^2}{2}\right),\ y_{3,1} = B_3 t,\ y_{4,1} = B_{041}t + B_{042}\frac{t^2}{2}$$ (57)

$$y_{5,1} = \left(B_{051}t + B_{052}\frac{t^2}{2}\right),\ y_{6,1} = \left(B_{061}t + B_{062}\frac{t^2}{2}\right),\ y_{7,1} = B_7 t$$

Where

$$B_{011} = \left(\frac{\alpha_1}{N_c}S_{c0}I_{c0} + \frac{\alpha_2}{N_a}S_{a0}I_{a0} + A_1S_{c0}\right), B_{012} = \left(\frac{\alpha_1}{N_c}\Lambda_c I_{c0} + \frac{\alpha_2}{N_a}\Lambda_c I_{a0} + A_1\Lambda_c\right)$$

$$B_{021} = \left(\frac{\alpha_1}{N_c}S_{c0}I_{c0} + \frac{\alpha_2}{N_a}S_{c0}I_{a0} - A_2I_{c0}\right), B_{022} = \left(\frac{\alpha_1}{N_c}\Lambda_c I_{c0} + \frac{\alpha_2}{N_a}\Lambda_c I_{a0}\right)$$ (58)

$$B_3 = (\tau I_{a0} - A_3Q_{c0}), B_{041} = (\varepsilon S_{a0} + \gamma_c Q_{c0} - \alpha R_{a0}), B_{042} = \varepsilon\Lambda_c$$

$$B_{051} = \left(\frac{\alpha_3}{N_a}S_{a0}I_{a0} + \mu_a S_{a0}\right), B_{052} = \left(\frac{\alpha_3}{N_a}\Lambda_a I_{a0} + \mu_a\Lambda_a\right)$$

$$B_{061} = \left(\frac{\alpha_3}{N_a}S_{a0}I_{a0} - A_4I_{a0}\right), B_{062} = \frac{\alpha_3}{N_a}\Lambda_a I_{a0}$$

$$B_7 = (\gamma_a I_{a0} - \mu_a R_{a0})$$

Solving (56) for $n = 1$ gives

$$y_{1,2} = \left(C_{011}\frac{t^2}{2} + C_{012}\frac{t^3}{6} + C_{013}\frac{t^4}{8}\right), y_{2,2} = \left(C_{021}\frac{t^2}{2} + C_{022}\frac{t^3}{6} + C_{023}\frac{t^4}{8}\right),$$

$$y_{3,2} = \left(C_{031}\frac{t^2}{2} + C_{032}\frac{t^3}{6}\right), y_{4,2} = \left(C_{041}\frac{t^2}{2} + C_{042}\frac{t^3}{6}\right)$$ (59)

$$y_{5,2} = \left(C_{051}\frac{t^2}{2} + C_{052}\frac{t^3}{6} + C_{053}\frac{t^4}{6}\right), y_{6,2} = \left(C_{061}\frac{t^2}{2} + C_{062}\frac{t^3}{6} + C_{063}\frac{t^4}{6}\right),$$

$$y_{7,2} = \left(C_{071}\frac{t^2}{2} + C_{072}\frac{t^3}{6}\right)$$

Where,

$$C_{011} = \left[\frac{\alpha_1}{N_c}(S_{c0}B_{021} - B_{011}I_{c0}) + \frac{\alpha_2}{N_a}(S_{c0}B_{061} - B_{011}I_{a0}) - A_1B_{011}\right]$$

$$C_{012} = \left[\frac{\alpha_1}{N_c}(S_{c0}B_{022} - 2B_{021}\Lambda_c - B_{012}I_{c0}) + \frac{\alpha_2}{N_a}(S_{c0}B_{062} - 2B_{061}\Lambda_c - B_{012}I_{a0}) - A_1B_{012}\right],$$

$$C_{013} = \left[\frac{\alpha_1}{N_c}B_{022}\Lambda_c + \frac{\alpha_2}{N_a}B_{062}\Lambda_c\right]\frac{t^4}{8}$$ (60)

$$C_{021} = \left[\frac{\alpha_1}{N_c}(S_{c0}B_{021} - B_{011}I_{c0}) + \frac{\alpha_2}{N_a}(S_{c0}B_{061} - B_{011}I_{a0}) - A_2B_{021}\right],$$

$$C_{022} = \left[\frac{\alpha_1}{N_c}(S_{c0}B_{022} - 2B_{021}\Lambda_c - B_{012}I_{c0}) + \frac{\alpha_2}{N_a}(S_{c0}B_{062} + 2B_{061}\Lambda_c - B_{012}I_{a0}) - A_2B_{022}\right],$$ (61)

$$C_{023} = \left[\frac{\alpha_1}{N_c}B_{022}\Lambda_c + \frac{\alpha_2}{N_a}B_{062}\Lambda_c\right], C_{031} = (\tau B_{061} - A_3B_3), C_{032} = \tau B_{062}$$

$$C_{041} = (-\varepsilon B_{051} + \gamma_c B_3 - \alpha_a B_{041}), C_{042} = (\varepsilon B_{052} + \alpha_a B_{042})$$

$$C_{051} = \left[\frac{\alpha_3}{N_a}(S_{a0}B_{061} - B_{051}I_{a0}) - \mu_a B_{051}\right]$$

$$C_{052} = \left[\frac{\alpha_3}{N_a}(S_{a0}B_{062} + 2B_{061}\Lambda_a - B_{052}I_{a0}) - \mu_a B_{052}\right]$$

$$C_{053} = \frac{\alpha_3}{N_a}B_{062}\Lambda_a\frac{t^4}{8}$$

(62)

$$C_{621} = \left[\frac{\alpha_1}{N_r}(S_{r8}B_{611} - B_{311}I_{r5}) - A_4 B_{621}\right],\ C_{622} = \left[\frac{\alpha_1}{N_r}(S_{r8}B_{612} - 2B_{611}\Lambda_r - B_{312}I_{r8}) - A_4 B_{622}\right],$$
$$C_{623} = \frac{\alpha_1}{N_r}\Lambda_r B_{612}\frac{t^3}{8},\ C_{721} = (\gamma_r B_{611} - \mu_r B_{711}),\ C_{723} = \gamma_r B_{612} \qquad (63)$$

Solving (56) for $n = 2$ gives

$$y_{1,3} = \left(D_{131}\frac{t^3}{6} + D_{132}\frac{t^4}{24} + D_{133}\frac{t^5}{40}\right),\ y_{2,3} = \left(D_{231}\frac{t^3}{6} + D_{232}\frac{t^4}{24} + D_{233}\frac{t^5}{40}\right) \qquad (64)$$
$$y_{3,3} = \left(D_{331}\frac{t^3}{6} + D_{332}\frac{t^4}{24} + D_{333}\frac{t^5}{40}\right),\ y_{4,3} = \left(D_{431}\frac{t^3}{6} + D_{432}\frac{t^4}{24} - D_{433}\frac{t^5}{40}\right)$$
$$y_{5,3} = -\left(D_{531}\frac{t^3}{6} + D_{532}\frac{t^4}{24} + D_{533}\frac{t^5}{40}\right),\ y_{6,3} = \left(D_{631}\frac{t^3}{6} + D_{632}\frac{t^4}{24} + D_{633}\frac{t^5}{40}\right)$$
$$y_{7,3} = \left(D_{731}\frac{t^3}{6} + D_{732}\frac{t^4}{24} + D_{733}\frac{t^5}{40}\right)$$

Where,

$$D_{131} = \left[\frac{\alpha_1}{N_r}(S_{r8}C_{621} - 2B_{111}B_{611} - C_{121}I_{r5}) + \frac{\alpha_2}{N_h}(S_{h8}C_{221} - 2B_{111}B_{211} - C_{121}I_{h8}) + A_1C_{121}\right] \qquad (65)$$
$$D_{132} = \left[\begin{array}{l}\frac{\alpha_1}{N_r}(S_{r8}C_{622} - 3B_{111}B_{612} - 3B_{112}B_{611} - C_{122}I_{r5}) \\ + \frac{\alpha_2}{N_h}(S_{h8}C_{222} - 3B_{111}B_{212} - 3B_{112}B_{211} - C_{122}I_{h8}) + A_1C_{122}\end{array}\right]$$
$$D_{133} = \left[\begin{array}{l}\frac{\alpha_1}{N_r}(S_{r8}C_{623} - 2B_{112}B_{612} - C_{123}I_{r5}) \\ + \frac{\alpha_2}{N_h}(S_{h8}C_{223} - 2B_{112}B_{212} - C_{123}I_{h8}) + A_1C_{123}\end{array}\right]$$

$$D_{231} = \left[\frac{\alpha_1}{N_r}(S_{r8}C_{621} - 2B_{111}B_{611} - C_{121}I_{r8}) + \frac{\alpha_2}{N_h}(S_{h8}C_{221} - 2B_{111}B_{211} - C_{121}I_{h8}) - A_2C_{221}\right]$$
$$D_{232} = \left[\begin{array}{l}\frac{\alpha_1}{N_r}(S_{r8}C_{622} - 3B_{111}B_{612} - 3B_{112}B_{611} - C_{122}I_{r5}) \\ + \frac{\alpha_2}{N_h}(S_{h8}C_{222} - 3B_{111}B_{212} - 3B_{112}B_{211} - C_{122}I_{h8}) - A_2C_{222}\end{array}\right]$$
$$D_{233} = \left[\begin{array}{l}\frac{\alpha_1}{N_r}(S_{h8}C_{623} - 2B_{112}B_{612} - C_{123}I_{r8}) \\ + \frac{\alpha_2}{N_h}(S_{h8}C_{223} - 2B_{112}B_{212} - C_{123}I_{h8}) - A_2C_{223}\end{array}\right]$$

$$D_{331} = (\tau C_{221} - A_3C_{321}),\ D_{332} = (\tau C_{222} - A_3C_{322}),\ D_{333} = \tau C_{223} \qquad (66)$$
$$D_{431} = (-\varepsilon C_{121} + \gamma_h C_{321} - \mu_h C_{421}),\ D_{432} = (-\varepsilon C_{122} + \gamma_h C_{322} - \mu_h C_{422}),\ D_{433} = \varepsilon C_{123} \qquad (67)$$

$$D_{531} = \left[\frac{\alpha_1}{N_r}(S_{r8}C_{621} - 2B_{111}B_{611} - C_{521}I_{r8}) + \mu_r C_{521}\right] \qquad (68)$$
$$D_{532} = \left[\frac{\alpha_1}{N_r}(S_{r8}C_{622} - 3B_{511}B_{612} - 3B_{512}B_{611} - C_{522}I_{r8}) + \mu_r C_{522}\right]$$
$$D_{533} = \left[\frac{\alpha_1}{N_r}(S_{r8}C_{623} - 2B_{512}B_{612} - C_{523}I_{r8}) + \mu_r C_{523}\right]$$

$$D_{631} = \left[\frac{\alpha_1}{N_r}(S_{r8}C_{621} - 2B_{611}B_{611} - C_{521}I_{r5}) - A_4C_{621}\right] \qquad (69)$$
$$D_{632} = \left[\frac{\alpha_1}{N_r}(S_{r8}C_{622} - 3B_{611}B_{612} - 3B_{612}B_{611} - C_{622}I_{r5}) - A_4C_{622}\right]$$
$$D_{633} = \left[[illegible]\ 2B_{612}B_{612} - C_{623}I_{r5}) - A_4C_{623}\right]$$
$$[illegible]\ (\gamma_r C_{[illegible]} - \mu_r C_{[illegible]}),\ D_{7[illegible]} = \gamma_r C_{[illegible]}\frac{t}{40}$$

Solving (56) for $n = 3$ gives

$$\text{[illegible]} = \left(E_{141}\frac{t^4}{24} + E_{142}\frac{t^5}{120} + E_{143}\frac{t^6}{720} + E_{144}\frac{t^7}{112}\right)$$

$$\text{[illegible]} = \left(E_{241}\frac{t^4}{24} + E_{242}\frac{t^5}{120} + E_{243}\frac{t^6}{720} + E_{244}\frac{t^7}{112}\right)$$

$$\text{[illegible]} = \left(E_{341}\frac{t^4}{24} + E_{342}\frac{t^5}{120} + E_{343}\frac{t^6}{720}\right)$$

$$\text{[illegible]} = \left(E_{441}\frac{t^4}{24} + E_{442}\frac{t^5}{120} - D_{443}\frac{t^6}{720}\right)$$

$$\text{[illegible]} = -\left(E_{541}\frac{t^4}{24} + E_{542}\frac{t^5}{120} + E_{543}\frac{t^6}{720} + E_{544}\frac{t^7}{112}\right)$$

$$\text{[illegible]} = \left(E_{641}\frac{t^4}{24} + E_{642}\frac{t^5}{120} + E_{643}\frac{t^6}{720} + E_{644}\frac{t^7}{112}\right)$$

$$\text{[illegible]} = \left(E_{741}\frac{t^4}{24} + E_{742}\frac{t^5}{120} + E_{743}\frac{t^6}{720}\right) \tag{70}$$

Where,

$$E_{141} = \left[\begin{array}{l}\frac{\alpha_1}{N_1}(S_{60}D_{631} - 3B_{111}C_{621} - 3B_{611}C_{121} - D_{111}I_{r0}) \\ + \frac{\alpha_2}{N_2}(S_{60}D_{221} - 3B_{111}C_{221} - 3B_{211}C_{121} - D_{111}I_{r0}) + A_1D_{111}\end{array}\right]$$

$$E_{142} = \left[\begin{array}{l}\frac{\alpha_1}{N_1}(S_{60}C_{612} - 4B_{111}C_{622} - 6B_{112}C_{621} - 4B_{611}C_{122} - 6B_{612}C_{121} - D_{112}I_{r0}) \\ + \frac{\alpha_2}{N_2}(S_{60}C_{212} - 4B_{111}C_{222} - 6B_{112}C_{221} - 4B_{211}C_{122} - 6B_{212}C_{121} - D_{112}I_{r0}) + A_1D_{112}\end{array}\right]$$

$$E_{143} = \left[\begin{array}{l}\frac{\alpha_1}{N_1}(3S_{60}C_{613} - 18B_{111}C_{623} - 10B_{112}C_{622} - 18B_{611}C_{123} - 10B_{612}C_{122} - 3C_{113}I_{r0}) \\ + \frac{\alpha_2}{N_2}(3S_{60}C_{213} - 18B_{111}C_{223} - 10B_{112}C_{222} - 18B_{211}C_{123} - 10B_{212}C_{122} - 3C_{113}I_{r0}) + 3A_1C_{113}\end{array}\right]$$

$$E_{144} = \left[\frac{\alpha_1}{N_1}(B_{112}C_{623} + B_{612}C_{123}) + \frac{\alpha_2}{N_2}(B_{112}C_{223} + B_{212}C_{123})\right] \tag{71}$$

$$E_{241} = \left[\begin{array}{l}\frac{\alpha_1}{N_1}(S_{60}D_{631} - 3B_{111}C_{621} - 3B_{611}C_{121} - D_{111}I_{r0}) \\ + \frac{\alpha_2}{N_2}(S_{60}D_{221} - 3B_{111}C_{221} - 3B_{211}C_{121} - D_{111}I_{r0}) - A_2D_{211}\end{array}\right] \tag{72}$$

$$E_{242} = \left[\begin{array}{l}\frac{\alpha_1}{N_1}(S_{60}D_{612} - 4B_{111}C_{622} - 6B_{112}C_{621} - 4B_{611}C_{122} - 6B_{612}C_{121} - D_{112}I_{r0}) \\ + \frac{\alpha_2}{N_2}(S_{60}D_{212} - 4B_{111}C_{222} - 6B_{112}C_{221} - 4B_{211}C_{122} - 6B_{212}C_{121} - D_{112}I_{r0}) - A_2D_{212}\end{array}\right]$$

$$E_{243} = \left[\begin{array}{l}\frac{\alpha_1}{N_1}(3S_{60}D_{613} - 18B_{111}C_{623} - 10B_{112}C_{622} - 18B_{611}C_{123} - 10B_{612}C_{122} - 3C_{113}I_{r0}) \\ + \frac{\alpha_2}{N_2}(3S_{60}D_{213} - 18B_{111}C_{223} - 10B_{112}C_{222} - 18B_{211}C_{123} - 10B_{212}C_{122} - 3C_{113}I_{r0}) - 3A_2C_{213}\end{array}\right]$$

$$E_{244} = \left[\frac{\alpha_1}{N_1}(B_{112}C_{623} + B_{612}C_{123}) + \frac{\alpha_2}{N_2}(B_{112}C_{223} + B_{212}C_{123})\right]$$

$$E_{341} = (\tau D_{211} - A_3D_{311}),\ E_{342} = (\tau D_{212} - A_3D_{312}),\ E_{343} = (3\tau D_{213} - 3A_3D_{313}) \tag{73}$$

$$E_{441} = (-\varepsilon D_{311} + \tau_w D_{411} - \mu_w D_{411}),\ E_{442} = (-\varepsilon D_{312} + \tau_w D_{412} - \mu_w D_{412})$$

$$E_{443} = (-3\varepsilon D_{313} + 3\tau_w D_{413} - 3\mu_w D_{413})$$

$$I_{s1} = \left[\frac{\alpha_1}{N_1}(S_{v0}D_{s11} - 3B_{111}C_{421} - 3B_{s11}C_{321} - D_{311}I_{v5}) + \mu_1 D_{s11}\right]$$

$$I_{s2} = \left[\frac{\alpha_1}{N_1}(S_{v0}D_{s12} - 4B_{111}C_{422} - 6B_{112}C_{421} - 4B_{s11}C_{322} - 6B_{s12}C_{321} - D_{312}I_{v6}) + \mu_1 D_{s12}\right]$$

$$I_{s3} = \left[\begin{array}{l}\frac{\alpha_1}{N_1}(3S_{v0}D_{s13} - 18B_{111}C_{421} - 10B_{112}C_{422} - 18B_{s11}C_{321} - 10B_{s12}C_{322} - 3C_{311}I_{v6})\\ +3\mu_1 C_{311}\end{array}\right]$$

$$I_{s4} = \left[\frac{\alpha_1}{N_1}(B_{112}C_{421} + B_{s12}C_{321})\right]$$

(74)

$$E_{s1} = \left[\frac{\alpha_1}{N_1}(S_{v0}D_{s11} - 3B_{111}C_{421} - 3B_{s11}C_{321} - D_{311}I_{v5}) - A_1 D_{s11}\right]$$

(75)

$$E_{s2} = \left[\frac{\alpha_1}{N_1}(S_{v0}D_{s12} - 4B_{111}C_{422} - 6B_{112}C_{421} - 4B_{s11}C_{322} - 6B_{s12}C_{321} - D_{312}I_{v6}) - A_1 D_{s12}\right]$$

$$E_{s3} = \left[\begin{array}{l}\frac{\alpha_1}{N_1}(3S_{v0}D_{s13} - 18B_{111}C_{421} - 10B_{112}C_{422} - 18B_{s11}C_{321} - 10B_{s12}C_{322} - 3C_{311}I_{v6})\\ -3A_1 C_{311}\end{array}\right]$$

$$E_{s4} = \left[\frac{\alpha_1}{N_1}(B_{112}C_{421} + B_{s12}C_{321})\right]$$

$$E_{t1} = (\gamma_1 D_{s11} - \mu_1 D_{111}),\ E_{t2} = (\gamma_1 D_{s12} - \mu_1 D_{112}),\ E_{t3} = (3\gamma_1 D_{s13} - 3\mu_1 D_{113})$$

$$\begin{aligned}
S_h(t) &= y_{1,0} + y_{1,1} + y_{1,2} + y_{1,3} + y_{1,4}\\
I_h(t) &= y_{2,0} + y_{2,1} + y_{2,2} + y_{2,3} + y_{2,4}\\
Q_h(t) &= y_{3,0} + y_{3,1} + y_{3,2} + y_{3,3} + y_{3,4}\\
R_h(t) &= y_{4,0} + y_{4,1} + y_{4,2} + y_{4,3} + y_{4,4}\\
S_v(t) &= y_{5,0} + y_{5,1} + y_{5,2} + y_{5,3} + y_{5,4}\\
I_v(t) &= y_{6,0} + y_{6,1} + y_{6,2} + y_{6,3} + y_{6,4}\\
R_v(t) &= y_{7,0} + y_{7,1} + y_{7,2} + y_{7,3} + y_{7,4}
\end{aligned} \qquad (76)$$

Substituting (55), (57), (59), (64) and (70) into (76) gives,

$$\begin{aligned}
S_h(t) &= S_{h0} + S_{h1}t + S_{h2}\frac{t^2}{2} + S_{h3}\frac{t^3}{6} + S_{h4}\frac{t^4}{24} + S_{h5}\frac{t^5}{120} + S_{h6}\frac{t^6}{720} + S_{h7}\frac{t^7}{112}\\
I_h(t) &= I_{h0} + I_{h1}t + I_{h2}\frac{t^2}{2} + I_{h3}\frac{t^3}{6} + I_{h4}\frac{t^4}{24} + I_{h5}\frac{t^5}{120} + I_{h6}\frac{t^6}{720} + I_{h7}\frac{t^7}{112}\\
Q_h(t) &= Q_{h0} + Q_{h1}t - Q_{h2}\frac{t^2}{2} + Q_{h3}\frac{t^3}{6} + Q_{h4}\frac{t^4}{24} + Q_{h5}\frac{t^5}{120} + Q_{h6}\frac{t^6}{720}\\
R_h(t) &= R_{h0} + R_{h1}t - R_{h2}\frac{t^2}{2} + R_{h3}\frac{t^3}{6} + R_{h4}\frac{t^4}{24} + R_{h5}\frac{t^5}{120} + R_{h6}\frac{t^6}{720}\\
S_v(t) &= S_{v0} - S_{v1}t + S_{v2}\frac{t^2}{2} + S_{v3}\frac{t^3}{6} + S_{v4}\frac{t^4}{24} + S_{v5}\frac{t^5}{120} + S_{v6}\frac{t^6}{720} + S_{v7}\frac{t^7}{112}\\
I_v(t) &= I_{v0} + I_{v1}t + I_{v2}\frac{t^2}{2} + I_{v3}\frac{t^3}{6} + I_{v4}\frac{t^4}{24} + I_{v5}\frac{t^5}{120} + I_{v6}\frac{t^6}{720} + I_{v7}\frac{t^7}{112}\\
R_v(t) &= R_{v0} - R_{v1}t - R_{v2}\frac{t^2}{2} + R_{v3}\frac{t^3}{6} + R_{v4}\frac{t^4}{24} + R_{v5}\frac{t^5}{120} + R_{v6}\frac{t^6}{720}
\end{aligned} \qquad (77)$$

where,

$$\begin{aligned}
&S_{h1} = (A_1 - B_{11}),\ S_{h2} = -(B_{111} - C_{121}),\ S_{h3} = -(C_{121} + D_{111}),\\
&S_{h4} = -(3C_{111} - D_{121} - E_{111}),\ S_{h5} = -(3D_{111} + E_{121}),\ S_{h6} = -E_{111},\ S_{h7} = -E_{111}\\
&I_{h1} = B_{111},\ I_{h2} = (B_{111} - C_{211}),\ I_{h3} = (C_{121} + D_{211}),\ I_{h4} = (3C_{111} + D_{211} + E_{211}),\\
&I_{h5} = (3D_{211} - E_{221}),\ I_{h6} = E_{211},\ I_{h7} = E_{2h}\\
&Q_{h1} = B_{311},\ Q_{h2} = C_{311},\ Q_{h3} = (C_{321} - D_{311}),\ Q_{h4} = (D_{312} + E_{311}),\\
&Q_{h5} = (3D_{311} + E_{321}),\ Q_{h6} = E_{311}\\
&R_{h1} = B_{41},\ R_{h2} = (B_{411} + C_{411}),\ R_{h3} = (D_{411} - C_{421}),\ R_{h4} = (D_{411} - E_{411})\\
&R_{h5} = (D_{411} + 3D_{421}),\ R_{h6} = E_{411}
\end{aligned} \qquad (78)$$

[illegible] (79)

3. Result and Discussions

3.1 Numerical Solution

In Table 3.1 are the variables and parameters of the model equation and their definitions. The values of table 3.1 were estimated for the purpose of numerical and graphical solutions. The tables 3.2 to 3.8 are the numerical solutions of each of the solution in equations (52) and (77). The HPM and ADM solutions were validated with Runge – Kutta to see the agreement between the solutions.

Table 3.1: Definition of Variables and Parameters

Variables/Parameters	Definition	Values
S_h	Susceptible Humans	10000
I_h	Infected Humans	500
Q_h	Quarantine Infected Humans	1000
R_h	Recovered Humans	300
S_r	Susceptible Rodents	500
I_r	Infected Rodents	200
R_r	Recovered Rodents	50
Λ_h	Recruitment Rate of Humans	65000
Λ_r	Recruitment Rate of Rodents	5000
α_1	Contact Rate of Rodents to Humans	0.001
α_2	Contact Rate of Humans to Humans	0.1
α_3	Contact Rate of Rodents to Rodents	0.01
μ_h	Natural Death Rate of Humans	0.015
δ_h	Disease Induced Death Rate of Humans	0.0001
γ_h	Recovery Rate of Humans	0.25
γ_r	Recovery Rate of Rodents	0.2
τ	Progression Rate from Infected to Quarantine	0.50
ε	Effectiveness Public Enlightenment Campaign	0.25
μ_r	Natural Death Rate of Rodents	0.01
δ_r	Disease Induced Death Rate of Rodents	0.001
N_h	Total Population of Humans	100000000
N_r	Total Population of Rodents	10000

Table 3.2: Numerical Solution for Susceptible Humans

T	ShRKM	ShHPM	ShADM
0.0	10000.0000	10000.0000	10000.0000
0.1	16153.0850	16153.0850	16153.1214
0.2	22145.2435	22145.2426	22145.5026
0.3	27980.6848	27980.6786	27981.4527
0.4	33663.5081	33663.4822	33665.0766
0.5	39197.7055	39197.6268	39200.2773
0.6	44587.1643	44586.9697	44590.7573
0.7	49835.6706	49835.2519	49840.0176
0.8	54946.9112	54946.0984	54951.3562
0.9	59924.4760	59923.0180	59927.8655
1.0	64771.8618	64769.4036	64772.4275

Table 3.3: Numerical Solution for Infected Humans

T	IhRKM	IhHPM	IhADM
0.0	500.0000	500.0000	500.0000
0.1	474.9230	474.9230	474.9230
0.2	451.1161	451.1162	451.1162
0.3	428.5149	428.5153	428.5155
0.4	407.0581	407.0597	407.0601
0.5	386.6874	386.6923	386.6931
0.6	367.3478	367.3597	367.3614
0.7	348.9867	349.0121	349.0151
0.8	331.5545	331.6036	331.6085
0.9	315.0040	315.0915	315.0990
1.0	299.2903	299.4373	299.4480

Table 3.4: Numerical Solution for Quarantine Humans

T	QhRKM	QhHPM	QhADM
0.0	1000.0000	1000.0000	1000.0000
0.1	997.8831	997.8831	997.8828
0.2	994.6159	994.6158	994.6117
0.3	990.2896	990.2889	990.2682
0.4	984.9899	984.9870	984.9216
0.5	978.7972	978.7886	978.6289
0.6	971.7872	971.7660	971.4348
0.7	964.0308	963.9853	963.3719
0.8	955.5945	955.5066	954.4605
0.9	946.5408	946.3838	944.7087
1.0	936.9282	936.6647	934.1122

Table 3.5: Numerical Solution for Recovered Humans

t	RhRKM	RhHPM	RhADM
0.0	300.0000	300.0000	300.0000
0.1	651.5348	651.5348	651.5349
0.2	1154.1676	1154.1685	1154.1692
0.3	1803.6770	1803.6835	1803.6874
0.4	2595.9540	2595.9811	2595.9938
0.5	3526.9986	3527.0811	3527.1135
0.6	4592.9177	4593.1217	4593.1920
0.7	5789.9211	5790.3599	5790.4958
0.8	7114.3195	7115.1712	7115.4126
0.9	8562.5224	8564.0497	8564.4518
1.0	10131.0334	10133.6081	10134.2443

Table 3.6: Numerical Solution for Susceptible Rodents

T	SrRKM	SrHPM	SrADM
0.0	500.0000	500.0000	500.0000
0.1	999.2489	999.2489	999.2489
0.2	1497.9978	1497.9978	1497.9984
0.3	1996.2473	1996.2473	1996.2496
0.4	2493.9980	2493.9980	2494.0034
0.5	2991.2503	2991.2503	2991.2610
0.6	3488.0049	3488.0049	3488.0233
0.7	3984.2622	3984.2622	3984.2915
0.8	4480.0228	4480.0228	4480.0666
0.9	4975.2872	4975.2871	4975.3496
1.0	5470.0559	5470.0559	5470.1417

Table 3.7: Numerical Solution for Infected Rodents

t	IrRKM	IrHPM	IrADM
0.0	200.0000	200.0000	200.0000
0.1	195.8257	195.8316	195.8256
0.2	191.7394	191.7624	191.7388
0.3	187.7394	187.7900	187.7372
0.4	183.8237	183.9119	183.8184
0.5	179.9906	180.1254	179.9803
0.6	176.2383	176.4284	176.2206
0.7	172.5651	172.8184	172.5372
0.8	168.9693	169.2933	168.9278
0.9	165.4492	165.8509	165.3905
1.0	162.0032	162.4891	161.9232

Table 3.8: Numerical Solution for Recovered Rodents

T	RrRKM	RrHPM	RrADM
0.0	50.0000	50.0000	50.0000
0.1	53.9061	53.9062	53.9061
0.2	57.7258	57.7261	57.7258
0.3	61.4609	61.4619	61.4608
0.4	65.1131	65.1155	65.1130
0.5	68.6842	68.6887	68.6839
0.6	72.1759	72.1836	72.1753
0.7	75.5899	75.6019	75.5888
0.8	78.9279	78.9454	78.9260
0.9	82.1914	82.2158	82.1883
1.0	85.3820	85.4147	85.3772

3.2 Graphical Solutions

Figures 3.1 to 3.7 are the graphical solutions of the approximate solution (2.86). Figures 3.1 to 3.4 are for the human population and figures 3.5 to 3.7 are for the rodent population.

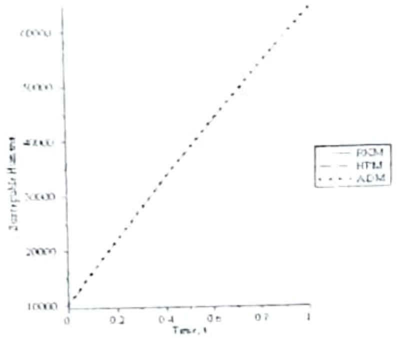

Figure 3.1: Graphical Solution of Susceptible Human Population

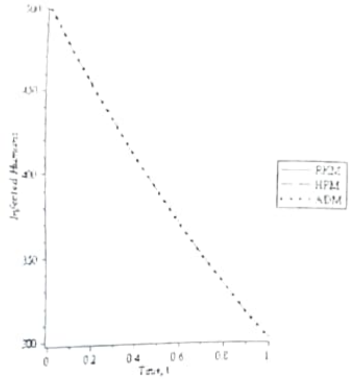

Figure 3.2: Graphical Solution of Infected Human Population

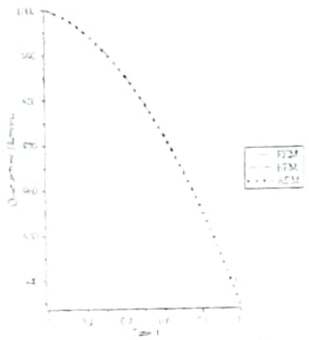

Figure 3.3: Graphical Solution of Quarantine Human Population

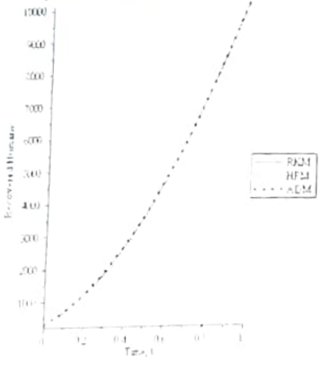

Figure 3.4: Graphical Solution of Recovered Human Population

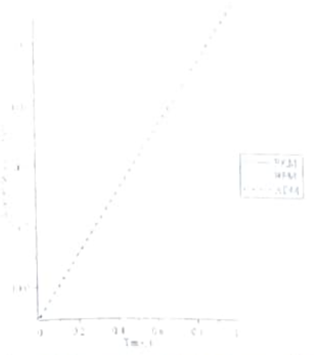

Figure 3.5: Graphical Solution of Susceptible Rodent Populaion

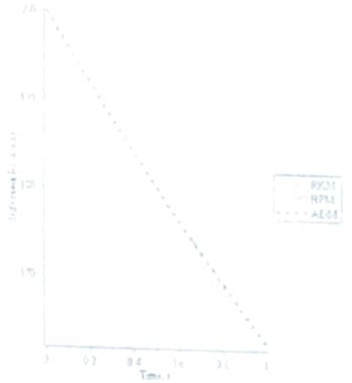

Figure 3.6: Graphical Solution of Infected Rodent Population

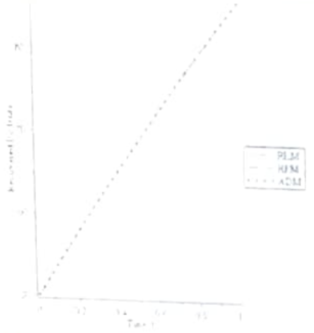

Figure 3.7: Graphical solution of Recovered Rodent Population

Discussions

It is observed from the numerical solution that in table 3.2 to 3.6 the HPM solution is closer to RKM than ADM while in table 3.7 and 3.8 the ADM solution is closer to RKM than HPM.

In figure 3.1 the Susceptible Human population increases with time as result of increase in contact rate of rodents to humans and contact rate of humans to humans. Figure 3.2 shows that the infected human decreases with time as a result of high progression rate from infection to quarantine. The movement of infected individuals to quarantine class will reduce the number of infected population. It is observed from figure 3.3 that the quarantine human population decreases with time as people recovered and some died from the disease. The individuals that are treated in quarantine class and recovered moved to recovered class. In figure 3.4 the recovered human population increase with time as the recovered individuals from quarantine and the enlightened susceptible individuals moved into the class. Figure 3.5 shows that the susceptible rodent population increases with time as a result of high contact rate of rodents to rodents. It is assumed that the rodents have no any control measure once the virus breakout in their population. In figure 3.6 the infected rodent population decreases with time as they died due to the disease and as some of them recovered naturally. Figure 3.7 shows that the recovered rodent population increases with time as the recovered infected rodents moved to recovered class.

The effort to quarantine and treat the infected people and carrying out public enlightenment campaign is the key to eradicate monkeypox virus from the population. It is shown from all the solutions that the HPM and ADM are in agreement with the Runge Kutta that was used as the basis for validation

4. Conclusion

Homotopy Perturbation Method (HPM) and Adomian Decomposition Method (ADM) were used to solved systems of seven ordinary differential equations of mathematical modeling of monkeypox virus. The solutions obtained are presented numerically and graphically. They are also validated with the Runge Kutta in Maple software. The numerical solutions revealed that in table 3.2 to 3.6 the HPM solution is closer to RKM than ADM while in table 3.7 and 3.8 the ADM solution is closer to RKM than HPM, hence we can deduce that HPM performs better than ADM.

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