APPLICATION OF ECONOMIC ORDER QUANTITY MODEL FOR THE PURCHASE AND GROWTH OF ORGANIC POULTRY WITH INCREMENTAL QUANTITY DISCOUNTS

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A THESIS SUBMITTED TO THE POSTGRADUATE SCHOOL FEDERAL UNIVERSITY OF TECHNOLOGY, MINNA, NIGERIA IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE AWARD OF THE DEGREE OF MASTER OF TECHNOLOGY IN APPLIED MATHEMATICS.

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ABSTRACT

The purpose of the study is to develop a lot sizing model for growing chickens if the supplier of the products offers incremental quantity discounts. A mathematical model was derived to determine the Optimal Inventory Policy which minimizes the total inventory cost in both the owned and rented facilities. A solution procedure for solving the model is developed and illustrated through a numerical example. Sensitivity Analysis was performed to demonstrate the response of the Order Quantity and total costs to some key parameters. Incremental quantity discounts in reduced purchasing costs. However, ordering very large quantities has downsides as well. The biggest downsides includes the increase holding cost, the risk of running out of storage capacity and item deterioration since the cycle time increases if larger quantities are purchased. Owing to the importance of growing items in the food chains, the model presented in this thesis can be used by procurement and inventory managers when making purchasing decisions.

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GLOSSARY OF ABBREVIATIONS

Abbreviation	Full Meaning
EOQ	Economic Order Quantity
EPQ	Economic Production Quantity
HCU	Holding Cost per Unit Time
FCU	Feeding Cost per Unit Time
PCU	Purchasing Cost per Unit Time
TCU	Total Cost per Unit Time
SCBZ	Satisfying Clock Back to Zero (a policy of distribution of Items by Uduman, 2016)
SCU	Setup Cost per Unit Time

CHAPTER ONE

1.0 INTRODUCTION

1.1 Background to the Study

Economic Order Quantity (EOQ) is the quantity that sets the holding cost and the ordering cost at equilibrium. It is the quantity of stock that must be ordered in order to minimizes total variable cost.

Growth model methodology has been widely used in the modeling of organisms such a plants and animals growth. Growth study in many branches of science have demonstrated that more complex nonlinear functions are justified and required, if the range of the independent variable encompasses juvenile, adolescent, matured and senescent stages of growth (Khamis & Sakomura 2005). The growth potential has been described by the Gompertz function in several studies (Sakomura *et al.,* 2005). The Richards' models have been shown to give good descriptions of growth (Goonewardene, 2003). The Richards' function, first introduced in 1959 (Amanulla *et al.*, 2007). The Richard's model is a more generalized, four parameter function with a variable inflection point that provides a more complete description of the growth process.

Inventory management is concerned with ensuring the right quantity of goods is available at the right time that is, when customer demand the goods. The two major decisions in inventory management are the quantity and the timing of the orders. These decisions were first addressed by Harris (1913). Harris proposed a model popularly referred to as the Economic Order Quantity (EOQ), which seeks to balance the fixed cost of ordering items against the variable cost of keeping stock, thereby determining the best quantity to order per procurement cycle. While the basic EOQ model has found some practical applications, it makes a number of assumptions which do not reflect most real-life inventory systems. In order to model more realistic systems, various researchers have revised the classic EOQ model by relaxing the model assumptions in some ways (Holmbom and Segerstedt, 2014). In an attempt to create a new variant of the EOQ model.

In poultry breeding, selection of strain for meat production is based on weight at the age given (Mignom-Gasteau and Beamond, 2016). They suggested that, the improvement of weight at a certain age would alter heavily the entire growth curve and after induce side effect on to fattening stage, the reproduction, the movement troubles or also sexual dimorphism, thereby necessitating consideration of the totality of growth curve.

Many mathematical functions like Richards model (Knizetova *et al.*, 1991). Janoschek model (Gille and Salomon, 1994), logistic model (Grossman and Bohren, 1985), Gompertz model (Barbato, 1991; N'dri *et al.,* 2006), were used for describing growth of poultry. Indeed, the mathematical model permits to recap the information in some parameters and strategic points (Knizetova *et al*, 1997), to describe the range of weights according to age.

Thus it is possible to compare animals at the same physiological stage where the growth speed is maximal, which is not possible to measure through the traditional body weight study (Mignom-Gasteau and Beamond, 2000). Moreover, the non-linear investigation of the growth process has some advantages in not only mathematical explaining growth, but also estimating the relationship between feed requirements and body weight, and plays a crucial role in animal husbandry (Sengul and Kiraz, 2005).

Many broiler growth data analyses have been conducted using the well-known Gompertz growth function, which describes a single sigmoidal growth phase (Wang and Zuidhof, 2004). In recent years, there are many studies that have been performed with respect to growth analyses in slow-growing broilers. (Santos *et al.,* 2005),used the Gompertz model to analyze growth in two slow-growing broiler lines housed in two different systems. (Dourado *et al.,* 2009), also used the Gompertz model to examine growth of slow- growth broilers reared in the free range system. Indeed, (N'dri *et al.,* 2006), made estimates of genetics parameters for Gompertz model parameters in slow-growing broilers reared in the label range system. It is cleared that Gompertz, Logistic and Richards models were used in the analyses of the growth of living organisms.

Modern poultry breeding would be an interesting solution for mitigating the problem of animal protein supply in every town having increase demography. Chicken production in developing countries serve as important source of animal protein and source of income especially for women (Zaman *et al.,* 2004). Many mathematical functions like Richards model and others were used for describing growth of poultry. However, despite the growing demand for poultry products, poultry farmers worldwide face numerous problems. In Nigeria and Ghana, for example, poultry farmers have suffered setbacks in poultry production due to rising costs of farm inputs and some other challenges that have hampered the production and growth process.

This work proposes an inventory system where the items being ordered grows during the course of the inventory replenishment cycle and the supplier offers incremental quantity discount.

1.2 Statement of the Research Problem

In recent times, the poultry industry in Nigeria has been experiencing a steep decline in output. The decline has been attributed to soaring cost of production driven some farmers out of the business and prospective investors increasingly unwilling to invest in the industry.

Poultry farming occupies a vital place in the economy of Nigeria. As human population increases, the poultry continues to grow to meet the demand for meat and eggs. The significant of poultry production lies in the quality of products that are provided to humans. Some of the factors that are responsible for successful poultry keeping are selection of proper breeds and site, economic housing, feeding and management policy. Our study focuses on inventory management when the unit purchasing cost decreases with the order quantity Q, in other words, a discount is given by a seller if the buyer purchases large number of units. Our objective is to determine the optimal ordering policy for the buyer when dealing with such items. We will discuss two types of quantity discount contracts; all units' discounts and incremental quantity discounts.

Quantity discounts are usually offered by suppliers as a means of encouraging buyers to purchase larger volumes. Most inventory models which consider incremental quantity discounts assume that demand is deterministic.

1.3 Aim and Objectives

1.3.1 Aim

The aim of this work is to apply Economic Order Quantity model in order to determine the optimal number of live newborn items (chicks) at the beginning of the growth cycle as well as to minimize total inventory costs.

1.3.2 Objectives

The objectives are to:

I. Apply a lot sizing model for growing items if the supplier of the items offers incremental quantity discounts.

ii. Determine the optimal inventory policy which minimizes the total inventory cost.

iii. Develop solution procedures for solving the model through numerical example.

iv. Economic order quantity with quantity discount will be modeled to achieve optimal level of inventory. This implies cost saving in inventory control and achievement of maximum profit, carrying cost will be reduced to the lowest possible value.

1.4 Significance of the Study

Most price discount models are very useful in food chain. This is because a number of food items like livestock and fish products are greatly influenced by time. It may be necessary to consume the food items within a limited time period, usually the shelf life. This is motivated by the inherent nature of most food items. In addition, most food items are functional products, and for each product categories, profit is usually driven by sales volume rather than margins. This also means that, most food items are, therefore, produced in volumes in order to take advantage of economy of scale to drive down the unit cost as result of the fairly large over head costs. This is enough motivation for suppliers within this chain to provide quantity discounts in so many instances so that the food items are moved away from them to the next level of the supply chain as quickly as possible in order to avoid losses due to spoilage and deterioration. It has, however, been observed that there seems to have been no study that has consider the implication of marginal discount on the lot sizing policy of growing items. This is probably because growing items models in inventory management is relatively young area and researchers are just beginning to study it. Also, it is important to focus on incremental discount, because all quantity discounts are more straight forward with standard algorithm, and hence, more commonly studied than the marginal discount pricing models.

This study seeks to fill this gap as such lot sizing model may be important for the procurement manager in charge of decisions in the supply chain of fresh items especially, due to the fact that quantity discount is not uncommon in this area.

1.5 Scope and Limitation of the Study

The study covers purchasing and growth of items, such as chickens with incremental quantity discount, the weight of items is determined, the consumption period is used to determine the cycle time (slaughtered age) and the objective function, which is the total cost per unit cycle.

The study is limited to purchasing and growth of items with incremental quantity discounts and the possible price breaks of ordered items.

1.6 Organization of the Study

The study consists of five chapters, with chapter one being the introduction, chapter two deals with the literature review, that is, the chapter reviews the literature of the past researches that had been conducted by various scholars, chapter three is the methodology(formulation of mathematical model). A numerical example is presented in chapter four illustrating the proposed solution procedure and to provide managerial insights through a sensitivity analysis on the major input parameters, while chapter five contains summary of the findings, conclusions and recommendations.

1.7 Definition of terms

i. Feasibility: is an assessment of the practicality of a proposed plan or method.

ii. **Replenishment cycle:** A term used in inventory management that describes the process by which stocks are resupplied from some central location.

iii. **Holding cost**: is the cost of maintaining inventory in stock. It includes the interest on capital and cost of storage, maintenance and handling.

iv. **Setup cost**: Represents the fixed charge incurred when an order (no matter the size) is placed.

v. **Inventory**: Is commonly thought of as the finished goods a company accumulates before selling them to end users, it can also describe as the raw materials used to produce the finished goods.

vi. **Constraints**: Limitation or restriction (something that imposes a limit or restriction)

vii**. Price breaks**: It is a reduction in price, especially for bulk purchase.

viii. **Optimal quantity**: An efficient quantity of items when its marginal benefit equals its marginal cost.

ix. **Algorithm**: An algorithm is a step by step method of solving problems.

x. **Discounts**: A deduction from the usual cost of items

CHAPTER TWO

2.0 LITERATURE REVIEW

This chapter reviews the literature in the past researches that had been conducted by various scholars on poultry production as well as growing items. In the past, most production planning-related research has focused on problem of the livestock industry. Stygar and Malkulska (2010) pointed out that mathematical models were usually used to derive production planning decisions for livestock management. The methodology used to generate these models can be divided into optimization approaches.

Poultry production as an aspect of livestock production is important to the economic and social development and biological needs of the people of any nation because it assists in alleviating food security, creates employment opportunities for the people who are engaged and creates incomes to the people who are engaged in the projects. It is a process that involve rearing of chicks from day one to the time they matured by using some farm inputs, capital, labour and entrepreneurial talent (Oladeebo and Lamidi, 2007). Ngoupayou (2013), points out that the cost of inputs determines the size of the poultry business that a poultry farmer is able to setup. When the cost is high, many farming business will either opt to reduce the size of the business or close the business altogether which will results to decreased output.

For growing items Rezaei (2014) was the first researcher known to have incorporated item growth into inventory theory by developing an EOQ model for growing items. Rezaei (2014)'s proposed inventory system had two distinct periods, namely, growth and consumption periods. During the growth period, the ordered live items are fed and raised until they reach an acceptable weight for sale. The items are then slaughtered and put on sale during the consumption period. The increase weight experienced by growing items during the growth period is what differentiate them from conventional items whose weights do not changed if they are not consumed or more items are added to the system. In this study growth is quantified only through an increase in weight.

Zhang *et al.,* (2016), formulated an inventory model for growing items in a carbon-constrain environment. Their model used the basic assumption, including the growth and feeding functions, as Rezaei (2014)'s model, and they extended that model by assuming that the company under study in a country where carbon taxes are legislated.

the carbon tax is based on the amount of emission released into the atmosphere as a result of the company's inventory holding, ordering and transportation activities. Building on Rezaei's work, (Nobil *et al,* 2018), studied an inventory system for growing items where shortages are allowed and fully backordered. The mode presented by Nobil *et al,* (2018) differed from Rezaei, (2014)'s model in two ways. Firstly, in the former, shortages are allowed and fully back ordered, and secondly, the growth function of the items approximated by a linear function in former model as opposed to using Richards, (1959)'s growth curve as was the case in the latter.

Sebatjane and Adetunji,(2018) extended Rezaei (2014)'s model by incorporating item quality. Their model was formulated under the assumption that a certain proportion of the order growing items is of inferior quality. In addition, this model investigated three different growth functions, namely, logistic, linear and split linear.

2.1 Incremental Quantity Discounts

Economic Order Quantity (EOQ) model with quantity discounts were proposed by Hadley and Whitin (1963). Quantity discounts are usually offered by suppliers as a means of encouraging buyers to purchase larger volumes. In inventory theory, suppliers, usually offer one or two types of quantity discounts. These are all-units quantity discounts, which reduced purchasing cost for the entire order if the quantity order is above a particular quantity called the break point, and incremental quantity discounts where the reduced purchasing cost only applies to items bought above the break point. Most inventory models which consider incremental quantity discounts assumed that demand is deterministic. This is not always true in most real-life inventory system. This prompted Ahad (1988) to develop inventory models with incremental quantity discounts under two non-constants demand patterns, namely constant-price elasticity and linear demand functions.

Taleizadeh *et al,* (2015) developed an inventory model with incremental quantity discounts under two different shortage conditions. In the first case, shortage are considered to be fully backordered, that is, all the customers are willing to wait for the backorder stock to arrive, and in the second case , partial backordering. It was assumed that some of the customers are willing to wait for the backorder stock and in this case a lost sales cost is taking into account.

Mohammadivojdan and Geunes (2018) studied the news vendor problem, that is, single period inventory model with multiple vendors and various types of discounts. The model assumed that the vendors offered incremental and all-units discounts for purchasing the items and carload discount for transporting the items to the buyer. In addition, the capacity of each of the suppliers was assumed to be limited.

Abdullah (2010) investigates the effect of strain on performance, and age at slaughter and duration of post-chilling aging on meat quality traits of broiler. Average weekly body weight was comparable between strains.

Peng-Sheng and Yi Chih (2018), a study of production and harvesting planning for the chicken industry. This study proposes a mathematical programming model to develop a production planning and harvesting schedule for chicken farmers. The production planning comprises of batches of chickens to be raised in each henhouse, the number of chicks to be raised, what breed of chicken to raise, when to start raising and the duration of the raising period.

Aiping and Jialing (2018),this model states that if inventory is larger than demand, the remaining will be sold at the discounted price or disposed at the end of period; however, if inventory is smaller than demand, some profit will be lost. Thus, this model focuses on optimizing cost or profit and develops relevant inventory policy. Subsequently, a great number of scholars makes extensions to this traditional model which can be divided into three categories. Firstly, traditional models usually assume a fixed purchase price which departs from the reality. Thus price sensitive demand is introduced.

Xiao *et al* (2016), study the price-dependent demand in a multi-product Newsvendor model and propose corresponding methods of obtaining the optimal order quantity and optimal sales price.

Kebe *et al.* (2015), consider a two echelon inventory lot-sizing problem with price-dependent demand and develop a mixed-integer programming formulation as well as Lagrangian relaxation solution procedure for solving this problem. Secondly, consider the profound influence of demand variation on expected profits or costs, many scholars focuses on the modeling of demand when studying Newsvendor model. (Uduman, 2016) employs demand distributions satisfying the clock back to zero (SCBZ) property to model the single demands for a single product, namely; Newspaper, and subsequently obtain the optimal order quantity.

Faithma *et al* (2017), study the generalization of Uduman's model with several individual demands for a single product, followed by numerical illustrations. Furthermore, Faithma and Uduman (2018) proposed an approximation closed-form optimal solution for three cases of single period inventory model, in which the demand varies with SCBZ property. In addition to SCBZ property, Kamburowski (2019) developed a Newsboy problem due to incomplete information.

Axsater (2014) reveals that a majority of inventory control models including NewsVendor are built on same assumptions, for instance, that lead-time demand follows the normal distribution. However, they pointed out that, although this assumption is reasonable in most cases, it is inconsistent with the reality in certain circumstances and leads to a waste of capital or low level service.

Literature on inventory management for growing items suggests that incremental quantity discounts have not been incorporated into the EOQ model for growing items. This work aimed to address the gap in the literature by developing an EOQ model for growing items with incremental quantity discounts.

Table 2.1 Gap analysis of Related works in the Literature

References characteristics	Major inventory system characteristics			Additional contribution
Author(s)/year Of publication	**Conventional items**	**Growing items**	**Incremental quantity discounts**	**to knowledge**
Harris(1913)	yes	x	yes	Balancing the fixed Cost of ordering items
Hadley and Whitin (1963)	yes	x	yes	Vendor- buyer system
Gasteau and Beamon (2000)	yes	yes	x	Nonlinear investigation of growth.
Sengul & Kiraz (2005)	yes	yes	x	Body weight requirement.
Zhang *et al* (2014)	yes	x	x	Advance payments.
Taleizadeh *et al* (2015)	yes	x	x	Limited budget and storage
Nobil *et al* (2018)	x	yes	x	Inventory model with Storage
Hossein Z. and Goshani (2018)	yes	yes	x	Pricing policy
Pen-Sheng & Y-Chih(2018)	yes	x	x	Production and Harvesting Planning For Chickens
This work	yes	yes	yes	Cost minimization

2.2 Problem Definition, Assumptions and Notations

Problem Definition

An organic poultry farmer runs a farm business raising different kinds items, such as chickens, ducks, turkeys and so on. A situation is considered where a company orders a certain number of items which are capable of growing during the course of inventory planning cycle, for example livestock. The supplier of the new born items offers the purchasing company incremental quantity discounts over a fixed price breaks. Under the incremental quantity discount pricing structure discounted purchasing cost only apply to the incremental quantity. Figure 2.1 represents the typical behavior of an inventory system for growing items.

In order for growth to occur, the company needs to feed the items. Every replenishment cycle can be divided into periods, namely; the growth and the consumption periods. During the growth period (shown as period t in figure 2. 1), ordered newborn items are fed and raised until they grow to a certain target weight, once the weight of items reaches the target weight, the growth period ends and the items are slaughtered. During the consumption as shown as period T in figure 1, the slaughtered items are kept in stock and sold to the market. The company incurs feeding cost during the growth period, and it incurs holding cost for keeping the slaughtered items in stock. All the inventories items are consumed within the cycle time T, at which point the items in the next inventory cycle would have completed their growth phase (i.e, the items in the next cycle would have grown to the target weight and are ready for sale). The company want to determine the optimal number of newborn items to order at the beginning of the growth cycle in order to minimize total inventory costs (i.e, the sum of the purchasing, setup, feeding and holding costs).

The company needs to determine the optimal number of newborn chicks to order at the beginning of the growing cycle and the frequency of placing order which minimizes the total cost (i.e, the sum of the setup, purchasing, feeding and holding costs). The proposed inventory model is studied as a cost minimization problem, with the total cost being the objective function and the cycle time and the order quantity as the decision variable(s), since both are determined together.

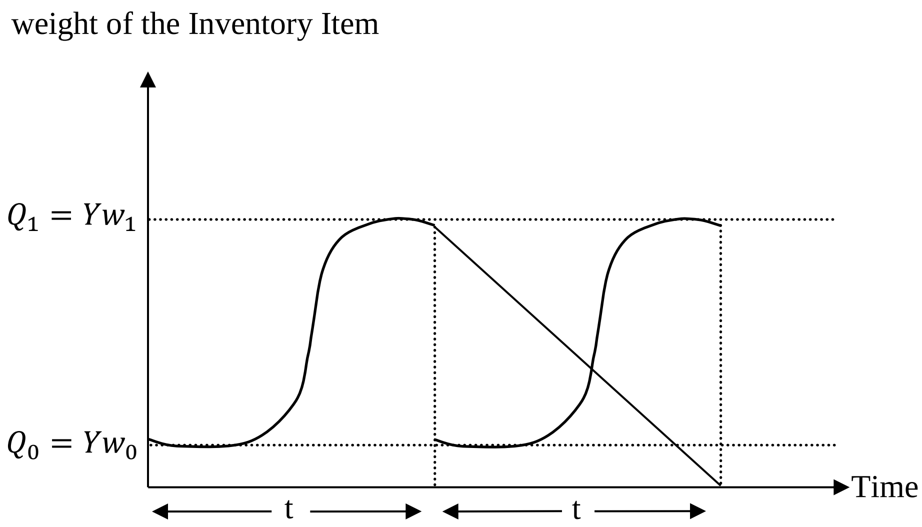

Figure 2.1 Behaviours of Inventory System for Purchasing and Growing Items (chicks)

Assumptions

The following assumptions are made when formulating the mathematical model:

i. The ordered items are capable of growing prior being slaughtered

ii. A single type of item is considered (chickens).

iii. Feeding cost are incurred for feeding and growing the chickens during the growth period. These costs are proportional to the weight gained by the chickens.

iv. Holding costs are incurred for the duration of the consumption period.

v. The supplier of the live newborn chicks offers incremental quantity discounts.

vi. Demand is deterministic constant

vii. The production (feeding) costs varies overtime as the new-born birds are grows. Assumed the total feeding cost as $c_f y\int_0^t w_t dt$ where t is the length of the growing cycle.

Notations

The notations employed in the formulation of the mathematical models are given below;

Y Number of ordered newborn chicks per cycle

W_o Initial weight of newborn chicks

W_1 Final weight of each grown chicken at the time of slaughtering

Q_o Total weight of newborn chicks

Q_1 Total weight of inventory at time t

α Asymptotic weight of the chicken to maturity

b integration constant

λ Exponential growth rate of the chickens

p_j purchasing cost per weight unit at jth break points

m Number of break points

y_j Order quantity for price j

h Holding cost per weight unit per unit time

k	Setup cost per cycle
D	Demand in weight unit per unit time
C_f	Feeding cost per weight unit per unit time
t	Growing period
T	cycle length

CHAPTER THREE

3.0 MATERIALS AND METHODS

3.1 Formulation of Mathematical Model

When a new growing cycle begins, a company purchases Y new born chickens, each weighing w_o, which are capable of growing during the replenishment cycle. The total weight of the inventory at this point, Q_o is determined by multiplying the weight of each of the chicks by the number of chickens ordered (i.e, $Q_o = Yw_o$). The company feeds the chickens and they grow to a target weight of w_1. This marks the end of the growth period, and at this point the chickens are slaughtered or sold out. The total weight of the inventory at the time of slaughter is $Q_1=Yw_1$. The behavior of the inventory over time is depicted in **Figure 2.1**. As a way of larger order sizes, the company's supplier offers incremental quantity discount. The discount cost is given as:

$$P_j = \left\{ \begin{array}{l} p_1 y_1 = 0 \le Y \prec y_2 \\ p_2 y_2 \le Y \prec y_3 \\ \\ p_m y_m \le Y \end{array} \right\} p_1 \ge p_2 \ge p_m$$

Where Y represents the number of new born chicks purchased and $y_1, y_2 \ldots.. y_m$ represent the order quantities at which the unit purchasing cost changes, for example the price breaks.

Growth and consumption occur over the periods t and T respectively. Hence, the company incurs feeding and holding costs over those respective time periods. The demand rate, D, and the weight of the inventory level at the beginning of the consumption period, $Q_1 = Qw_1$, are used to determine the cycle time as

$$T = \frac{Yw_1}{D} \tag{3.1}$$

Item growth followed a pattern which can be represented by four stages. In the first stage, which occurs when a new growth cycle begins, the items experience slower growth. The second stage is characterized by faster growth, while in the third stage, growth is slower as the chicks approach maturity. In the final stage, the chickens are fully matured and they have reached peak weight and they do not experience considerable weight gain.

This pattern of growth is common in most growing items and depicted by fig.1, can be represented by the logistic function (Hossein *et al.,* 2016). The logistic growth function relates the weight of the items with time. It has three parameters, denoted by $\alpha, b \text{ and } \lambda$, which represent the asymptotic weight of the chickens, the integration constant and the exponential growth rate, respectively. The weight of the chickens as a function of time is given by

$$W_t = \frac{\alpha}{1 + be^{-\lambda t}} \tag{3.2}$$

The items are slaughtered when their weight reaches the target weight (i.e, w_1) following the conclusion of the growth period t. From equation (3.2), the duration of the growth period (i.e, the slaughter age) is determined as

$$t = -\frac{\ln\left[\frac{1}{b}\left(\frac{\alpha}{w_1} - 1\right)\right]}{\lambda} \tag{3.3}$$

The equation(3.3) represents the slaughtered age of the birds or the target age.

3.2 Purchasing Cost Per Unit Time (All Units Discount)

Let $y_i = 0, y_1, y_2, y_3 \ldots\ldots y_m$ as the order quantities at which the purchase cost per weight unit changes and there are m such changes, called price breaks. When a supplier offers incremental quantity discounts, the purchasing cost per weight unit, p_j, is the same for all Y values in (y_j, y_{j+1}). The purchasing cost per weight unit decreases from one price break to the next (i.e, $p_1 \succ p_2 \ldots\ldots \succ p_j \succ p_{j+1} \succ \ldots . p_m$)

Y is the quantity in jth price break (i.e, $y_j \leq Y \prec y_{j+1}$)). The purchasing cost per cycle for Y items, each weighing w_o, in this price break is given by

PC= $p_1(y_2 - y_1)w_o + p_2(y_3 - y_2)w_o + \ldots . p_{j-1}(y_j - y_{j-1})w_o p_j(Y - y_j)w_o$ (3.4)

Define A_j as the sum of the terms in equation (3.4) which are independent of Y, and thus

$A_j = \{_o\, p_1(y_2 - y_1)w_o + p_2(y_3 - y_2)w_o + \ldots . + p_{j-1}(y_j - y_{j-1})w_o + p_j(Y - y_j)w_o$, j= 0,1,2,. (3.5)

Equation (3.4) can be rewritten as

$$PC = A_j + p_j w_o (Y - y_j). \quad (3.6)$$

Dividing eq.(3.6) by the cycle time, as given in eq.(3.1), gives an expression for the average unit purchasing cost per unit time, denoted by PCU, as

$$PCU = D\left[\frac{A_j}{Yw_o} + \frac{p_{jw_o}}{w_o} - \frac{p_j w_o y_j}{Yw_1}\right] \quad (3.7)$$

Equation (3.7) is the different between the annual demand cost of the newborn chicks and that of matured chickens.

3.3 Food Procurement Cost Per Unit Time

The feeding cost per cycle is computed as the product of the numbers of items to be fed, the feeding cost per weight unit and the area under the growth/feeding period as given in figure 2.1. Thus

$$FC = c_f y \int_o^t w_t dt \tag{3.8}$$

Substituting equation (3.2) into equation (3.8) gives

$$FC = c_f y \left\{ \alpha t + \frac{\alpha}{\lambda} \left[\ln\left(1 + be^{-\lambda t}\right) - \ln\left(1 + b\right) \right\} \right. \tag{3.9}$$

Dividing equation (3.9) by the cycle time, as given in equation (3.1), yields an expression for the feeding cost per unit time, FCU, given by

$$FCU = \frac{c_f D\alpha}{w_1} \left\{ t + \frac{1}{\lambda} \left[\ln\left(1 + be^{-\lambda t}\right) - \ln\left(1 + b\right) \right] \right\} \tag{3.10}$$

Equation (3.10) represents food procurement cost of the matured birds (slaughter age)

3.4 Setup Cost Per Unit Time

Every time the company places an order for live newborn items (i.e, birds), it incurs a cost of K for setting up the growth and feeding facilities. The setup cost per cycle is thus

$$SC = K \tag{3.11}$$

The setup cost per unit time, SCU, is determined by dividing the setup cost by the cycle time as

$$SCU = \frac{KD}{Yw_1} \tag{3.12}$$

Equation (3.12) represents the setup cost times the annual demands of the matured birds

3.5 Holding Cost Per Unit Time

The holding cost per cycle, HC, is computed from fig. 1 using the area under the consumption period (since the holding cost is incurred for the slaughtered inventory) and thus

$$HC = h\left[\frac{y^2 w_1{}^2}{2D}\right] \tag{3.13}$$

The holding cost per unit time, HCU, is computed by dividing equation (3.13) by equation (3.1), that is

$$HCU = h\left[\frac{Yw_1}{2}\right] \tag{3.14}$$

Equation(3.14) represents the holding cost of items during replenish cycle.

3.6 Total Cost Per Unit Time (Incremental Quantity Discount)

The total cost per unit time, TCU, is computed by summing equation(3.7), (3.10),(3.12)and(3.14)togiv

$$TCU = D\left[\frac{A_j}{Yw_1} + \frac{p_j w_0}{w_1} - \frac{p_j w_0 y_j}{Yw_1}\right] + \frac{KD}{Yw_1} + h\left[\frac{Yw_1}{2}\right] + \frac{c_{fD\alpha}}{w_1}\left\{t + \frac{1}{\lambda}\left[\ln\left(1 + be^{-\lambda t}\right) - \ln(1+b)\right]\right\}$$

(3.15)

Through rearranging the times, equation (3.15) becomes

$$TCU = \frac{p_j w_0 D}{w_1} + \frac{D}{Yw_1}\left[A_j - p_j w_0 y_{j+K}\right] + h\left[\frac{Yw_1}{2}\right] + \frac{c_f D\alpha}{w_1}\left\{1 + \frac{1}{\lambda}\left[\ln\left(1 + be^{-\lambda t}\right) - \ln\left(1 + b\right)\right]\right\}$$

(3.16)

Equation (3.16) represents the overall measure of performance(goal) we need to minimize the cost and maximize the profit of ordered items.

3.7 Model Constraints

There are two constraints that ensure the feasibility of this inventory system, they are categorize as case 1and case 2, thus;

Case 1: in order to ensure that the slaughtered chickens are ready for sale during the consumption phase, the duration of the growth phase should be less than or equal to the duration of the consumption phase. This results in a constraint (on the duration of the consumption phase) being formulated as

$$T \geq t \qquad (3.17)$$

By substituting t from equation (3.3) into equation (3.17) to get

$$T \geq \frac{\ln\left[\frac{1}{b}\left(\frac{\alpha}{w_1} - 1\right)\right]}{\lambda} \qquad (3.18)$$

Equation(3.18) represents the consumption phase of the Birds.

Case 2: in order to ensure that the order quantity determined falls within the range of the giving price break, a constraint on the order quantity is formulated as

$$y_j \leq Y \leq y_{j+1} \tag{3.19}$$

The optimal order quantity for the proposed inventory system is determined by setting the first derivative of the objective function in equation (3.16) to zero. Thus;

$$\frac{\partial TCU}{\partial Y} = -\left(A_j - p_j w_0 y_j + K\right) \frac{D}{Y^2 w_1} + \frac{h w_1}{2} = 0$$

$$Y* = \sqrt{\frac{2(A_{j-} p_j w_o y_j + K)D}{h {w_1}^2}} \tag{3.20}$$

The optimal cycle time is obtained by substituting equation (3.20) into equation (3.1), and the result is

$$T = \sqrt{\frac{2(A_{j-} p_j w_o y_j + K)D}{hD^2}}$$

Therefore, optimal cycle time(T∗) is given as

$$T* = \sqrt{\frac{2(A_{j-} p_j w_o y_j + K)D}{hD^2}} \tag{3.21}$$

To show the measure of nonlinear relationship on the duration of price break and the falls within the range of the given price, a second partial derivative of the objective function, as given in equation (3.16), gives

$$\frac{\partial^2 TCU}{\partial Y^2} = A_j - p_j w_0 y_j + K \frac{D}{Y^3 w_1} \geq 0 \tag{3.22}$$

From equation (3.6), total purchasing cost for any range $y_{j-1} \leq Y \leq y_j$ is governed by

$$PC = A_j + p_j w_0 \left(Y - y_j\right)$$

Where A_j is already defined in equation (3.5), it can be rewritten as

$$A_j = A_{j-1} + p_{j-1} w_0 \left(Y - y_{j-1}\right) \tag{3.23}$$

Similarly, the total purchasing cost, PC, can also be rewritten as

$$PC = A_{j-1} + p_{j-1}w_0\left(y_j - y_{j-1}\right) + p_j w_0\left(Y - y_j\right) \tag{3.24}$$

Equation (3.24) can also be rewritten to look like equation(3.22),that is;

$$PC = \left(A_{j-1} - p_{j-j}w_0 y_{1-1}\right) + \left(p_{j-1} - p_j\right)w_0 y_j + p_j w_0 Y \tag{3.25}$$

Equation (3.25) is the general form of the purchasing cost function for j $\geq$ 2Base case is j =2

By definition, A_1=0 and y_1=0, therefore

$$PC = 0 + \left(p_1 - p_2\right)w_0 y_2 + p_2 w_0 Y \tag{3.26}$$

Observe also that $p_1 > p_2$, since by definition $p_{j-1} > p_j$ for all j's.

Hence, all terms are zero or positive. Therefore, the nonnegative hold for the base case. Let assume that the case holds for any j then we show that it holds for any j+1 for which $y_j \leq Y \leq y_{j+1}$ is true, then

$$PC = A_{j+1} + p_{j+1}w_0\left(Y + y_{j+1}\right)$$

$$= A_j + p_j w_0\left(y_{j+1} - y_j\right) + p_{j+1}w_0\left(Y - y_{j+1}\right)$$

$$= A_J + P_J W_0 Y_{J+1} - P_J W_0 Y_J + P_{J+1}W_0 y - P_{J+1}W_0 Y_{J+1} \tag{3.27}$$

Equation (3.27) can be rewritten for the range j+1 as

$$PC = \left(A_j - p_j w_0 y_j\right) + \left(p_j - p_{j+1}\right)w_0 y_{j+1} + p_{j+1}w_0 y_{j+1} \tag{3.28}$$

This is the desired form. Therefore, the function also holds and this proves the nonnegativity of equation (3.22) which is the Hessian function of the total cost function.

3.8 Computational Algorithms

The EOQ model for the purchasing and growing items with incremental quantity discounts is determined using the following algorithms:

Step 1: Compute Y for each j using equation (3.20).Denote this as Y_j. **Step 2:** Check each Y_js feasibility. They are feasible if $y_j \leq Y \leq y_{j+1}$. Infeasible Y_js are disregarded and only the feasible ones proceed to step 3.

Step 3: For each feasible Y, compute the corresponding T using equation (3.21). **Step 4:** Check the feasibility of each computed Y_j with regard to the cycle time. Each Y_j is feasible if $T \geq t$.Infeasible Y_js are disregarded and only the feasible ones proceed to step 5.

Step 5: Compute TCU using equation (3.16) for all the feasible Y_js. The Y_j value which results in the lowest TCU is the EOQ. Step 6 End (Taha, 2011)

CHAPTER FOUR

4.0 NUMERICAL RESULTS

4.1 Numerical Example

The propose inventory model is applied to numerical example which considers a company that purchase new born chicks (one day old) chickens, feed/grows them until they reach a targeted weight, w_1, and then put them on sales. From data which were obtained from Songhai farm located at River Basin Development Authority Minna Niger State, the following parameters are used to analyze the proposed model.

i. Demand Rate (D) = ₦100, 000 kg/year

ii. Setup Cost (K) = ₦75, 000 kg/cycle

iii. Holding Cost (L) = ₦10 kg/year

iv. Feeding Cost (c_f) = ₦50 kg/cycle

Initial weight of new born chick, $w_o = 6.8\, kg/chick$ final weigh of chicken at the time of Slaughtering $w_1 = 32\ kg/chicken.$

Asymptotic Weight $(\propto) = \ 65kg$

Integration constant (b) = 8

Exponential Growth Rate $\lambda = 9.5/cycle$

The purchasing Cost structure is given in table 4.1

Table 4.1: Purchase Cost Structure under Incremental Quantity Discounts.

Quantity of birds Purchased	Price per unit weight(₦)/kg
0-1000	250
1001-1500	200
1501-2000	150
2001 and above	100

By applying the algorithm, the following were obtained, thus:

Step 1: Compute y for each j using equation (3.20)

Let consider the equation $A_j = 2A_{j-1} + p_{j-1}w_o(y_j - y_{j-1})$ to find the value of A_j where j= 1,2,3…..

$$A_1 = 0$$

$$A_2 = 250(1001 - 0)6.8 = 170170$$

$$A_3 = 340340 + 200(1501 - 1001)6.8 = 2382312$$

$$A_4 = 4764624 + 150(2001 - 1501)6.8 = 32909443$$

$$Y_1 = \sqrt{\frac{2(0-250\times 6.8\times 0+75000)100,000}{10\times 35^2}} = 1106.6$$

$$Y_2 = \sqrt{\frac{2(170170-200\times 6.8\times 201+75000)100,000}{10\times 35^2}} = 1334.2$$

$$Y_3 = \sqrt{\frac{2(238231-150\times 6.8\times 301\times 75,000)100,000}{10\times 35^2}} = 1617.6$$

$$Y_4 = \sqrt{\frac{2(329094-100\times 6.8\times 401\times 75{,}000)100{,}000}{10\times 35^2}} = 1932.8$$

Step 2: check each y_{js} feasibility. They are feasible if $y_j \leq y < y_{j+1}$. infeasible y_{js} are disregarded and only the feasible area proceed to step 3.

$$0 \leq y_1 = 1106.6 > 1001$$

$$1001 \leq y_2 = 1334.2 < 1501$$

$$1501 \leq y_3 = 1617.6 < 2001$$

$$2001 \leq y_4 = 1932.8.$$

Thus y_2 *and* y_3 are feasible.

Step 3: for each feasible y, using equation (3.21) to compute the corresponding T.

$$T_2 = \sqrt{\frac{2(170170-200\times 6.8\times 201\times 75{,}000)100{,}000}{10\times (100{,}000)^2}} = 0.2670$$

$$T_3 = \sqrt{\frac{2(238170-150\times 6.8\times 301\times 75{,}000)100{,}000}{10\times (100{,}000)^2}} = 0.5658$$

Step 4: check the feasibility of each computed y_j with regard to cycle time. Each y_j in feasible if $T \geq t$ *and* y_{js} are disregarded and only the flexible ones proceed to Step 5.

$$t = \frac{\left[\frac{1}{8}\left(\frac{65}{32}-1\right)\right]}{9.5} = 0.2157$$

Thus y_2 *and* y_3 are feasible since $T_2 \geq t$ *and* $T_3 \geq t$

Step 5: to compute total cost per unit time TCU, in the EOQ.

$$Tcu_2 = \times\ (170170 - 1001\ \times\ 6.8\ \times\ 1001\ + 75{,}000) +$$

$$100\left[\frac{1334.2\times 35}{2}\right]+\frac{2.5\times 100000\times 41}{32}\left\{0.4621+\frac{1}{9.5}\left[ln(1+5e^{-7.3\times 0.4621})-\ ln(1+5)\right]\right\}$$

$$= ₦925332.82$$

$$Tcu_3 = \frac{1501\times 6.8\times 100{,}000}{32}+\frac{100{,}000}{1616.6\ x\ 35}\times(2382312-1501x6.8x1501+75{,}000)+100\left(\frac{1616.6\times 35}{2}\right)+\frac{2.5\times 100{,}00\times 41}{32}\left\{0.4621+\frac{1}{9.5}\left[ln(1+5e^{-7.3\times 0.4621})-\ ln(1+5)\right]\right\}$$

$$= ₦927018.08$$

$Y^{*} = y_2\ since\ Tcu_2 < Tcu_3$

End

Based on the results of the numerical example, some of which are summarized in table 4.2, the company should place and order for 1335-day-old chickens, at the beginning of each cycle.

This orders quantity lies in the 1501-2001 price break, and therefore the company will pay 250, 200 and 150 naira per kg, for the first 1000, the next 500 and the remaining 117, respectively. Based on the target slaughter weight, the chickens should be grown for 0.21565 years (79) days. Orders should be replenished every 0.2670 years (97) days.

Following this optimal inventory policy, the company will incur a total cost of ₦925, 332.82 per year.

4.2 Comparison with the basic EOQ for Production and Growing Items with or without Incremental Quantity Discounts.

In order to investigate the cost savings, if any, resulting from a supplier offering incremental quantity discounts, the proposed model is compared with the basic EOQ model for growing items as proposed by Rezaei (2014). Since Rezaei used a different growth function than the one used in this work, the feeding costs between the two models will be different. To clarify it, Rezaei (2014) model is reinstated using the same growth function as used in this work, that is, the total cost per unit time in the basic model without incremental quantity discounts is given as

$$TCU = PD + \frac{KD}{Yw_1} + h\left[\frac{Yw_1}{2}\right] + \frac{cD\alpha}{w_1}\left\{1 + \frac{1}{\lambda}\left[\ln\left(1 + be^{-\lambda t}\right) - \ln\left(1 + b\right)\right]\right\} \quad (3.29)$$

And the EOQ is given by

$$Y^* = \sqrt{\frac{2KD}{hw_1^2}} \quad (3.30)$$

Table 4.2: Shows the summary of the results of numerical example

Variable	Unit	Quantity
t	2.6 months	0.2157
T^*	3.2 months	0.2670
Y^*	Chickens	1335
TCU^*	Amount/year	925,332.82

The variables used for the numerical example are applied in the basic model in order to achieve a fair comparison, and the results are summarized in table 4.1 Although, the quantity of newborn items ordered at the beginning of a cycle increased by 2.2% as a result of incremental quantity discounts, the total cost decreased by 2.0%. The effect of incremental discounts on the holding costs was negative, that is, the increase, and it was positive on both the setup and the purchasing costs.

Consequently, having a supplier who offers incremental quantity discounts reduced, the total costs of managing inventory, and therefore, quantity discounts are viable alternative for reducing procurement costs.

The table below shows comparison between a model with a quantity discounts and a model without quantity discounts.

Table 4.3 Comparisons between Models with or without QuantityDiscounts

Cost components & EOQ	**Rezaei (2014)**	**This thesis**	**%change**
PCU	485,714.29	476,543.78	1.9
SCU	193,649.17	175,642.30	9.3
FCU	69,783.89	69,781.86	2.9
HCU	193,649.17	231,487.75	-19.5
TCU	942,796.51	935,323.82	0.8
EOQ	1107	1335	-20.6

4.3 Sensitivity Analysis

Sensitivity analyses are performed on the major input parameters in order to investigate their impart on the decision variables and to provide managerial insights for improving management decisions.

Increasing the setup cost, both the EOQ and the total costs as shown in **Table 4.4**, the EOQ shifted into different price breaks as a result of the changes to the setup cost. Managers can offset the increase in the total costs, by purchasing larger quantities, that is, placing orders less frequently.

However, this should be done in moderation because, if too much stock is ordered, the holding cost will increase, and this will lead to an increase in total costs.

Table 4.5. Increasing the holding cost increases the total and also reduces the EOQ as illustrated in the table. The effect of decreasing the holding cost is substantial because the EOQ shift into different price breaks. As result of the shift in price breaks, the total costs decreases because of the lower purchasing cost per unit weight in the new price break. This shift into a lower price breaks offset the increased holding cost, due to increase in the EOQ as result of ordering larger quantities.

Table 4.4 changes to Y* and TCU* due to changes in Setup Cost

% change in setup cost	EOQ Chicks	EOQ % change	TCU Amount/year	TCU % change
-40	782	-32.3	829,359	-10.4
-27.3	1149	13.9	860,621	-7.0
0	1335	0	925,333	0
12.5	1663	24.6	943,352	2.0
14	1709	27.5	959,329	3.7

Table 4.5, changes to Y* and TCU* due to changes in Holding Cost

% change in Holding cost	EOQ Chicks	EOQ % change	TCU Amount/year	TCU % change
-40	2729	104.6	742,670	-18.4
-27.3	2441	78.0	798,043	-13.0
0	1335	0	925,333	0
12.5	1258	-6.7	943,352	2.0
14	1193	-11.4	959,329	3.7

Table 4.6 shows that the total costs increase with the increase in feeding costs, whereas the EOQ is not affected by changes in the feeding costs. The feeding is essentially the cost of procuring feedstock for the items, and it is very difficult for managers to reduce

This cost, since it is set by the feed stock suppliers. Nonetheless, managers can reduce this cost through procuring larger volumes of item's feedstock which normally have discounted pricing.

Table 4.6: changes to Y* and TCU* due to changes in Feeding Cost(c_f)

% change in	EOQ		TCU	
Feeding cost	Chicks	% change	Amount/year	% change
-40	1335	0	890,441	-3.8
-27.3	1335	0	899,164	-2.8
0	1335	0	925,333	0
12.5	1335	0	934,056	0.9
14	1335	0	951,329	1.0

Table 4.7: changes to Y^* and TCU^* due to changes in lower bound for Order Quantity for price j

% change in lower bound for order for price j	EOQ		TCU	
	Chicks	% change	Amount/year	% change
-40	11573	17.9	814,617	-12.0
-27.3	1669	5.1	848,345	-8.3
0	1335	0	925,333	0
12.5	1360	-1.9	934,356	1.0
14	1116	-7.1	942,797	1.9

Table 4.8: changes to Y^* and TCU^* due to changes in Purchasing Cost at jth price breaks

% change in purchasing cost at price breaks	EOQ		TCU	
	Chicks	% change	Amount/year	%change
-40	1226	104.6	693,061	-25.1
-27.3	1254	78.0	751,438	-18.8
0	1335	0	925,333	0
12.5	1558	-6.7	974,056	6.1
14	1669	-11.4	1036,329	12.0

Increasing the lower bound for the order quantity in each price break increases both the EOQ and the total costs, as shown in table 4.7, The changes to the total costs are minimal most likely because the EOQ remains in the same price break.

Managers do not have much control over the discount quantity structure as it is determined by the supplier, but if it happens that the suppliers reduce the lower bounds on the order quantities in each price breaks, it is beneficial for managers to order less items.

Table 4.8 shows that increasing the purchasing cost decreases the EOQ and increase the total costs. Invariably, the EOQ shifts into different price breaks, and consequently, the effect on the average cost is significant as well. By reducing purchasing costs, managers can save significantly on their average total costs by ordering larger quantities. While this will result in the increase in the holding cost, the savings which result from lower purchasing cost may outweigh the impart of the increased holding cost.

CHAPTER FIVE

5.0 CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

The result indicates that, the order quantity lies between 1501-2001 price breaks and therefore the company will pays 250,200 and 150 naira per kg for the first 1000, the next 500 and the remaining 117 respectively. Based on the targeted slaughter weight, the items (chickens) should be grown for 0.2157 years (79 days) and the replenishment is every 0.2670 years (97 days).

Following the optimal inventory policy, the company will incur a total cost of ₦925,332.82 per year. There is also a comparison with the basic Economic Order Quantity model for production and growing items without incremental quantity discounts and that of incremental quantity discounts, refer to **table 4.3**. Rezaei (2014) proposed a general mathematical model which was used for various types of growing products, followed by a particular mathematical model for specific type of poultry.

Here, this work is a modification of Rezaei (2014) model of which weight of the items and incremental quantity discounts are incorporated, that is, for every living organism weight is a determine factor for growth.

In the numerical example, the feasibility of the ordered items are determined, that is, from the computational algorithm of y_1 to y_4, we observed that, as the results of computation shows, y_1 and y_4 are infeasible, while y_2 and y_3 are feasible, as such y_1 and y_4 are discarded. Here, since y_2 and y_3 are feasible and their corresponding T_2 and T_3 are greater or equal to t, meaning, only the feasible proceeds to next step of the computations.

Below are the summary results from the numerical example:

i. slaughter weight of the chickens is grown for about 0.2157 in a year (79 days)

ii. The optimal cycle time of chickens is 0.2670 in a year (97 days)

iii. Optimal quantity of newborn chickens is 1335

iv. The total costs per unit time is ₦925,332.82

From the analysis, the computation of the total cost per unit time, TCU, in economic order quantity model, only TCU_2 and TCU_3 are considered and as result, the optimal quantity $Y^* = y_2$. Hence $TCU_2 < TCU_3$.

The economic order quantity (EOQ) model of (Harris, 1913) is the foundation of recent inventory models. Following this model, many researchers have contributed to the development of economic order quantity (EOQ) model with addition of shortage cost which was developed by (Nobil *et al.*, 2018). EOQ/EPQ models for perishable products such as food, vegetables, milk are also proposed.

Various inventory models have been developed in recent years. In 2014, Rezaei provided an inventory model for growing items. Distinct from the literature, this thesis exhibits an inventory model for purchasing and growing items (chickens) with incremental quantity discounts especially focusing on organic poultry production.

The contribution made by this work to the work of Rezaei (2014), is the incorporation of incremental quantity discounts and weight of the items as well as cost minimization. This addition to the literature is very important because supplier often offer discounts for purchasing larger volumes of items.

The cost structure, in terms of both the purchasing cost in each price breaks and the lower bounds for the order quantities in each price break was shown to have significant impact on the order quantity and the average total cost of the managing inventory.

5.2 Recommendations

1. Due to considerable impact of the incremental quantity discounts on inventory, it presents operation managers with opportunities to reduce costs through better procurement practices.

2. Certain factors need to be consider when purchasing large volumes of items (chickens), these factors includes accommodations, available procurement budget, age and the breed.

3. The proposed inventory model did not take into account issues like deterioration, growing and storage facility capacity and budget constraints. These factors should be consider when purchasing large quantities because in certain instances, management might be forced to lease extra capacity if they purchased more chickens than can be grown and stored in their own facilities..

5.3 Contribution to Knowledge

In this study, the major inventory system characteristics is on the purchasing and growth of organic poultry, the conventional items, the growing items as well as incremental quantity discount which focuses on cost minimization and maximization of profit. The mathematical model was derived to determine the optimal inventory policy which maximize the total inventory cost in both the owned and rented facilities. Its also help the inventory manager when making purchasing decisions.

Meaning that when you have total number of items such as y=1335, you will now pay ₦890,441 that is the amount per year instead of ₦951,329 of the same number of items, discount of 3.8%.

5.4 Suggestions for Further Research

It came out that, there are numerous factors that influenced poultry production and growth in Nigeria. If the factors are taken into consideration, it will certainly increase costs and negate the benefit of purchasing larger quantities.

The model presented in this work can be extended to includes; capacity limits, budget limits, among other popular EOQ extensions. Deterioration becomes increasingly important if larger quantities are purchased, extensions which account for deterioration during the consumption period are another possible area for future research as they represent more realistic inventory system.

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APPENDIX

RESPONDENT(S) QUESTIONAIRE

Dear Respondent,

My name is Audu Sunday EKU, who is currently undertaking a master of Technology in applied Mathematics at the Federal University of Technology Minna. Iam conducting a study on the purchasing and growing of organic poultry as a production process.

I am requesting you to spare some time to help me respond to some questions that I will like to ask you. The information provided will be treated strictly as confidential and will only be used for the purpose of this study. The findings of the study and the recommendations arrived at, will be of benefit to the poultry farmers in Songhai Farm centre, Niger state and Nigeria at large.

Thank you for your cooperation.

Audu Sunday Eku.

Please, answer the following questions below; Answer each question by writing in the spaces provided or tick against the boxes for the choice provided. Information provided is considered strictly confidential and will only be used for the purpose of this research. Do not write your name while answering any question. Thus;

1. How does farm inputs influence poultry production ?.

2.What type of newborn birds do you ordered?

3. At what age do you order ?

4. How do you as a farm manager ensure that, the objective of organic farm system is achieved ?

5. What is the total revenue accrued annually in production and sells of the products.

6. What is the demand level by the consumers?

7. Is there any strong demand for organically reared table birds, provided that they are of good conformation?

8. What measure is adopted to enhance biosecurity?

9. How many kilograms of feed do you use per day?

10. What is your flock size? Less than 50 ☐ 50-250 ☐ above 250 ☐

Thank you for the responds!